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SUM RULE APPROACH TO THE NUCLEAR RESPONSE
IN THE ISOVECTOR SPIN CHANNEL

W.M. Alberico*)

CERN -- Geneva

M. Ericson

Université C. Bernard Lyon I, 69621 Villeurbanne

and

CERN -- Geneva

and

A. Molinari

Istituto di Fisica Teorica dell'Università
di Torino

and

INFN, Sezione di Torino, Torino

ABSTRACT

We study the global features of the response of infinite nuclear matter in the spin-isospin channel through the energy weighted sum rules S_1 and S_{-1} . In particular we compare the outcome of the ring approximation with the exact RPA evaluation of the sum rules. We also investigate the influence of the collective character of the response, induced by the particle hole force for a longitudinal and transverse spin couplings. We show that S_1 is insensitive to the collectivity of the response, as long as the Δ degree of freedom is ignored. The inverse energy weighted sum rule on the other hand, which is linked to the paramagnetic susceptibility, always reflects the hardening or softening of the nuclear response, due to the repulsive or attractive character of the p-h force. This quantity is well suited to the comparison with the experiments, which we perform for ^{12}C and ^{56}Fe .

*) On leave of absence from Istituto di Fisica Teorica dell'Università di Torino.

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1. - INTRODUCTION

The response of nuclei to a spin-isospin sensitive probe is a subject of particular interest. One reason is that the spin-isospin channel is the one where the phase transition of pion condensation may occur. It is also the channel which couples the nucleon to its lowest excitation, the Δ isobar.

The force responsible for the spin-isospin excitations has been the subject of extensive investigations¹⁾⁻⁴⁾ which have unravelled its strong repulsive character at small momenta. It is thus able to produce a collective nuclear response which is observed in the giant Gamow-Teller resonances⁵⁾ and also in the quenching of the Gamow-Teller⁶⁾ and magnetic⁷⁾ strengths due to the mixing of nucleon-hole and Δ hole excitations. When the momentum increases, the force remains repulsive in the transverse channel, while it becomes attractive in the spin longitudinal one, owing to the growing influence of the pion exchange.

We have studied in previous works^{8),9)} the collective nuclear response in the region of moderate momenta. We have shown that it manifests different features: the transverse response is quenched and hardened, whereas the longitudinal one is enhanced and softened (a precursor of pion condensation). The existing data on the transverse response in (e,e') scattering are compatible with these predictions.

Our approach was based on the ring approximation which neglects the Hartree-Fock (HF) effects as well as the antisymmetrization of the interaction matrix elements. In this paper we study with the sum rule method¹⁰⁾ the response of the infinite nuclear matter to a probe which interacts with a longitudinal $(\vec{\sigma} \cdot \vec{q})$ and a transverse $(\vec{\sigma} \times \vec{q})$ coupling with the nucleonic spins. Although the sum rules are insensitive to finer details of the nuclear response, we will see that they are suitable for displaying the hardening or softening.

One of our aims is also to improve upon the previous approach, by obtaining exact RPA expressions which include HF and exchange effects.

We focus our attention on the energy weighted and the inverse energy weighted sum rules. The first one in the spin transverse channel has already been considered by several authors¹¹⁾⁻¹⁵⁾ with the commutators' technique, which does not easily accommodate the nucleonic excitation (Δ). On the other hand this procedure has the advantage of allowing for tensor correlations in the ground state^{14),15)} which, in the frame of the linear response theory, correspond to

2p - 2h excitations. It should be kept in mind however that these excitations affect mostly the high energy side of the quasi-elastic peak, which is beyond the scope of our present description. We concentrate instead on the collective aspects of the nuclear response which, according to our ring approximation, should already appear on the low energy side. The pertinent results are then consistently tested with the double commutator method on an uncorrelated basis¹³⁾.

For comparing our present approach with the existing experimental data the inverse energy weighted sum rule is more suitable, being mostly sensitive to the low energy side of the nuclear response.

Accordingly, our paper is organized as follows: in Section 2 we investigate the energy weighted sum rule S_1 , discussing the role of HF and exchange diagrams. We explore whether the softening or hardening effects are reflected in S_1 and the crucial role played by the Δ degree of freedom. We also study these effects through the average excitation energy.

In Section 3 we examine the inverse energy weighted sum rule S_{-1} . Here an exact RPA expression can only be obtained for q small by an extension of a method recently proposed¹⁶⁾. In this limit we test our ring approximation looking at the dependence of S_{-1} on the Migdal parameter g' to which, instead, S_1 is totally insensitive when the Δ is ignored.

While the sum rule S_1 lends itself to a proper theoretical description for any momentum, S_{-1} , as mentioned, is more suitable for comparing with the experimental data. In this connection we remind ourselves of the relationship between S_{-1} and the paramagnetic isovector susceptibility χ_p ¹⁷⁾. At $q = 0$ this quantity has been recently measured in several nuclei and found to be strongly quenched by the RPA correlations¹⁸⁾. We extend the investigation of χ_p to finite q and suggest that in this kinematical region the shell model effects, dominant at $q = 0$, are washed out and only the "bulk" magnetic properties are left.

Summarizing the outcome of the present investigation, in the last section we conclude that the main results of our previous paper on the contrast between the spin longitudinal and the spin transverse responses essentially survive the test of the sum rule approach, in spite of the substantial corrections to the ring approximation of Refs. 8) and 9).

2. - THE ENERGY WEIGHTED SUM RULE

The n -energy weighted sum rule for the excitations induced by an operator $\hat{O}$ is defined as

$$S_n = \sum_f |\langle f | \hat{O} | 0 \rangle|^2 (E_f - E_0)^n \quad (2.1)$$

where n is a positive or negative integer. For $n = 1$ the energy weighted sum rule can be expressed as follows¹⁰⁾:

$$S_1 = \frac{1}{2} \langle 0 | [[\hat{O}, \hat{H}], \hat{O}] | 0 \rangle. \quad (2.2)$$

By Thouless' theorem¹⁹⁾, if the expectation value (2.2) is taken in the HF ground state, S_1 is actually evaluated in RPA.

We consider isovector spin excitations in the nucleonic sector induced by the operators

$$\hat{O}_T = \sum_{k=1}^A \frac{\tau_3^k}{2} (\hat{q} \times \vec{\sigma}_k) e^{i\vec{q} \cdot \vec{r}_k} \quad (2.3)$$

in the transverse spin channel ($\sigma_z = \pm 1$) and

$$\hat{O}_L = \sum_{k=1}^A \frac{\tau_3^k}{\sqrt{2}} (\hat{q} \cdot \vec{\sigma}_k) e^{i\vec{q} \cdot \vec{r}_k} \quad (2.4)$$

in the longitudinal one ($\sigma_z = 0$), $\vec{q}$ being the momentum transfer. Since scalar interactions commute with the operators (2.3) and (2.4), we keep in the Hamiltonian only the π and ρ exchange potentials. The RPA transverse and longitudinal S_1 are then, respectively:

$$\begin{aligned} \frac{1}{Z} S_{1,T}^{RPA}(q) &= \frac{\hbar^2 q^2}{2m} + f_\pi^2 \frac{k_F}{\pi^2} [4 I_c(q, \mu_\pi) - I_T(q, \mu_\pi)] + \\ &+ f_\rho^2 \frac{k_F}{\pi^2} [8 I_c(q, \mu_\rho) + I_T(q, \mu_\rho)] \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} \frac{1}{Z} S_{1,L}^{RPA}(q) &= \frac{\hbar^2 q^2}{2m} + f_\pi^2 \frac{k_F}{\pi^2} [4 I_c(q, \mu_\pi) + 2 I_T(q, \mu_\pi)] + \\ &+ f_\rho^2 \frac{k_F}{\pi^2} [8 I_c(q, \mu_\rho) - 2 I_T(q, \mu_\rho)] \end{aligned} \quad (2.6)$$

where

$$I_c(q, \mu) = \int_0^{\infty} \frac{dr}{r} e^{-\mu r} [1 - j_0(qr)] j_1^2(k_F r) \quad (2.7a)$$

and

$$I_T(q, \mu) = \int_0^{\infty} \frac{dr}{r} e^{-\mu r} \left[1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right] j_2(qr) j_1^2(k_F r) \quad (2.7b)$$

k_F being the Fermi momentum and $j_\ell(x)$ the spherical Bessel functions.

Expression (2.5) was given by Suzuki¹³⁾. Note that a zero range force does not contribute to S_1 since the integrands in (2.7) vanish at $r = 0$, hence we omitted the Migdal force in the Hamiltonian. Moreover, the Pauli correlation factor $j_1^2(k_F r)$ clearly indicates that π and ρ exchanges affect S_1 only through antisymmetrization.

It is interesting to display the validity of Thouless' theorem, by showing the identity of (2.5) and (2.6) with the results obtained with an exact RPA polarization propagator. We do that in the $q \rightarrow 0$ limit, where the above integrals can be performed analytically, yielding

$$\begin{aligned} \frac{1}{Z} S_{1,T}^{RPA}(q \rightarrow 0) = & \frac{\hbar^2 q^2}{2m} + \frac{q^2}{(2\pi)^2} \frac{1}{10 k_F} \left\{ f_\pi^2 \left[13 Q_1(\alpha) + Q_0(\alpha) - \frac{1}{\alpha-1} \right] + \right. \\ & \left. + f_\rho^2 \left[27 Q_1(\beta) - Q_0(\beta) + \frac{1}{\beta-1} \right] \right\} \end{aligned} \quad (2.8)$$

in the transverse channel and

$$\begin{aligned} \frac{1}{Z} S_{1,L}^{RPA}(q \rightarrow 0) = & \frac{\hbar^2 q^2}{2m} + \frac{q^2}{(2\pi)^2} \frac{1}{5 k_F} \left\{ f_\pi^2 \left[7 Q_1(\alpha) - Q_0(\alpha) + \frac{1}{\alpha-1} \right] + \right. \\ & \left. + f_\rho^2 \left[13 Q_1(\beta) + Q_0(\beta) - \frac{1}{\beta-1} \right] \right\} \end{aligned} \quad (2.9)$$

in the longitudinal one, with $\alpha = 1 + \mu_\pi^2/2k_F^2$, $\beta = 1 + \mu_\rho^2/2k_F^2$. The $Q_\ell(x)$ are the Legendre functions of the second kind.

On the other hand, the expressions (2.8) and (2.9) can be cast, as shown in the Appendix, into the form

$$\frac{1}{Z} S_1^{RPA}(\sigma_z, \tau) = \frac{\hbar^2 q^2}{2m} (1+\sigma) (1+u_{11}^{\sigma_z, \tau}) \quad (2.10)$$

where the generalized Landau parameters $u_{\ell\ell'}^{\sigma_z, \tau}$ account for the exchange particle hole diagrams (for $\ell = \ell' = 0$ a direct contribution is also present), whereas σ is related to the effective mass, $m^*/m = (1+\sigma)^{-1}$, and embodies the HF effects. It is given by

$$\sigma = \frac{1}{1+u_{11}^{0,0}} - 1. \quad (2.11)$$

Expression (2.10) is indeed the result one obtains with an exact RPA polarization propagator [by extending the method of Ref. 16) to include tensor forces]. It also shows that the potential contributions to S_1 stem exclusively from anti-symmetrization, being related to p-wave Landau parameters.

We now test against the exact $S_1^{RPA}(q)$ our ring approximation, which utilizes the following polarization propagator

$$\Pi^{ring}(q, \omega) = \frac{\Pi^0(q, \omega)}{1 - V_{ph}^{\sigma, \tau}(q, \omega) \Pi^0(q, \omega)}; \quad (2.12)$$

$V_{ph}^{\sigma, \tau}$ includes, as in Refs. 8) and 9), π and ρ exchange plus the Migdal zero range force. Since the double commutator method has only nucleonic degrees of freedom, for the moment we restrict Π^0 to the nucleon-hole polarization propagator.

We define the n-energy weighted sum rule in the ring approximation as

$$\frac{1}{Z} S_n^{ring}(q) = -\frac{6\pi}{k_F^3} \int_0^{\hbar\omega_{max}} d(\hbar\omega) (\hbar\omega)^n \text{Im} \Pi^{ring}(q, \omega) \quad (2.13)$$

where we limit the integral to the nucleonic sector: thus $\hbar\omega_{max}$ coincides with the upper boundary of the particle-hole continuum, including the eventual contribution of the collective mode (in this case $\omega_{max} = \omega_{coll}$).

For $n = 1$ we find that S_1^{ring} is practically independent of the force and coincides with the free value $S_1^{free} = \hbar^2 q^2 / 2m$ (but for a small effect due to the frequency dependence of our particle-hole interaction). This was to be expected since the effects of the interaction, as shown by the double commutator approach,

stem only from antisymmetrization which it totally neglected in Π^{ring} . The accuracy of the ring approximation is displayed in Figs. 1 and 2 where the quantity S_1/S_1^{free} is shown both in the ring and in the exact RPA. The error induced by neglecting HF and exchange diagrams is of the order 50 ÷ 60% and tends to decrease as q becomes larger, particularly in the $\sigma_z = 0$ channel. It is density dependent and clearly decreases when the density is lowered.

Since S_1^{ring} is independent of g' it is obviously impossible to simulate the effect of antisymmetrization by adjusting the value of the Migdal parameter. Notice that the ρ contribution is dominant in the transverse channel, whereas in the longitudinal one the π and the ρ contribute essentially by an equal amount.

We now focus our attention on the hardening and softening of the nuclear response, introducing the mean excitation energy $E_{\text{av}} = S_1(q)/S_0(q)$. We utilize the ring approximation since an RPA expression for S_0 is not easily attainable, and reintroduce the Δ degree of freedom. This last fact alters S_1^{ring} , which now differs markedly from the free value owing to the LLEE effect²⁰⁾⁻²⁶⁾: there is a quenching when the force is repulsive and an enhancement when it is attractive. These features are illustrated in Fig. 3 where S_1^{ring} is displayed with and without the inclusion of the Δ .

The mean energy is shown in Fig. 4 together with the corresponding free quantity, i.e.,

$$\left(\frac{S_1}{S_0}\right)_{\text{free}} = \begin{cases} \frac{\hbar^2 k_F}{2m} \cdot \frac{4q/3}{1 - q^2/12 k_F^2} & \text{for } q \leq 2k_F \\ \frac{\hbar^2 q^2}{2m} & \text{for } q > 2k_F. \end{cases} \quad (2.14)$$

For small values of q the transverse and longitudinal $E_{\text{av}}^{\text{ring}}$ stay substantially above the free one, reflecting the hardening of the nuclear response. This effect persists up to large values of q in the transverse channel. In the longitudinal one, beyond $q \sim 1 \text{ fm}^{-1}$, $E_{\text{av}}^{\text{ring}}$ falls below the free value, reflecting the softening of the response, a precursor effect of pion condensation⁸⁾.

We remind ourselves that when the Δ is ignored, S_1^{ring} is insensitive to the force (cf. Fig. 3), thus the dynamical effects in $E_{\text{av}}^{\text{ring}}$ are totally embodied in S_0^{ring} . Even in an exact RPA treatment S_1 is not suited to deal with the softening and hardening since it is completely blind to a Migdal force, which largely governs these effects. Therefore one is not justified in drawing conclusions from S_1 alone, as long as the Δ is ignored.

This was implicitly done in a recent paper by Suzuki¹³⁾: he considers the scaling variable $y = \omega/q - \hbar q/2m$, which has the mean value

$$\bar{y} = \frac{1}{\hbar q} \left[\frac{S_1}{S_0} - \frac{\hbar^2 q^2}{2m} \right]. \quad (2.15)$$

The magnitude of $\bar{y}$ measures the violation of scaling invariance in deep inelastic electron scattering. According to Suzuki, who utilizes the free asymptotic value $S_0(q \rightarrow \infty) = 1$, $\bar{y} > 0$ would entail a hardening of the nuclear response, whereas $\bar{y} < 0$ would be associated with a softening, the occurrence of which he questions. We disagree on this conclusion for a number of reasons:

- i) we always stressed that the softening is a feature of the spin longitudinal channel only, whereas Suzuki explores the transverse one;
- ii) by using the fixed value $S_0 = 1$, his $\bar{y}$ measures in fact the deviation of S_1 from S_1^{free} . Therefore he is not investigating the mean excitation energy S_1/S_0 , which is the quantity of significance for detecting the softening or hardening. In fact, in our description softening goes along with enhancement and therefore we find a softening with $S_1(q)$ larger than its free value;
- iii) moreover, the commutator method without Δ is totally insensitive to the Migdal force, as previously stated. Therefore S_1 , when the Δ excitation is ignored, is not the appropriate quantity to investigate the dynamical effects we are concerned with.

The comparison of our previous predictions with the experimental data is plagued, in particular for S_1 , by the difficulty of disentangling, in the high energy side of the experimental response, the $1p - 1h$ component from more complicated processes. Indeed, experimentally there is no clear separation between the low energy (nuclear) sector and the high energy sector; the filling of the dip between the two is generally attributed to $2p - 2h$ processes, not incorporated in our theory^{27),28)}.

For this reason we shall restrict the comparison with the experiment to the inverse energy weighted sum rule which is practically insensitive to the high energy tail.

3. - THE INVERSE ENERGY WEIGHTED SUM RULE

This quantity is of particular significance since it is related to the nuclear paramagnetic susceptibility, which has been measured at small momenta by Knüpfer and Richter¹⁸⁾ all over the periodic table. At finite momenta we extract it from the inelastic electron scattering.

The inverse energy weighted sum rule cannot be evaluated with commutator techniques. Nevertheless, for small q values, the following approximate (but accurate) RPA expression can be obtained (see the Appendix):

$$\frac{1}{Z} S_{-1}^{RPA}(\sigma_z, \tau) \approx \frac{1}{2} \frac{3m}{\hbar^2 k_F^2} \frac{1}{(1+\sigma)(1+u_{00}^{\sigma_z, \tau} + 2 u_{02}^{\sigma_z, \tau}/\sqrt{5})}. \quad (3.1)$$

We have evaluated the corresponding generalized Landau parameters using the full π and ρ exchange plus the spin-isospin Migdal force (the explicit expressions for the interaction particle hole matrix elements are reported in the Appendix). We have also explored the influence of the scalar interaction f_0 . We neglect the contributions associated with the other Migdal parameters, f'_0 and g_0 , since they are partially accounted for by the π and ρ exchange. We recall that the Δ degree of freedom is ignored in the RPA approach.

Unlike S_1 , S_{-1}^{RPA} gets contributions from diagonal and off-diagonal s -wave Landau parameters which bring into play the $q \rightarrow 0$ limit of the direct particle-hole interaction: therefore S_{-1}^{RPA} is quite sensitive to the value of g' .

In Fig. 5a we display $S_{-1}^{RPA}(q=0)$ as given by (3.1) together with $S_{-1}^{ring}(q=0)$ (Δ omitted) as a function of the Migdal parameter g' . Also shown is S_{-1}^{free} , which is identical to the static limit of $\text{Re}\Pi^0(q, \omega)$ without the (Δ, N^{-1}) piece:

$$S_{-1}^{free}(q) = \frac{m}{6\hbar^2 k_F^2} \left\{ \left(\frac{k_F}{q} - \frac{q}{4k_F} \right) \ln \left| \frac{1+q/2k_F}{1-q/2k_F} \right| + 1 \right\}. \quad (3.2)$$

The ring approximation obviously coincides with the free value for $g' = 0$. This is not the case for the RPA result, quenched by a factor of two due to HF and antisymmetrization which play a substantial role when the repulsive Migdal force is weak. However, for realistic values of g' ($g'=0.6 \div 0.7$), S_{-1}^{ring} and S_{-1}^{RPA} come closer and thus our ring approximation is rather good. The inclusion of f_0 , which affects the spin-isospin channel only through the exchange matrix element, makes the differences between ring and RPA even smaller, as is shown in the figure.

Although the present method does not allow us to make a comparison with the exact RPA at finite q , we hope that the same conclusions hold true at moderate momenta. It is therefore worth while confronting our predictions for $S_{-1}^{\text{ring}}(q)$ with the experimental data. Since the Δ degree of freedom has to be included in this comparison, we illustrate in Fig. 5b its effect at $q = 0$. The LLEE quenching clearly increases with g' and is of the order of 50% for $g' = 0.6 \div 0.7$.

We remind ourselves that S_{-1} is related to the nuclear paramagnetic susceptibility¹⁷⁾ as follows

$$\frac{1}{2} \chi_p^{\tau=1} = \mu_0^2 (\mu_p - \mu_n)^2 S_{-1} \quad (3.3)$$

where μ_0 is the nuclear Bohr magneton and $\mu_p = 2.79$, $\mu_n = -1.91$ the proton and neutron magnetic moments, respectively.

In Fig. 6 we display the q dependent susceptibility per proton for the free nuclear matter and in ring approximation^{*)} (Δ included). One sees the strong quenching, partly associated with the Δ , reflecting the reshaping of the response induced by the RPA correlations. Also shown are the experimental values for ^{12}C ²⁹⁾ and ^{56}Fe ³⁰⁾ as well as the $q = 0$ points for some nuclei, given by Knüpfner and Richter¹⁸⁾, who actually observed the quenching due to the Δ . With the exception of a few light nuclei, the latter are well below the static nuclear matter susceptibility

$$\frac{1}{Z} \chi_p^{\tau=1} (k_F=1.36) = \begin{cases} 13.7 \times 10^{-3} \text{ fm}^3/Z & (\text{free}) \\ 4.5 \times 10^{-3} \text{ fm}^3/Z & (\text{ring approx.}) \end{cases} \quad (3.4)$$

which are finite owing to the continuous nature of the excitation spectrum of an infinite system. Instead the paramagnetic susceptibility of a spin saturated nucleus like ^{16}O and ^{40}Ca is zero in the shell model and indeed is found to be quite small, although finite (an effect associated with $2p - 2h$ excitations).

Moreover, the χ_p of finite nuclei is strongly fluctuating, at $q = 0$, as a consequence of the shell structure. For q between 1 and 2 fm^{-1} , however, it is remarkable that the susceptibilities per proton of ^{12}C and ^{56}Fe practically

^{*)}The force of Speth et al.⁴⁾ has actually been employed for the comparison with the experiment.

coincide and approach the ring approximation of infinite nuclear matter. This suggests that the fluctuations associated with the finite size have faded away at these momenta, allowing the nuclear matter description to be valid.

It should however be emphasized that any conclusions based on two nuclei is premature. The coincidence could be accidental also because the detailed response of ^{12}C is badly reproduced, and experiments on other nuclei are needed. On the theoretical side it is not clear whether the nuclear matter description, which neglects the orbital contribution to χ_p , is relevant for light nuclei, even in the kinematical situation where $qR > 1$.

To complete our analysis we have also explored $S_{-1}(q)$ in the spin longitudinal channel. At $q = 0$ we have found that it practically coincides with the corresponding quantity in the transverse channel - an obvious outcome in ring approximation, but not so in the full RPA. When q increases the two quantities deviate and the longitudinal S_{-1} becomes enhanced with respect to the free one³¹⁾ (indeed it would become infinite at the critical density for pion condensation).

Therefore we advocate that, once the proper probes are selected, S_{-1} is the most convenient quantity to display the contrast between the collective features of the two spin channels⁹⁾.

In this connection a further quantity well suited to exhibit the above-mentioned contrast is the characteristic frequency

$$E_c(q) = \frac{S_0(q)}{S_{-1}(q)}. \quad (3.5)$$

Fig. 7 shows the behaviour of $E_c(q)$ in ring approximation, compared with the corresponding free quantity. We notice that the characteristic frequency (3.5) is quite similar to the average energy displayed in Fig. 3, but has the advantage of being far more accessible experimentally than $E_{av}(q)$.

4. - CONCLUSION

In this paper we have explored the nuclear response of infinite nuclear matter in the spin-isospin channel. We have studied its global features through the sum rules, focusing our attention on S_1 and S_{-1} . For the present purpose we have restricted the sum to the nucleonic sector.

We have utilized for the particle hole interaction the π and ρ exchange potentials plus the zero range repulsive force of Migdal.

The classical method of the double commutator allows an exact RPA evaluation of the energy weighted sum rule S_1 at any q . We have thus been able to test the accuracy of the ring approximation previously employed to predict hardening or softening in the nuclear response. We have found S_1^{ring} to be only moderately good.

It is thus natural to give up the ring approximation and use the double commutator technique for the evaluation of S_1 . However, this method does not easily incorporate the Δ degree of freedom. As a consequence, S_1 is totally insensitive to the dynamical effects of softening or hardening, being unaffected by the Migdal force.

In addition, the experimental S_1 is subject to large uncertainties owing to the difficulty of disentangling, in the high energy part of the response, the $1p - 1h$ component - the only one we consider - from more complicated processes. Therefore S_1 is not a quantity of particular significance for the investigation of the dynamical effects we are interested in.

The inverse energy weighted sum rule S_{-1} (which gives the susceptibility) however is much less affected by these experimental problems. On the other hand, an exact RPA expression for S_{-1} can, at present, only be achieved at $q = 0$. We have obtained it utilizing a generalization of the RPA equations in the Landau limit, which is suitable for including tensor forces. We have found that the neglect of HF and exchange effects in the ring approximation does not entail significant errors for realistic values of the Migdal parameter g' . Extrapolating this conclusion at finite q we believe that S_{-1} is a good tool for investigating the dynamical effects of hardening and softening, to which it is quite sensitive.

For this purpose we have extracted from the inelastic electron scattering experiments on ^{12}C and ^{56}Fe the corresponding q dependent susceptibility per proton. We have found that they are surprisingly similar and approach, as q becomes large ($\sim 2 \text{ fm}^{-1}$), our ring approximation of the nuclear matter susceptibility, which is quenched with respect to the free value. Further experimental information in this connection would be most valuable.

We have also considered the mean frequency S_1/S_0 and a characteristic frequency defined by S_0/S_{-1} . Both measure the hardening or softening of the nuclear response but, as previously noted, the second one, which does not involve the experimentally uncertain S_1 , is more interesting.

In summarizing the results of the present paper, we can conclude that our predictions concerning the collective features of the nuclear response in the spin longitudinal and transverse channels survive the test of the sum rules. Indeed we have found that the susceptibility sum rule S_{-1} , while being only slightly affected by antisymmetrization, is sizeably quenched with respect to the free one, thus reflecting the quenching and hardening of the response at low momentum.

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APPENDIX: TENSOR FORCES IN THE LANDAU LIMIT OF RPA

An exact expression for the RPA response function of infinite nuclear matter and for the odd sum rules related to it is found in Ref. 16) when the momentum $\vec{q}$ transferred to the system is vanishingly small and the interacting particles and holes are on the Fermi surface.

This method applies to central forces with a finite number of partial waves, once they are expanded in terms of Legendre polynomials. Therefore it cannot be directly utilized when dealing with the one pion (or rho) exchange potentials, which we utilize in the present paper.

Indeed we consider the following antisymmetrized particle hole matrix elements:

$$V_{ph}^{\sigma_z=\pm 1, \tau=1}(\vec{q}, \vec{p}-\vec{k}) = -4 \frac{f_\rho^2}{\mu_\rho^2} \frac{q^2}{q^2 + \mu_\rho^2} + 3 \frac{f_\pi^2}{\mu_\pi^2} q' + \frac{f_\rho^2}{\mu_\rho^2} - \frac{f_\rho^2}{\mu_\rho^2} \frac{(p_z - k_z)^2 + \mu_\rho^2}{(\vec{p}-\vec{k})^2 + \mu_\rho^2} + \frac{f_\pi^2}{\mu_\pi^2} \frac{(p_z - k_z)^2}{(\vec{p}-\vec{k})^2 + \mu_\pi^2} \quad (A.1)$$

in the transverse channel and

$$V_{ph}^{\sigma_z=0, \tau=1}(\vec{q}, \vec{p}-\vec{k}) = -4 \frac{f_\pi^2}{\mu_\pi^2} \frac{q^2}{q^2 + \mu_\pi^2} + 3 \frac{f_\pi^2}{\mu_\pi^2} q' + \frac{f_\pi^2}{\mu_\pi^2} - \frac{f_\pi^2}{\mu_\pi^2} \frac{2(p_z - k_z)^2 + \mu_\pi^2}{(\vec{p}-\vec{k})^2 + \mu_\pi^2} + 2 \frac{f_\rho^2}{\mu_\rho^2} \frac{(p_z - k_z)^2}{(\vec{p}-\vec{k})^2 + \mu_\rho^2} \quad (A.2)$$

in the longitudinal one. The momenta $\vec{p}$, $\vec{q}$ and $\vec{k}$ are shown in Fig. 8, which illustrates the particle-hole interaction. The direction of $\vec{q}$ has been chosen as the z axis.

In the Landau limit, $q \rightarrow 0$, $|\vec{p}| = |\vec{k}| = k_F$, one can write (A.1) and (A.2) as functions of $\xi \equiv \hat{q} \cdot \hat{p}$ and $\xi' \equiv \hat{q} \cdot \hat{k}$ by taking

$$u^{\sigma_z, \tau}(\xi, \xi') = \frac{1}{2\pi} \int_0^{2\pi} d(\varphi_p - \varphi_k) \lim_{q \rightarrow 0} V_{ph}^{\sigma_z, \tau}(\vec{q}, \vec{p} - \vec{k}). \quad (A.3)$$

For the sake of illustration let us consider explicitly the transverse channel; according to (A.3) we then get

$$u^{1,1}(\xi, \xi') = \frac{f_p^2}{\mu_s^2} \left\{ 1 - \left[\frac{\mu_s^2}{2k_F^2} + \frac{1}{2} (\xi - \xi')^2 \right] \frac{1}{\sqrt{\xi^2 + \xi'^2 - 2\beta\xi\xi' + \beta^2 - 1}} \right\} + \quad (A.4)$$

$$+ \frac{f_\pi^2}{\mu_\pi^2} \left\{ 3g' + \frac{(\xi - \xi')^2}{2\sqrt{\xi^2 + \xi'^2 - 2\alpha\xi\xi' + \alpha^2 - 1}} \right\}$$

where $\beta = 1 + \mu_p^2/2k_F^2$ and $\alpha = 1 + \mu_\pi^2/2k_F^2$.

A diagonal expansion of (A.4) in terms of $P_\ell(\xi)P_\ell(\xi')$, the P_ℓ being Legendre polynomials, is no longer feasible owing to the presence of the tensor interaction (indeed it would require V_{ph} to depend upon $\cos\theta \equiv \hat{p} \cdot \hat{k}$ only). So the following, more general, expansion should replace the one used in Ref. 16) [cf. Eq. (2.21)]

$$u^{\sigma_z, \tau}(\xi, \xi') = (1+\sigma) \frac{(2\pi\hbar)^2}{2mk_F} \sum_{\ell, \kappa} \sqrt{2\ell+1} \sqrt{2\kappa+1} u_{\kappa\ell}^{\sigma_z, \tau} P_\kappa(\xi) P_\ell(\xi') \quad (A.5)$$

where, for the interaction (A.1),

$$u_{\kappa\ell}^{1,1} = \frac{mk_F}{(2\pi\hbar)^2} \frac{2}{1+\sigma} \left\{ \frac{f_p^2}{\mu_s^2} \left[\delta_{\kappa,0} \delta_{\ell,0} - \frac{\mu_s^2}{2k_F^2} Q_\ell(\beta) \delta_{\kappa,\ell} - b_{\kappa\ell}(\beta) \right] + \right. \quad (A.6)$$

$$\left. + \frac{f_\pi^2}{\mu_\pi^2} \left[3g' \delta_{\kappa,0} \delta_{\ell,0} + b_{\kappa\ell}(\alpha) \right] \right\}.$$

The $b_{k\ell}(x)$ are generated by the π and ρ tensor forces and are non-vanishing for $k = \ell$ and $k = \ell \pm 2$ only. Their explicit expression is reported in a forthcoming paper.

We can then evaluate the energy weighted sum rule starting from the equation [cf. Ref. 16)]

$$S_1^{RPA}(\sigma_z, \tau) = \frac{\hbar^2 q^2}{2m} \frac{3}{2} \int_{-1}^{+1} d\xi \int_{-1}^{+1} d\xi' \xi \xi' \left[(1+\sigma) \delta(\xi - \xi') + \frac{mk_F}{(2\pi\hbar)^2} \mathcal{U}^{\sigma_z, \tau}(\xi, \xi') \right] \quad (A.7)$$

which, utilizing the expansion (A.5), is easily cast into the form (2.10). For completeness, we write here the generalized p -wave Landau parameter corresponding to the p - h interaction (A.1):

$$u_{11}^{\sigma_z=1, \tau=1} = -\frac{2}{1+\sigma} \frac{mk_F}{(2\pi\hbar)^2} \left\{ \frac{f_\rho^2}{\mu_\rho^2} \left[\frac{\mu_\rho^2}{2k_F^2} Q_1(\beta) + b_{11}(\beta) \right] - \frac{f_\pi^2}{\mu_\pi^2} b_{11}(\alpha) \right\} \quad (A.8)$$

with

$$b_{11}(x) = \frac{3}{5} Q_1(x) - \frac{4}{15} Q_2(x) - \frac{1}{3} Q_0(x). \quad (A.9)$$

Let us now focus our attention on the parameter σ , which is defined in Eq. (2.11). It is given by the derivative of the HF nucleon self-energy with respect to momentum, evaluated at the Fermi surface; hence the relationship with the effective mass immediately follows. In the present context one should note the following:

- i) according to (2.11), σ stems from the particle hole interaction in the channel $\sigma = 0, \tau = 0$, thus being unaffected by tensor forces;
- ii) in principle, σ also gets contributions from scalar forces (through the exchange matrix elements entering into the p -wave Landau parameters) which are not included in our model. Their presence, however, would not alter S_1 since in Eq. (2.10) the contributions of a scalar force to σ and $u_{11}^{\sigma_z=1, \tau=1}$ cancel exactly. This result remains obviously true for finite momenta, as shown by the evaluation of S_1 in terms of commutators.

More delicate problems arise in connection with the inverse energy weighted sum rule. It is still possible [see Ref. 16) for more details] to express it as

$$S_{-1}^{RPA} = \frac{3m}{2\hbar^2 k_F^2} \frac{1}{1+\sigma} \oint d\xi_n \frac{[a_1(\xi_n)]^2}{\xi_n N^2(\xi_n)} \quad (A.10)$$

where the $a_k(\xi_n)$ are the Fourier coefficients of the solution of the RPA equations, with norm $N^2(\xi_n)$, corresponding to the eigenvalue ξ_n .

However, tensor forces do not allow a simple analytical expression for the solution of the RPA equations; in particular the $a_k(\xi_n)$ are no longer polynomials. Nevertheless, an approximate (and accurate) solution can be obtained by truncating the expansion (A.5) (actually for π and ρ exchange, $7 \div 8$ partial waves reproduce the particle-hole interaction almost exactly). Even then the corresponding expressions for the coefficients $a_k(\xi_n)$ are far more complicated than in Ref. 16) owing to the couplings between partial waves induced by the tensor force. This fact would prevent any further utilization of the previous formalism, but we have found that in all the cases we are interested in, the "exact" $a_k(\xi)$ are fairly well approximated by polynomials of degree k . In particular, within at most an error of $2 \div 3\%$, it is

$$a_1(\xi_n) \cong \xi_n \left\{ 1 + u_{00}^{\sigma_z \tau} + \frac{2}{\sqrt{5}} u_{02}^{\sigma_z \tau} \right\}^{-1} \quad (A.11)$$

which, inserted into (A.10), leads to Eq. (3.1).

We can thus conclude that the tensor character of the particle hole interaction prevents an analytical solution of the RPA equations. Yet a fairly accurate calculation of S_{-1} which yields, at least in the Landau limit, the amount of correction to the ring approximation, which comes from HF and exchange diagrams, is possible.

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FIGURE CAPTIONS

- Fig. 1 : The energy weighted sum rule in the longitudinal spin channel divided by S_1^{free} as a function of q . The continuous line is the ring approximation (with $g' = 0.7$), and the dashed line the exact RPA result. The separate RPA contributions of the π (dotted line) and of the ρ (dot-dashed line) are also shown. The Fermi momentum is $k_F = 1.36 \text{ fm}^{-1}$.
- Fig. 2 : The same as in Fig. 1, in the transverse channel.
- Fig. 3 : The ratio $S_1^{\text{ring}}(q)/S_1^{\text{free}}(q)$ when the Δ excitation is neglected (dashed line for $\sigma_z = \pm 1$, dotted line for $\sigma_z = 0$) and when it is included in the polarization propagator (continuous line for $\sigma_z = \pm 1$, dot-dashed line for $\sigma_z = 0$). In the figure $k_F = 1.36 \text{ fm}^{-1}$, $g' = 0.7$.
- Fig. 4 : The quantity $E_{\text{av}}(q)$ divided by $\hbar c q$ in ring approximation (Δ included) both in the transverse (continuous line) and in the longitudinal (dashed line) channel. The free $E_{\text{av}}(q)$ is also shown (dot-dashed line).
- Fig. 5 : The inverse energy weighted sum rule at $q = 0$ in the transverse channel as a function of the Migdal parameter g' .
a) Comparison of the ring approximation without Δ (dot-dashed line) with the exact RPA, evaluated with $f_0 = 0$ (continuous line) and $f_0 = 1$ (dashed line).
b) Comparison of the ring approximation with Δ (dashed line) and without it (continuous line).
In both cases the constant free value is also shown; the Fermi momentum is $k_F = 1.36 \text{ fm}^{-1}$.
- Fig. 6 : The quantity $S_{-1}(q)/Z$ in ring approximation both in the longitudinal (dashed line) and in the transverse channel (continuous line). The latter is the susceptibility per proton in units of $[2\mu_0^2(\mu_p - \mu_n)^2]^{-1} \text{ MeV}^{-1}$. The experimental points at finite q in ^{56}Fe [bars, from Ref. 31] and in ^{12}C [crosses, from Ref. 30] are displayed, as well as the $q = 0$ values for $^6\text{Li}(\circ)$, $^{28}\text{Si}(\square)$, $^{90}\text{Zr}(\bullet)$ and $^{208}\text{Pb}(\blacktriangle)$ [taken from Ref. 18]. In the figure $k_F = 1.15 \text{ fm}^{-1}$.

Fig. 7 : The characteristic frequency $E_c(q)$ divided by $\hbar c q$ in ring approximation (Δ included), both in the transverse (continuous line) and in the longitudinal channel (dashed line). The free quantity is also shown (dot-dashed line). In the figure $k_F = 1.36 \text{ fm}^{-1}$ and $g' = 0.7$.

Fig. 8 : The particle hole interaction: a) direct, b) exchange matrix elements.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5a

FIG. 5b

FIG. 6

FIG. 7

FIG. 8