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HADRON SPECTROSCOPY IN LATTICE QCD

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ABSTRACT

A Monte Carlo computation of meson and hadron spectroscopy within lattice QCD is made. We give a detailed discussion of the statistical and systematic errors of the results and analyze the present limitations of our approach. The results are in agreement with the observed spectrum. We also estimate the values of up, down and strange quark masses.

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1. - INTRODUCTION

After the first promising results of computer simulations of hadronic states in the quenched approximation¹⁾ (no fermionic vacuum polarization diagrams) we have started a more detailed analysis in order to have both the systematic and statistical errors under control.

The general pattern of this kind of computation consists of first finding the quark propagator in the presence of an external gauge field and building up from it various species of operator-operator correlations. In this paper we will not deal with all possible correlations which can be formed: we have focused our attention on the masses of the lightest pseudoscalar and vector mesons and of the lowest lying states of the spin 1/2 octet and of the spin 3/2 decuplet of both intrinsic parities.

The purpose of this paper is twofold: on the one hand we want to see if there is agreement, within errors, between our results and the experimental values; on the other hand we aim to clarify the data analysis procedure and the role of systematics and volume effects which limit the accuracy of the results. We believe that the experience gained by this work will be useful for future developments of this kind of computations.

The paper is organized as follows. Section 2 contains the basic notations and formulae; in Section 3 we discuss the method of analysis of the data. The results are presented in Section 4 and concluding remarks in Section 5.

2. - BASIC FORMULAE AND NOTATIONS

In this Section we present the basic formalism we used to evaluate correlation functions and hadron masses on the lattice.

In the Wilson formulation²⁾, the action for interacting quarks and gluons on a Euclidean lattice is given by

$$S = \sum_f \bar{\psi}^f \Delta(U, k_f) \psi^f + S_{\text{gluon}}(U) =$$

$$\sum_f \sum_x \left\{ \sum_{\hat{\mu}} \left[k_f \bar{\psi}^f(x) (1 - \gamma_{\hat{\mu}}) U_{\hat{\mu}}(x) \psi^f(x + \hat{\mu}) + \right. \right.$$

$$\left. \left. + k_f \bar{\psi}^f(x + \hat{\mu}) (1 + \gamma_{\hat{\mu}}) U_{\hat{\mu}}^{\dagger}(x) \psi^f(x) \right] - \bar{\psi}^f(x) \psi^f(x) \right\} +$$
(2.1)

$$+ \beta/6 \sum_{\text{Plaquettes}} \text{Tr} [U_P + U_P^\dagger] \quad (2.1) \text{ cont.}$$

All colour and spinor indices have been omitted for simplicity. The quantity $\psi_\alpha^{f,A}(x)$ represents the quark field with flavour f , colour A and spinor index α ; $(\gamma_{\alpha\beta}^\mu)$ are Dirac matrices in Euclidean space-time defined as

$$\vec{\gamma} = -i\vec{\gamma}_{BD} ; \gamma_0 = -\gamma_{0BD} \quad (\gamma_\mu^\dagger = \gamma_\mu)$$

where the subscript BD indicates the basis defined in Ref. 3), $U_\mu^{AB}(x)$ is the $SU(3)$ gauge field defined on the link with direction μ and U_P is the product of gauge fields belonging to the elementary plaquette P . The quantity β is related to the inverse of the lattice bare coupling constant g_0 : $\beta = 6/g_0^2$. In the limit where the lattice spacing $a \rightarrow 0$, the action (2.1) describes the interactions of n_f coloured quarks, where f denotes the type of flavour ($u, d, s, \dots$). The action we used avoids the problem of fermion doubling⁴⁾ but contains an explicit breaking of the chiral symmetry. In the free case, for $a \rightarrow 0$ one gets $K_f \rightarrow 1/8$ and correspondingly $m_f = (1-8K)/2a \rightarrow 0$ (m_f is the bare quark mass for the flavour f): the chiral symmetry is restored. We expect that also in the interacting case, in the limit $a \rightarrow 0$, there will be a value of K_f for which chiral symmetry is restored, corresponding to a zero physical mass m_f^{phys} for the quark. However, if dynamical spontaneous symmetry occurs, for $m_f^{\text{phys}} = 0$ we should observe a Goldstone boson in the spectrum, which is in fact what happens.

We are interested in computing from the action (2.1) vacuum expectation values of operators containing quark and gluon fields. Since the action is quadratic in the fermionic fields, these can be formally integrated out.

$$\begin{aligned} \langle \hat{O}(\psi, \bar{\psi}, U) \rangle &\equiv \frac{\int d[U] d[\psi] d[\bar{\psi}] \exp -S \, O(\psi, \bar{\psi}, U)}{\int d[U] d[\psi] d[\bar{\psi}] \exp -S} \\ &= \frac{\int d[U] [\Pi_f \det(\Delta(U, K_f))] \exp -S_G(U) \, \tilde{O}(U)}{\mathcal{Z}} \end{aligned} \quad (2.2)$$

If $\Pi_f \det(\Delta(U, K_f))$ has a definite sign (which definitely happens for two equal mass flavours) one can write

$$\pi_f \det [\Delta(U, K_f)] \cdot \exp -S_G(U) \rightarrow \exp -S_G(U) + \sum_f \text{Tr} [K_f |\Delta(U, K_f)|] \quad (2.3)$$

$$\equiv \exp -S_{\text{eff}}(U)$$

The problem is then to compute, with standard Monte Carlo methods, the expectation values of functions depending on the gauge fields U only, but with an effective action $S_{\text{eff}}(U)$. In a Monte Carlo simulation the functional integral over U variables in Eq. (2.2) is replaced by a sum over independent link configurations extracted by the measure $\exp -S_{\text{eff}}(U) d[U]$ using the standard Metropolis algorithm.

The quantity $\det[\Delta(U, K_f)]$ represents the feedback of closed quark loops on the gluon Green functions as shown in Fig. 1. The computation of $\det[\Delta(U, K_f)]$ at each Monte Carlo step is very time (or computer memory) consuming because it requires the evaluation of a non-local quantity. For the purposes of this paper, we will make the approximation $\det[\Delta(U, K_f)] = 1$. If the number of flavours n_f is small compared to N_c , this is a reasonable approximation: it gives an exact Zweig rule and hadrons made of valence quarks and gluons only, which is not so far from the real world. The result becomes exact when the number of colours N_c tends to infinity at fixed n_f ($n_f/N_c \rightarrow 0$). Moreover, a recent paper based on the hopping parameter expansion⁵⁾ shows that the inclusion of quark loops (with $n_f = 3$) only produces small corrections to the relevant physical quantities. It has been argued¹⁾ that the effect of including quark loops roughly amounts to a shift of the value of β .

The hadron masses are computed by measuring the expectation values of correlation functions for operators carrying the same quantum numbers of the hadrons. A possible operator corresponding to a meson, is for example,

$$M(x) = \sum_{f_1, f_2} C_{f_1 f_2} \bar{\psi}_{f_1, A}(x) \Gamma \psi_{f_2, A}(x) \quad (2.4)$$

where Γ is one of the 16 Dirac matrices. $C_{f_1 f_2}$ are the appropriate Clebsch-Gordan coefficients and the sum over colour indices A is performed. For a spin 3/2 baryon one can write^{6)*)}

*) Note that the field $B_{\mu, \sigma}(x)$ also contains a spin 1/2 state.

$$B_{\mu,\delta}(x) = \sum_{f_1 f_2 f_3} C_{f_1 f_2 f_3} [\psi_{f_1}^{f_1,A}(x) \mathbb{C} \gamma_\mu \psi_{f_2}^{f_2,B}(x)] \psi_{f_3}^{f_3,C}(x) \epsilon_{ABC} \quad (2.5)$$

where ϵ_{ABC} is the colour antisymmetric tensor and $\mathbb{C}$ is the charge conjugation matrix.

Finally, for a spin 1/2 baryon

$$B_\delta(x) = \sum_{f_1 f_2 f_3} C_{f_1 f_2 f_3} [\psi_{f_1}^{f_1,A}(x) \mathbb{C} \gamma_5 \psi_{f_2}^{f_2,B}(x)] \psi_{f_3}^{f_3,C}(x) \epsilon_{ABC} \quad (2.6)$$

Note that B_δ is antisymmetric for the exchange $f_1 \leftrightarrow f_2$. Using Eqs. (2.4) - (2.6) we write

$$\begin{aligned} \pi^+(x) &= \bar{u}^A(x) \gamma_5 d^A(x) \\ \rho_\mu^+(x) &= \bar{u}^A(x) \gamma_\mu d^A(x) \\ P_\delta(x) &= [u^A(x) \mathbb{C} \gamma_5 d^B(x)] u_\delta^C(x) \epsilon_{ABC} \\ \Delta_{\mu,\delta}^{++}(x) &= [u^A(x) \mathbb{C} \gamma_\mu u^B(x)] u_\delta^C(x) \epsilon_{ABC} \end{aligned} \quad (2.7)$$

Starting from the meson and baryon operators, it is straightforward to compute their Green functions in the quenched approximation. In the case of a meson

$$\begin{aligned} G(x) &\equiv \langle M(x) M(0)^+ \rangle = \\ &= \frac{1}{Z} \sum_{f_1 \dots f_4} C_{f_1 f_2}^* C_{f_3 f_4}^* \int d[U] d[\psi] d[\bar{\psi}] \exp -S \cdot \\ &\quad \cdot \bar{\psi}_{f_1}^{f_1,A}(x) \Gamma \psi_{f_2}^{f_2,A}(x) \bar{\psi}_{f_3}^{f_3,B}(0) \Gamma^+ \psi_{f_4}^{f_4,B}(0) \\ &= \frac{1}{Z} \sum_{f_1 f_2} |C_{f_1 f_2}|^2 \int d[U] \{ \pi_f \det [\Delta(U, k_f)] \} \exp -S_q(U) \\ &\quad \cdot \text{Tr} [G^{f_1}(0, x) \Gamma G^{f_2}(x, 0) \Gamma^+] \\ &\rightarrow \frac{1}{Z} \sum_{f_1 f_2} |C_{f_1 f_2}|^2 \int d[U] \exp -S_q(U) \text{Tr} [G^{f_1}(0, x) \Gamma G^{f_2}(x, 0) \Gamma^+] \end{aligned} \quad (2.8)$$

where

$$G_{\alpha\beta}^{f,AB}(x,0) = \Delta^{-1}(v, k_f, x, 0)_{\alpha\beta}^{AB} \quad (2.9)$$

and

$$G_{\alpha\beta}^{AB}(x,0) = [\gamma^5 G^{BA}(0,x)^* \gamma^5]_{\beta\alpha}$$

Notice that in the last term of Eq. (2.8) the contribution of quark loops has been dropped. At the lowest order in n_f/N_c and for flavour singlet mesons, we neglected a term proportional to

$$\text{Tr}[G^f(x,x)\Gamma] \cdot \text{Tr}[G^f(0,0)\Gamma^+] \quad (2.10)$$

corresponding to the diagram shown in Fig. 2.

In this approximation the pion and the η' are degenerate. It is interesting to note that even in the quenched case it is possible to compute the mass difference between π and η' because the diagram of Fig. 2 is the first non-trivial diagram responsible for the mass difference.

In a perfect analogy with mesons we can compute the baryon Green functions. The knowledge of the propagator gives us the mass of the corresponding particle. In fact, if there is only one pole corresponding to some stable meson, its Green function, integrated over the space hyperplane, behaves as

$$G(t) \equiv \sum_{\vec{x}} G(\vec{x},t) \sim \exp -mt \quad (2.11)$$

which is the Euclidean propagator for a scalar particle at rest. On a periodic lattice with period T in the time direction, one finds

$$G(t) \sim \cos k [m(t - \frac{T}{2})] + \dots \quad (2.12)$$

where the dots indicate terms which are down by at least a factor $\exp -mT$. In the case of a baryon, the most general form of the propagator is

$$G(t) = (1-\gamma_0) \{ \Theta(t) c_+ \exp[-m_+ t] + \Theta(-t) c_- \exp m_- t \} \\ + (1+\gamma_0) \{ \Theta(-t) c_+ \exp m_+ t + \Theta(t) c_- \exp[-m_- t] \} \quad (2.13)$$

which, with periodic boundary conditions, becomes

$$G(t) = (1-\gamma_0) [C_+ \exp -m_+ t + C_- \exp -m_- (T-t)] + (1+\gamma_0) [C_+ \exp -m_+ (T-t) + C_- \exp -m_- t] \quad (2.14)$$

$m_{\pm}$ corresponds to particles with opposite intrinsic parity [e.g., the proton and the $N^*(1535)$].

By fitting the meson and baryon propagators with the expressions (2.12) and (2.14), one measures the masses in units of the lattice spacing. In general, we expect that more than only one [Eq. (2.12)] or two [Eq. (2.14)] particle corresponding to the lowest lying states will contribute to the propagator. In fact we can have other contributions due to higher mass particles or many particle states. In practical cases, the best one can do in order to give an estimate of the systematic error on the mass induced by these effects is to parametrize the propagator by adding new terms of the same form as in Eq. (2.12) and (2.14). For example, in the case of the meson propagator

$$G(t) = C \cosh m(t - \frac{T}{2}) + C' \cosh m'(t - \frac{T}{2}) \quad (2.15)$$

The relevance of these corrections will be discussed in the section on numerical results (Section 5). By neglecting them, one systematically increases the value of the lowest lying state mass. At large distances ($t \rightarrow \infty$) these effects are expected to become completely negligible [$m' > m$ in Eq. (2.15)].

In order to give predictions in physical units, after the determination of the hadron masses in units of the lattice spacing, one has to fix a fundamental hadronic scale and a mass parameter (in our case K_f) for each quark flavour. We decided to set the scale and the masses of the quarks by fixing the squared mass difference between the ρ and the $\pi(m_\rho^2 - m_\pi^2)$ and the mass of the lightest pseudoscalar states containing one quark of a certain flavour ($m_\pi, m_K, \dots$)*). In the Wilson formulation, the bare mass of the quark can be defined as

$$\exp(m_f^B a) - 1 = \frac{1}{2} \left(\frac{1}{K_f} - \frac{1}{K_{crit}} \right) \quad (2.16)$$

K_{crit} is the value of K for which the pseudoscalar meson becomes massless. The value of the lattice coupling constant g_0 is another free parameter as long as the renormalization group equation relating the lattice spacing and Λ_{MOM} is not checked:

*) We work in the approximation of an exact isospin symmetry.

$$\Lambda_{\text{MOM}} \simeq \frac{\pi}{a} \left[\frac{8\pi^2}{33} (\beta - 2.75) \right]^{54/121} \exp \left[-\frac{4\pi^2}{33} (\beta - 2.75) \right] \quad (2.17)$$

Λ_{MOM} is a fixed physical quantity which can be extracted from the experiments. The relation (2.17) ensures that the lattice spacing $a \rightarrow 0$ in physical units when $\beta \rightarrow \infty$. The scaling behaviour predicted by Eq. (2.17) has been measured in several Monte Carlo experiments⁷⁾; in this paper we will work at a fixed value of $\beta = 6$.

3. - THE EXPERIMENT AND THE ANALYSIS OF THE DATA

We have carried out a Monte Carlo experiment using a $5^3 \times 10$ lattice obtained by duplicating the gauge fields in the time direction. We computed the quark propagator at $\beta = 6$ and $K = 0.130, 0.145, 0.1475, 0.150, 0.1525$ for 32 gauge field configurations using the Gauss-Seidel algorithm. The average number of iterations required to obtain a 1% accuracy for the quark propagator at any distance, ranges from 20 ($K=0.130$) to 100 ($K=0.1525$). In the case of $K = 0.1525$, for some configurations, it was not possible to make the Gauss-Seidel method converge with any number of iterations.

Each link configuration was obtained by the previous one by performing 100 Monte Carlo sweeps ($\times 10$ upgradings of the same link) after an initial thermalization of the system of 500 sweeps. We found a long-range correlation of the order of $400 \div 500$ sweeps, between successive link configurations for the value of hadron masses. This is illustrated in Fig. 3 where we report on the mass squared of the ρ extracted by a single configuration at $K = 0.1475$ as a function of the number of Monte Carlo sweeps^{*)}. The knowledge of the correlation between different link configurations avoids underestimating the size of statistical errors. In general, with a given number of configurations, one can reduce the statistical errors by increasing the number of Monte Carlo sweeps between two successive configurations. Most of the total computer time used (about 90 hrs. on a CDC 7600) was spent on computing the quark propagator ($\sim 4/5$ of the total).

^{*)} To generate SU(3) configuration and to compute the quark propagator, we have basically used the programme written by H. Hamber¹⁾.

The estimate of the Green function of some operators like those of Eq. (2.7) is given by the average of the corresponding correlators functions over the gauge fields configurations for which we have computed the quark propagator. In Figs. 4 and 5 we show the results for the Green function of the "pion" and "proton" operator obtained by the average over the full set of configurations at $K = 0.1475$.

We fit the curves in the figures with expressions like those of Eqs. (2.12), (2.13) and (2.15) in order to derive the experimental values of hadron masses. The estimate of statistical errors is obtained by dividing the full set of configurations into various clusters and by computing the average masses and their dispersions over the clusters. Each of these clusters must be large enough to be considered a good estimate of the functional integral. In practice, because of the existing correlations between the different configurations, the number of such clusters ranges between one and four.

Given the small number of clusters, a better estimate of the average value of the masses is obtained by averaging over different partitions of the configurations at a fixed number of clusters. Of course, in this case the error is given by the average error. In Fig. 6 we plot the results for the proton, Δ^{++}, ρ and pion-like masses as a function of K averaged over two clusters of 16 configurations with the corresponding errors. The values of ρ and pion masses are very stable against variations of the number of clusters as is reported in Table 1. For the proton and the Δ^{++} the fluctuations among different clusterizations are larger, but still within the estimated statistical errors.

Following the discussion in Section 2, we set the mass scales for the lightest (u,d) quarks by fixing the pion and ρ masses at a given β by the equations

$$m_\rho^2 - m_\pi^2 = \frac{1}{a^2} F(K) \quad (3.1)$$

$$m_\pi^2 = \frac{1}{a^2} f(K)$$

where the functions F and f are experimentally measured. The value one obtains by solving these equations is $K \sim 0.16$ which is rather far from the accessible region: we cannot go to values of K larger than ~ 0.1525 because of volume effects due to nearly massless excitations^{*)}. Already at $K = 0.1525$ we note a clear set up of these volume effects, as one can see from Fig. 6, especially for the proton and the Δ^{++} . In the rest of our analysis this point will be neglected, also because of the bad convergence on the Gauss-Siedel method.

*) The points of Ref. 1) for $K = 0.1525$ and 0.155 are affected by the same convergence problems of the Gauss-Siedel method; see also Ref. 8).

It is clear that to get the physical values of the masses of hadrons made of light quarks ($K_f = K_{u,d}$) we are forced to extrapolate far away from the experimental points. The best we can do with the available data is to make a linear fit to the masses of the form:

$$\begin{aligned} m_M^2 a^2 &= A_M \frac{1}{K} + B_M \\ m_B a &= A_B \frac{1}{K} + B_B \end{aligned} \quad (3.2)$$

K is the value of the Wilson parameter taken to be equal for all the quarks contained in a meson/baryon and the subscripts M and B denote mesons and baryons respectively. Starting from the points and the errors given in Fig. 6, the standard error propagation theory would produce errors for the values of physical light hadron masses of the order of hundred per cent. This happens if one does not take into account the strong correlation existing, for a given cluster, between the slope and the intercept of the linear fit as can be seen from Fig. 7 where, for the squared pion mass, the behaviour of three clusters is separately displayed. The procedure which minimizes the statistical errors entails first making the fit of Eqs. (3.2) cluster by cluster, computing the masses for each cluster at $K = K_{u,d}$ and then averaging the masses over the clusters. In order to apply such a procedure, the same configuration set must be used for all values of K . There are quantities, such as the ratio of the baryon $(3/2)^+$ over the corresponding vector meson, which are found to be almost independent of the mass of the quark; for them we can directly extract the physical values from our data without any extrapolation. We used this method as an independent estimate of the p , Δ^{++} and Ω^- masses.

4. - RESULTS

In this section we present our numerical results. All the results refer to a subdivision into two clusters of the configuration set.

In Table 2 we give the values of A_M , B_M , A_B , B_B for the pseudoscalar and the vector mesons and for the $(3/2)^+$ and $(1/2)^+$ baryons. In the same table we give the value of K_{crit} . In the hypothesis of a linear dependence of the meson mass near K_{crit} , even in the case where they are different, Eq. (3.2) can be generalized as follows:

$$m_M^2 a^2 = \frac{A_M}{2} \left[\frac{1}{K_{f_1}} + \frac{1}{K_{f_2}} \right] + B_M \quad (4.1)$$

where K_{f_1}, K_{f_2} refer to two different flavours.

By fixing the values of ρ , π and κ one can get the value of Λ_{lattice} , m_p , $m_{\Delta^{++}}$, m_{N^*} , $m_{\Delta(3/2^+)}$, m_{Ω^-} , m_{K^*} , m_ϕ and the bare quark masses [see Eq. (2.16)]. In lattice units we get

$$\begin{aligned} m_p a &= 0.89 \pm 0.31 \\ m_{\Delta(3/2^+)} a &= 0.96 \pm 0.41 \\ m_{\Omega^-} a &= 1.18 \pm 0.38 \\ m_{K^*}^2 a^2 &= 0.39 \pm 0.06 \\ m_\phi^2 a^2 &= 0.48 \pm 0.05 \end{aligned} \quad (4.2)$$

and for the bare quark masses m_f^B *)

$$\begin{aligned} m_{u,d}^B a &= (3.2 \pm 0.4) * 10^{-3} \\ m_s^B a &= (8.17 \pm 0.50) * 10^{-2} \end{aligned}$$

by using the central values of the results reported above, we get

$$\begin{aligned} \frac{m_\phi^2 - m_\pi^2}{2(m_K^2 - m_\pi^2)} &= 0.82 \quad (\text{exp} \sim 0.96) \\ \frac{m_\Omega^2 - m_\Delta^2}{3(m_K^2 - m_\pi^2)} &= 2.01 \quad (\text{exp} \sim 1.89) \end{aligned} \quad (4.3)$$

As we have already pointed out in the previous section, it is possible to give an independent estimate of baryon masses by making their ratio with the vector meson mass under the verified hypothesis of K independence. In this case we get

$$\frac{m_p}{m_\phi} = 1.46 \pm 0.23 \quad (\text{exp} \sim 1.20) \quad (4.4)$$

$$\frac{m_{N^*}}{m_\phi} = 2.41 \pm 0.26 \quad (\text{exp} \sim 1.97)$$

from which follows $\frac{m_{N^*}}{m_p} = 1.65 \quad (\text{exp} \sim 1.65)$

$$\frac{m_{\Delta(3/2^+)}}{m_\phi} = 1.55 \pm 0.24 \quad (\text{exp} \sim 1.58)$$

*) Using the relation between the bare mass on the lattice and the renormalization group invariant mass ($\hat{m}$) in the continuum, one finds⁸⁾: $\hat{m}_{u,d} = (9.3 \pm 1.2)$; $\hat{m}_s = (239 \pm 14)$ MeV.

$$\frac{m_{\Delta(3/2^-)}}{m_\pi} = 2.44 \pm 0.14 \quad (\text{exp} \sim 2.14) \quad (4.4) \text{ cont.}$$

from which follows $\frac{m_{\Delta(3/2^-)}}{m_{\Delta(3/2^+)}} = 1.57 \quad (\text{exp} \sim 1.36)$

$$\frac{m_{\Omega^-}}{m_\phi} = 1.60 \pm 0.25 \quad (\text{exp} \sim 1.64)$$

and $a^2 [m_{\Delta(3/2^+)}^2 - m_P^2] = 0.25 \pm 0.11$

With this method we get better results for the baryon masses. In particular, the ratio m_N^*/m_P turns out to be the right one in spite of the fact that the absolute values are rather large. This is also the only way we can give a reasonable error to the mass difference between the proton and the Δ^{++} . The translation of the results in to physical units is given in Tables 3 and 4. The inverse of the lattice spacing a turns out to be

$$a^{-1} = [1.42 \pm 0.12] \text{ GeV} \quad (4.5)$$

corresponding to a value for $\Lambda_{\overline{\text{MS}}} = 0.075 \pm 0.008 \text{ GeV}$ in reasonable agreement with the results from deep inelastic scattering.

The results which have been presented have been obtained by fitting the meson propagator by Eq. (2.12) using only the five central points in the plot in Fig. 4. Using Eq. (2.15) and all the points (except that at zero distances) one obtains masses shifted downwards by less than 13%. This is an estimate of the systematic error for meson masses and for the value of Λ . For the baryons, a similar analysis has been made: in this case it is more difficult to disentangle the proton contribution to the propagator because, besides the effects of higher excitations, there is a contamination of the opposite parity hadron. All these effects are reduced by using larger lattices. In fact, as can be seen from the fit to the "proton" propagator in Fig. 5, it is only at a distance of about five lattice spacings that the proton pole stems out. This kind of effect can partially account for the systematically large values we obtained for baryons and for the larger statistical errors. We also computed the masses of

the mesons $\delta(980)$, A_1 and $B(1235)$. We observe that for small values of K there is a clear splitting between these states and the pion and the ρ , which decreases very quickly for increasing K . For $K = K_{u,d}$ we would extrapolate for these states a value for the mass smaller than the ρ mass. The reason for this behaviour is not clear to us: it could be connected to the fact that the operators we used to estimate their masses are zero in the non-relativistic limit. Another possible reason is that our lattice is too coarse-grained to resolve the structure of more extended objects ($L = 1$ states).

5. - CONCLUDING REMARKS

From the analysis of the results of our Monte Carlo experiment we learned the following lesson. First of all, the statistics required to obtain reasonable errors for the masses is rather large and many configurations are needed for a precision of the order of the percentage. Furthermore, all the difficulties one encounters are connected with volume effects; i) the strong correlation between different configurations may be due to finite temperature effects (or to the existence of metastable states); ii) the anomalous behaviour of the baryon masses for large values of K which forbids us to go close to the critical value of K ; iii) the contamination in the determination of the masses of the lowest lying states due to higher excited states (remember that the effect is stronger for baryons). We finally observe that an efficient and fast algorithm for computing the quark propagator is needed: in fact most of the computer time was spent on the propagator and, close to K_{crit} , it was impossible to get a sensible result with the Gauss-Seidel method.

In spite of all the difficulties mentioned, the results are in agreement, within errors, with the observed pattern of hadron spectroscopy and confirm the validity of the lattice technique for the investigation of the hadronic world in the low-energy regime.

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K	$(m_{\pi}^2 \pm \delta m_{\pi}^2) \cdot a^2$			$(m_{\rho}^2 \pm \delta m_{\rho}^2) \cdot a^2$		
	$N_C = 1$	$N_C = 2$	$N_C = 3$	$N_C = 1$	$N_C = 2$	$N_C = 3$
0.1450	0.96	0.97 ± 0.07	0.98 ± 0.07	1.06	1.07 ± 0.08	1.09 ± 0.07
0.1475	0.77	0.78 ± 0.07	0.79 ± 0.07	0.89	0.91 ± 0.08	0.92 ± 0.07
0.1500	0.62	0.64 ± 0.08	0.65 ± 0.07	0.79	0.81 ± 0.08	0.82 ± 0.08

Table 1

K_{crit}	A_{PS}	B_{PS}	A_V	B_V	$A_{\frac{3}{2}^+}$	$B_{\frac{3}{2}^+}$	$A_{\frac{1}{2}^+}$	$B_{\frac{1}{2}^+}$
0.1590 ± 0.0010	1.22 ± 0.09	-7.6 ± 0.6	1.01 ± 0.07	-6.1 ± 0.5	1.01 ± 0.36	-5.4 ± 2.7	0.98 ± 0.24	-5.2 ± 1.8

Table 2

m_p	$m_{\Delta(\frac{3}{2}^+)}$	m_{Ω^-}	m_K^*	m_ϕ	$m_{u,d}^B$	m_S^B
1.27 ± 0.44	1.37 ± 0.60	1.68 ± 0.50	0.89 ± 0.07	0.99 ± 0.05	$(4.5 \pm 0.6) 10^{-3}$	0.116 ± 0.007

Table 3

m_p	m_{N^*}	$m_{\Delta(\frac{3}{2}^+)}$	$m_{\Delta(\frac{3}{2}^-)}$	m_{Ω^-}	$m_{\Delta(\frac{3}{2}^+)-m_p}$
1.14 ± 0.18	1.88 ± 0.20	1.21 ± 0.19	1.90 ± 0.11	1.58 ± 0.24	0.22 ± 0.09

Table 4

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FIGURE CAPTIONS

- Figure 1 : Example of a typical diagram representing the feedback of closed quark loops on gluon Green functions.
- Figure 2 : Example of diagram contributing, even in the quenched approximation, to the $\eta' - \pi$ mass difference.
- Figure 3 : The value of m_ρ^2 in lattice units is plotted at $K = 0.1475$ as a function of the number of sweeps made on the gauge configurations.
- Figure 4 : The crosses on the data point to the pion propagator as a function of time. The full curve represents the fit of Eq. (2.12) to the five central points and the dashed curved the fit of Eq. (2.15).
- Figure 5 : The crosses for $t \leq 5$ refer to the experimental data for the "proton" propagator as a function of time. The corresponding curve is obtained with the fit of Eq. (2.14). For $t \geq 6$ the scale is on the right-hand side: the corresponding curve is obtained with the fit of Eq. (2.14) and represents the opposite intrinsic parity hadron propagating backwards in time.

Figure 6 : The experimental points and their linear fit (in $1/K$ for pion, ρ , proton and Δ_{++} states. The point at $K = 0.1525$ has been plotted for completeness but not included in the fit.

Figure 7 : The experimental values of m_{π}^2 in lattice units for three different clusters of configurations as a function of K . The full lines are only to guide the eyes.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig.6

Fig. 7