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REVISITING THE SUPERLUMINAL LORENTZ TRANSFORMATIONS
AND THEIR GROUP-THEORETICAL PROPERTIES

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REVISITING THE SUPERLUMINAL LORENTZ TRANSFORMATIONS
AND THEIR GROUP-THEORETICAL PROPERTIES^(°)

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Abstract- We briefly revisit the introduction of Generalized Lorentz transformations (GLT) in four dimensions, by writing them in a new form which satisfies the requested group-theoretical properties. The new GLT's, in the particular case of boosts, reduce to the ones by Mignani and Recami (the latter, in fact, forming a group only in the case of collinear boosts). Some other aspects are briefly clarified.

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REVISITING THE SUPERLUMINAL LORENTZ TRANSFORMATIONS
AND THEIR GROUP-THEORETICAL PROPERTIES (°)

G.D.MACCARRONE and Erasmo RECAMI

Summary - We briefly revisit the introduction of Generalized Lorentz transformations (GLT) in four dimensions, by writing them in a new form which satisfies the requested group-theoretical properties. The new GLT's, in the particular case of boosts, reduce to the ones by Mignani and Recami (the latter, in fact, forming a group only in the case of collinear boosts). Some other aspects are briefly clarified.

Introduction - It is wellknown that the extension of the Principle of Relativity to Superluminal inertial frames is straightforward only in the pseudo-Euclidean ("Minkowski") space-times $M(n,n)$ having the same number n of space and time axes. In the two-dimensional case ($n=1$) the Superluminal Lorentz transformations were first introduced —even if in a preliminary form (not yet satisfying the group-theoretical properties)— in Ref.(¹) and then independently rediscovered(²) and completed(³) in Refs.(^{2,3}). As to the six-dimensional case ($n=3$), a preliminary suggestion first appeared in Ref.(⁴), followed by various contributions(⁵).

When facing the problem of extending the Lorentz transformations to faster-than-light frames in four-dimensions, one meets the "no-go" theorem by GORINI et al.(⁶), that essentially stated no such extensions to exist when they are required to satisfy all the following properties: (i) to refer to the 4-dimensional space-time M_4 ; (ii) to be real; (iii) to be linear; (iv) to preserve space isotropy; (v) to preserve the light-speed invariance; (vi) to possess the prescribed group-theoretical properties. It appears then necessary to release one (at least) of the above requirements.

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The first extensions of Lorentz transformations in the 4-dimensional Minkowski space M_4 tried to stick to the reality requirement (ii): cf. Refs.⁽⁷⁾; with the interesting exception of the early paper by SOMIGLIANA⁽⁸⁾.

According to us, however, it is essential to try to keep the characteristic postulates of Special Relativity (SR), i.e. the requirements (iii), (iv), (v), (vi), by releasing on the contrary the requirements (i) or (ii). This line of research has been first adopted in Refs.^(9,10), leading to the Generalized Lorentz transformations (GLT) by MIGNANI and RECAMI^(10,11). Such GLT's appeared till now, however, in a form suitable only for boosts: in the sense that they formed a group only in the case of collinear boosts. We want here to rederive them in a more general form, so to satisfy also the prescribed group-theoretical properties (requirement (vi)).

The new form of SLT's - The Superluminal Lorentz transformations (SLT) must transform —in the language of SR— time-like into space-like quantities, and vice-versa. Therefore, we require the GLT's (both sub- and Super-luminal), G_{μ}^{ν} , connecting two generic inertial frames f, f'

$$dx'_{\mu} = G_{\mu}^{\nu} dx_{\nu},$$

to have the following properties: 1) to transform inertial motion into inertial motion; 2) to form a (new) group $\mathbb{G}$; 3) to preserve space-isotropy; 4) to leave the chronotopical quadratic-form invariant, except for its sign^(*): $dx'_{\mu} dx'^{\mu} = \pm dx_{\mu} dx^{\mu}$, $[u^2 \leq c^2]$. The four-position x^{μ} is supposed of course to be a 4-vector even with respect to $\mathbb{G}$. In such an "Extended Relativity" the quadratic-form $dx_{\mu} dx^{\mu}$ is a scalar under subluminal Lorentz transformations (LT) and a pseudo-scalar under SLT's.

Stated differently, the GLT's, G_{μ}^{ν} , are unimodular, special, and such that the LT's are orthogonal whilst the SLT's are anti-orthogonal:

(*) To compare this fact with what ordinarily happens in General Relativity, see the following.

$$(1) \quad G^T G = + \mathbb{1} \quad u^2 < c^2 ;$$

$$(2) \quad G^T G = - \mathbb{1} \quad u^2 > c^2 .$$

At this point let us explicitly recall that, in two different bases, the scalar products define

$$x_\mu x^\mu \equiv x_\mu g^{\mu\nu} x_\nu ; \quad x'_\mu x'^\mu \equiv x'_\mu g'^{\mu\nu} x'_\nu ,$$

and the anti-orthogonal transformations operate in M_4 in such a way that⁽¹²⁾ either

$$(3a) \quad g'_{\mu\nu} = + \eta_{\mu\nu} \quad \text{and} \quad x_\mu x^\mu = - x'_\mu x'^\mu ,$$

or

$$(3b) \quad g'_{\mu\nu} = - \eta_{\mu\nu} \quad \text{and} \quad x_\mu x^\mu = + x'_\mu x'^\mu .$$

Notice that our anti-orthogonal transformations do not belong to the ordinary group of transformations usually adopted in General Relativity: See Ref.⁽¹³⁾, where this fact is clearly mentioned.

In other words, a SLT from a sub- to a Super-luminal frame, $s_0 \rightarrow S'$, can be identical with a suitable (ordinary) LT —let us call it the "dual" transformation— except for the fact that it must operate so as in eq.(3a), i.e. change time-like into space-like vectors.

Alternatively, one may also say that a SLT is identical with its dual sub-luminal LT, provided that we impose the primed observer S' to use the opposite metric-signature $g'_{\mu\nu} = - g_{\mu\nu}$, according to eq.(3b) [however, without changing the signs into the definitions of time-like and space-like quantities].^(10,14)

It follows that a generic SLT, corresponding to a Superluminal velocity $\vec{U}$, will be formally expressed by the product of the dual LT, corresponding to the subluminal velocity $\vec{u} \parallel \vec{U}$ with $u \equiv c^2/U$, by the matrix $S \equiv i \mathbb{1}$

$$(4) \quad \left\{ \begin{array}{l} \text{SLT}(\vec{U}) = \pm S [\text{LT}(\vec{u})] ; \\ S \equiv i \mathbb{1} , \end{array} \right. \quad \left[\begin{array}{l} \vec{u} \parallel \vec{U}; \quad u \equiv c^2/U \\ u^2 < c^2; \quad U^2 > c^2 \end{array} \right]$$

(5)

where $S \equiv S_4$ plays the rôle of the "transcendent SLT" since for $\vec{u} \rightarrow 0$ one gets $SLT(\vec{u} \rightarrow \infty) = \pm i \mathbb{I}$.

It is easy to recognize that the group-theoretical properties and space-isotropy are preserved only by an operation S which be representable by a 4×4 matrix symmetrical with respect to all possible axis-permutations, so as it is in eq.(5). Notice that our present operation S does not affect the speed u (i.e., does not operate any change $\beta \rightarrow 1/\beta$), differently from what stated in Refs.⁽¹¹⁾.

If we call $\underline{Z}(4)$ the discrete group of the 4-th roots of unity

$$(6) \quad \underline{Z}(4) \equiv \{\sqrt[4]{1}\} \equiv \{+1, -1, +i, -i\},$$

then the new group $\mathbb{G}$ of GLT's (both sub- and Super-luminal) is

$$(7) \quad \mathbb{G} = \underline{Z}(4) \otimes \mathcal{L}_+^\uparrow$$

where $\mathcal{L}_+^\uparrow$ is the ordinary (proper, orthochronous) Lorentz group.

If $\underline{G}$ is a GLT, then⁽¹⁰⁾ also $-\underline{G}$ belongs to $\mathbb{G}$:

$$(8) \quad \underline{G} \in \mathbb{G} \Rightarrow -\underline{G} \in \mathbb{G}, \quad \forall \underline{G} \in \mathbb{G};$$

so that, when we confine ourselves to subluminal LT's, we are left with the group $[\underline{Z}(2) \equiv \{\sqrt[2]{1}\} \equiv \{+1, -1\}]$

$$\mathcal{L}_+ \equiv \mathcal{L}_+^\uparrow \cup \mathcal{L}_+^\downarrow = \underline{Z}(2) \otimes \mathcal{L}_+^\uparrow$$

which is the true Lorentz group of Special Relativity, since it describes both particles and antiparticles within a purely relativistic scenario^(15,11).

At last, let us emphasize the important symmetry-property of the ordinary Lorentz boosts expressed (in natural units: $c=1$) by the identities

$$(9) \quad \left\{ \begin{array}{l} \frac{x - ut}{\sqrt{1-u^2}} \equiv -\frac{t - ux}{\sqrt{u^2-1}} ; \\ \frac{t - ux}{\sqrt{1-u^2}} \equiv -\frac{x - ut}{\sqrt{u^2-1}} . \end{array} \right. \quad [u \equiv 1/u]$$

Reduction to Mignani & Recami's transformations - In the particular case of boosts along x , eqs.(4)-(5) yield

$$(10) \quad \begin{cases} t' = \pm i \frac{t - ux}{\sqrt{1-u^2}} \equiv \mp i \frac{x - Ut}{\sqrt{U^2-1}} ; \\ x' = \pm i \frac{x - ut}{\sqrt{1-u^2}} \equiv \mp i \frac{t - Ux}{\sqrt{U^2-1}} ; \\ y' = \pm i y ; \\ z' = \pm i z , \end{cases} \quad \left[\begin{array}{l} \text{Superluminal case} \\ U \equiv c^2/u \\ u^2 < c^2 ; U^2 > c^2 \end{array} \right]$$

where we took advantage of eqs.(9). Let us observe that, under transformations (10), the $\mathbb{C}$ -fourvelocity $u_\mu \equiv dx^\mu/d\tau_0$ changes in such a way that $u'_\mu u'^\mu = -u_\mu u^\mu$, so that from $u^2 = +1$ it follows $u'^2 = -1$ (that is to say, bradyonic speeds are transformed into tachyonic speeds, notwithstanding the appearance of eqs.(10): cf. Ref.⁽¹²⁾).

We have to tackle the problem of physically interpreting our Superluminal boosts, eqs.(10). In a full paper to be published elsewhere⁽¹²⁾ —and in which recourse is temporarily made to an auxiliary six-dimensional space-time $M_6 \equiv M(3,3)$ — we show that one can deal with the first two eqs.(10) separately from the last two eqs.(10), and therefore split the reinterpretation-procedure in two steps.

As to the two-dimensional space-time $M(1,1) = (x,t)$, it can be regarded as a complex plane so that the imaginary unit $i \equiv \exp(\frac{1}{2}i\pi)$ operates there as a 90° -(pseudo)rotation. The same can be said of course for the "two-dimensional" operation

$$S_2 \equiv i \mathbb{1} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

which represents the transcendent SLT in $M(1,1)$; actually, in the case of a transcendent boost along x , both observers s_0, s' agree about the axes x', t' , x, t that: $t' \equiv x$; $x' \equiv t$, apart from their signs. This can be generali-

zed¹²): for instance, the transcendent SLT in six-dimensions ($S_6 = i \mathbb{I}$, where $\mathbb{I}$ is now the 6-dimensional identity-matrix) operates the "rotation" $\vec{x} \rightarrow \vec{x}'$.

As a consequence, the first two eqs.(10) can be reinterpreted, and eqs.(10) rewritten

$$(11) \quad \left\{ \begin{array}{l} t' = \pm \frac{x - ut}{\sqrt{1-u^2}} \equiv \mp \frac{t - Ux}{\sqrt{U^2-1}} ; \\ x' = \pm \frac{t - ux}{\sqrt{1-u^2}} \equiv \mp \frac{x - Ut}{\sqrt{U^2-1}} ; \\ y' = \pm i y ; \\ z' = \pm i z , \end{array} \right. \quad \left[\begin{array}{l} \text{Superluminal case} \\ U \equiv c^2/u \\ u^2 < c^2 ; \quad U^2 > c^2 \end{array} \right]$$

where the imaginary units do remain in the transversal coordinates (even after the "excursion" in M_6).¹² Let us observe that our (partially) reinterpreted SLT's, eqs.(11), yield for the speed of the first frame s_0 with respect to the second frame S' :

$$(12) \quad \vec{x} = 0 \Rightarrow \frac{x'}{t'} = \pm \left(-\frac{1}{u} \right) = \mp U , \quad \left[\begin{array}{l} u^2 < c^2 ; \quad U^2 > c^2 \\ U = c^2/u \end{array} \right]$$

where u, U are the speeds (along the boost direction: $\vec{u} // \vec{U}$) of the two dual frames s, S' ; remember that the dual frames possess always infinite relative speed. This shows once more that eqs.(11) are actually associated to Superluminal relative motion. The problem of suitably defining the tachyon velocity will be considered elsewhere¹².

We have thus shown that our new SLT's, eqs.(10), reduce to the "old" SLT's by MIGNANI and RECAMI^(10,11) in the case of Superluminal boosts. Actually, those SLT's formed a group only under collinear boosts.

Notice that our new SLT's together with the LT's form a group, namely the group $\mathbb{G}$ in eqs.(6)-(7), only in their original form (10); and not in their (partially) reinterpreted form (11). The reinterpretation (when necessary) is to be applied only at the end of any possible chain of GLT's. To act differently would mean¹² to use diverse metric-signatures (in our sense) during the GLT-chain, and this is of course forbidden: Cf. Ref.¹².

The second step in the interpretation of the SLT's has to do with the last

two eqs.(11). But these equations, as mentioned above, coincide with Mignani & Recami's. We can therefore refer to the work already performed in this direction in recent times; and we confine ourselves to quote Refs.⁽¹⁶⁾. Further details will appear elsewhere.⁽¹²⁾

Let us here mention, however, that we succeeded^(16,12) to geometrico-physically interpret —at least in some relevant cases— also the last two eqs.(11), thus identifying in a sense the space-time M_4 accessible to s_0 and the space-time M'_4 accessible to S' . For this reason we went on calling "transformations" (and not mappings) our SLT's, eqs.(11).

* * *

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