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LIGHT BOSON PRODUCTION IN REAL OR VIRTUAL
PHOTON-ELECTRON COLLISIONS

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ABSTRACT

We compute the exact cross-sections of light vector, axial, scalar or pseudoscalar boson production in the cases of real $\gamma e^{\pm}$ collisions and of radiation from electron beams in an external field. We emphasize the particular interest of photon beam electron beam collisions.

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1. - INTRODUCTION

Several kinds of very light bosons (we shall denote them by X) have recently been theoretically predicted: scalar or pseudoscalar particles [Higgs particles¹⁾, axions²⁾] as well as vector or axial particles [new gauge bosons, for example the U gauge boson of supersymmetric theories³⁾].

Experimental searches have been made [for recent reviews see Ref. 4)] with electron or proton beam dumps, during neutrino experiments, with nuclear reactions, and by looking to rare decay modes of kaons, narrow quarkonia ψ , T , No evidence has appeared up to now. However, the model dependence of the analysis of hadronic interactions, the difficulties of detection and the background leave many ambiguities and room for further searches.

Lepton physics offers several ways which are less ambiguous. The results can often be clearly expressed in terms of X masses and well-defined couplings to the leptons. Electron beam dump experiments have been carried out where high energy axions have been sought through the detection of Bethe-Heitler $\mu^+\mu^-$ pairs they would have created when going through matter⁵⁾⁻⁷⁾. One could also look for several other kinds of radiation of X bosons (similar to radiation of photons) from electron beams (radiation in Coulomb scattering, synchrotron-like radiation in $e^\pm$ storage rings⁸⁾, channelling radiation from $e^\pm$ going through crystals⁹⁾, ...). Low energy X bosons could be detected through their real decay modes ($X \rightarrow e^+e^-$, $\gamma\gamma$, $\gamma\gamma\gamma$, ...). Another possibility would be to produce X bosons in real photon-electron collisions ($\gamma e^\pm \rightarrow X e^\pm$). One way is to make a kind of photon beam dump, X particles being created by collisions of the γ beam with electrons at rest in matter. Another way is to make γ beam- $e^\pm$ beam collisions. There is a proposal^{10),11)} to send a laser beam against $e^\pm$ bunches of the SLC in order to produce high energy photons. During the same collision one should also produce high energy X particles along the incident electron direction (backward "Compton" scattering $\gamma(\text{laser}) + e^\pm \rightarrow X + e^\pm$). The use of γ beams from synchrotron radiation in e^+e^- storage rings and the search for $\gamma(\text{synchr.}) + e^\pm$ collisions has also been considered⁸⁾.

For all these purposes we think it is interesting to give the precise form of the amplitudes for vector, axial, scalar or pseudoscalar particle production in real $\gamma e^\pm$ collisions and in radiation from electron beams. Only approximate forms of pseudoscalar axion production neglecting mass effects were given in Ref. 6). We noticed that very important m_X effects ($q_{\min}^2$ and propagator effects) reduce the rates of X production in bremsstrahlung processes, especially

for low X momenta. These effects are in fact essential for estimating the rates of low energy X radiation in the processes mentioned above.

In Section 2 (and Appendix) we give the precise form of the transition amplitudes and probabilities for real or virtual $\gamma e \rightarrow X e$ reactions in a complete analytic form in the four cases of spin and parity. This is applicable to Higgs boson, axion and gauge boson production using the corresponding numerical value of the coupling constants predicted by theoretical models.

In Section 3 we treat the case of real $\gamma e^\pm$ collisions applicable to photon beam dump or to γ beam - e beam collisions. Results are presented in the form of ratios of cross-sections for X production to cross-sections for γ production $R = \sigma^X / \rho \sigma^\gamma$, where ρ is the ratio of couplings g_{Xee}^2 / e^2 . Cross-sections appear to be sizeable when one uses the standard g_{Xee} values for couplings of $X = V, A, S$ or P bosons to the electron. We also give an estimate of the $\gamma + \text{proton} \rightarrow X + \text{proton}$ cross-sections for comparison.

In Section 4 we treat the case of X radiation from electron beams and give the shape of the X spectrum relative to the case of γ radiation. The m_X effects [neglected in Ref. 6)] are very important for low X momenta. This is just the case which would allow the detection of X bosons through their real decay modes (like $P, S \rightarrow \gamma\gamma, e^+e^-, \dots$ or $V, A \rightarrow \gamma\gamma\gamma, e^+e^-, \dots$). Illustrations are given for $E_e = 15$ GeV and also for 300 GeV photon beams.

2. - THE RADIATION AMPLITUDES

We consider the process " γ " + $e^\pm \rightarrow X + e^\pm$ described by the two diagrams of Fig. 1. " γ " represents either a real photon or the pseudo-photon of an external electromagnetic field (Coulomb scattering, synchrotron radiation, ...). X is the light vector, axial, scalar or pseudoscalar boson. The electron can of course be replaced by any other charged point-like lepton ($\mu^\pm, \dots$) just by changing the value of the mass m .

We use the couplings

$$L = -e \bar{\psi} \gamma^\mu (a - b \gamma^5) \psi \quad \text{for the } V, A \text{ cases and}$$

$$L = e \bar{\psi} (c - id \gamma^5) \psi \quad \text{for the } S, P \text{ cases.}$$

The standard models give

$$C = \frac{m_e}{2 M_W \sin \theta_W} \quad \text{for scalar Higgs bosons}^{1)}$$

$$d = \frac{m_e}{2 x M_W \sin \theta_W} \quad \text{for axions } (0 \leq x \leq 1)^{2)}$$

$$a = \frac{m_U}{4 M_W \sin \theta_W} \cos \phi_e, \quad b = \frac{m_U}{4 M_W \sin \theta_W} \quad \text{for light U gauge bosons}^{3)}$$

For $X \equiv \gamma$, $a = -1$ and the ratio ρ defined in Section 1 is simply equal to $\rho = |a|^2, |b|^2, |c|^2, |d|^2$ in the V, A, S, P cases, respectively. $|a|^2$ and $|b|^2$ depend upon the value of m_U and upon the mixing angle ϕ_e ³⁾; $|c|^2 \approx 4 \times 10^{-11}$; $|d|^2$ is of the same order of magnitude up to the factor $1/x$ where x is the ratio of vacuum expectation values of Higgs fields²⁾ which is generally expected to be of order one.

The amplitude corresponding to Fig. 1 is obtained with

$$J^\mu = -e^2 \bar{u}(p') \left[\gamma^\mu \frac{q' + m_e}{q'^2 - m_e^2} \not{O} + \not{O} \frac{q'' + m_e}{q''^2 - m_e^2} \gamma^\mu \right] u(p)$$

$O = aV, -bA, -c, id\gamma^5$ in the V, A, S, P cases, respectively (V^μ and A^μ are spin one polarization vectors). In the case of a real incident photon beam with polarization vector $\epsilon^\mu(q)$, the amplitude is simply

$$R_{fi} = \epsilon^\mu(q) J_\mu.$$

In the case of a static external field $A^\mu(\vec{r})$ the amplitude can be written:

$$R_{fi} = \tilde{A}^\mu(q) J_\mu$$

with

$$\tilde{A}^\mu(q) = \int A^\mu(\vec{r}) e^{-i\vec{q} \cdot \vec{r}} d^3r.$$

The transition probabilities can be obtained using the quantity $T^{\mu\nu} = \sum_{\text{spins}} J^\mu J^{\nu*}$ given explicitly in the Appendix.

For a real photon beam, the cross-section of the $\gamma e \rightarrow X e$ process is given in the γe centre-of-mass by:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 k}{4p\sqrt{s}} T$$

with $T = -g_{\mu\nu} J^\mu J^{\nu*}$ given by Eq. (A.2) and

$$s = (q+p)^2, \quad p = \frac{s-m_e^2}{2\sqrt{s}}, \quad k^0 = \frac{s+m_X^2-m_e^2}{2\sqrt{s}}, \quad k = ((k^0)^2 + m_X^2)^{1/2}.$$

For the radiation of X particles from a high energy electron beam in the presence of a static external field, the differential cross-section in the laboratory frame is given by

$$d\sigma = \frac{\alpha^2}{4\pi^3} T^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu^* \frac{d_3 k}{k^0} \cdot \frac{d_3 p'}{p'^0} \delta(p^0 - k^0 - p'^0)$$

or

$$\frac{d\sigma}{dk d\Omega d\Omega'} = \frac{\alpha^2 \vec{k}^2 p'}{4\pi^3 k^0} T^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu^*.$$

For example, in the case of the Coulomb field of a heavy particle of charge Ze ,

$$T^{\mu\nu} \tilde{A}_\mu \tilde{A}_\nu^* = T^{00} |A^0|^2 \quad [\text{see Eq. (A.1)}]$$

with

$$|A^0|^2 = \left(\frac{4\pi Ze}{q^2} \right)^2 \quad \text{and} \quad q^2 = -\vec{q}^2 = -(\vec{p}' + \vec{k} - \vec{p})^2.$$

3. - APPLICATION TO REAL PHOTON-ELECTRON SCATTERING

We consider $\gamma e^\pm$ collisions either between photon beams and electrons at rest in matter or between photon beams (laser,...) and $e^\pm$ beams. We shall compare the cross-section of the process $\gamma e^\pm \rightarrow X e^\pm$ to the Compton scattering ($\gamma e^\pm \rightarrow \gamma e^\pm$) cross-section; its numerical values $\sigma^Y(s)$ are shown in Fig. 2. The cross-section for X production is written $\sigma^X = \rho R \sigma^Y$. In standard models (see Section 2) the ratio ρ of X couplings to γ couplings is of the order of 4×10^{-11} . We have calculated the ratio $R = \sigma^X / \rho \sigma^Y$ in terms of m_X at given values of s using Eq. (A.1) for the four cases $X = V, A, S, P$. Its values depend upon the kinematical conditions (m_X and s). The allowed range of mass m_X is given by phase space limitations:

$$0 \leq m_X \leq \sqrt{s} - m_e$$

In the case of V , A and S bosons with m_X well inside the allowed phase space, R is always of order one. At energies much larger than masses chiral symmetry leads to $R = 1$ for V and A bosons and $R = 1/2$ for S and P bosons (because of spin state number). At low energies, V , A and S bosons can be produced in S wave so R remains of order one, but P bosons are produced in P wave so that R is depressed by the centrifugal barrier factor $\vec{k}^2/s$. In Fig. 3 we give the result for $\sqrt{s} = 1.12$ MeV ($0 \leq m_X \leq 0.6$ MeV). These conditions can correspond to the collision of a laser beam with $q = 5$ eV against an electron beam with $E_e = 50$ GeV or to the collision of a low energy photon beam ($q = 1$ MeV) with electrons at rest. One gets σ^X of the order of $10^{-35} - 10^{-36}$ cm².

Let us quickly treat another example with $\sqrt{s} = 14.3$ MeV ($0 \leq m_X \leq 13.8$ MeV). It can correspond to the collision of synchrotron radiated photons ($q = 3.3$ keV) with an electron beam ($E_e = 15$ GeV) or to the collision of a higher energy photon beam ($q = 200$ MeV) with electrons at rest. In this case $\sigma^Y = 4.7 \times 10^{-27}$ cm², $R \approx 1$ for V , A and $\approx 1/2$ for S , P such that σ^X is of the order of 10^{-37} cm².

At low values of s the angular distribution is relatively isotropic (this is the case at $\sqrt{s} = 1.12$ MeV) whereas as the energy increases the dominance of the electron exchange diagram (Fig. 1a) gives progressively a strong peaking of X production along the direction of the incident electron beam ($\langle \theta_{kp} \rangle \approx m_e/p$ in the γe centre-of-mass). In the case of scattering of photon beams through matter, a complete treatment would involve multiple scattering effects depending upon precise experimental conditions.

The case of colliding beams seems clearer. The precise knowledge of the vertex location in space and time can be used for coincidences and background rejection. Using the conditions proposed for $\gamma e^\pm$ collisions at the SLC¹⁰⁾ and the ratio $\rho \approx 4 \times 10^{-11}$, one expects about one X particle per $e^\pm$ burst, i.e., $10^7 X$ per day.

Detection of X particles can be done after shielding along the incident $e^\pm$ direction. One can use either real decay modes ($\ell^+\ell^-$, $\gamma\gamma$ or $\gamma\gamma\gamma$ channels), lepton pair production or γ bremsstrahlung when the X particles produced go through matter as discussed previously^{6),7)}. One can think of making the same kind of collisions between photon and proton beams. The quantitative differences come from

- the mass range $0 \leq m_X \leq \sqrt{s} - m_p$ which is much smaller for low photon energies;
- the basic Compton cross-section which is smaller around $s \approx m_p^2$ than around $s \approx m_e^2$ (it behaves roughly like $1/s$) (see Fig. 2);

- the coupling constants of X particles to the proton which are more model-dependent (ambiguities on the quark masses, anomalies, ...); in Ref. 6) the value $|d|^2 = 1.45 \times 10^{-7}$ $g^2 \pi NN/e^2 \approx 3.4 \times 10^{-4}$ was quoted in the case of the axion;
- proton beams which are more intense than electron beams.

We give examples with $E_p = 300$ GeV (treating the proton as a point-like Dirac particle):

- for $q = 1$ keV, $\sqrt{s} = 0.939$ GeV, $0 \leq m_X \leq 0.6$ MeV, $\sigma^Y \approx 2 \times 10^{-31}$ cm² one gets $R \approx 1, 2.5, 0.5, 2 \times 10^{-7}$ and $\sigma^X \approx 0.7 \times 10^{-34}, 1.7 \times 10^{-34}, 3 \times 10^{-35}, 1.5 \times 10^{-41}$ cm² for V, A, S, P, respectively;
- for $q = 1$ MeV, $\sqrt{s} = 1.44$ GeV, $0 \leq m_X \leq 0.5$ GeV, $\sigma^Y \approx 10^{-31}$ cm² one gets $R \approx 1, 2.5, 0.4, 0.06$ and $\sigma^X \approx 0.3 \times 10^{-34}, 0.75 \times 10^{-34}, 10^{-35}, 2 \times 10^{-36}$ cm² for V,A,S,P, respectively.

4. - APPLICATION TO RADIATION OF X PARTICLES IN AN EXTERNAL FIELD

We now consider the scattering of an electron beam by an external electromagnetic field (for example the Coulomb field of a heavy particle). Because absolute cross-sections are strongly dependent upon experimental conditions here we shall only consider the ratio of X production to γ production. Absolute rates can then be obtained using measured or estimated rates for γ radiation in each particular case. In Ref. 6) this ratio was given in the case of a relativistic pseudoscalar axion neglecting its mass. We shall show that this approximation cannot be used for the radiation of low energy X particles. This point is essential if one wants to detect real X decays along a reasonable path length. Mass effects come essentially from $q_{\min}^2$ effects for the virtual photons of Fig. 1a,b; for massless X particles (for example, $X \equiv \gamma$) one has

$$q_{\min}^2 = - \frac{m_e^4 k^2}{4p_0^2 p_0'^2} + \dots$$

(i.e., $q_{\min}^2 \approx 3 \times 10^{-27}$ GeV² for $k = 100$ keV, $p_0 \approx p_0' = 15$ GeV) whereas for massive X particles:

$$q_{\min}^2 = - (k_0 - k)^2 + \frac{m_e^2 k_0 (k_0 - k)}{p_0 p_0'} + \dots$$

(i.e., $q_{\min}^2 \approx 1.6 \times 10^{-9}$ GeV² for $m_X = 100$ keV, $k = 100$ keV, $p_0 \approx p_0' = 15$ GeV).

There is also an important m_X effect in the electron propagators ($m_X^2 - 2p \cdot k$ and $m_X^2 + 2p' \cdot k$) especially for forward scattering. As in the case of real $\gamma e^\pm$ collisions, the production of very light ($m_X \rightarrow 0$) V, A and S bosons

differs from the production of P bosons mainly because of the centrifugal barrier factor $\vec{k}^2/s$. The final result is shown in Fig. 4a,b,c representing the ratio $R = \sigma^X/\sigma^\gamma$ in terms of k_X for fixed initial electron energy ($E_e = 15$ GeV). In the case of $m_X = 0$, for a pseudoscalar axion, one recovers the result of Ref. 6). For $m_X \neq 0$ the $q_{\min}^2$ and electron propagator effects appear to be stronger and stronger as m_X increases and k_X decreases. For a better appreciation of the consequences for X rates we show in Fig. 5 the separate spectra of γ radiation and X radiation with $m_X = 170$ keV. In Fig. 6a,b,c we illustrate the same effects in the case of radiation from a 300 GeV proton beam.

It appears that if one wants to observe X production in these kinds of processes, an optimum has to be found between two extreme cases. One can either make a profit of the more intense X production with high momenta but lose part of it through a difficult detection (virtual pair production, γ emission,...) or start with a weaker X production with low momenta but have more chance of observing the real decay modes.

Analytical formulae given in the Appendix can be used for computing the rates of X production in any other kinematical conditions and should be useful for future experimental plannings.

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APPENDIX: TRANSITION PROBABILITIES FOR RADIATION OF X PARTICLES

The tensor $T^{\mu\nu}$ defined in Section 2 and coming from the diagrams of Fig. 1a,b can be written

$$T^{\mu\nu} = \frac{N^{\mu\nu}}{D} + \frac{N'^{\mu\nu}}{D'} + \frac{N''^{\mu\nu}}{D''} \quad (A.1)$$

with

$$D = (m_X^2 - 2p \cdot k)^2, \quad D' = (m_X^2 + 2p' \cdot k)^2, \quad D'' = (m_X^2 - 2p \cdot k)(m_X^2 + 2p' \cdot k)$$

and (m being the electron mass):

$$\begin{aligned} N^{\mu\nu} = & a^2 \left\{ -4m^2(m^2 - p \cdot k + m_X^2)g^{\mu\nu} - (4m^2 + 2m_X^2)(p'^\mu p^\nu + p'^\mu p^\nu - p \cdot p' g^{\mu\nu}) \right. \\ & \left. + (4m^2 + 4p \cdot k)(p'^\nu k^\mu + p'^\mu k^\nu - p' \cdot k g^{\mu\nu}) \right\} \\ & + b^2 \left\{ 4m^2(m_X^2 + 3(m^2 - p \cdot k))g^{\mu\nu} + (12m^2 - 2m_X^2)(p'^\mu p^\nu + p'^\mu p^\nu - p \cdot p' g^{\mu\nu}) \right. \\ & \left. - 4(3m^2 - p \cdot k)(p'^\nu k^\mu + p'^\mu k^\nu - p' \cdot k g^{\mu\nu}) \right\} \\ & + 4iab \varepsilon^{\mu\nu\rho\sigma} \left\{ m_X^2 p'_\rho p'_\sigma + 2(m^2 - p \cdot k)p'_\rho k_\sigma \right\}. \end{aligned}$$

(in the V,A case; a and b being real by Hermiticity).

$$\begin{aligned} N'^{\mu\nu} = & |c|^2 \left\{ m^2(4m^2 - 4p \cdot k + m_X^2)g^{\mu\nu} + (4m^2 - m_X^2)(p'^\mu p'^\nu + p'^\mu p'^\nu - p' \cdot p' g^{\mu\nu}) \right. \\ & \left. + (2p \cdot k - 4m^2)(p'^\nu k^\mu + p'^\mu k^\nu - p' \cdot k g^{\mu\nu}) \right\} \\ & + |d|^2 \left\{ 2p \cdot k(p'^\nu k^\mu + p'^\mu k^\nu - p' \cdot k g^{\mu\nu}) - m_X^2(p'^\mu p'^\nu + p'^\mu p'^\nu - p' \cdot p' g^{\mu\nu} + m^2 g^{\mu\nu}) \right\} \\ & + 4i \operatorname{Im} cd^* \varepsilon^{\mu\nu\rho\sigma} \left\{ m^2(p'_\rho k_\sigma - p'_\rho p'_\sigma - p'_\rho k_\sigma) + p \cdot k p'_\rho k_\sigma + \frac{1}{2} m_X^2 p'_\rho p'_\sigma \right\}. \end{aligned}$$

(in the S,P case)

$$N'^{\mu\nu} = N^{\mu\nu}(p \leftrightarrow p', k \leftrightarrow -k; \text{ and in the V,A case : } b \leftrightarrow -b)$$

$$\begin{aligned}
 N''^{\mu\nu} = & (a^2 + b^2) \left\{ -4k \cdot p (2p'^\mu p'^\nu + p^\mu p'^\nu + p^\nu p'^\mu - 2p \cdot p' g^{\mu\nu}) + 4k p' (2p'^\mu p'^\nu + p'^\mu p'^\nu - 2p' \cdot p' g^{\mu\nu}) \right. \\
 & - 4m^2 (2p \cdot p' g^{\mu\nu} - 2k^\mu k^\nu) - 4p p' (2p'^\mu p'^\nu + 2p^\nu p'^\mu + k^\mu (p^\nu - p'^\nu) - k^\nu (p'^\mu - p^\mu) - 2p p' g^{\mu\nu}) \\
 & \left. - 4m_X^2 p \cdot p' g^{\mu\nu} \right\} \\
 & + b^2 \left\{ 16m^2 (p^\mu p^\nu + p^\nu p'^\mu + p^\mu p'^\nu + p'^\mu p'^\nu - k^\mu k^\nu) \right\} \\
 & - 4iab \left\{ \varepsilon^{\mu\nu\rho\sigma} (4m^2 p_\rho p'_\sigma + 2m^2 (p_\rho k_\sigma - k_\rho p'_\sigma) + p_\rho p'_\sigma (k \cdot (p' - p) - 2p \cdot p')) \right. \\
 & \left. + p_\rho k_\sigma p' \cdot (k - p) + k_\rho p'_\sigma p \cdot (p' - k) \right) + k^\nu \varepsilon^{\mu\tau\rho\sigma} p'_\tau k_\rho p_\sigma + k^\mu \varepsilon^{\nu\tau\rho\sigma} k_\tau p'_\rho p_\sigma \left\} .
 \end{aligned}$$

(in the V,A case)

$$\begin{aligned}
 N''^{\mu\nu} = & |c|^2 \left\{ m^2 (2g^{\mu\nu} [m_X^2 + 4m^2 + 4p' \cdot k - 4p \cdot k - 4p \cdot p'] + 4p^\mu k^\nu + 4p^\nu k^\mu + 8p^\mu p'^\nu \right. \\
 & + 8p^\nu p'^\mu - 4p'^\nu k^\mu - 4p'^\mu k^\nu - 4k^\mu k^\nu) + 2p \cdot k (k^\mu p'^\nu + k^\nu p'^\mu) + 2p' \cdot k (k^\mu p^\nu + k^\nu p^\mu) \\
 & \left. - 4p \cdot k p' g^{\mu\nu} - 4p \cdot p' k^\mu k^\nu + 2m_X^2 (p \cdot p' g^{\mu\nu} - p'^\mu p'^\nu - p'^\nu p'^\mu) \right\} \\
 & + |d|^2 \left\{ 2p \cdot k (k^\mu p'^\nu + k^\nu p'^\mu) + 2p' \cdot k (k^\mu p^\nu + k^\nu p^\mu) - 4p \cdot k p' g^{\mu\nu} - 4p \cdot p' k^\mu k^\nu \right. \\
 & \left. + 2m_X^2 (p \cdot p' g^{\mu\nu} - p^\nu p'^\mu - p^\mu p'^\nu) + m^2 (4k^\mu k^\nu - 2m_X^2 g^{\mu\nu}) \right\} \\
 & - 2i \gamma_{mc} d^* \left\{ \varepsilon^{\mu\nu\rho\sigma} (m^2 (k_\rho p_\sigma - k_\rho p'_\sigma) - p \cdot p' k_\rho (p - p')_\sigma) \right. \\
 & + p'^\mu \varepsilon^{\nu\tau\rho\sigma} p_\tau k_\rho p'_\sigma + p'^\nu \varepsilon^{\mu\tau\rho\sigma} p'_\tau k_\rho p_\sigma - p^\nu \varepsilon^{\mu\tau\rho\sigma} p'_\tau p_\rho k_\sigma \\
 & \left. - p^\mu \varepsilon^{\nu\tau\rho\sigma} p_\tau p'_\rho k_\sigma + k^\mu \varepsilon^{\nu\tau\rho\sigma} p'_\tau p'_\rho k_\sigma + k^\nu \varepsilon^{\mu\tau\rho\sigma} p'_\tau k_\rho p_\sigma \right\} .
 \end{aligned}$$

(in the S,P case).

We use the metric $g^{00} = 1$, $g^{ii} = -1$. The above expansion is suitable for all cases of external electromagnetic fields. For example, in the case of static Coulomb fields one just needs to take T^{00} (see Section 2).

For the case of real incident photons we need $T = -g_{\mu\nu} T^{\mu\nu}$ explicitly given below:

$$T = \frac{N}{D} + \frac{N'}{D'} + \frac{N''}{D''} \quad (A.2)$$

$$N = -4a^2 \{ 2p \cdot k p' \cdot k - 2m^2(p \cdot p' + 2p \cdot k - p' \cdot k - 2m^2) - m_X^2(p \cdot p' - 4m^2) \} \\ - 4b^2 \{ 2p \cdot k p' \cdot k - 2m^2(-3p \cdot p' + 3p' \cdot k - 6p \cdot k + 6m^2) - m_X^2(p \cdot p' + 4m^2) \}$$

(in the V,A case)

$$N = -|c|^2 \{ 4p \cdot k p' \cdot k + 4m^2(2p \cdot p' - 2p' \cdot k + 4p \cdot k - 4m^2 - m_X^2) - 2m_X^2 p \cdot p' \} \\ - |d|^2 \{ 4p \cdot k p' \cdot k + m_X^2(4m^2 - 2p \cdot p') \}.$$

(in the S,P case)

$$N' = N(p \leftrightarrow p', k \leftrightarrow -k)$$

$$N'' = -4a^2 \{ -4p \cdot p'(p \cdot p' + k \cdot (p - p')) + 2m^2(4p \cdot p' + k \cdot (p - p')) + 2m_X^2(2p \cdot p' - m^2) \} \\ - 4b^2 \{ -4p \cdot p'(p \cdot p' + k \cdot (p - p')) + 2m^2(-4m^2 + k \cdot (p - p')) + 2m_X^2(2p \cdot p' + m^2) \}.$$

(in the V,A case)

$$N'' = -4|c|^2 \{ 2p \cdot k p' \cdot k + m^2(-m_X^2 - 8m^2 - 6p' \cdot k + 6p \cdot k + 4p \cdot p') \} \\ - 4|d|^2 \{ 2p \cdot k p' \cdot k + m_X^2 m^2 \}.$$

(in the S,P case).

It may be useful for users unaware of this to notice that a high numerical precision is required in the computation of these expressions because of large cancellations between the three terms, especially at small angles.

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FIGURE CAPTIONS

- Figure 1 : Diagrams for real or virtual $\gamma e^\pm$ collisions giving X bosons.
- Figure 2 : Values of the Compton cross-section in terms of c.m. energy.
- Figure 3 : Values of the ratio R in terms of the X mass for $\sqrt{s} = 1.12$ MeV in the case of real $\gamma e^\pm$ collisions.
- Figure 4 : Values of the ratio R in terms of the X momentum in the case of radiation of X particles from electron beam ($E_e = 15$ GeV) in an external Coulomb field.
- 4a) $X \equiv V$ (the case $X \equiv A$ gets similar values which only become slightly larger than those of the case $X \equiv V$ when R becomes of order one).
- 4b) $X \equiv S$
- 4c) $X \equiv P$
- : $m_X = m_e/1000$
 — . — . — : $m_X = m_e/100$
 - - - - - : $m_X = m_e/10$
 — — — : $m_X = m_e$
- Figure 5 : Separate behaviours of γ radiation cross-section and X radiation cross-sections for $E_e = 15$ GeV, $m_X = m_e/3$ in terms of the X momentum.
- Figure 6 : Same caption as for Fig. 4 in the case of a proton beam ($E_p = 300$ GeV).

a)

b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4 a

Fig. 4b

Fig. 4c

Fig. 5

Fig. 6a

Fig. 6b

Fig. 6c