Ref.TH.3273-CERN/ADDENDUM

FURTHER REMARKS/ON THE DETERMINATION OF
EFFECTIVE POTENTIALS IN SUPERSYMMETRIC THEORIES

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In Ref. 1) we analyzed an $O(N)$ Wess-Zumino type model, concluding that supersymmetry is not broken despite an apparent negative energy stationary state identified by Zanon²⁾. Since then, two other papers^{3),4)} have appeared which cast some doubts (at least one of them⁴⁾) on the consistency of the model or its perturbative (or $1/N$) expansion.

In this Addendum we wish to clarify this difference of interpretation. We will indeed stress that the model may be consistently analyzed with a $1/N$ expansion up to a Landau pole position characteristic of a non-asymptotic free theory. Beyond that position the $1/N$ expansion is invalid and, in particular, the leading $1/N$ potential and the negative energy stationary state it has have no significance. Therefore, in its validity region the $1/N$ expansion shows a supersymmetry preserving ground state as is indeed expected from the index theorem¹⁾.

In Ref. 1) we remarked that the inter-relation between coupling constant and wave function renormalization suggests a renormalization prescription that keeps positive the kinetic term of the $O(N)$ singlet superfield (all other superfields being integrated out exactly). This was achieved by relating the renormalization point to the constant scalar field a around which one chooses to expand the effective action. The coupling constant showed then a dependence on a dictated by a renormalization group equation.

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In the leading $1/N$ approximation $g^2(a)$ showed a Landau-type pole (characteristic of non-asymptotic free theories) for a large value of a . Before the Landau pole, i.e., for small a , the $1/N$ expansion is consistent, $g^2(a)$ is positive and so is the effective potential which shows an absolute minimum for $a=0$ which corresponds to the supersymmetric ground state.

In approaching the Landau pole position the $1/N$ expansion breaks down. Indeed, this expansion implies the smallness of the parameter $g^2(a)/N$ and it is clear that for every large but finite N there is always a value of a , sufficiently near to the Landau pole position, for which $g^2(a)/N \sim 1$ thus invalidating the expansion.

In the usual renormalization prescription g^2 is a constant and g^2/N is independent of a . But in this case the renormalized kinetic term depends on a and has a zero ^{1),3)} related to our Landau pole position. This implies a singularity in a in the propagator of the superfield so that each internal line will contribute a factor depending on a and the effective expansion parameter will be again $g^2(a)/N$ as before. We see that what limits the validity of the $1/N$ expansion in this prescription [used in Refs. 2),3), 4)] is that for large values of a the kinetic term of the dynamical field has a zero and becomes successively a ghost ^{*}).

The Landau pole [or the zero of the kinetic term in the prescription of Refs. 2),3),4)] may, of course, be an artifact of the leading $1/N$ approximation. This has been shown to be the case in $O(N)$ $\lambda\phi^4$ theories ⁵⁾ where the next-to-leading contribution to the β function has an opposite sign as compared to the leading one. This generates an ultra-violet fixed point for finite coupling so that no Landau pole is left to next-to-leading approximation ⁵⁾. But, of course, subleading calculations represent only a hint for the possible elimination of the Landau pole disease. This may only be clearly established through non-perturbative (or $1/N$) approaches. There are indeed hints that some non-asymptotic free theories may be inconsistent ⁶⁾ and it is unclear to us whether or not the model under discussion may belong to this class. Anyhow, this is unrelated to the existence of an apparent negative energy minimum of the leading $1/N$ potential beyond the Landau pole. There, the $1/N$ expansion is invalid so that the leading potential and its minimum have not much significance.

*) Even if we forget that the leading $1/N$ potential has no meaning in this situation, a negative minimum of it would not imply a ground state of the theory. Indeed, at the negative minimum, the dynamical field is a ghost ^{1),3)}. Ghosts try to increase their potential energy so that a negative energy stationary point is energetically unfavourable as compared to the zero energy supersymmetric stationary state.

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Ref.TH.3273-CERN

ON THE DETERMINATION OF EFFECTIVE POTENTIALS
IN SUPERSYMMETRIC THEORIES

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ABSTRACT

We propose a renormalization procedure for dynamically generated effective actions. We show that, as expected, it leads to no spontaneous supersymmetry breaking if this is unbroken at the tree level. We also understand why the usually adopted renormalization prescription has led in some models to an apparent supersymmetry breaking for an unacceptable negative energy vacuum state.

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Application of the well-established $1/N$ expansion to some supersymmetric models seemed to indicate the possibility of spontaneous dynamical breaking of supersymmetry violating general properties such as positivity of the ground state energy or the index theorem ¹⁾. This would shed negative light on $1/N$ techniques which, nevertheless, seem applicable to supersymmetric theories. This puzzling result is well illustrated by Zanon's model ²⁾ where a non-supersymmetric minimum was found for a negative value of the effective potential.

In this note we wish to show that this apparent contradiction stems from an inappropriate renormalization procedure ³⁾ adopted in the evaluation of effective potentials. Moreover, if the renormalization is correctly performed, the effective potential, in terms of the dynamical fields, vanishes at the origin and is otherwise positive, thus confirming that supersymmetry will be preserved by radiative corrections if it is not broken at the tree level ⁴⁾.

We shall use Zanon's model to illustrate the correct renormalization procedure and find the loophole in that used in Ref. 2). The model consists of $N+1$ chiral supermultiplets ϕ and ϕ_i , $i=1, \dots, N$, described by the action ^{*)}

$$S = \int d^4x d^4\theta (\bar{\phi}_i \phi_i + \bar{\phi} \phi) - \int d^2x d^2\theta \left[\left(\frac{1}{2} m \phi^2 + \frac{1}{2} m_0 \phi_i^2 + \frac{g}{\sqrt{N}} \phi \phi_i^2 \right) + h.c. \right]$$

The ϕ_i fields appearing only bilinearly may be integrated over, thus leading to an effective action depending only on ϕ .

The well-known fact that these theories need only a wave function renormalization suggests a rescaling

$$\phi \rightarrow \sqrt{Nz} \phi, \quad g \rightarrow \frac{1}{\sqrt{z}} g, \quad m \rightarrow \frac{1}{z} m \quad (1)$$

Fields, coupling constants and masses will now represent renormalized quantities in terms of which the integration of S over ϕ_i leads to

*) We have introduced a mass m for the fields ϕ_i . It avoids infra-red problems. The final potential is infra-red free so that m may be set to zero if one wishes to make a comparison with Ref. 2).

$$S_{\text{eff}} = N \left(Z S_\phi - \frac{1}{2} \log \det A \right) \quad (2)$$

where

$$S_\phi = \int d^4x d^2\theta \bar{\phi} \phi - \frac{m}{2} \int d^4x d^2\theta \phi^2 \quad (3)$$

and

$$\det A = \det U \det^{-1} B$$

$$U = \begin{vmatrix} -2gF & -\square & -2gA-m_0 & 0 \\ -\square & -2gF^* & 0 & -2gA^*-m_0^* \\ -2gA-m_0 & 0 & 0 & -1 \\ 0 & -2gA^*-m_0^* & -1 & 0 \end{vmatrix} \quad (4)$$

$$B = \begin{vmatrix} -\frac{1}{2} \lambda g A \sigma_2 & \frac{1}{4} \lambda \sigma^\mu \partial^\mu \\ \frac{1}{4} \lambda \sigma^{\mu T} \partial^\mu & -\frac{1}{2} \lambda g^* A^* \sigma_2 \end{vmatrix}$$

$A(x)$ and $F(x)$ being the lowest component and the auxiliary field of the supermultiplet ϕ , respectively.

The usual way to evaluate the exact formal expression of Eq. (2) is to develop it around constant fields. In so doing, one has to regularize ultraviolet divergences. We adopt a Pauli-Villars prescription which suffices for the case under analysis, and call M the regulator mass.

We now evaluate the $\log \det$ term in Eq. (2) through its $1/N$ expansion around constant fields $A=a$, $F=f$, $\chi=0$. We shall see a singular behaviour in M for $M \rightarrow \infty$ not only in the potential (i.e., $-\mathcal{L}_{\text{eff}}$ for constant fields) but also in the coefficients of the A and χ kinetic terms. We thus write

$$\mathcal{L}_{\text{eff}} = (Z + C_A) A^* \square A + (Z + C_\chi) \frac{1}{2} \lambda \bar{\chi} \sigma^\mu \partial^\mu \chi - V + R \quad (5)$$

where R is regular in M and contains derivatives in the fields A , F and X higher than those explicitly written in Eq. (5). V will have an expression of the form

$$V = -Z f^* f + m a f + m^* a^* f^* + V_1 \quad (6)$$

where C_A , C_X and V_1 are explicit functions of M , a and f . V_1 is given by

$$V_1 = \frac{1}{64\pi^2} \left\{ (\alpha^2 + \beta)^2 \ln(\alpha^2 + \beta) + (\alpha^2 - \beta)^2 \ln(\alpha^2 - \beta) - 2\alpha^4 \ln \alpha^2 - 2\beta^2 \ln(M^2 + \alpha^2) - 3\beta^2 \right\} \quad (7)$$

with

$$\alpha^2 = |2ga + m_0|^2, \quad \beta = 2|gf| \quad (8)$$

C_A has the form

$$C_A = \frac{|g|^2}{8\pi^2} \ln \frac{M^2 + \alpha^2}{\alpha^2} + \text{terms regular in } M \quad (9)$$

The usual renormalization prescription ³⁾ consists of determining Z by imposing a condition on V at an arbitrary specific value of f and a that defines a renormalization scale μ^2 . This implies

$$Z = 1 - \frac{|g|^2}{8\pi^2} \ln \frac{M^2}{\mu^2} \quad (10)$$

Surely enough this is a legitimate renormalization condition in the sense that it eliminates all M^2 singularities in Eq. (5) and in particular in the potential V . But if the auxiliary field f is eliminated in terms of a by

$$\frac{\partial V}{\partial f} = 0 \quad (11)$$

then $V = V(a, f(a))$ acquires negative values, in particular for sufficiently large a . Moreover, if there is a stationary point

$$\frac{dV(a, f(a))}{da} = 0 \quad (12)$$

away from the origin, it happens for negative values of V . This was the stationary point detected in Ref. 2) and to which was assigned the responsibility of spontaneous symmetry breaking. In order to understand the reason for this apparent inconsistency with positivity of the ground state energy implied by supersymmetry, let us analyze the kinetic terms of Eq. (5). Using (9) and (10) we find, after setting $M = \infty$

$$Z + C_A = 1 + \frac{191^2}{8\pi^2} \ln \frac{\mu^2}{\alpha^2} + \dots \quad (13)$$

expression which becomes negative for large values of a . The same happens for $Z + C_X$ and it is possible to see that the negativity of both kinetic terms is correlated with the negativity of the potential. Thus the stationary point of Ref. 2) is a property of a ghost potential and cannot be identified with a stationary ground state. This rather unpleasant description may be circumvented by an alternative renormalization prescription showing clearly that supersymmetry is unbroken in this theory.

A wave function renormalization controls the normalization of the corresponding kinetic term. We are thus naturally led to normalize it at the background field values (a and f in our notation) that we wish to consider. The minimum conditions on the potential thus obtained determine those values for which linear terms in the fluctuations are absent, thus allowing the identification of a and f at the minimum with $\langle A \rangle$ and $\langle F \rangle$, respectively.

We could determine Z from $Z + C_A = 1$, but to avoid useless complications with the regular terms in Eq. (9), let us choose

$$Z = 1 - \frac{191^2}{8\pi^2} \ln \frac{M^2}{\alpha^2} \quad (14)$$

which eliminates from $Z + C_A$, and therefore from the A kinetic term, the logarithmic term which was at the origin of its negative values for the choice of Z in Eq. (10).

Equation (1) shows that if Z depends on a , the renormalized parameters g and m depend on a through Z . This dependence is the usual one, i.e.,

$$\frac{1}{g^2(\alpha)} = \frac{1}{g^2(\alpha_0)} + \frac{1}{8\pi^2} \ln \frac{\alpha^2}{\alpha_0^2} \quad (15)$$

Equations (6), (7) and (14) imply

$$V = -|f|^2 + maf + m^* a^* f^* + \frac{1}{64\pi^2} \left\{ (\alpha^2 + \beta)^2 \ln(\alpha^2 + \beta) + (\alpha^2 - \beta)^2 \ln(\alpha^2 - \beta) - 2\alpha^4 \ln \alpha^2 - 2\beta^2 \ln \alpha^2 - 3\beta^2 \right\} \quad (16)$$

It is easy to see that

$$V - f \frac{\partial V}{\partial f} - f^* \frac{\partial V}{\partial f^*} = |f|^2 + \frac{1}{64\pi^2} \left\{ (\alpha^4 - \beta^2) \ln \frac{\alpha^4 - \beta^2}{\alpha^4} + \beta^2 \right\} \geq 0 \quad (17)$$

for all α and $\beta \leq \alpha^2$, the equality sign in (17) holding only for $\beta = 0$. It is then clear that on the line

$$\frac{\partial V}{\partial f} = \frac{\partial V}{\partial f^*} = 0 \quad (18)$$

which expresses f in terms of a , the potential $V = V(a, f(a))$ is a positive function of a with its absolute minimum at $a = 0$ where V vanishes. Therefore $\langle A \rangle = \langle F \rangle = 0$ and supersymmetry is unbroken.

We see therefore that with our renormalization prescription, we succeeded in describing the theory for all choices of the background field a with a positive potential and in terms of fluctuations which have positive kinetic terms. On the other hand, in order to describe the same theory at different values of a , we found that the coupling constant depends on a , as described in Eq. (15).

If we call g_0 the coupling constant at $a = 0$, we find

$$g^2(\alpha) = \frac{g_0^2}{1 + \frac{g_0^2}{8\pi^2} \ln \frac{m_0^2}{\alpha^2}} \quad (19)$$

which shows a Landau-type pole at $\alpha^2 = m_0^2 \exp(8\pi^2/g_0^2)$ characteristic of a non-asymptotic free theory like the one analyzed here. All our discussions on the sign and values of kinetic terms and the potential are also valid for background fields beyond the Landau pole where g^2 becomes negative. This is due to the fact that V depends only on $|g|^2$.

To summarize, we have shown that the renormalization prescription of Refs. 2) and 3) generates negative potentials together with negative kinetic terms. Ghosts try to increase their potential energy so that a negative stationary point is energetically unfavourable as compared with a zero potential configuration. Thus, even in this language, we understand why supersymmetry is not broken in the model of Ref. 2). Moreover, we have shown how to avoid this pathological ghost interpretation through an alternative renormalization prescription that leads to bona fide fields with positive kinetic energy and with non-negative potentials as required by supersymmetry.

We may wonder why the generally adopted prescription of renormalizing the interaction at a fixed scale does not work for supersymmetric theories while it is perfectly applicable to usual theories as $\lambda\phi^4$ 5). In conventional theories the coupling constant is renormalized independently of the wave function and thus any prescription for the first one cannot influence the kinetic terms which are controlled by the second. In supersymmetry the only renormalization is the wave function one and therefore is determined by the kinetic terms. Or, at least if a definition of the interaction is introduced which leads to negative kinetic energy, it is impossible to appeal to another independent renormalization to correct that sign.

We wish to acknowledge fruitful discussions with L. Girardello, J. Iliopoulos, R. Cahn, G. Veneziano and S. Yankielowicz.

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