RL-82-024

Science and Engineering Research Council

Rutherford Appleton Laboratory

CHILTON, DIDCOT, OXON, OX11 0QX

GRAND UNIFICATION WITH LARGE SUPERSYMMETRY BREAKING

J Ellis L Ibanez and G G Ross

March 1982

RL-82-024

© Science and Engineering
Research Council 1982

The Science and Engineering Research Council does not accept any responsibility for loss or damage arising from the use of information contained in any of its reports or in any communication about its tests or investigations.

ABSTRACT

We give general criteria for the magnitudes of mass splittings between supersymmetric partners compatible with the desired hierarchy of mass scales in GUTs. These splittings arise naturally when masses are generated radiatively from a Higgs sector breaking supersymmetry through the O'Raifeartaigh mechanism. We construct explicit GUTs in which the supersymmetry breaking scale may easily be as large as the Planck mass.

GRAND UNIFICATION WITH LARGE SUPERSYMMETRY BREAKING

J Ellis

All Souls College and Dept of Theoretical Physics,
University of Oxford, Oxford

Permanent Address: Theory Division, CERN, 1211 Geneva 23,
Switzerland

L Ibáñez

Departamento de Física Teórica, Universidad Autónoma de Madrid

G G Ross

Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX11 0QX, England

and

Pembroke College,
University of Oxford, Oxford

Grand Unified Theories (GUTs) with a simple $N = 1$ supersymmetry have been intensively studied recently (1,2,3,4) because they offer a solution to the hierarchy problem. Through supersymmetry, scalar masses can be forbidden by the chiral symmetries of the associated fermion fields, and thus one can arrange that some scalar fields remain massless naturally even in the presence of a superheavy mass scale. However it remains to discover why the hierarchy of gauge symmetry breaking scales takes the numerical value it does: $m_w/m_{\text{Planck}} \approx O(10^{-17})$, $m_w/m_{\text{GUT}} = O(10^{-14})$. Also, since supersymmetry is not manifest at low energies, there is the question how large the supersymmetry breaking scale is, and how it is related to the gauge symmetry breaking scales. Ideally one would like to find a GUT where the only primordial scale was the Planck mass, and all other dynamical scales were numerically related to it by calculable factors.

In this paper we discuss what magnitudes of mass splittings between partners in a supermultiplet are compatible with the observed hierarchy of mass scales, pointing out that the splittings between non-Abelian gauge singlet partners may be arbitrarily large. We then discuss alternative mechanisms for supersymmetry breaking, emphasising phenomenological problems with D-breaking of the Fayet-Iliopoulos type (5) and the natural appearance of an increasing sequence of supersymmetry breaking mass differences in models with F-breaking of the O'Raifeartaigh type (6).

Finally we exhibit explicit SU(5) GUTs in which the primordial supersymmetry breaking scale is around the Planck mass and the observed hierarchy of gauge symmetry breaking emerges naturally. ($m_w/m_{\text{Planck}} = O(10^{-17})$) (7)

The hierarchy problem is that of arranging that the scalar fields assumed to be responsible for the spontaneous breaking of the weak SU(2) $\times$ U(1) gauge symmetry naturally have (negative) mass-squared $\lesssim O(1) \text{ TeV}^2$. Exact supersymmetry would maintain scalars massless if they were set to zero at the tree level. The essential breaking of supersymmetry can generate scalar masses even if they are initially zero. One can organize the calculation of the light scalar masses in two steps. First one integrates out the heavy degrees of freedom to obtain an effective low-energy theory, and then calculates corrections to the scalar masses in this effective theory. In a supersymmetric theory there need not be any direct coupling between light scalars and heavy scalars, but there will be an indirect connection via heavy gauge and possibly other supermultiplets as shown in Fig 1a, which would vanish if supersymmetry were exact. These diagrams would contribute less than 1 TeV^2 to the light scalar mass-squared if

$$O\left(\frac{1}{4\pi^2}\right) g_L^2 g_H^2 \left(\frac{\Delta m_H^2}{m_H^2}\right) m_H^2 \lesssim m_L^2 \lesssim O(1) \text{ TeV}^2 \quad (1)$$

where Δm_H^2 is the supersymmetry breaking (mass)² splitting in the heavy sector g_L and g_H are three-point couplings to light and heavy scalar fields in fig 1b, and we assume that

the heavy scalars may have vacuum expectation values of $O(m_H)$. The hierarchy condition (1) is clearly satisfied if

$$\Delta m_H^2 \lesssim O(1) \text{ TeV}^2 \times \frac{4\pi^2}{g_L^2 g_H^2} \quad (1)'$$

When calculating the diagrams of fig 1b we encounter an incipient quadratic divergence which is imperfectly cancelled due to the supersymmetry mass splitting Δm_2^2 in the light sector. If we demand

$$O\left(\frac{1}{4\pi^2}\right) g_L^2 \Delta m_2^2 \lesssim O(1) \text{ TeV}^2 \quad (2)$$

or

$$\Delta m_2^2 \lesssim O(1) \text{ TeV}^2 \times \frac{4\pi^2}{g_L^2} \quad (2)'$$

the hierarchy of gauge symmetry breaking scales can be maintained. If we consider the light gauge bosons and their gaugino partners G, whose couplings $g_L^2/4\pi = O(\alpha)$, equation (2)' tells us that

$$\Delta m_{LG}^2 \lesssim O(1) \text{ TeV}^2 \times \frac{\pi}{\alpha} \quad (3)$$

so that their breaking scale can be somewhat larger than 1 TeV. If we consider chiral supermultiplets, their Yukawa couplings g_L may be arbitrarily small, and their supersymmetry breaking mass splittings Δm^2 could therefore be arbitrarily large. Indeed, if they were sufficiently well protected by small chiral coupling constants, their splittings could easily be as large as the Planck mass m_P . In the absence of an R-invariance⁽⁸⁾ mass splittings between gauge non-singlet chiral superpartners would however feed through to the gauge supermultiplet by virtue of their

gauge couplings: so that their supersymmetry breaking mass splittings are subject to an extra constraint

$$\Delta m_{GS}^2 \lesssim \Delta m_{LG}^2 \times \left(\frac{\pi}{\alpha}\right) \lesssim O(1) \text{ TeV}^2 \times \left(\frac{\pi}{\alpha}\right)^2 \quad (4)$$

and could not be as large as the Planck mass. However, Yukawa shielding could easily enable gauge singlet chiral superfields to have mass splittings as large as m_P and we will later construct explicit GUTs in which a chain of symmetry breaking starts in this way. First we review the pros and cons of different mechanisms for spontaneous breaking of supersymmetry.

There are two mechanisms on the market which avoid scalar partners of quarks and leptons which are lighter than the observed fermions. One is the D-breaking scheme of Fayet and Iliopoulos⁽⁵⁾ in which a new U(1) gauge interaction is introduced with axial current couplings to quarks and leptons and an un-GUT-^{*}ted model of this type with large supersymmetry breaking has recently been proposed⁽⁸⁾.

However, models constructed with this new current tend to have axial anomalies. If the spectrum is complicated enough to avoid these, there are dangers that the gauge couplings will become strong before the grand unification scale and/or the resulting scalar potential will not break supersymmetry. Furthermore, coupling a Fayet-Iliopoulos term consistently to gravity requires the imposition of^(8,9) R-invariance which forces the gluino to be massless in all orders of perturbation theory - unless R-invariance is

* In this model the D term triggers a non zero value for the F term as well

spontaneously broken in which case there must be an (eaten?) Goldstone boson coupling to quarks and leptons. It has been pointed out⁽⁸⁾ that the gluino may get a mass from some gravitational effect, but the D-breaking mechanism seems generally so beset with problems that we turn to F-breaking of the O'Raifeartaigh type.⁽⁶⁾

In this scheme unacceptable mass relations are avoided because of radiative corrections^(2,3,4). O'Raifeartaigh supersymmetry breaking starts in a gauge-singlet sector, as required by our general reasoning about the scales of supersymmetry breaking. When it reaches the gauge supermultiplet, radiative correction diagrams like fig 2a naturally make gauginos unprotected by an R-invariance become heavier than gauge bosons, because the latter are still protected by gauge invariance. Then radiative corrections as in fig 2b naturally make squarks and sleptons heavier than the quarks and leptons, whose chirality prevents them from acquiring masses until weak SU(2) x U(1) is broken. Finally, the radiative corrections of fig 2c can give negative mass-squared^(2,3) to the complex doublets of Higgs fields breaking SU(2) x U(1), eventually giving masses to the quarks, leptons $W^\pm$ and Z^0 . Thus F-breaking imposes very naturally the pattern of mass splittings between superpartners required by phenomenology. Fig 3 illustrates the sequence of super- and gauge symmetry breaking we will be exploiting, starting off with the gauge singlet structure which we will introduce in a moment. The possibility of sending the primordial O'Raifeartaigh supersymmetry breaking scale off to the

Planck mass has been discussed in the context of an SU(3) x SU(2) x U(1) model by Alvarez-Gaumé, Claudson and Wise⁽³⁾. They needed a chiral coupling constant of order 10^{-15} , whereas we will get away with $O(10^{-1})$, and since they did not discuss explicitly grand unification they also did not address the gauge hierarchy problem. Furthermore, their gluinos are massless because of R-invariance.

After all this preamble, we finally discuss an explicit SU(5) GUT which embodies the long food chain of fig 3. In addition to the adjoint gauge supermultiplet we consider left-handed chiral supermultiplets interacting via a superpotential which starts with a classic O'Raifeartaigh gauge singlet piece:

$$P_{OR} = \lambda_1 \bar{A}^2 B + \lambda_2 (A^2 - M^2) C \quad (5)$$

where M is $O(M_P)$. We then couple the O'Raifeartaigh sector into the gauge world either directly

$$P_{GD} = \lambda_3 \phi_1^a \phi_{2a} A + \lambda_4 A^3 \quad (6a)$$

where ϕ_1^a and ϕ_{2a} are $\bar{5}$ and $\bar{5}$ representations of SU(5), or else indirectly through an additional gauge singlet field X:

$$P_{GI} = \lambda_3' \phi_1^a \phi_{2a} X + \lambda_4' X^2 A + \lambda_5 A^3 \quad (6b)$$

The additional terms in equations (6) are required to break R-invariance. More complicated examples are possible, but P_{GD} and P_{GI} will suffice to indicate how one may obtain any desired degree of Yukawa suppression. After equations (6)

we are in the gauge sector and need a term in the superpotential to break SU(5):

(7)

$$P_{52} = \beta_3^2 \Sigma^3 + \frac{\beta_2}{2} \mu \Sigma^2 + T(\gamma_1 Z + \gamma_2 \Sigma) S$$

Here μ is a dimensional parameter of order the grand unification mass which may eventually either become $O(m_P)$ or else be generated radiatively. The fields Σ , T , Z and S are in $\underline{24}$, $\underline{5}$, $\underline{1}$ and $\underline{5}$ representations of SU(5) respectively. The purpose of the last term in P_{52} is to solve the "second hierarchy problem" in the manner previously suggested by two of the present authors (L.I. and G.G.R.).⁽²⁾ In addition to the superpotential terms (5), (6), (7) there are also conventional terms coupling $N_G \rightarrow 3$ generations of quark and lepton superfields to the Higgs supermultiplets T and S . These complete our model and we now analyse the pattern of symmetry breaking it exhibits.

From the F_B and F_C terms arising from $P_{OR}(S)$ we find

$$\langle A \rangle = \frac{|\lambda_2|^2}{(|\lambda_1|^2 + |\lambda_2|^2)} M^2 \quad (8)$$

where we will use $\langle \chi \rangle$ to denote the vacuum expectation value of the scalar field component of any chiral supermultiplet χ . From the F_{ϕ_1} and F_{ϕ_2} terms associated with P_{GD} (6a) we infer $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$, and the F_A term from $P_{OR} + P_{GD}$ (5), (6a) imposes

$$3\lambda_4 \langle A \rangle^2 + 2\lambda_1 \langle A \rangle \langle B \rangle + 2\lambda_2 \langle A \rangle \langle C \rangle = 0 \quad (9)$$

The superpotential term P_{52} (7) has degenerate minima with residual SU(5), SU(4) x U(1) and SU(3) x SU(2) x U(1) and other¹¹⁾

symmetries. We assume for now that God in her infinite wisdom chooses the third of these, thereby splitting the Σ , T and S supermultiplets into $\Sigma_{3,2,1}$, $T_{3,2}$ and $S_{3,2}$ respectively. Later on we will see that $\langle T_2 \rangle$, $\langle S_2 \rangle \neq 0$ and hence the F_T and F_S terms vanish only if

$$\gamma_1 \langle Z \rangle + \gamma_2 \langle \Sigma_2 \rangle = 0 \quad (10)$$

The condition (10) ensures that the T_2 and S_2 scalars are naturally massless at the first stage of SU(5) symmetry breaking, which is the solution to the "second hierarchy problem" proposed in ref (2).

After this tree level analysis of the superpotential (5), (6a), (7) we now turn to the radiative corrections. In this model the appropriate variant of the mass feed down mechanism of fig 2a for light gauginos is that shown in Fig 4a, while Fig 4c and 4d are other contributing two-loop diagrams.

At the tree level the scalar components of ϕ_1 and ϕ_2 acquire masses $\lambda_3 \langle A \rangle$ while there is a mixing between the fermionic components ψ_1 and ψ_2 which gives each of them the same mass $\lambda_3 \langle A \rangle$. There is no tree-level mixing between the ϕ_1 and ϕ_2 scalars because of condition (9) and the first contribution to the round blob in fig 4a is of one loop order as in fig 4b. Thus a gluino mass occurs at two loop order through the graphs of Fig 4(a). This gives a term $\propto \lambda_3^2$ of the form

$$m_{\tilde{g}} = \frac{\alpha_s}{3\pi} \frac{\lambda_3^2 \lambda_4}{16\pi^2} \ln \left(\frac{m_{Y_A}^2}{m_{\phi_A}^2} \right) \langle A \rangle \quad (11)$$

In eq(11) m_{ψ_A} and m_{ψ_f} refer to the masses of the fermion and scalar parts of A and there are other terms cubic in other Yukawa couplings λ from figs 4c,4d. The other light gauginos $\tilde{W}$ and $\tilde{B}$ acquire masses similarly. In the more disconnected model of (6b) we obtain eq (11) with the argument of the log, $m_{\psi_x}/m_{\phi_x}^2$. Since this differs from unity only in order $\frac{\lambda^2}{4\pi^2}$, now

$$m_{\tilde{g}} \approx \frac{\alpha_s \lambda^6}{256\pi^3} \langle A \rangle \quad (12)$$

The diagram of fig 2b, imperfectly cancelled by its superpartners, then gives in leading logarithmic order

$$m_{\tilde{g}} = \frac{4\alpha_s}{3\pi} \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{g}}^2}\right) m_{\tilde{g}}^2 \quad (13)$$

and

$$m_{\tilde{g}}^2 = \delta m_{T_2, S_2}^2 = \frac{3\alpha_s}{4\pi} \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{g}}^2}\right) m_{\tilde{g}}^2 + (\tilde{B} \text{ term}) \quad (14)$$

While the contribution (14) to the light Higgs and slepton masses is positive, there are also negative contributions to the Higgses (but not the sleptons) from the squark loops in fig 2c which are of order

$$\delta m_{T_2, S_2}^2 = -\frac{3}{8\pi^2} |h_{u,3}|^2 \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{g}}^2}\right) m_{\tilde{g}}^2 \quad (15)$$

where $h_{u,D}$ are the quark-Higgs Yukawa coupling matrices.

It is easy to check that the negative contribution (15) overwhelms the positive piece (14) and triggers spontaneous symmetry breaking by the Higgses (and not the sleptons) if we choose

$$|h_{u,3}|^2 > 0 \left(\frac{1}{2 \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{g}}^2}\right)} \right) \text{ i.e. } m_t \gtrsim 20 \text{ GeV} \quad (16)$$

As was pointed out in ref 2, a crucial element in the analysis of the light Higgs potential is the quartic term $\gamma_1^2 |\text{TS}|^2$ induced coming from $|F_Z|^2$.

One might be nervous that radiative corrections could affect the hierarchy imposed by the last term in p 5z (7). However, it is easy to verify that to first order in supersymmetry breaking for the T and S multiplets coupling Z and Σ the only change in the (Z, Σ) potential is $\propto |\gamma_1 Z + \gamma_{24} \Sigma|^4$. It is only in second order in supersymmetry breaking that one encounters a term not proportional to $|\gamma_1 Z + \gamma_{24} \Sigma|^{n \geq 2}$, and this is too small to upset the natural solution to the "second hierarchy problem" proposed in ref 2.

We now examine our general constraints on supersymmetry breaking in the context of this SU(5) GUT. Clearly the gaugino condition (3) is exemplified by the relation(14) in the absence of a conspiratorial cancellation with the term (15) of opposite sign. We therefore require $m_{\tilde{W}} \lesssim 10 \text{ TeV}$ and $m_{\tilde{g}} \lesssim 30 \text{ TeV}$. Since the O'Raifeartaigh sector is only indirectly coupled to the light Higgses, we can impose $\langle A \rangle \sim 10^{19} \text{ GeV}$ as desired, as long as

$$\frac{\alpha_s \lambda_3^2 \lambda_4}{64\pi^3} \lesssim O(10^{-12}) \quad (17)$$

It follows that generic Yukawa couplings λ_i of order 10^{-2} to 10^{-3} would suffice to obtain the desired numerical magnitude of gauge hierarchy. On the other hand, in the more devious model (6b) one sees from equation (12) that λ_i of order 10^{-1} would probably suffice. For comparison

theory. But enough of these quibbles: these are the first "natural" GUTs with large supersymmetry breaking.

we note that the Yukawa couplings in the minimal Weinberg-Salam model vary all the way from $O(10^{-6})$ to $O(10^{-1})$. We therefore feel that the desired numerical magnitude of gauge hierarchy can be obtained with Yukawa couplings in a very plausible range: no individual coupling has to be $O(10^{-15})$.

We conclude by listing some of the salient predictions of this model. The grand unification m_x is slightly below the previous supersymmetric GUT prediction⁽¹¹⁾ for 2 light Higgs doublets: $m_x = O(10^{15} \text{ to } 10^{16})$ GeV. We expect m_b to be essentially unchanged from conventional and supersymmetric GUT predictions^(2,13), while $\sin^2 \theta_w$ is slightly closer to the present central experimental value than it was in previous supersymmetric GUTs with two light Higgs doublets⁽¹¹⁾. Nucleons decay predominantly into $\bar{\nu} K^{(13)}$ with a lifetime close to the present experimental limit. We have a goldstino associated with spontaneous symmetry breaking on a scale of $m_{F'}$ which is presumably eaten by the gravitino and gives it an unobservably large mass. The lightest supersymmetric particle is a neutral Higgsino with a mass a few GeV, which is essentially stable. It is not clear that it is heavy enough to be compatible with conventional cosmology. Another question for our model is how the Universe at finite temperature picks out the $SU(3) \times SU(2) \times U(1)$ symmetric minimum of the $SU(5)$ potential. We expect that this will be done by radiative corrections and/or finite temperature effects and it seems that the class of models discussed here is sufficiently rich to admit a phenomenologically realistic

REFERENCES

1. E. Witten, Nucl. Phys. B186 (1981) 513
- M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B189 (1981) 575
- S. Dimopoulos and S. Raby, Nucl. Phys. B192 (1981) 353
- S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150
- N. Sakai, Z. Phys. C11 (1982) 153
2. L. E. Ibáñez and G. G. Ross, Phys. Lett. (in press)
3. L. Alvarez-Gaumé, M. Claudson and M. Wise, Harvard preprint HUTP-81/AO63 (1982)
4. M. Dine and W. Fischer, Princeton preprint (1981)
- C. R. Nappi and B. A. Ovrut, Princeton preprint (1982)
5. P. Fayet and J. Iliopoulos, Phys. Lett. 51B (1974) 461
6. L. O'Raifeartaigh, Nucl. Phys. B96 (1975) 331
7. E. Gildener, Phys. Rev. D14 (1976) 1667; Phys. Lett. 92B (1980)111
- E. Gildener and S. Weinberg, Phys. Rev. D15 (1976) 3333
8. R. Barbieri, S. Ferrara and D. V. Nanopoulos, CERN preprint TH-3226 (1982)
- R. Barbieri, S. Ferrara, D. V. Nanopoulos and K. S. Stette, CERN preprint TH-3243 (1982)
9. M. Sohnius and P. West, King's College preprint (1982)
10. P. Fayet and S. Ferrara, Phys. Reports 32C (1977) 249
- P. Fayet, Phys. Lett. 78B (1978) 417
11. L. Ibáñez and G. G. Ross, Phys. Lett. 105B (1981) 439
- M. B. Einhorn and D. R. T. Jones, University of Michigan preprint UM HE 81-55 (1981)
12. N. V. Dragon, University of Heidelberg preprint HD-THEP-82-3
- S. Buccella, J. P. Derendinger, C. A. Savoy and S. Ferrara CERN TH 3212 and to be published.
13. J. Ellis and D. V. Nanopoulos, S. Rudaz CERN preprint TH3199 (1981)

FIGURE CAPTIONS

- Fig 1. Radiative contributions to scalar masses.
- Fig 2. Graphs contributing to the masses of (a) Gluinos
(b) Squarks and sleptons (c) Higgs scalars.
- Fig 3. "Food chain" generating masses in a supersymmetric
GUT.
- Fig 4. Graphs contributing to the gluino mass in two loop
order.

(a)

(b)

Fig. 1

(a)

(b)

(c)

Fig. 2

Fig. 3

(a)

(b)

(c)

(d)

Fig. 4