Ref.TH.3266-CERN

THE QCD GENERALIZED PARTON MODEL AND A NEXT-TO-LEADING
APPROXIMATION OF THE GLUON DENSITY

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ABSTRACT

Using recent neutrino data, we calculate in the framework of the generalized parton model the modification of the gluon density caused by next-to-leading logarithmic effects. We find that they are of the same order of magnitude as the experimental uncertainties, and that the gluon density tends to be more peaked towards small x .

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Ref.TH.3266-CERN

23 March 1982

1. - INTRODUCTION

The CDHS collaboration has recently determined the nucleon gluon density from deep inelastic data¹⁾. Their analysis was carried out in the leading logarithmic approximation using the Altarelli-Parisi equations²⁾. Here we examine the validity of this approximation and give the correction to their gluon density due to next-to-leading logarithmic effects. The framework of the generalized parton model³⁾ is particularly well suited to such an analysis since it allows a generalization to all orders in $\alpha(Q)$ (running coupling constant) of the Altarelli-Parisi equations. Before presenting our analysis (Section 3), we briefly describe the generalized parton model and introduce the Q dependent parton densities which parametrize the deep inelastic structure functions (Section 2).

2. - QCD GENERALIZED PARTON MODEL

In this model one parametrizes the deep inelastic structure functions with Q -dependent parton densities. (As usual $-Q^2$ is the squared momentum of the scattered current and x is the Feynman variable $0 \leq x \leq 1$.) We work in the limit where all masses are neglected and we consider that the range of Q is such that f flavours of quarks should be taken into account. (In practice, $2m_c < Q < 2m_b$, thus we take four flavours.) The factorization theorem⁴⁾, renormalization group, and also the symmetries of charge conjugation, colour and flavour (exact in the zero mass limit) tell us that one can define f quark densities $q_i(x, Q)$, f antiquark densities $\bar{q}_i(x, Q)$ and a gluon density $G(x, Q)$, which parametrize all symmetric deep inelastic structure functions. These densities satisfy the following coupled equations, true to all orders in perturbation of $\alpha(Q)$.

$$\dot{q}_i = P_{qq} \otimes q_i + P_{q\bar{q}} \otimes \bar{q}_i + \frac{1}{f} P_{q_1 q_2} \otimes \sum_{i=1}^f q_i + \frac{1}{f} P_{q_1 \bar{q}_2} \otimes \sum_{i=1}^f \bar{q}_i + \frac{1}{2f} P_{gq} \otimes G \quad (1a)$$

$$\dot{\bar{q}}_i = P_{q\bar{q}} \otimes \bar{q}_i + P_{q_1 \bar{q}_2} \otimes q_i + \frac{1}{f} P_{q_1 q_2} \otimes \sum_{i=1}^f \bar{q}_i + \frac{1}{f} P_{q_1 \bar{q}_2} \otimes \sum_{i=1}^f q_i + \frac{1}{2f} P_{g\bar{q}} \otimes G \quad (1b)$$

$$\dot{G} = P_{Gq} \otimes \sum_{i=1}^f (q_i + \bar{q}_i) + P_{Gg} \otimes G \quad (1c)$$

where the $\dot{}$ means $d/d \log Q$, and the convolution $\otimes$ is defined by

$$A(x) \otimes B(x) = \int_x^1 \frac{dy}{y} A(y) B\left(\frac{x}{y}\right)$$

The kernels P_{ij} are functions of x and $\alpha(Q)$. They are the inverse Mellin transforms of space-like cut vertices⁴⁾ (or minimal twist Wilson operators⁴⁾) anomalous dimensions. They are computable in perturbation of $\alpha(Q)$, and have been calculated up to second order $O(\alpha^2(Q))$ ⁵⁾⁻⁷⁾. The Altarelli-Parisi equations are recovered at first order in $\alpha(Q)$ since at this level of approximation $p_{q\bar{q}}$ and $P_{q_1 q_2}^{(-)}$ vanish. If one takes into account terms of order $\alpha^2(Q)$, one must consider the flavour non-singlet transition quark-antiquark ($P_{q\bar{q}}^{(-)}$) and the singlet transition $q_i \rightarrow q_j, \bar{q}_j$, respectively, characterized by $P_{q\bar{q}}$, $P_{q_1 q_2}$ and $P_{q_1, \bar{q}_2}$ ⁸⁾.

The integro-differential equations (1) allow the computation of the densities at any value of Q once they have been measured at a particular value of $Q = Q_0$. Furthermore, one can show that the parton densities defined above satisfy the same constraints as those of the naive parton model (for instance the well-known sum rules).

Because of the charge conjugation symmetry, it is very convenient to define the following combinations of quark and antiquark densities⁹⁾

$$q_{ij}^{NS-} = (q_i - \bar{q}_i) - (q_j - \bar{q}_j) \quad , \quad 1 \leq i, j \leq f \quad (2a)$$

$$q_{ij}^{NS+} = (q_i + \bar{q}_i) - (q_j + \bar{q}_j) \quad , \quad 1 \leq i, j \leq f \quad (2b)$$

$$\Delta = \sum_{i=1}^f (q_i - \bar{q}_i) \quad , \quad \Sigma = \sum_{i=1}^f (q_i + \bar{q}_i) \quad (2c)$$

Indeed, from Eqs. (1) and (2) one sees immediately that $q_{ij}^{NS\pm}$ and Δ satisfy diagonal equations

$$\dot{q}_{ij}^{NS\pm} = P_{qq}^{NS\pm} \otimes q_{ij}^{NS\pm} \quad (3a)$$

$$\dot{\Delta} = P_{q\bar{q}} \otimes \Delta \quad (3b)$$

with

$$P_{qq}^{NS\pm} = P_{qq} \pm P_{q\bar{q}} \quad , \quad P_{q\bar{q}} = P_{q\bar{q}} - P_{q\bar{q}} + P_{q_1, q_2} - P_{q_1, \bar{q}_2} \quad (3c)$$

The flavour singlet quantity Σ and the gluon density G satisfy coupled equations

$$\dot{\Sigma} = P_{qq}^s \otimes \Sigma + P_{qG} \otimes G \quad (4a)$$

$$\dot{G} = P_{Gq} \otimes \Sigma + P_{GG} \otimes G \quad (4b)$$

where

$$P_{qq}^S = P_{qq} + P_{q\bar{q}} + P_{q_1 q_2} + P_{q_1 \bar{q}_2} \quad (4c)$$

One may observe that $q_i^{NS\pm}$, Δ and (Σ, G) do not mix together. The reason is that q_i^{NS-} , Δ is odd under charge conjugation where as q_{ij}^{NS+} , Σ and G are even, q_{ij}^{NS+} is a flavour non-singlet contrary to Σ, G which are flavour singlet. Let us stress that the difference $q_i - q_j$, sometimes referred to in the literature as a non-singlet quantity, does not satisfy^{8),9)} a diagonal equation beyond the first order in $\alpha(Q)$.

The kernels $P_{qq}^{NS\pm}$, P_{qq}^- , P_{qq}^S , P_{qG} , P_{Gq} and P_{GG} have been given to second order in $\alpha(Q)$ in Refs. 6)-8). In the second order approximation $P_{q_1 q_2} = P_{q_1 \bar{q}_2}$. In this approximation one can see that the quantity $q_i^{NS-} = q_i - \bar{q}_i$ also satisfies a diagonal equation

$$\dot{q}_i^- = P_{qq}^{NS-} \otimes q_i^- + O(\alpha(Q^3)) \quad (5)$$

Any structure function can be parametrized in terms of the densities q_i^{NS-} , q_{ij}^{NS+} , Σ and G . If it is odd under charge conjugation, it depends only on q_i^{NS-} and Δ for instance

$$x F_3(x, Q) = x [E_3(x, \alpha(Q)) \otimes \Delta(x, Q)] \quad (6)$$

If the structure function is even under charge conjugation, it is parametrized with q_{ij}^{NS+} or Σ and G depending on whether or not it is a flavour non-singlet or a flavour singlet. For instance,

$$F_2^{vp+v\eta}(x, Q) = x [E_{2q}(x, \alpha(Q)) \otimes \Sigma(x, Q) + E_G(x, \alpha(Q)) \otimes G(x, Q)] \quad (7)$$

In Eqs. (5) and (6), E_{aq} and E_G can be interpreted as effective charges between the quarks and gluons and the scattered current. They are functions of x , computed in perturbation of $\alpha(Q)$.

$$E_{aq} = \delta(1-x) + \alpha(Q) f_q^a(x) + O(\alpha^2) \quad (8a)$$

$$E_G = 0 + \alpha(Q) f_G(x) + O(\alpha^2) \quad (8b)$$

By using Eqs. (4) and (7) one can eliminate the singlet quark density and obtain coupled equations between F_2 and G ^{7),8)},

$$\dot{F}_2 = x \left[K_{FF} \otimes F_2/x + K_{FG} \otimes G \right] \quad (9a)$$

$$\dot{G} = K_{FG} \otimes F_2/x + K_{GG} \otimes G \quad (9b)$$

Similarly, one can eliminate Δ from Eq. (6) and obtain the following equation for $x F_3$ ⁶⁾⁻⁸⁾.

$$\dot{F}_3 = K_{FF}^3 \otimes F_3 \quad (10)$$

Ther kernels $K_{ij}(x, \alpha)$ and $K_{FF}^3(x, \alpha)$ are computable by convoluting the effective charges E_i^a and evolution kernels P_{ij} . They are known up to second order in $\alpha(Q)$ for all electroweak structure functions ^{7),8)}. Equations (8) and (9) could be used to test the theory.

We have therefore briefly described the content of the generalized parton model and shown how it allows one to compute the deep inelastic structure functions and their scaling violation in terms of $2f+1$ Q dependent parton densities. Let us now show how it can be used as a guideline for computing the corrections to the gluon density. We must note here that beyond the leading logarithmic approximation, the gluon density defined above is in general a renormalization scheme-dependent quantity ¹⁰⁾. As we have already noted, our definition for the gluon density is chosen in such a way that the naïve parton model sum rules are automatically satisfied. In addition, we performed the $\overline{MS}$ renormalization scheme to define the renormalized cut vertices.

3. - THE GLUON DENSITY

It was shown in Ref. 11) how to extract the nucleon gluon density from the knowledge of the $F_2(x, Q)$ structure function. The analysis was done in the leading logarithmic approximation. More recently, the CDHS collaboration presented an estimation of the gluon density ¹⁾ as the solution of the following coupled equations which are the leading logarithmic approximation ¹¹⁾ of Eq. (8a,b).

$$\dot{F}_2(x, Q) = x \cdot \frac{\alpha^0(Q)}{4\pi} \left[P_{qg}^0(x) \otimes F_2/x + P_{qq}^0(x) \otimes G \right] \quad (11a)$$

$$\dot{G}(x, Q) = \frac{\alpha^0(Q)}{4\pi} \left[P_{gg}^0(x) \otimes F_2/x + P_{gq}^0(x) \otimes G \right] \quad (11b)$$

Here $(\alpha^0/4\pi)P_{ij}^0(x)$ is the first order approximation of the evolution kernel $P_{ij}(x, \alpha(Q))$ and, for consistency, $\alpha^0(Q)$ is the leading logarithmic approximation of the running coupling constant $\alpha(Q)$. In order to eliminate the correlation between the Λ parameter and the gluon shape, the anti-quark distribution $\bar{q} = \sum_{i=1}^f \bar{q}_i$ is also used in Ref. 1). Indeed, in the leading logarithmic approximation, $\bar{q}$ satisfies

$$\dot{\bar{q}}(x, Q) = \frac{\alpha^0(Q)}{4\pi} \left[P_{q\bar{q}}^0(x) \otimes \bar{q} + \frac{1}{2} P_{qG}^0(x) \otimes G \right] \quad (12)$$

The result of the CDHS collaboration analysis is then¹⁾

$$x G^0(x, Q) = 2.62 (1 + 3.5x) (1-x)^{5.9}, \quad Q^2 = 5(\text{GeV})^2 \quad (13a)$$

with

$$F_2(x, Q) = 1.10 (1 + 3.7x) (1-x)^{3.19}, \quad Q^2 = 5(\text{GeV})^2 \quad (13b)$$

In order to estimate how higher order effects modify the gluon density, we observe that the next-to-leading approximation $G^1(x, Q)$ of the gluon density should satisfy the equation

$$\dot{F}_2(x, Q) = x \left[K_{FF}'(x, \alpha'(Q)) \otimes F_2/x + K_{FG}'(x, \alpha'(Q)) \otimes G' \right] \quad (14)$$

where K_{FF}^1 and K_{FG}^1 are the second order approximations of the kernels K_{FF} and K_{FG} in Eq. (8a). From Refs. 7) and 8) we have

$$\begin{aligned} K_{FF}^1(x, \alpha) = & \frac{\alpha}{4\pi} \left[8\delta(1-x) + \frac{16}{3} \frac{1+x^2}{(1-x)_+} \right] + \left(\frac{\alpha}{4\pi} \right)^2 \left[433.8714 \delta(1-x) \right] \\ & + \left(\frac{\alpha}{4\pi} \right)^2 \frac{16}{9} \left[\frac{1}{3} \left(\frac{208}{x} - 830 - 11x - 64x^2 \right) + 4(1-3x+24x^2) \log x \right. \\ & - 4(1+x) \left(12 \text{Li}_2(x) + 13 \log^2 x - \frac{11}{4} \pi^2 \right) + \left(\frac{32}{x} + 49 + x - 32x^2 \right) \log(1-x) \\ & + \frac{1+x^2}{1-x} \log x (38 + 9 \log x - 16 \log(1-x)) + 85.6157 \frac{1}{(1-x)_+} - 50 \left(\frac{\log(1-x)}{1-x} \right)_+ \\ & \left. - 2 \frac{1+x^2}{1+x} \left(\frac{1}{2} \log^2 x - 2 \text{Li}_2(-x) - 2 \log x \log(1+x) - \frac{1}{6} \pi^2 \right) \right] \end{aligned} \quad (15)$$

and

$$\begin{aligned}
 K_{FG}^1(x, \alpha) = & \frac{\alpha}{4\pi} 16(1-2x+2x^2) + \left(\frac{\alpha}{4\pi}\right)^2 \frac{16}{3} \left[\frac{52}{x} - 157 - 304x + 432x^2 \right. \\
 & + 2(2-208x+201x^2) \log x - (41+134x+2x^2) \log^2 x \\
 & + 2\left(\frac{12}{x} - 13 + 156x - 177x^2\right) \log(1-x) - 4(13+28x+8x^2)(Li_2(x) - \frac{\pi^2}{6}) \\
 & - 2(1-2x+2x^2)(26 \log x \log(1-x) - 9 \log^2(1-x) + \frac{17}{6} \pi^2) \\
 & \left. + 18(1+2x+2x^2)\left(\frac{1}{2} \log^2 x - 2 Li_2(-x) - 2 \log x \log(1+x) - \frac{\pi^2}{6}\right) \right]
 \end{aligned} \quad (16)$$

The difference $\Delta G \equiv G^1 - G^0$ satisfies the following equation.

$$\Delta G(x) = [K_{FG}^1(x)]^{-1} \otimes R(x) \quad (17a)$$

where

$$\begin{aligned}
 R(x) = & \left[K_{FF}^1(x, \alpha^1(Q)) - \frac{\alpha^0(Q)}{4\pi} P_{99}^0(x) \right] \otimes \frac{F_2(x, Q)}{x} \\
 & + \left[K_{FG}^1(x, \alpha^1(Q)) - \frac{\alpha^0(Q)}{4\pi} P_{96}^0(x) \right] \otimes G^0(x, Q)
 \end{aligned} \quad (17b)$$

The inverse kernel $[K_{FG}^1]^{-1}$ is defined such that $K_{FG}^1 \otimes (K_{FG}^1)^{-1} = \delta(1-x)$. The first and second order approximations to the running coupling constant $\alpha^0(Q)$ and $\alpha^1(Q)$ are given for $f = 4$ as

$$\frac{4\pi}{\alpha^0(Q)} = \frac{25}{3} \log \frac{Q^2}{\Lambda_{LO}^2} \quad (18a)$$

$$\frac{4\pi}{\alpha^1(Q)} = \frac{25}{3} \log \frac{Q^2}{\Lambda_{MS}^2} + \frac{154}{25} \log \log \frac{Q^2}{\Lambda_{MS}^2} \quad (18b)$$

Using the results of the CDHS analysis for $F_2(x, Q)$ and $G^0(x, Q)$ [Eqs. (13a), (13b)], and the fact that $\Lambda_{LO} \approx \Lambda_{MS} \approx 200$ MeV, it is easy to compute $R(x, Q_0)$ by numerical integration. ΔG is then determined by Eq. (17) and we have again performed a numerical integration. In this last equation we have made the approximation

$$\left[K_{FG}^1(x, \alpha^1(q)) \right]^{-1} \simeq \frac{4\pi}{\alpha^1(q)} \left[P_{qG}^0(x) \right]^{-1} \quad (19)$$

where P_{qG}^{0-1} is the inverse of the P_{qG}^0 which was first computed in Ref. 9).

$$\left[P_{qG}^0(x) \right]^{-1} = \frac{1}{16} \left[-x \delta'(1-x) + 2 \delta(1-x) + \sqrt{x} \left(3\sqrt{\frac{4}{3}} \sin\left(\sqrt{\frac{2}{3}} \log x\right) - 2 \cos\left(\sqrt{\frac{2}{3}} \log x\right) \right) \right] \quad (20)$$

We have checked that this approximation is numerically justified in our analysis.

Our result for xG^1 is displayed in Fig. 1 together with xG^0 . One can observe that xG^1 is more peaked towards small x than xG^0 . In Fig. 2 one can see that the second order effects are of the same order of magnitude as the experimental uncertainties. The integral of xG^1 , which represents the fraction of longitudinal momentum carried by gluons, is also different from that of xG^0 . This is in fact not a surprise since in the next-to-leading approximation the sum rule

$$\int_0^1 dx \, x G^0(x, Q) = 1 - \int_0^1 dx \, F_2(x, Q) \quad (21)$$

is still not valid at the next-to-leading order. The correct sum rule is

$$\int_0^1 dx \, x G^1(x, Q) = 1 - \int_0^1 dx \, x \Sigma(x, Q) \quad (22)$$

Indeed, beyond the leading logarithmic approximation, $x\Sigma$ cannot be identified with F_2 . We found that

$$\int_0^1 dx \, x (G^1(x, Q) - G^0(x, Q)) = -0.03, \quad Q^2 = 5(\text{GeV})^2$$

This correction is very small compared with its experimental uncertainties.

4. - CONCLUSION

We have formulated the QCD generalized parton model, and introduced all necessary parton densities which allow a convenient parametrization of the structure functions. In this framework we have estimated the corrections, due to the next-to-leading logarithmic effects, to the gluon density extracted by the CDHS collaboration. These corrections are not too big, but exhibit the interesting feature that the "second order" gluon density seems more peaked towards small x than the "first order" gluon density.

ACKNOWLEDGMENTS

We would like to thank K. Ellis and R. Petronzio for a critical reading of this manuscript. One of us (L.B.) thanks the Theoretical Physics Division at CERN for its hospitality while this work was being completed.

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FIGURE CAPTIONS

- Figure 1 The gluon distribution of Ref. 1) $xG^0(x)$ at $Q^2 = 5(\text{GeV})^2$ and the corrected one $xG^1(x)$ using the next-to-leading logarithmic analysis.
- Figure 2 The gluon distribution $xG^0(x)$ and its experimental uncertainties plotted with $xG^1(x)$. It is clear that the next-to-leading logarithmic analysis likes the gluon distribution to be more peaked toward small x .

Fig. 1

Fig. 2