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A STUDY OF SECOND ORDER INTERFERENCE
FOR PIONS PRODUCED IN VARIOUS HADRONIC INTERACTIONS

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Abstract

Second-order interference of pion pairs of equal charge is observed in good statistics samples of 16 GeV/c π^+p and K^-p interactions (2×10^6 and 10^6 pion pair combinations, respectively) and in a smaller sample of $\bar{p}p$ annihilations at rest. The effective radius R of the pion source is found to be approximately 1.4 fm for all three types of interaction. The interference appears to be not "total", about 60 - 80% of the like-pairs interfering. It is shown that the numerical results for the radius R , for the depth ct of the source "photosphere" and for the fraction λ of the interfering pairs are sensitive to any dynamical correlation in the background, and hence depend on the background chosen. It is argued that further refinements in the theoretical formulation of the phenomenon and in the experimental approach are needed for truly quantitative results.

1. INTRODUCTION

The existence of second-order interferences in the production of pion pairs of equal charge, i.e. of Bose-Einstein (BE) condensation effects, has been demonstrated by several experimental groups [1-19], including this collaboration [2], both in hadronic and leptonic interactions. The phenomenon is seen as an enhancement in the number of these pairs when the difference of the four-momenta of the two members of the pair tends to zero. These effects allow - in principle - the determination of the effective radius R of the region from which pions are emitted, as suggested by G. Kopylov and M. Podgoretski [20] and by G. Cocconi [21], in 1974.

All experiments agree in indicating values of R ranging from 1 to a few fm. However, the information accumulated in the past seven years is still to be considered more qualitative than quantitative, because of severe statistical and systematic experimental limitations.

The work reported here had the following aims:

- (a) Study the BE effect on large data samples.
- (b) Compare the results obtained for different hadronic interactions, namely π^+p , K^-p and $p\bar{p}$.
- (c) Study the "background problem", i.e. how the results are affected by different choices of the "non-interfering" pion pairs needed as reference.

The data available were $\sim 140\,000$ events of π^+p interactions and $\sim 71\,000$ events of K^-p interactions, both at 16 GeV/c, with at least six charged secondaries, and $\sim 43\,000$ four-prong events of $p\bar{p}$ annihilations at rest. This yielded $\sim 2 \times 10^6$, $\sim 10^6$ and $\sim 2.5 \times 10^5$ pairs of charged "pions" (all secondaries not otherwise identified were called pions) in the π^+p , K^-p and $p\bar{p}$ experiments, respectively.

While in our previous investigation only events containing no neutral particles (4C fits) were used, the present samples, to improve statistics, accepted events of all types, i.e. with no, one or more than one neutral particles (4C, 1C and 0C fits).

2. METHOD OF ANALYSIS

2.1 Variables and formulation

The BE interference is generally investigated by studying the variation in the number of like (L) pions, i.e. pions of equal charge, $N_L = N(\pi_i^\pm \pi_j^\pm)$ relative to non-interfering pairs (background), as a function of difference $\Delta p = (p_i - p_j)$ of the four-momentum vectors of the two pions in the pair, or equivalently of the space and time-like components, $\Delta \vec{p} = (\vec{p}_i - \vec{p}_j)$ and $q_0 = |E_i - E_j|$.

The effect is then described by

$$N_L / N_{BGD} = 1 + f(\Delta p) \quad (1)$$

where, in the case of total interference, the function f has the value 1 at the origin and tends to zero as Δp increases. In most of the works reported in the literature, including our own [2], the function f has been parametrized as suggested by Kopylov and Podgoretski [20], in the framework of a simple model of a radiating spherical surface of radius R with incoherent point-like oscillators of life-time τ , namely

$$\frac{N_L}{N_{BGD}} = 1 + \frac{I^2(q_t R)}{1 + (q_0 \tau)^2} \quad (2)$$

where the variable q_0 is as defined above and

$$q_t = \frac{|(\vec{p}_i - \vec{p}_j) \times (\vec{p}_i + \vec{p}_j)|}{|\vec{p}_i + \vec{p}_j|}$$

is the projection of $\Delta \vec{p}$ on the plane perpendicular to $(\vec{p}_i + \vec{p}_j)$ and is closely related to the effective mass $M_{\pi\pi}$ of the two pions (for $q_0 \rightarrow 0$, $q_t^2 = M_{\pi\pi}^2 - 4m_\pi^2$). The function $I(q_t R) = 2J_1(q_t R)/q_t R$, where J_1 is the first order Bessel function.

For each pion pair, the values of the variables q_t and q_0 were computed in the c.m.s. of the interaction and the number of like pairs relative to a background was determined, bin by bin, as a function of q_t^2 , for successive intervals of q_0 .

2.2 The background problem

In quantitative studies of the interference signal, the question of what to take as the background, i.e. as the non-interfering pairs, N_{BGD} , is of fundamental importance.

The "ideal" background should consist of pions that

- (a) being not identical, cannot interfere,
- (b) are totally incoherent, i.e. in particular, free of dynamical correlations.

As such pairs do not exist in nature, the ideal conditions can only be approximated, experimentally, and any shortcoming of the background chosen will affect the results obtained. Also, it is possible that a background suitable for one experiment will be inadequate for another based on results of different type, multiplicity, etc.

In most of the works reported, N_{BGD} has been equated to the number of pairs of unlike (U) charge, $N_U = N(\pi^+\pi^-)$ in the same event as the like pairs, and hence the quantity studied is AN_L/N_U , where A is the ratio between the numbers of like and unlike pair combinations that can be made with the total number of pions present. This "standard" background is obviously far from satisfying condition (b), i.e. of being free of dynamical correlations. In fact, the $(\pi^+\pi^-)$ mass spectrum, and hence the q_t distribution, is strongly affected by resonances.

In our paper of ref. [2], based on 4C fit events (no neutral particles in the final state), it had been verified that the effect of the ρ^0 , the dominant resonance, was negligible if the kinematic region used was limited to $q_t^2 < 0.2 \text{ (GeV/c)}^2$ and $q_0 < 0.2 \text{ GeV}$ (i.e. $M_{\pi\pi} < 0.5 \text{ GeV}$).

The situation is, a priori, more delicate when events of all types (and hence containing in general one or more neutral particles) are used. In this case, the reflections in the $\pi^+\pi^-$ spectra due to the η and ω resonances may cause correlations in the N_U background affecting also the kinematic region near the origin, where the effects of the ρ^0 are small.

For these reasons, in the investigation reported here, in addition to using the "standard" background $N_{BGD} = N_U$, we have searched for other backgrounds that would better approximate the condition of being free of dynamical correlations. Several methods have been tried, which were, in fact, variants of two main types consisting either in taking the two members of the pion pairs from different events with the same final state, or in creating fictitious pairs, in a given event, by mixing at random the transverse momenta of the secondaries, while leaving the longitudinal momenta unchanged.

The quality of the various backgrounds so constructed was gauged by asking that the ratio N_U/N_{BGD} , as a function of the effective mass $M_{\pi\pi}$, be as flat as possible, apart from the resonance signals.

This condition was best met by a background based on fictitious pions created from the real ones in a given event, by interchanging at random, and separately, the two components of the transverse momentum (note that the locations of pions on the DST are according to charge, so that the longitudinal charge distribution remains unchanged). Thus, in this method, the total three-momentum is rigorously conserved, event by event, and energy conservation is only slightly violated, longitudinal phase-space and leading particle effects are preserved.

From these "reshuffled" pions, pairs of like or unlike charge can be formally constructed, which will be called N_{LR} and N_{UR} , respectively, where "R" stands for "reshuffled".

Fig. 1(a) shows the effective mass distribution of the experimental pairs of unlike charge, N_U , for the events of the reaction $\pi^+ p \rightarrow p 3\pi^+ 2\pi^- \pi^0$ compared with that for the reshuffled pairs, N_{UR} . In fig. 1(b) is shown the ratio N_U/N_{UR} . It can be seen that clear enhancements are present for η and ω reflections and for the ρ^0 resonance, over a reasonably flat curve.

In the following, this method is thus adopted as an acceptable way of constructing pairs of (hopefully) uncorrelated non-interfering particles, to be used as a background.

In the next section, results are presented for both the quantities AN_L/N_U and AN_L/N_{UR} , where N_U is the "standard" background of unlike pion pairs and N_{UR} is the background obtained as described above. In both cases, the kinematic region is that defined by

$$q_t^2 = 0.002 - 0.2 \text{ (GeV/c)}^2$$

$$q_0 = 0 - 0.2 \text{ GeV.}$$

The q_t^2 interval $0 - 0.002 \text{ (GeV/c)}^2$ has been excluded because of a contamination of Dalitz pairs in the OC fit events.

3. RESULTS

The number of events and of like and unlike pion pair combinations in the kinematic region considered are listed in table 1, for the three experiments.

The results obtained for the ratios AN_L/N_U and AN_L/N_{UR} are presented in figs 2 and 3 respectively, as functions of q_t^2 , for four intervals of q_0 , and lead to the following conclusions:

- (a) In both figs 2 and 3, and for all three types of interaction, a clear enhancement is seen for $(q_t, q_0) \sim 0$, which vanishes, as expected, with increasing q_t and q_0 . Such enhancement is evidence for the occurrence of the BE interference for the pion pairs of like charge.
- (b) The amplitude of the enhancements at the origin is different, in general, in different interactions.
- (c) The amplitude of the effect is strongly affected by the choice of the background.
- (d) With both $N_{BGD} = N_U$ and $N_{BGD} = N_{UR}$, the values of AN_L/N_{BGD} do not reach the value of 2 expected in the case of total interference (except for the $\bar{p}p$ results in fig. 2). The maximum value reached is ~ 1.3 for π^+p and K^-p interactions with the standard background and $1.5 - 1.6$ with the "uncorrelated" background. With this latter background the maximum reached in $\bar{p}p$ interactions is ~ 1.5 .

The remark made in (d) applies to all the results reported in the literature (except for one experiment of $\bar{p}p$ interactions, of low statistical significance (4393 events) [6]): the expected value of 2 is never reached and the maximum observed lie between ~ 1.1 and ~ 1.5 . Whether this is due to a real physical situation remains to be understood and will be discussed in sect. 4. In any case, the conclusion must be drawn that the results behave as if the interference were not total, i.e. did not involve all the pairs of equal charge, but only a fraction of them.

Under these circumstances, the Kopylov-Podgoretski formula of eq. (2), which describes the total interferences of fully incoherent bosons, is inadequate to describe the experimental results. The acceptable fits to the data reported in the literature were possible only because the data had large enough errors to accommodate it. The fit to our π^+p and K^-p data of fig. 2 with formula (2) gave totally unacceptable results, with χ^2 probabilities of $10^{-6} - 10^{-7}$.

To cope with this situation, the Kopylov-Podgoretski formula (2) has been modified as follows:

$$\frac{N_L}{N_{BGD}} = C \left(1 + \lambda \frac{I^2 (q_t R)}{1 + (q_0 \tau)^2} \right), \quad (3)$$

where C is a normalization constant and the new parameter λ , to be determined in the fits to the data along with C, R and τ , represents the "effectively interfering" fraction of all pairs of like particles, taken as pions.

The solid lines in figs 2 and 3 are fits with formula (3). The values of the parameters R, $c\tau$, C and λ , with their statistical errors, and the χ^2/ND values are listed in tables 2 and 3, corresponding to figs 2 and 3, respectively.

The following comments can be made:

- (a) Table 2 - The values of R and $c\tau$ obtained with the standard background, N_U , are found to be similar in the three experiments, irrespective of the nature of the particle inducing the interaction. The values

$$R = 1.85 \pm 0.15 \text{ fm} \quad (4)$$

are consistent with the three results. The errors given here reflect the dispersion of the values obtained with different binnings, and hence are larger by a factor of ~ 3 than the statistical errors given in table 2.

The values of $c\tau$ are always smaller than R and vary between 1 and 1.5 fm. The values of the parameter λ are of 0.4 - 0.5 for π^+p and K^-p interactions, while λ is close to unity for $p\bar{p}$ annihilations.

- (b) Table 3 - The values of R and $c\tau$ obtained with the hopefully uncorrelated background based on pion pairs with their transverse momenta reshuffled, are all lower than those obtained with the "standard" background. For all three experiments, the value

$$R = 1.40 \pm 0.15 \text{ fm} \quad (5)$$

is consistent with the results of the fits. The values of $c\tau$ vary between 0.9 and 1.3 fm. The parameter λ is now higher for π^+p and K^-p (0.8 - 0.9) and lower for $p\bar{p}$ (~ 0.6). The $p\bar{p}$ results are those most affected by the background change.

If the method used to construct uncorrelated pions is valid, then the results for R , $c\tau$ and λ obtained from the ratio AN_L/N_{UR} , should be obtained, for all interactions, also if one studied the ratio N_L/N_{LR} , where N_{LR} is the number of fictitious pairs of like charge constructed with the method described in sect. 2. This has been verified to be true and the results are listed in table 4. For these reasons, the results obtained with the "uncorrelated" background are considered as reliable, while those obtained with the "standard" background are judged as being affected by the dynamical correlations in the distributions of pairs of unlike charge, due to resonances.

In our earlier work of ref. [2] based on a smaller sample of 4C fit events of π^+p interactions at incident momenta between 4 and 25 GeV/c, the fit to the AN_L/N_U distributions with formula (2) had yielded two χ^2 minima, corresponding to R values of 1.0 and 1.9 fm, respectively, the former with a better χ^2 , and leading to a combined value of $1.0 \pm {}^{0.4}_{0.2}$ fm. Refitting now those distribution with formula (3), a single χ^2 minimum is obtained corresponding to $R = 1.5 \pm 0.2$ fm, $c\tau = 1.1 \pm 0.3$ fm and $\lambda = 0.6 \pm 0.1$, consistent with the present results obtained with "uncorrelated" background.

4. DISCUSSION

Notwithstanding the encouraging features of the results obtained with the "uncorrelated" background based on pion pairs with reshuffled transverse momenta, two difficulties must be pointed out:

- (a) The q_t distributions obtained with both N_{UR} and N_{LR} as backgrounds, do not seem to exhaust the interference effect in the kinematic region selected, as indicated by the fact that the ratios N_L/N_{BGD} do not flatten out completely, but keep on decreasing slightly, at large q_t values. It is impossible to establish whether this is a physical phenomenon or a spurious effect caused by the background. The consequence of it are a slight decrease of R and increase of λ as the kinematic region is extended.
- (b) The χ^2 probabilities of the fits to the π^+p data, where the statistics are best, are from fair to poor (tables 2 and 3). This may be caused, in part, by the effect described in (a), but another plausible reason for it may be sought in the inadequacy of the theoretical formulation to describe the real physical situation. The pion source is far from being an ideal sphere of radius R . One should stress that the radius R measured experimentally cannot be identified with the radius of the interacting region. In fact, pions are produced both directly in the interaction and indirectly from resonance decay, this latter being the dominant source [22]. The radius one measures is an average of some sort between that of the direct pion source and that of the sphere containing the resonance decay points.

P. Grassberger [23] has pointed out that a resonance should give rise to a peak in the N_L/N_{BGD} distributions at $q_t, q_0 = 0$, with width related to its mean path before decay. Our results seem consistent with the observation of the effects due to short-lived resonances like, e.g. the ρ . However, the width of the peaks due to long-lived resonances ($\eta, \omega, \dots$) would be too narrow to detect: for the ω , for example, the interference region would be confined to $q_t^2 < 0.003 \text{ GeV}^2/c^2$, too small for our resolution and thus resulting in a flat N_L/N_{BGD} distribution which would simulate non-interference. The slight decrease of N_L/N_{BGD} at large q_t , discussed in (a), if real, could reflect the smaller radius of the direct pion source.

As shown by the results in sect. 3, the values of R and λ depend strongly on the non-interfering background used.

There are several other factors which can also affect the fitted values of R and λ and they will be examined here in turn:

- (a) Resolution - i.e. the precision in the energy and angle measurements determining the values of the variables q_t and q_0 . Insufficient resolution will tend to flatten the distribution of N_L/N_{BGD} , and hence will reduce the maximum value of this ratio, produce $\lambda < 1$ and give values of R and σ smaller than the real ones.

We have studied the resolution in our three experiments and have found that in the π^+p and K^-p experiments, for the pion pairs in the small kinematic region considered, the mean error in the determination of q_0 is of ~ 0.01 GeV, and between 0.002 and 0.01 (GeV/c)² for q_t^2 . Our resolution is adequate to measure R values smaller than 3 fm. Pions from much larger sources would be observed as non-interfering.

- (b) Contamination - i.e. particle misidentifications that cause calling a "pion" a particle of different nature. In 4C fit events, contamination is negligible, but in 1C and 0C fit events, $\pi^+/K^+/p$ and π^-/K^- ambiguities are present. As a consequence, not all "like" pairs will interfere, and hence λ will be smaller than unity.

In the π^+p , and even more, in the K^-p events, some contamination must be present. Several estimates based on the number of ambiguous hypotheses, indicate that, in the kinematic region considered, contamination does not account for a reduction of λ by more than 0.1 - 0.2^(*).

- (c) Normalization - the maximum value that the ratio N_L/N_{BGD} will reach for $q_t, q_0 = 0$ is $C(1 + \lambda)$, where C is the normalization constant, i.e. the value of N_L/N_{BGD} at very large q_t and q_0 value, where interference no longer operates. The value of C will in practice be

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(*) As a check, the BE effect was investigated separately for the pairs of negative pions of 6-pronged π^+p interactions, where contamination should be negligible. The values obtained for R , σ and λ were all compatible with those reported for the total sample.

the value of the ratio at the far end of the kinematic region investigated, and may depend on the background chosen. As the normalization procedure brings the N_L and N_{BCD} distributions to the same area over the entire kinematic range, then the excess of like pairs near the origin must result in a deficiency at large q_t and q_0 values: thus, the value of C will be smaller than 1. Furthermore, larger deviations of C from unity may occur if dynamic constraints - e.g. presence of resonances - affect the pion pairs used for the background.

In all fits to our distributions (tables 2 and 3), the values of C are always close to 0.9 which indicates that our normalization procedure is reasonable.

- (d) Dynamics of pion production - the BE interference effect is calculated under the assumption that one is dealing with identical bosons incoherently produced. The phenomenon we are studying, i.e. pion production in hadronic interactions, is quite likely at variance with this ideal situation. In fact, the assumption that the pion source is fully incoherent is probably incorrect, especially when leading particle effects, diffraction dissociation, two-body reactions and multiperipheral processes may create dynamical correlations that reduce or enhance the BE interference.

Several authors [24-26] have pointed out that the BE interference effect may be a sensitive test of the relative importance of coherent and incoherent pion production, and have made predictions for different interactions (pp , $p\bar{p}$, e^+e^- etc.). The authors of ref. [25] stated that the BE effects can be cancelled or damped by dynamical short-range repulsion between like pairs. They predict that this cancellation should be complete in e^+e^- annihilations, of comparable importance in πp and Kp processes and significantly smaller in $p\bar{p}$ annihilation. The results obtained with the "standard" background seem to be consistent with these predictions. This is not the case when we consider the results obtained with the "uncorrelated" background. Also the prediction of ref. [25] concerning the cancellation of BE effects in e^+e^- annihilation is in contradiction with the observations of G. Goldhaber [17].

5. CONCLUSIONS

- (i) The results presented here provide clear evidence for the existence of second-order interference between two pions π_i and π_j of equal charge, based on data of good statistical significance.
- (ii) The effect is observed in three hadronic interactions, namely π^+p , K^-p at 16 GeV/c and $\bar{p}p$ annihilations at rest. The effective radius of the pion emitting source is found to be approximately the same for the three types of interaction. This radius is to be understood as a sort of average between the dimensions of the region of direct pion emission and of the region containing the mean path of the short-lived resonances decaying into pions.
- (iii) The quantity $c\tau$, related to the time-like component $q_0 = E_i - E_j$ of the four-momentum vector $p_i - p_j$, is found to be always smaller than R , which is consistent with its interpretation as the depth of the "photo-sphere" of the pion emitting source.
- (iv) The second-order interference observed is not total, i.e. only a fraction λ of the pairs of equal charge appear to interfere. This may be caused both by experimental limitations (poor resolution, contamination of particles other than pions, etc.) and by dynamical correlations between the pions, which hinder the interference.

The qualitative results (i) and (iv) listed above are independent of the systematic uncertainties caused by the "background problem", i.e. by the choice made for the non-interfering pairs that must be used as reference. Evidence has been presented here that any correlation between the two particles used for background distorts the results obtained and that different values of R , $c\tau$ and λ are obtained with different backgrounds. With samples consisting of events of relatively low multiplicities (4-10) and containing neutral particles, as is the case in this work and in many other investigations reported in the literature, the "standard" background of pairs of pions of unlike charge is unsatisfactory, because of the distortions caused by resonances (η , ω , ρ^0 ...). With a background of fictitious pairs created by interchanging at random the two components of

the transverse momenta of all pions in every event, more reliable results are obtained. The value of the radius R , for the three interactions investigated here is found to be of ~ 1.4 fm, $c\tau \sim 1$ fm and $\lambda \sim 0.8$. It is felt that more precise determinations of these quantities require deeper understanding of the problems involved, both experimentally and theoretically.

The background problem may become less severe with events of very high multiplicity (20-30 tracks), where the very high combinatorial rate reduces the effect of resonances. However, if high multiplicity means high average pion energies, the problem of precision of measurements (absolute, not relative precision is needed) becomes critical.

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TABLE CAPTIONS

Table 1   Number of events and of pion pair combinations in the π^+p , K^-p and $\bar{p}p$ experiments.

Table 2   Results of the fits when using the standard background of pairs of unlike charge, N_U .

Table 3   Results of the fits when using as background the pairs of unlike charge, N_{UR} , constructed by reshuffling at random the transverse momentum components of the pions in each event.

Table 4   Results of the fits when using as background the pairs of like charge, N_{LR} , constructed by reshuffling at random the transverse momentum components of the pions in each event.

TABLE 1

Incident particle	Incident momentum GeV/c	Number of prongs	Total number of events	No. of like pion pairs in the kinematic region used	No. of unlike pion pairs in the kinematic region used
π^+	16	6	112 496	104 664	152 659
		8	24 740	74 631	94 074
		≥ 10	2 836	22 601	23 689
		≥ 6	140 072	201 896	270 422
K^-	16	≥ 6	71 536	104 776	151 470
$\bar{p}$	0	4	43 024	35 642	66 288

TABLE 2

Incid. part.	No. of prongs	R, fm	$c\tau$, fm	λ	C	χ^2/NDF
π^+	≥ 6	1.84 ± 0.06	1.08 ± 0.11	0.49 ± 0.02	0.89 ± 0.01	212/196
K^-	≥ 6	1.85 ± 0.10	0.96 ± 0.15	0.39 ± 0.03	0.92 ± 0.01	113/96
$\bar{p}$	4	1.88 ± 0.06	1.54 ± 0.14	1.20 ± 0.08	0.92 ± 0.01	105/96

TABLE 3

Incid. part.	No. of prongs	R, fm	$c\tau$, fm	λ	C	χ^2/NDF
π^+	≥ 6	1.45 ± 0.04	0.96 ± 0.05	0.88 ± 0.02	0.86 ± 0.01	142/96
K^-	≥ 6	1.36 ± 0.06	0.92 ± 0.07	0.83 ± 0.04	0.84 ± 0.02	79/96
$\bar{p}$	4	1.44 ± 0.09	1.27 ± 0.20	0.63 ± 0.05	0.92 ± 0.02	104/96

TABLE 4

Incid. part.	No. of prongs	R, fm	$c\tau$, fm	λ	C	χ^2/ND
π^+	≥ 6	1.43 ± 0.05	0.86 ± 0.06	0.76 ± 0.02	0.89 ± 0.01	164/196
K^-	≥ 6	1.27 ± 0.08	0.63 ± 0.09	0.75 ± 0.05	0.85 ± 0.03	105/96
$\bar{p}$	4	1.45 ± 0.12	1.39 ± 0.26	0.64 ± 0.06	0.91 ± 0.02	100/96

FIGURE CAPTIONS

- Fig. 1 (a) Effective mass distribution, in the reaction $\pi^+ p \rightarrow p 3\pi^+ 2\pi^- \pi^0$ at 16 GeV/c, of the experimental pairs of pions of unlike charge, N_U , and of the fictitious pairs of unlike charge constructed by reshuffling randomly, event by event, the two components of the transverse momentum of all pions, N_{UR} .
- (b) Dependence of the ratio N_U/N_{UR} on the $\pi\pi$ effective mass.
- Fig. 2 q_t^2 dependence of the quantity AN_L/N_{BGD} , with $N_{BGD} = N_U$, the number of pion pairs of unlike charge, for four intervals of q_0 . The curves are fits with the formula (3) of the text.
- Fig. 3 q_t^2 dependence of the quantity AN_L/N_{BGD} with $N_{BGD} = N_{UR}$, for four intervals of q_0 . N_{UR} are the pairs of pions of unlike charge constructed by reshuffling randomly the two components of the pion transverse momenta, event by event. The curves are fits with the formula (3) of the text.

Fig. 1

$q_T^2 \cdot (\text{GeV}/c)^2$

Fig. 2

Fig. 3