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SOME ASPECTS OF THE U(1) PROBLEM
AND THE PSEUDOSCALAR MASS SPECTRUM

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ABSTRACT

A discussion of some aspects of the U(1) problem is given and
the large N point of view is considered. The necessity for a vector
ghost gluonic particle, in order to resolve the U(1) problem, is stressed.
Scalar ghosts are shown to be insufficient. An analysis of the n-n'
mixing problem is made and it is suggested that the singlet decay constant
may be nearly twice as large as the other octet decay constants. PCAC
corrections are then considered and the above conclusion is reaffirmed.
Quark mass values are given and it is pointed out that in the usual scheme,
\[\langle As \rangle \cong 1.48 \langle Au \rangle.\] The compatibility of the above with the \[n \to 3\pi\]
decays is also checked.

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1. INTRODUCTION: THE U(1) PROBLEM

Recently the large N point of view [1-4] has provided a consistent
picture for handling the U(1) problem [5]. In particular, the connection
with phenomenology has been made [2,6]. In this note we examine some aspects
of the pseudoscalar mass spectrum. The rest of this section is devoted to
highlighting the U(1) problem while in Sec.2 the large N point of view
and mechanism is considered. Sec.3 concerns an application of these ideas
to the \[n-n'\] mixing problem. Some discussion and motivation is deferred to the
latter part of that section. PCAC corrections are considered in Sec.4.

Assuming there is no anomaly, the mass squared matrix for the \[n-n'\] system
(in the octet-singlet basis) can be written as

\[
\begin{bmatrix}
\frac{1}{3} \xi^2 - \frac{1}{3} \eta^2 \\
\frac{2}{3} \xi^2 - \frac{2}{3} \eta^2 \\
\end{bmatrix}
\]

In writing the above we have taken explicit breaking due to the quark masses
to be of the form \[\xi^2 \propto \xi_{\nu}, \xi_{\bar{\nu}} \propto \xi_{\nu}, \xi_{\bar{\nu}} \propto \xi_{\nu}, \xi_{\bar{\nu}} \propto \xi_{\nu}.

\[\langle \bar{u}u \rangle \propto \langle \bar{d}d \rangle \propto \langle \bar{s}s \rangle \]
and have made use of standard soft "pion" theorems.

The eigenvalues and eigenvectors of (1) are

\[
\begin{align*}
\lambda_1 & = 0; & |1\rangle & = \frac{1}{\sqrt{3}} (|\bar{d}\rangle + \sqrt{2}|\bar{s}\rangle) \\
\lambda_2 & = 2\xi^2 - \eta^2 = 0; & |2\rangle & = \frac{1}{\sqrt{3}} (|\bar{d}\rangle - \sqrt{2}|\bar{s}\rangle) \\
\end{align*}
\]

As \[|1\rangle\] is predominantly singlet and \[|2\rangle\] predominantly octet it is
usual to associate them with \[|n\rangle\] and \[|n'\rangle\], respectively. Defining
the mixing angle as

\[
\begin{align*}
|n\rangle & = \cos \theta |\bar{d}\rangle + \sin \theta |\bar{s}\rangle \\
|n'\rangle & = -\sin \theta |\bar{d}\rangle + \cos \theta |\bar{s}\rangle \\
\end{align*}
\]

results in

\[
tan \theta = \frac{1}{\sqrt{2}} (\theta \cong 35.3^\circ).
\]

This situation is known as ideal mixing.

*) Some justification and explanation of PCAC terminology can be found in
Sec.4. Otherwise see Ref.7.
The U(1) problem concerns itself with the absence of any such light isoscalar particle with mass $m$. Such a particle is simply not observed experimentally. The existence of such a light isoscalar particle can also be connected with substantial isospin violations [8,9]. Roughly speaking, isospin violations go via $\mathcal{Q}_{1/2}^3$ insertions and are proportional to

$$
\epsilon \left< \mathcal{Q}_{1/2}^3 \right>_{\text{isoscalar}} \frac{\delta}{\epsilon^2} \text{ mass}^2 \text{ of isoscalar} \text{ mass}^2 \text{ of isoscalar} \quad \epsilon = \frac{m_1 - m_2}{2}
$$

The existence of a light isoscalar implies isospin violations of $O(\epsilon/m_0)$; $m_0 = \frac{m_1 + m_2}{2}$. Otherwise they are $O(\epsilon/m_3)$. The U(1) problem can further be connected with the suppression of $n \rightarrow 3\pi$ decays in the soft $s^0$ limit. Amazingly, this is a case where the predicted isospin violation is not as large (actually vanishing) as what is observed experimentally. The amplitudes for the $n \rightarrow 3\pi$ decays are proportional to

$$
\left< s^0 \pi^+ \right| \int_x \frac{m_1 - m_2}{2} \mathcal{Q}_{1/2}^3(x) \mid n \rangle
$$

where the operator inside the matrix element is taken to be the interaction Hamiltonian (with massive pions and eta). Using a soft pion theorem this can be reduced to

$$
\frac{1}{\pi} \left< s^0 \pi^+ \right| \int_x \mathcal{Q}_{1/2}^3(x) \mid n \rangle = \frac{1}{\pi} \left< s^0 \pi^+ \right| \left< \frac{m_1 - m_2}{2} \right| n \rangle
$$

$$
= \frac{1}{\pi} \left< s^0 \pi^+ \right| \int_x \left( \frac{1}{21(m_0^2 - t)} \right) \left( m_0^2 \mathcal{Q}_{1/2}^3(x) + 2e \alpha g_{\pi N N}^2 \right) \mid n \rangle
$$

The above assertion then follows by noting that $\frac{3}{2e}$ amounts to a connected insertion of $\int_x \mathcal{Q}_{1/2}^3(x)$, as can easily be seen from the path integral.

The gauge current is denoted by $J_{\mu5}$ and $J_{\mu5, sym}$ there is no difference between $J_{\mu5}$ and $J_{\mu5, sym}$

The gauge current will be denoted by $J_{\mu5}$. For the present discussion (no anomaly) there is no difference between $J_{\mu5}$ and $J_{\mu5, sym}$

The usefulness of the large $N$ picture relies on the fact that as $N = \infty$ the anomaly can be turned off.

Although this appears to be a positive norm propagator it is necessarily an antihermitean field. The corresponding hermitean field is a real ghost.
in the leading order of the $\frac{1}{\hbar}$ expansion (i.e. pure Yang-Mills theory). This is done by introducing a vector ghost that couples to $K_\mu$. If we assume $^{13}$ that there are no zero mass poles coupled to the gauge invariant $U(1)$ current $J_{L}^{\mu}$ ($L$ = number of flavours) then the anomalous Ward identity $^{12}$ (in the chiral limit)

\[ 0 = \int x d^4 x \, T \left< J_{L}^{\mu}(x) K_\mu(0) \right> = 2 \int x d^4 x \, T \left< K_\mu(x) K_\mu(0) \right> \]

implies that $\left< J_{L}^{\mu} \right>_{QCD}$ should vanish, i.e. when we include fermions. This then specifies the ghost singlet coupling in terms of $\left< J_{L}^{\mu} \right>_{YM}$.

In order for the mesonic and the gluonic contributions to $\left< J_{L}^{\mu} \right>$ to cancel it is necessary that they be of opposite signs, i.e. one of the contributing intermediate states must be of negative norm. This is the reason for introducing a vector gluonic ghost; this results in a positive value $^{13}$ for $\left< J_{L}^{\mu} \right>_{YM}$. The correct sign propagator for the gluonic state would necessarily give the wrong sign shift in the singlet mass $^{2}$. To ensure that no zero mass pole couples to the gauge invariant current $J_{L}^{\mu} = J_{L}^{\mu,\text{sym}} + 2iK_\mu$, this vector ghost must couple equally and oppositely to $J_{L}^{\mu,\text{sym}}$ and $2iK_\mu$. We will refer to this as the Veneziano mechanism $^{2}$.

Before continuing it should be noted that the Kogut-Susskind mechanism $^{14}$ is insufficient in resolving the $U(1)$ problem. A particular version $^{15}$ of the K-S mechanism consists of two massless scalar fields $\phi_+$ and $\phi_-$, with positive and negative norm, respectively. Gauge invariant quantities are supposed to couple to the sum $\phi_+ + \phi_-$, which has zero propagator, thus ensuring no singularities in matrix elements of gauge invariant operators. On the other hand, the gauge variant current, $J_{L}^{\mu,\text{sym}}$, is supposed to couple to the difference $\phi_+ - \phi_-$. Goldstone bosons can then contribute to matrix elements containing one $J_{L}^{\mu,\text{sym}}$ and a string of gauge invariant operators. The problem with this particular version is that

\[ \int T \left< J_{L}^{\mu,\text{sym}}(x) J_{L}^{\mu,\text{sym}}(0) \right> \]

is also vanishing. It is well known by now that a consistent resolution to the $U(1)$ problem requires $^{12}$ a non-vanishing positive value (in the non-chiral limit) for $\left< J_{L}^{\mu} \right>_{QCD}$ that is $O(\hbar)$ with respect to the light quark masses. This requirement essentially follows from a Ward identity $^{12}$ which relates $\left< J_{L}^{\mu} \right>$ to $\left< \bar{q} q \right>$. Assuming no zero mass poles coupled to $J_{L}^{\mu}$ (a necessary requirement so to solve the $U(1)$ problem) $\left< J_{L}^{\mu} \right>$ can be rewritten as

\[ \left< J_{L}^{\mu} \right> = \frac{\alpha}{\Lambda^2} \int x d^4 x \, T \left< J_{L}^{\mu,\text{sym}}(x) J_{L}^{\mu,\text{sym}}(0) \right> \]

As already noted the particular version of the K-S mechanism stated above cannot give (8) and hence $\left< J_{L}^{\mu} \right>$, a non-zero value and thus cannot solve the $U(1)$ problem.

One may think of remediying this deficiency of the K-S mechanism by allowing only $^{**}$ $\phi_+$ (or $\phi_-$) to couple to $J_{L}^{\mu,\text{sym}}$. This is still not enough, as by Lorentz invariance alone, a scalar pole cannot give (8) a non-zero value

\[ \left< J_{L}^{\mu} \right>_{\text{scalar}} \sim \lim_{q \to 0} \int x d^4 x \, T \left< J_{L}^{\mu,\text{sym}}(x) J_{L}^{\mu,\text{sym}}(0) \right> \]

\[ = \lim_{q \to 0} \frac{\alpha}{\Lambda^2} \delta_q q_x q_y q_z = 0 \]

It should have been noted above that in the K-S mechanism the scalars are supposed to remain massless even with explicit chiral symmetry breaking. In other words, they are "trapped" particles. It is hard to imagine how then a genuine light Goldstone boson singlet can appear when the anomaly is turned off (by taking $N \to \infty$ say). The Veneziano mechanism appears then to be the only viable alternative.

The K-S mechanism $^{14}$ was originally constructed so as to give a non-vanishing value for the $n + 3\pi$ decays (see Eq.(6))

\[ \text{Amp}(n + 3\pi) \sim \lim_{q \to 0} \frac{\alpha}{\Lambda^2} \delta_q q_x q_y q_z \neq 0 \]

\[ \text{[K-S]} \]

\[ \lim_{q \to 0} \frac{\alpha}{\Lambda^2} \delta_q q_x q_y q_z \neq 0 \]

\[ \text{[Ven.]} \]

*) It was found, in Ref.12, that instantons give too small a contribution ($O(\hbar^3)$) to this quantity to offer a realistic solution to the $U(1)$ problem. The large $N$ picture and the Veneziano mechanism can however be shown to remain consistent with the anomalous Ward identities in the non-chiral limit $^{2}$.

**) This seems to be the way it was actually presented in Ref.14.
Even though the K-S mechanism was successful in accounting for these decays we saw above that it failed when it came to an examination of the anomalous Ward identities. Regardless of this, the idea was crucial in arriving at the correct mechanism which is consistent with the U(1) problem.

3. **The n-n’ Mixing Problem**

The important thing [1] learnt from the large N point of view was that the observed n’ (for three light flavours) is just as much a relic of the U(1) axial symmetry as the pion is of SU(2) axial symmetry.

With the modifications of the previous section the n-n’ mass matrix can now be written as *4* [2]:

\[
M^2 = \begin{pmatrix}
\frac{4}{3}m_K^2 - \frac{1}{3}m_n^2 & -\frac{2}{3}\frac{q^2}{m_K^2} (m_K^2 - m_n^2) \\
-\frac{2}{3}\frac{q^2}{m_K^2} (m_K^2 - m_n^2) & \frac{2}{3}m_K^2 + \frac{1}{3}m_n^2 + \frac{1}{2} \chi^2/N
\end{pmatrix} \approx \begin{pmatrix}
-2139 & -3213 \\
-3213 & -2139 + 1401 + \frac{\chi^2}{N}
\end{pmatrix}.
\]

(9)

The extra \(\frac{q^2}{N}\) contribution comes from the replacement

\[
\text{singlet} \rightarrow S + \text{ghost} S
\]

i.e.

\[
\frac{1}{q^2 - m_S^2} \rightarrow \frac{i}{q^2 - m_S^2} \left[ 1 + \left( \frac{i}{\chi/N} \frac{q^2}{m_S^2} \right) \left( \frac{1}{q^2 - m_S^2} \right) \right] + \ldots
\]

\[
\left( \frac{\chi}{N} \right) \frac{i}{q^2 - m_S^2}
\]

(10)

This corresponds to giving the singlet an additional mass through gluon annihilation diagrams. In writing (10) it has been assumed that \(\frac{q^2}{N}\) corrections are small, but in fact they will turn out to be large. In essence an assumption about pole dominance with an \(n'\) has been made.

*) Average values for \(m_K^2\) and \(m_n^2\) have been used in (9),

\[m_K^2 \approx 0.2437 \text{ GeV}^2; \quad m_n^2 \approx 0.1185 \text{ GeV}^2.\]

The inclusion of \(\chi \neq 0\) effects now upset ideal mixing. The eigenvalues of (9) are

\[
m_{n',n}^2 = (m_K^2 + \frac{\chi^2}{2N}) \pm \frac{1}{2} \sqrt{(2m_K^2 - 2m_n^2 - \frac{\chi^2}{3N})^2 + \frac{8\chi^2}{N^2}}.
\]

(11)

In Eq.(11) we have identified the larger of the two solutions with \(m_{n'}^2\).

Note that in the limit \(\frac{q^2}{N} \rightarrow 0\) this solution becomes \(m_{n'}^2 \approx m_n^2\), which corresponds to the solution of (1) that we previously identified with the \(n\). In the above \(\frac{q^2}{N}\) remains a free parameter. Taking the trace of (9) gives the relation

\[
m_n^2 = m_{n'}^2
\]

(12)

From (9) and (11), the mixing angle \(\theta\) (defined by (3)) is found to be *

\[
\tan \theta = \frac{\frac{1}{3} \frac{q^2}{N} \frac{m_n^2 - 3m_n^2 - m_n^2}{2N}}{2 \lambda \left( \frac{m_n^2 - m_n^2}{2N} \right)}
\]

(13)

Following Veneziano [2] \(\frac{q^2}{N}\) can be determined from (12) by inputting the experimental value for the sum \(m_n^2 + m_n^2 \approx 1.2382 \text{ GeV}^2\). The result is

\[
\frac{q^2}{N} \approx 0.7268 \text{ GeV}^2
\]

Inserting this back into Eqs.(11) and (13) results in

\[
\begin{align*}
m_{n'}^2 &= 0.2677 \text{ GeV}^2 \quad \text{(exp: 0.2717)} \quad (14a) \\
m_n^2 &= 0.2505 \text{ GeV}^2 \quad \text{(exp: 0.2502)} \quad (14b) \\
\theta &\approx 18.4^\circ \quad \text{(exp: 10^\circ)} \quad (14c)
\end{align*}
\]

Veneziano quotes the following values:

\[
\begin{align*}
m_{n'}^2 &\approx 0.291 \quad (15a) \\
m_n^2 &\approx 0.267 \quad [\text{Veneziano}] \quad (15b) \\
\theta &\approx 18^\circ \quad (15c)
\end{align*}
\]

*) Eqs.(11), (12) and (13) can be found in Veneziano, Ref.2. -8-
They differ from (14) because in his estimation Weinberg kept only the
largest terms. The agreement with experiment is at least encouraging. The
important thing to note is that in order to get a reasonable fit for the
pseudoscalar mass spectrum it is in practice necessary that $\frac{2}{3}$ corrections
be large; at least for the pseudoscalar channel. That is, Zweig forbidden
processes have to be violated by a large amount in this case. Large $\Sigma$

In order to improve the analysis we now redo the calculations with
$F_s \approx F_0 \approx F_K$ but allow $F_S$ to differ from them. The reason for doing
this will be motivated later. Defining $\xi = F_S/F_0$, Eqs. (9), (11), (12) and
(13) are respectively replaced by

$$\mathcal{L} = \left( \frac{2}{3} m_K^2 - \frac{1}{3} m_\pi^2 \right) - \xi \frac{2 \sqrt{2}}{3} \left( m_K^2 - m_\pi^2 \right) + \frac{\xi}{2} \left( \frac{2}{3} m_K^2 + \frac{1}{3} m_\pi^2 \right) + \frac{\chi}{N} \right) \right)$$

$$m_{\pi^0,\eta} = \frac{1}{2} \left[ \frac{\chi}{N} + 3213 + 1701 \xi \right]$$

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$$m_{\pi^0,\eta} = \frac{1}{2} \left[ \frac{\chi}{N} + 3213 + 1701 \xi \right]$$

The best fit is found for

$$\xi \approx \frac{F_S}{F_0} \approx 0.51.$$  

In the above, a fit to the $\pi^0-n$ mixing analysis was made by adjusting two
parameters, $\frac{\chi}{N}$ and $\xi$. One prediction is thus obtained, the mixing angle
say. Alternatively, one could view the above analysis as an estimation
for $\xi \approx \frac{F_S}{F_0}$.

We remark once again that in order to get a reasonable fit for the
pseudoscalar mass spectrum it is necessary that large $\Sigma$ corrections be large.
This can be summarized by the following numerical estimates:

$$1 \text{ GeV}^2 \frac{\chi}{N} \approx 0.8 \gg \frac{\chi}{N} \approx 0.25 \gg m_p^2 \approx 0.01.$$  

On the basis of (20) alone it may be expected that Zweig forbidden processes
should be more important for pseudoscalar channels than for vector channels; 
simply because

$$\left( \frac{\chi^2}{N} \right) / m_p^2 \approx 3.2 \quad \text{while} \quad \left( \frac{x^2}{N} \right) / m_p^2 \approx 1.0.$$  

It is amusing to actually perform a similar analysis for the $u+d$ system.  

Adding a possible gluonic term to the singlet channel and fixing it by the
sum $m_0^2 + m_1^2$ results in

$$\left( \frac{x^2}{N} \right) / \text{vector channel} \approx \frac{0.0622 \text{ GeV}^2}{m_p^2}$$

$$m_0^2 \approx 0.6321 \quad (\text{exp: 0.613})$$

$$m_1^2 \approx 1.021 \quad (\text{exp: 1.04})$$

$$\tan \theta \approx \frac{\left( \frac{x^2}{N} \right) / \text{pseudoscalar}}{\left( \frac{x^2}{N} \right) / \text{vector}} \approx \frac{3213 - m_p^2}{2.139 \xi}$$

$$\tan \theta \approx \frac{3213 - m_p^2}{2.139 \xi}$$

The experimental values of $m_0^2$ and $m_1^2$ have been used in (17), (18) and (19).

By fixing $\xi$ at various values and following the same procedure as
before the predictions of Table I are found.

---

*Here we simply mean; using (9) with $m_0^2$ and $m_1^2$ replaced by $m_0^2$ and $m_0^2$.
Let us now motivate why we chose to improve the $\eta$-$\eta'$ analysis by allowing $F'_8$ to differ from the other octet of decay constants. In the leading order of $1/2$, $F'_8$ is expected to be the same as the other decay constants; $1/2$ corrections are expected to split isospin symmetry because now the singlet can proceed through gluonic intermediate diagrams while the flavour bearing mesons cannot. As $1/2$ corrections are necessarily large it is reasonable to allow for such corrections.

In the effective Lagrangian treatment of $F'_8$  

$$\frac{a}{2F'_8} \text{tr}(U \bar{\eta} U^*) \text{tr}(U \bar{\eta} U).$$

(21)

Such a term is due to an additional power of $1/2$ with respect to a usual $\eta$-meson vertex because the double flavour trace requires two quark loops. Hence as $F'_8 = 0(\sqrt{\mathcal{N}})$, $a = 0(1/2)$. In going over to the non-linear version, in which QCD has been assumed and massive scalars removed

$$\mathcal{L} = \frac{a}{\sqrt{2}} \exp(i \frac{\beta}{\beta_F} \sigma) \left( \begin{array}{c} 1 \\ i \end{array} \right), \quad \frac{a}{\sqrt{2}} = \gamma_5$$

(21) becomes

$$\frac{2a}{\sqrt{2}} \left( \frac{\sigma \eta}{a} \right)^2$$

which then adds to the usual kinetic term $1/2 \left( \frac{\sigma \eta}{a} \right)^2$ and thus results in

$$\langle \eta_0 | J_{\mu} \eta_0 \rangle = F'_8 (1 + \frac{2a}{\sqrt{2}}) q_0$$

The singlet decay constant is then shifted from its leading value of $F'_8$ to $F'_8 (1 + \frac{2a}{\sqrt{2}})$. Comparing this with the estimate

$$F'_8 \approx 1.95 F_8 \approx 1.95 F'_8$$

(22)

gives a value of $a \approx 0.63$.

Di Vecchia et al. [6] have also improved the mixing analysis, however, by employing a possible term in the effective Lagrangian

$$\sim (\bar{\eta} \eta) \text{tr}(\mathcal{M}(1 - \bar{U} U)).$$

After eliminating the $K^0$ field (by its equation of motion) they arrived at a term $\sim \text{tr}(\not{\phi}) \text{tr}(\mathcal{M}(\phi^0))$. Such a term then induces additional octet-singlet and singlet-singlet couplings. These corrections are $0(\frac{1}{3})$ and may appear to be non-leading with respect to the $0(1/2)$ corrections to $F'_8$. In the mass $\mathcal{M}$ matrix however

$$\frac{F'_8}{F_8} \approx \frac{\eta_0^2}{\eta_0^2} + O(\frac{1}{3})$$

and we see that the two different sorts of corrections are in fact compatible. In the scheme of Di Vecchia et al. [6] the mass $\mathcal{M}$ matrix [9] was supplemented by the following addition (in the notation of Ref.6, $\mathcal{N} = 3$; $\mathcal{E}$ is a free parameter):

$$\begin{pmatrix}
0 & -\mathcal{E} & \mathcal{E} \mathcal{M} \\
-\mathcal{E} & \mathcal{M} & \mathcal{E} \\
\mathcal{E} & \mathcal{E} & \mathcal{M}
\end{pmatrix}$$

Comparing with (16) the two are compatible if $\mathcal{E} = (1 + \frac{2a}{\sqrt{2}})$. In other words, the above corrections can be reabsorbed into a redefinition of $F'_8$. The result found in Ref.6, i.e. $a = 1.08 (\frac{1}{3} = 0.36)$ implies $\mathcal{E} = 0.46$ and is compatible with (22).

Defining

$$\langle 0 | J_{\mu} \eta_0 | \eta_0, \eta_0' \rangle$$

and

$$\langle 0 | J_{\mu} \eta_0 | \eta_0, \eta_0' \rangle$$

where

$$F'_8 \approx F_8 \quad \text{and} \quad F'_8 \approx F_8$$

and using (3) (together with $F'_8 \approx 0.51$ and $a \approx 0.63$) we find that

$$p_0 = \cos \theta_F F'_8 \quad \text{and} \quad p_0 = \cos \theta_F F_8$$

$$\eta'_0 = \cos \theta_F F'_8 \quad \text{and} \quad \eta'_0 = \cos \theta_F F_8$$

$$\eta_0 = \sin \theta_F F'_8 \quad \text{and} \quad \eta_0 = \sin \theta_F F_8$$

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$$\eta'_0 = \sin \theta_F F'_8 \quad \text{and} \quad \eta'_0 = \sin \theta_F F_8$$

-12-
In this section we consider FCAC corrections generally and their effect on the work of the previous section. A brief discussion on current quark mass values is also given.

By considering the Ward identity (in the non-chiral limit)

\[
O = \int_x \langle \gamma_\mu \gamma_5 S(x) \gamma_\nu \gamma_5 S(0) \rangle = \int_x \langle \gamma_\mu \gamma_5 S(x) \gamma_\nu \gamma_5 S(0) \rangle + \langle [F_5^3, \gamma_\mu \gamma_5] \rangle
\]

(23)

where

\[
\gamma_\mu \gamma_5 \equiv \frac{i}{2} \gamma_5 \gamma_\mu \gamma_5 \quad \frac{i}{2} \gamma_\mu \gamma_5 \quad \text{and} \quad F_5^3 \equiv \int_x \langle \gamma_\mu \gamma_5 \rangle
\]

it is easy to show that

\[
u_0^2 \nu_0^2 = -\langle \frac{1}{4} \{n_\mu, n_5\} q \rangle \quad (24a)
\]

In obtaining the above we have evaluated the equal-time commutator, with \(n_\mu^3 = n_\mu n_\nu n_\rho \) to order \(\langle n_0 \rangle^2\), in the second term on the right-hand side of (23) and have saturated the first term on the right-hand side of (23) with one pion pole. The 3 pion pole contribution can easily be shown to be \(O(n_0^4)\).

Eq. (24a) can thus be considered to be true with corrections up to \(O(n_\mu n_\nu n_\rho)\); i.e., with \(O(n_\mu^3, n_0^2)\) corrections included in the symmetry values for \(F_0 \langle \bar{q}n_\mu \rangle\), etc. We follow Langacker and Pagels [16] in keeping terms with logarithms in preference to terms without logarithms. They are obviously dominant near the chiral limit, although their importance in the real world is not immediately obvious. Eq. (24a) should be considered as the leading chiral contribution to the \(m_0^2\) mass term; electromagnetic corrections and contributions from \(\pi_0 - \eta - \eta'\) mixing would have to be included as extra effects.

In what follows we will ignore the anomaly again; for the time being anyhow. Just as we derived Eq. (24a) the Ward identities for the other \(U(3)\) axial currents can be used to derive *

\[
m_0^2 F_0^2 = \frac{1}{3} \langle \bar{q} m_0 m_3 \rangle q \rangle
\]

(24b)

\[
m_0^2 F_0^2 = \frac{2}{3} \langle \bar{q} m_0 m_3 \rangle q \rangle
\]

(24c)

\[
m_0^2 F_0^2 = \frac{1}{2} \langle \bar{q} m_0 m_3 \rangle q \rangle
\]

(24d)

\[
m_0^2 F_0^2 = \frac{1}{2} \langle \bar{q} m_0 m_3 \rangle q \rangle
\]

(24e)

\[
m_0^2 F_0^2 = \frac{1}{2} \langle \bar{q} m_0 m_3 \rangle q \rangle
\]

(24f)

In writing Eqs. (24) we have considered the \(\eta - \eta'\) system in the octet-singlet basis prior to mixing. With this clarification \(m_0^2\) should be understood as \(\langle \bar{q}^0 | D | q^0 \rangle\); the off-diagonal mass matrix element. The physical basis is always restored after diagonalization of the mass matrix. In other words, the 5 ordered product \(\int_x \langle \bar{q}_5 \nu_5 \nu_5 \rangle \), for example, is saturated as follows:

\[
\langle 0 | \bar{q}_5 \nu_5 | 18 \rangle = \frac{1}{m_0^2} \langle \bar{q}_5 | s \chi S | 18 \rangle = \frac{1}{m_0^2} \langle \bar{q}_5 | s \chi S | 18 \rangle
\]

(24g)

(24h)

\[
(-i) F_0 F_5 \langle s | \chi S | 18 \rangle
\]

(24i)

*) The formula for \(m_0^2 F_0^2\) and \(m_0^2 F_0^2\) are quite often misquoted in the literature, with e.g. \(m_0^2 F_0^2 = \langle \bar{q} m_0 m_3 \rangle q \rangle\). The expression for the charged pion has not been included in the above because it will not be needed later.

** As the contributions from \(\pi_0 - \eta - \eta'\) mixing to the \(m_0^2\) mass term are indeed small \([8,9]\) we will totally ignore this in the subsequent discussion.

---

\(-14-\)
Using the pseudoscalar density \( \bar{\psi} \gamma_5 \psi \) in place of \( \bar{\psi} \gamma_5 \psi \) in Eq.(23) and following the same procedure as before we obtain

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{\langle \bar{d}d \rangle + \langle \bar{s}s \rangle} \right),
\]

where

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \langle 0 | i \bar{\psi} \gamma_5 \gamma_5 \psi | 0 \rangle.
\]

Eq.(25a), rewritten as

\[
\langle 0 | i \bar{\psi} \gamma_5 \gamma_5 \psi | 1^0 \rangle = \left( \frac{\alpha_5}{\alpha_0} \right)^{1/2} \langle 0 | i \bar{\psi} \gamma_5 \gamma_5 \psi | 1^0 \rangle
\]

is just what is usually referred to as a soft pion theorem. Here we simply note that it remains true up to \( O(\alpha_5^2 \alpha_0^2) \) corrections.

In a hopefully obvious notation the following additional equations can also be derived:

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{d}d \rangle + \langle \bar{s}s \rangle}{\langle \bar{d}d \rangle + \langle \bar{s}s \rangle} \right),
\]

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{\langle \bar{d}d \rangle + \langle \bar{s}s \rangle} \right).
\]

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{u}u \rangle + \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle + \langle \bar{s}s \rangle} \right),
\]

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle} \right),
\]

\[
\frac{\alpha_5}{\alpha_0}^{1/2} = \left( \frac{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle} \right).
\]

PCAC logarithm corrections to the \( F \)'s and the \( Z \)'s have been calculated by Langacker and Pagels [16,7] with the following numerical results:

\[
\frac{F_K}{F_0} \approx 1.21, \quad \frac{F_K}{F_0} \approx 1.28, \quad \frac{F_K}{F_0} \approx 1.14, \quad \frac{F_K}{F_0} \approx 1.28, \quad \frac{F_K}{F_0} \approx 1.14, \quad \frac{F_K}{F_0} \approx 1.28,
\]

\[
\langle \bar{u}u \rangle \approx 1.023 \frac{F_0}{F_3}, \quad \langle \bar{s}s \rangle \approx 1.033 \frac{F_0}{F_3}, \quad \langle \bar{d}d \rangle \approx 1.017 \frac{F_0}{F_3}, \quad \langle \bar{u}u \rangle \approx 1.0065 \frac{F_0}{F_3}, \quad \langle \bar{s}s \rangle \approx 1.0072 \frac{F_0}{F_3}, \quad \langle \bar{d}d \rangle \approx 1.0146 \frac{F_0}{F_3}.
\]

A few remarks are in order. The minute difference between \( F_0 \) and \( F_3 \) (and between \( Z^{1/2} \) and \( Z^{1/2} \)) will be crucial in estimating \( \alpha_5 \). The above values follow from allowing an electromagnetic correction to \( \alpha_5 \) which is roughly twice as large as the value predicted by Dashen's theorem [17]. We will return to this question later. The results for the singlet have been "extrapolated" from the calculations of Langacker and Pagels by consistency conditions such as:

\[
\left( \frac{F_0}{F_3} - 1 \right) = \frac{2}{3} \left( \frac{F_0}{F_3} - 1 \right).
\]

Eqs.(25) can now be used to express the ratios of \( \langle \bar{s}s \rangle \), \( \langle \bar{u}u \rangle \) and \( \langle \bar{d}d \rangle \) in terms of the \( F \)’s and \( Z \)’s. The result is

\[
\frac{2 \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle} = \frac{Z_{K^+} F_{K^+} + Z_{K^0} F_{K^0} - Z_{\pi^0} F_{\pi^0}}{Z_{K^0} F_{\pi^0}},
\]

\[
\frac{\langle \bar{d}d \rangle - \langle \bar{u}u \rangle}{\langle \bar{d}d \rangle + \langle \bar{u}u \rangle} = \frac{Z_{K^0} F_{K^0} - Z_{K^+} F_{K^+}}{Z_{\pi^0} F_{\pi^0}}.
\]

Using the values in (26) we arrive at the estimates

\[
\frac{2 \langle \bar{s}s \rangle}{\langle \bar{u}u \rangle + \langle \bar{d}d \rangle} \approx 1.48,
\]

\[
\frac{\langle \bar{d}d \rangle - \langle \bar{u}u \rangle}{\langle \bar{d}d \rangle + \langle \bar{u}u \rangle} \approx 0.0072,
\]

i.e.

\[
\langle \bar{s}s \rangle \approx 1.48 \langle \bar{u}u \rangle, \quad \langle \bar{d}d \rangle \approx 1.0146 \langle \bar{u}u \rangle.
\]

Due to the consistency conditions the other equations in (25) also lead to the estimates (26).

Defining

\[
\alpha_0^a = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}, \quad \alpha_0^a = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle},
\]

Eqs.(24) can be rewritten as

\[
\alpha_0^a = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}.
\]

\[
\alpha_0^a = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle}.
\]

\[
\alpha_0^a = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}.
\]

\[
\alpha_0^a = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle}.
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\]

\[
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\]

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\]

\[
\alpha_0^a = \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle}.
\]

\[
\alpha_0^a = \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle}.
\]
\[ m_{\pi_0}^2 = \left( m_i + \alpha_u^d m_z + \frac{\langle \alpha_u \rangle}{F_{\pi}} \right) \left( m_i + m_z \right) \]  

\[ m_{\pi_0}^2 = \frac{2}{3} \left( \frac{F_{\pi}}{F_{\pi}} \right)^2 \left( m_i + m_z \right) \]  

\[ m_{\pi_0}^2 = \left( m_i + m_3 \right) \left( \alpha_u^d + \frac{\langle \alpha_u \rangle}{F_{\pi}} \right) \left( m_i + m_z \right) \left( m_i + m_z \right) \]  

\[ m_{\pi_0}^2 = \frac{\sqrt{2}}{3} \left( m_i + \alpha_u^d m_z - 2 \alpha_u^d m_3 \right) \left( m_i + m_z \right) \left( m_i + m_z \right) \]  

The masses on the left-hand side of Eqs. (29a), (29d) and (29e) should be considered as the physical masses; for this reason an electromagnetic contribution has been subtracted from \( m^2_{\pi_0} \). Electromagnetic contributions to the decay constants, etc., are understood to be included.

Eqs. (29a), (29d) and (29e) can be used to solve for the quark mass ratios. The result is

\[ \frac{m_i - m_z}{m_i + m_z} = \frac{2}{3} \left( \frac{F_{\pi}}{F_{\pi}} \right)^2 \left( \frac{F_{\pi}}{F_{\pi}^0} \right)^2 \frac{m_{\pi_0}^2}{m_{\pi_0}^2} \left( \frac{m_i + m_z}{m_i + m_z} \right) \]  

Inserting the experimental values for \( m^2_{K^0}, m^2_{K^+} \) and \( m^2_{\pi_0} \) and the chiral estimates from Eqs. (26a), (26b), (26c) and (26d), together with \( (\delta m^2_{K^0})_{em} \approx 2975 \text{ MeV}^2 \) results in

\[ \frac{m_i - m_z}{m_i + m_z} \approx 0.54 \]  

\[ \frac{m_{\pi_0}^2}{m_{\pi_0}^2} \approx 30.8 \]  

These essentially correspond to the values found by Langacker and Pagels [18]. We have even made use of their value for \( (\delta m^2_{K^0})_{em} \approx 2974 \text{ MeV}^2 \) \( (\delta m^2_{K^+})_{em} \approx 3.0 \text{ MeV} \). The usual value predicted by Dashen's theorem is

\[ (\delta m^2_{K^0})_{em} = (\delta m^2_{K^+})_{em} = (\delta m^2_{\pi_0})_{em} = (\delta m^2_{\pi_0})_{em} \approx 1265 \text{ MeV}^2 \]  

or \( (\delta m^2_{K^0})_{em} \approx 1.3 \text{ MeV} \).

The value for \( \frac{m_i - m_z}{m_i + m_z} \) is fairly dependent on the precise choice of \( R \) and \( \beta \), but the estimates for \( R \) and \( \beta \) should be underestimated by about 1%.

The corresponding values for \( \frac{m_{\pi_0}^2}{m_{\pi_0}^2} \), \( \frac{m_{\pi_0}^2}{m_{\pi_0}^2} \approx 1.0 \), \( \alpha_u^d \) and \( \frac{m_{\pi_0}^2}{m_{\pi_0}^2} \) are also given.
\[
\frac{Z_{K^+}}{Z_{K^0}} \simeq 1 + \frac{\left( m_{K^+} - m_{K^0} - (8m_{K^0})_{	ext{em}} \right)}{142 \pi^2 \frac{F_π^2}{F_\pi}} \left[ \frac{1}{4} \right] \frac{G_F V}{G_F V^2}, \quad m^2 \simeq 1.7 G_F V^2
\]

have been used. The quark mass values in parentheses follow from allowing for the \(1\%\) correction mentioned in the very last footnote.

The values given in Table II should be compared to the lowest order estimates of Weinberg [19],

\[
\begin{align*}
\frac{m_3 - m_1}{m_3 + m_1} & \approx 0.287 \\
\frac{m_3}{m_3 + m_1} & \approx 0.258 \\
m_3 & \approx 150 - 200 \text{ MeV} \\
\end{align*}
\]

Substituting (33), (28c) and (28a) into (29b), (29c) and (29f)

gives

\[
\begin{align*}
\frac{m_3}{m_3 + m_1} & \approx 0.3416 \text{ GeV}^2 \\
\frac{m_3}{m_3 + m_1} & \approx 0.2889 \text{ GeV}^2 \quad \text{[WITH PCAC CORRECTIONS]} \\
\frac{m_3}{m_3 + m_1} & \approx 0.2991 \text{ GeV}^2 \\
\end{align*}
\]

These should be compared to the previous values (used in Sec.3) of

\[
\begin{align*}
\frac{m_3}{m_3 + m_1} & \approx 0.3213 \text{ GeV}^2 \\
\frac{m_3}{m_3 + m_1} & \approx 0.1701 \text{ GeV}^2 \quad \text{[WITHOUT PCAC CORRECTIONS]} \\
\frac{m_3}{m_3 + m_1} & \approx 0.2139 \text{ GeV}^2 \\
\end{align*}
\]

Because there is a substantial difference between the values in (36) and (35), it is worthwhile repeating the previous analysis. The best fit to the \(n \rightarrow \eta'\) masses is found for

\[
\xi \approx 0.51 \quad (F_{\pi} = 1.26 F_{\pi}) \quad \Rightarrow \quad 0.55 \pm 0.05 \quad (\text{exp:} 0.50) \text{ MeV}
\]

The value for \(\theta\) is however shifted to

\[
\theta \approx 14.8^0
\]

We thus conclude that a value of \(F_{\pi}\), which is nearly twice as large as the other octet of decay constants is required in obtaining a reasonable fit to the \(n-n'\) mass spectrum. Just as is the case with ordinary PCAC corrections it might be suggested that not all the large \(N\) corrections can be absorbed into a redefinition of \(F_{\pi}\). In such a case some theoretical uncertainty remains with our result. It may be hoped that by treating \(\frac{G}{F}\) analogously to the quark masses \(m_1, m_2, m_3\) we may be able to actually calculate the leading corrections. Unfortunately logarithmic dominance is not justified in this case.

A value of \(F_{\pi}\) (or \(F_{\eta'}\)) which is larger than \(F_h\) (or \(F_{\eta}\)) is not totally unreasonable. The decay constants are essentially a measure of the probability of finding a particle at the origin and should be larger the larger the mass of the particle. It should then be expected that

\[
F_\pi < F_h < F_{\eta'} < F_{\eta}
\]

Before concluding, we make a brief comment on the \(n \rightarrow 3\pi\) decays. Using \(|n\rangle = \cos \theta |\bar{\pi}\rangle + \sin \theta |\bar{\eta}\rangle\) and assuming the validity of soft octet and singlet theorems we can rewrite the expression (5) as

\[
\left( \begin{array}{c}
\frac{m_3 - m_1}{m_3 + m_1} \\
\end{array} \right) \frac{G_F^{2}}{4 F_{\pi}} \left( \begin{array}{c}
\cos \theta + \frac{G_F^{2}}{F_{\pi}} \\
\sin \theta \\
\end{array} \right) \left( \begin{array}{c}
0.54 \pm 0.05 \quad \text{MeV} \\
0.64 \pm 0.05 \quad \text{MeV} \\
\end{array} \right)
\]

The values given on the right-hand side of Eq. (36) are determined from the experimental data coming from the \(n \rightarrow 3\pi^0\) decay (upper value) and the \(n \rightarrow \pi^+ \pi^- \pi^0\) decay (lower value).

Inserting \(F_{\pi} \approx 1.28 F_{\pi} \quad \Rightarrow \quad 0 \approx 10^2\) and the values for \(m_3^2\) and \(F_{\pi}\) (\(\approx 95\) MeV) we arrive at an equation relating the ratio \(R = m_3^2/m_{\pi} m_{\eta}\) and \(\xi \approx F_{\pi}/F_{\eta}\)

\[
R (0.51 \pm 0.206) \xi = \left( \begin{array}{c}
0.47 \pm 0.05 \\
0.55 \pm 0.05 \\
\end{array} \right)
\]

(*. The Weinberg values follow from Eqs.(30) and (31) with

\[
F_h = F_h \quad (4m_K^2 = 1256 \text{ MeV}^2, \quad a^d = a^u = 1, \quad F_{\eta} = F_{\eta}^*).
\]

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Eq. (37) has been plotted in Fig. 1. The upper solid curve comes from the $\eta = \sqrt{3}, \sqrt{3}$ data and the lower solid curve from the $\eta = 3\sqrt{3}$ data. The dotted curves represent the experimental bounds. The following fairly inconclusive conclusions are drawn:

1) $R < 0.73$. This means $\frac{n_0}{\bar{n}_1} < 5.7$ (or $\bar{n}_1 > 0.176 n_0$).

2) $R \simeq 0.56$ (the best estimate we have [18]) implies $\xi < 1.05$. This at least confirms the compatibility of our result of $\xi \simeq 0.5$ and the Langacker-Fagles estimate of $R$ (Eq. (38)).

3) $\xi \simeq 0.5$ implies $0.45 \lesssim R \lesssim 0.64$. It is interesting to note that this corresponds to a value of $\left(\frac{\Delta m}{m}\right)^2$ in the range

$$1650 \text{ MeV}^2 \lesssim \left(\frac{\Delta m}{m}\right)^2 \lesssim 4100 \text{ MeV}^2$$

The value coming from a use of Dashen's theorem is just outside this range. Credence is given to the value anticipated by Langacker and Pagels [18, 20].

ACKNOWLEDGMENTS

The author takes much pleasure in thanking Professors C.H. Llewellyn-Smith and R.J. Creether for many interesting discussions. Thanks also to Professor G. Furian for reading the manuscript and his comments. The author would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, and Professor R.H. Dalitz for hospitality at the Department of Theoretical Physics, Oxford.

* On the basis of the crude approximations used in obtaining (37) and the experimental uncertainties involved.

REFERENCES

[8] For a review of POMC see [18].
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**Table II**

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**Table I**

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