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Decays of Heavy Quark Systems: Effects of Gluon Condensate

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ABSTRACT

We compute to first order in α_s the perturbative and nonperturbative contributions to three-point functions. The results are applied to the decay $\eta_c \rightarrow 2\gamma$ and large corrections are found compared to the nonrelativistic result.

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DECAYS OF HEAVY QUARK SYSTEMS: EFFECTS OF GLUON CONDENSATE

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In the last few years a large amount of work has been able to reproduce the spectrum of hadrons by means of QCD sum rules (1,2,3). The agreement with experiment is in general very good. In this approach the short distance expansion is applied to two-point functions in QCD including effects beyond perturbation theory which are parametrized by quark and gluon condensate matrix elements. The values for these parameters are in agreement with independent calculations using Monte Carlo methods (4).

The purpose of this paper is to present the results on the application of QCD sum rules to three-point functions, including nonperturbative effects. The early calculations of the ITEP group (5) of some two photon decays of heavy mesons are based on the bare loop only without taking perturbative and/or nonperturbative corrections into account. Other papers (6) have tackled radiative transitions in charmonium but always within the bare loop approximation. perturbative corrections (to first order in α_s) to some of these decays have been calculated before (7) using different methods than ours, and large corrections have been obtained.

In this paper we will present perturbative (to first order in α_s) and nonperturbative (gluon condensate) contributions to the three-point function

$$\begin{aligned}
 A_{\mu\nu}(q, q_1, q_2) &= e^2 Q^2 \int d^4x d^4y e^{-i(qx + q_2y)} \times \\
 &\times \langle 0 | T(j_\mu(0) j_\nu(x) j_\nu(y)) | 0 \rangle \\
 &= 3\alpha Q^2 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta A(q^2, q_1^2, q_2^2)
 \end{aligned} \quad (1)$$

The currents in eq(1) are the heavy quark vector ($\bar{c}\gamma_\mu c$) and pseudoscalar ($i\bar{c}\gamma_5 c$) currents, and Q is the charge of the quark. Here we will only consider the case where the external momenta q_1^2 and q_2^2 of the vector currents are equal to zero. In this way, eq(1) describes the decay of heavy quark pseudoscalar mesons into two (real) photons, e.g. $\eta_c \rightarrow 2\gamma$. Calculations for other cases (different combinations of currents and the external momenta $q_i^2 \neq 0$ (for decays like $J/\psi \rightarrow \eta \gamma$)) are in progress. The case considered here is the simplest possible for several reasons:

- a) the heavy quark mass ensures asymptotic freedom for the quarks even at $q^2 = 0$ for the pseudoscalar current.
- b) the masslessness of the physical photons simplifies the calculations considerably.
- c) only nonperturbative effects due to the gluon condensate give a contribution, for the same reasons as in the case of two-point functions of heavy quark currents discussed in detail in Refs. 1 and 2.

The diagrams to be calculated are given in Fig.1.

Diagram (a) is the bare loop contribution which can easily be calculated and gives (5,6) in terms of a dispersion integral

$$A_0(q^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds \frac{2m^2}{s(s-q^2)} \ln \frac{1+v}{1-v} \quad (2)$$

where $v^2 = 1 - \frac{4m^2}{s}$ with m the quark mass.

The other six diagrams in Fig.1 give the first order α_s correction to the amplitude. The calculation of these diagrams is complicated and can be done most easily by considering the imaginary part in the q^2 channel. The dominant contribution is the Coulombic piece ($\sim 1/v$) (diagram (b)). We have only taken the Coulombic contribution plus the next order ($\sim v^0$) into account and neglected all higher order corrections in v . In this case we get

$$\text{Im} A_1(s) = \frac{2m^2}{s} \frac{\alpha_s}{\pi} (2v) \left[\frac{v^2}{2v} - 2 + 2C \right] \quad (3)$$

where $C \approx -0.3$ is the sum of all contributions of zeroth order in v , not contained in the Coulombic piece.

The nonperturbative gluon condensate contributions

i.e. the Wilson coefficient $C_G(q^2)$ of the operator

$G_{\mu\nu}^a G_{\mu\nu}^a$ (is the gluon field tensor) in the operator product expansion of the T-ordered product in eq.(1) are obtained from the diagrams in Fig.1 by cutting the gluon line as explained in Refs.1 and 2. Since only one loop integration is involved the calculations although complicated can be performed analytically. We have made extensive use of the MACSYMA program at MIT to perform our calculations. We get for the Wilson coefficient

$$C_G(q^2) = \frac{(u^2 - 1)}{8\pi u^4} \left[\frac{33u^6 - 13u^4 - u^2 - 3}{2u} \ln \frac{1+u}{1-u} - \right. \quad (4)$$

$$\left. - 33u^4 + 2u^2 + 3 \right] \\ \text{where } u^2 = 1 - \frac{4m^2}{q^2}.$$

And the contribution to $A(q^2)$ is given by

$$C_G(q^2) \phi = C_G(q^2) \frac{4\pi^2}{9} \langle O | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | O \rangle / (4m^2)^2. \quad (5)$$

The universal gluon condensate matrix element in (5) has been determined from the charmonium spectrum in Refs. 1 and 2 and is equal to

$$\langle O | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | O \rangle \approx (340 \text{ MeV})^4 \quad (6)$$

which gives $\phi \approx 0.13 \times 10^{-2}$ with $m = m_c(p^2 = -m_c^2) \approx 1.28 \text{ GeV}$.

To apply the formulas obtained to the decay $\eta_c \rightarrow 2\gamma$ we follow the same procedure as in the two point function case and define the moments by differentiating $A(Q^2)$ at $Q^2 = -q^2 = 0$

$$M_n = \frac{1}{n!} \left(-\frac{d}{dQ^2} \right)^n A(Q^2) \Big|_{Q^2=0}, \quad Q^2 = -q^2. \quad (7)$$

This leads to the following moment equation

$$M_n = \frac{2}{\pi} \frac{1}{2n+1} \frac{n! n!}{(2n+2)!} \left[1 + a_n^{(3)} \alpha_s + b_n^{(3)} \phi \right] \quad (8)$$

with

$$a_n^{(3)} = \frac{4}{3} \left\{ \pi \frac{(2n+1)!}{n! n! 2^{2n+1}} + \frac{4}{\pi} \frac{n+1}{2n+3} (-1+C) - \frac{3(2n+1)\ln 2}{2\pi} \right\} \quad (8a)$$

$$b_n^{(3)} = -\frac{n+1}{2n+3} n(n^2 + n+2) \quad (8b)$$

The last term in $a_n^{(3)}$ results from the mass renormalization at the point $p^2 = -m^2$, consequently m in (8) is $m(p^2 - m^2)$. It can be seen from eqs. (8) that the nonperturbative contributions b_n grow very fast with n . As in the two-point function case, increasing n means moving closer to the resonance region in the q^2 (pseudoscalar) channel.

To construct the phenomenological side of our sum rule we adopt a narrow resonance approximation in this channel

$$\text{Im}A(q^2) = \pi g h_- M_{\eta_C} \delta(s - M_{\eta_C}^2) + \pi g' h_- M_{\eta_C'} \delta(s - M_{\eta_C'}^2) \quad (9)$$

where $g(g')$ are the couplings of the $\eta_C(\eta_C')$ to the pseudo-scalar currents $\langle \alpha | i \bar{\gamma}_5 c | P \rangle = M_P^2 g$ and $h_-(h_-')$ determines the $\eta_C \rightarrow 2\gamma$ ($\eta_C' \rightarrow 2\gamma$) width (see Fig. 2) (we adopt the notation of Ref.2):

$$\Gamma(\eta_C \rightarrow 2\gamma) = (3Q^2 \alpha)^2 \frac{h_-^2 M_{\eta_C}}{64\pi} \quad (10)$$

From (9) we can calculate phenomenological expressions for the moments M_n . Equating with (8) expresses the product of the couplings gh_- in α_s, ϕ and the quark mass m .

$$gh_- + g'h_-' \left(\frac{M_{\eta_C}}{M_{\eta_C'}} \right)^{2n+1} = \frac{2}{\pi} \left(\frac{M_{\eta_C}}{m} \right)^{2n+1} \frac{n!n'}{(2n+2)!} \times \left[1 + a_n^{(3)} \alpha_s + b_n^{(3)} \phi \right] \quad (11)$$

To eliminate the couplings g we return to the two-point function case, (2) which gave a similar moment equation for the coupling g^2 . Saturating the imaginary part with two resonances we got

$$g^2 + g'^2 \left(\frac{M_{\eta_C}}{M_{\eta_C'}} \right)^{2k} = \frac{3}{8\pi^2} \left(\frac{M_{\eta_C}}{m} \right)^{2k} \frac{k!(k-1)!}{(2k+1)!} \left[1 + a_k^{(3)} \alpha_s + b_k^{(3)} \phi \right] \quad (12)$$

with a_n^P and b_n^P given in Table 1 of Ref.8.

Experimentally both M_{η_C} and $M_{\eta_C'}$ are known ($M_{\eta_C} = 2.981 \text{ GeV}$ and $M_{\eta_C'} = 3.592 \text{ GeV}$) (9) and one can easily estimate the

contamination of the η_C' resonance in the sum rules (11) and (12) as a function of n . In analogy to the vector meson case where the coupling to the current is directly related to the e^+e^- width we expect that $g'^2 \approx \frac{1}{3}g^2$, and it can easily be verified that for $n \geq 4$ the contamination of the η_C' in (11) and (12) is less than 10%. From Ref.2 we know that moments four and five are suitable to determine the coupling (and/or mass) of the resonance. The contamination by higher resonances is small at these moments while at the same time the perturbative and nonperturbative contributions are small compared to the bare loop. If we want to use higher moments we have to evaluate the moments (7) at spacelike $Q^2 \neq 0$.

We neglect the η_C' contributions in (11) and (12), take the square of (11), divide by (12) with $k = n+1$ and expand:

$$|R_{1S}(0)|^2 = 0.30 \text{ GeV}^3 \quad (15a)$$

while from $J/\psi \rightarrow e^+ e^-$ one has

$$|R_{3S}(0)|^2 = 0.55 \text{ GeV}^3 \quad (15b)$$

Due to the high power of m in (13) our result (14) is very sensitive to the value of the quark mass. We have chosen $m_c = 1.28 \text{ GeV}$ as determined (very accurately up to 10 MeV) in Ref.2 from the application of QCD sum rules to the charmonium spectrum. A value $m_c = 1.27 \text{ GeV}$ would bring $\Gamma(\eta_c \rightarrow 2\gamma)$ up to about 4.5 keV still far short of the 6 keV which is expected from the ratio with $\Gamma(J/\psi \rightarrow e^+ e^-)$. Moreover, one should keep in mind that the value we obtain is an upper limit because of contamination with higher resonances in the sum rule. Therefore, we conclude that

nonperturbative effects may greatly upset the simple nonrelativistic results for these decays.

In future publications we intend to report in more detail on the calculations of the nonperturbative effects extending the method to include radiative decays of the charmonium P-states, and on decays like $J/\psi \rightarrow \eta_c \gamma$ based on the three-point function (1) with q_1^2 or $q_2^2 \neq 0$.

Finally we want to make the following remark. It has been conjectured by 't Hooft (11) (in the context of light quarks) that the Adler theorem remains unchanged when nonperturbative corrections are included. For $q^2=0$ (i.e., $n=0$ in the moment) the nonperturbative gluon condensate

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$$h_-^2 = \frac{32}{3} \left(\frac{M_{\eta_c}}{m} \right)^{2n} \frac{n!n!}{(2n+2)!} \left\{ \frac{2n+3}{n+1} \left[1 + (2a_n^{(3)} - a_{n+1}^{(P)})\alpha_s + (2b_n^{(3)} - b_{n+1}^{(P)})\phi \right] \right\} \quad (13)$$

We chose $k = n+1$ in (12) since as can be seen from Table 1. then the coefficients a_n and b_n in the two cases are similar. Substituting $n = 4, 5$ in (13) we find ($\alpha_s = 0.3$)

$$h_-^2 = 3.0 \pm 0.3$$

and for the $\eta_c \rightarrow 2\gamma$ widths using eq. 10

$$\Gamma(\eta_c \rightarrow 2\gamma) = (4.2 \pm 0.4) \text{ keV} \quad (14)$$

which is less than one would naively expect from the ratio

$$\frac{\Gamma(\eta_c \rightarrow 2\gamma)}{\Gamma(J/\psi \rightarrow e^+ e^-)} = \frac{4}{3}$$

In this ratio the wavefunctions at the origin of the η_c and J/ψ have been assumed to be equal and α_s corrections have been neglected. It is known (10) that for some of these decays large corrections should exist in view of the fact that the experimental value for $J/\psi \rightarrow \eta_c \gamma$ is much less than one would expect from these simple considerations. We hope to come back to $J/\psi \rightarrow \eta_c \gamma$ when we have calculated the three-point functions with one of the photons massive. Our result (14) for the $\eta_c \rightarrow 2\gamma$ indicates that the wave functions at the origin for the J/ψ and η_c can differ by 40%. We find from (14)

contribution to $\eta_c \rightarrow 2\gamma$ vanishes as can be seen from (8). We want to point out that this result (at least for finite quark mass and up to $G_{\mu\nu} G_{\mu\nu}$ power corrections) supports 't Hooft's conjecture.

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REFERENCES

1. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
2. L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B186 (1981) 109.
3. B. L. Ioffe, Nucl. Phys. B188 (1981) 317.
4. T. Banks, R. Horsley, H. R. Rubinstein, and Ulli Wolf Nucl. Phys. B190 (1981) 692.
5. V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Phys. Rep. 41C (1978) 1.
6. A. Yu. Khodjamirian, Phys. Lett. 90B (1980) 460.
7. R. Barbieri, M. Caffo, R Gatto, and E. Remiddi, Phys. Lett. 95B (1980) 93.
8. L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. 94B (1980) 203.
9. D. L. Scharre, in Proc. of the 1981 Intern. Symp. on Lepton and Photon Interactions at High Energies, Bonn, 1981 ed W. Pfeil.
10. M. A. Shifman, in Proc. of the 1981 Intern. Symp. on Lepton and Photon Interactions at High Energies, Bonn, 1981 ed W. Pfeil.
11. G. 't Hooft, Cargese Summer School Lectures (1979).

FIGURE CAPTIONS

Fig 1. Diagrams that contribute to the three-point function to first order in α_g . Wavy lines depict vector currents, dashed lines pseudoscalar currents, solid lines quarks, and curly lines gluons.

Fig 2. Graphical representation of the phenomenological side of the sum rule for $\eta_c \rightarrow 2\gamma$; g is the coupling of the meson to the pseudoscalar current and h_- the coupling to the photons.

TABLE CAPTIONS

Table 1 Values for the coefficients a_n^P , b_n^P , $a_n^{(3)}$, and $b_n^{(3)}$ for $n = 1 - 8$.

Table 1

n	a_n^P	b_n^P	$a_n^{(3)}$	$b_n^{(3)}$
1	0.28	2.4	0.94	-1.6
2	0.74	3.4	0.79	-6.9
3	0.75	0.0	0.52	-18.7
4	0.57	-10.9	0.19	-40.0
5	0.28	-32.3	-0.19	-73.8
6	-0.07	-67.2	-0.61	-123.2
7	-0.47	-118.6	-1.06	-191.1
8	-0.90	-189.5	-1.55	-280.4

Figure 1

Figure 2