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R Marshall

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THE ELECTROWEAK INTERACTION IN e^+e^- ANNIHILATIONS

K Marshall
Rutherford Appleton Laboratory
Chilton
Didcot
Oxon, OX11 0QX
England

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1. Introduction

The role of e^+e^- annihilation in studying the weak neutral current has taken on an increased significance during the last two or three years due to the operation of PETRA and PEP which provide values of α_s (C. M. energy²) in excess of 1000 GeV². Although it was clear from the outset that several years would be needed to collect enough data to make an accurate determination of the neutral current parameters, it was not long after PETRA had started operation that it was realised how even a modest amount of data could put tight limits on some of the lepton couplings [1]. The data also restricted the scope of possible extended gauge models.

The purpose of these lectures is to provide a general description of electroweak effects in e^+e^- annihilations guided perhaps in places by my own prejudices. The relationship with other areas of study, like neutrino polarised electron scattering will also be considered.

The amount of data is now quite substantial and it is possible to distinguish between problems caused by statistics or lack of energy, which time will solve, and problems caused by fundamental obstacles, which may survive until LEP experiments, or even longer.

The first theoretical discussion of possible weak effects in e^+e^- processes was made by Cabibbo and Gatto [2] in 1961. This was at the time when e^+e^- colliding beams were being proposed at Frascati and Stanford, proposals which led to the birth of the e^+e^- era. Now over 20 years later, we are about to see an e^+e^- machine take over the leading role in experimental particle physics research in Europe.

Although the work of Cabibbo and Gatto contains much that is still relevant and is well worth reading today, a more comprehensive treatment is contained in the paper by Ellis and Gaillard [3].

The main area of interest at present accelerators (PEP and PETRA) is in the production of a fermion-antifermion pair via s-channel e^+e^- annihilation:

where the process is mediated by photon or neutral weak (Z^0) boson exchange. To first order (single photon or single weak gauge boson exchange) the relative strengths of the two contributions are obtained from the respective constants α and G_F and the e^+e^- energy. Single photon exchange has a $\frac{1}{s}$ dependence whereas Z^0 exchange rises linearly with s (neglecting propagator effects). The γ - Z^0 interference term is therefore constant with beam energy; it is however swamped at small s by the γ term and overwhelmed at large s by the pure weak term. In between, there is an energy region where the interference term alone provides information about the weak process. Moreover, some of the information (eg the sign of the weak charges) can only be obtained in the interference region. This region starts at the present PETRA energies and ends at an energy determined by the weak gauge boson mass.

2. Classification of fermions and their electroweak couplings

We start by assuming that the fermion neutral current coupling has a "Fermi type" vector component (V) and a "Gamow-Teller type" axial vector component (A).

The neutrino is believed to have V-A coupling since right handed neutrinos decouple, ie $(1-\gamma^5)$ only; the couplings of fermions with mass has to be established by experiment. Let us now consider the currently known "elementary" fermions, including the t quark which has not yet been observed but which is nevertheless suspected to exist. This gives a list of 12 which we can arrange according to electric charge and mass (or

association in the case of ν) into three families or generations (I, II and III):

Table 1

Q_f	I	II	III
0	ν_e	ν_μ	ν_τ
-1	e	μ	τ
$-\frac{1}{3}$	d	s	b
$\frac{2}{3}$	u	c	t

We therefore have in general 24 possible different coupling constants (real numbers only if T invariance holds) if we admit a different V and A coupling for each fermion. I don't consider for the moment additional parameters like the mass of the gauge field or the mass of any associated scalar fields (eg Higgs). This maximal number of 24 can be successively reduced by a series of assumptions thus:

1. All the neutrinos are massless, eg V-A defined for ν . This reduces the number to 21.
2. Generation universality. By assuming that each fermion in generation I has a counterpart in generation II and III with the same electric and weak charges. The number of independent numbers is now reduced by a factor 3 to 7.
3. If the gauge group is assumed to be $SU(2)_U \times U(1)$, spontaneously broken with the simplest Higgs structure, then the 7 independent parameters are reduced to only one, namely $\sin^2 \theta_w$, which defines all the couplings and at the same time the mass of the gauge boson. The mass of the Higgs field would still remain undetermined. The weak charges which arise from this maximal assumption ie the standard $SU(2)_U \times U(1)$ model of Glashow, Weinberg and Salam can now be listed as follows.

Table 2

	Electric charge	Weak axial charge	Weak vector charge
	Q_f	a	v
ν_e, ν_μ, ν_τ	0	1	1
e, μ , τ	-1	-1	$-1 + 4 \sin^2 \theta_w$
d, s, b	$-\frac{1}{3}$	-1	$-1 + \frac{4}{3} \sin^2 \theta_w$
u, c, t	$+\frac{2}{3}$	1	$1 - \frac{8}{3} \sin^2 \theta_w$

At this stage, we can note that there are certain "special" values of $\sin^2 \theta_w$ which lead to a symmetric array of the electroweak charges (Q_f, v_f, a_f). In fact, the currently accepted value for $\sin^2 \theta_w$, namely 0.23, is rather close to one of these "special" values eg ($\sin^2 \theta_w = \frac{1}{4}$)

Table 3

	Q_f	a_f	v_f
$\nu \dots$	0	1	1
e, μ , τ	-1	-1	0
d, s, b	$-\frac{1}{3}$	-1	$-\frac{2}{3}$
u, c, t	$\frac{2}{3}$	1	$\frac{1}{3}$

suggesting (perhaps) an underlying $SU(3)$ symmetry or sub-structure of even more fundamental particles with EW charges 0 and $\pm \frac{1}{3}$.

Note that there is an alternative convention of defining the coupling constants, eg for electrons, $g_v = \frac{1}{2}v$ and $g_a = \frac{1}{2}a$.

The range of possible numbers of independent parameters, from 24 down to one, represents a scale of options for the physicist. The experimentalist might wish to go to one end of the scale and measure as many of the 24 parameters as possible. This would among other things be a direct check of generation universality. At the other end of the scale, one would hope

for an economic theory and if all 24 parameters can be derived from the single number $\sin^2 \theta_w$, this would be a successful theoretical outcome.

However, e^+e^- processes at present accelerator energies are influenced by two additional factors, namely the value of Q^2 ($=s$) is higher than ever before and heavy quarks are produced democratically (no sea/valence suppression). This means that it could be more profitable to assume as little as possible, to measure as many fermion couplings as possible and thus maximise the possibility of learning something new.

Such an ambitious program might have to be curtailed in the face of reality. The neutrino couplings are hard to measure since it requires the detection of the process $e^+e^- \rightarrow \nu\bar{\nu}$, not impossible but tricky since the method would be to rely on initial state radiation to provide a unique signature: $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. We shall see later that the measured spectrum of hard photons from initial state radiation agrees very well with the calculations of Berends and Kleiss [4] in the case of $e^+e^- \rightarrow \mu^+\mu^-\gamma$ when the muon pair is also observed. The γ spectrum for the neutrino process can therefore be reliably calculated. The measured yield of single photons therefore defines the number of possible neutrinos contributing to the process. Even if there is a series of heavier leptons with masses above the current accelerator energies, their associated neutrinos being massless (or at least light) can be produced. This reaction can therefore be used to measure, or put a limit on the number of neutrino flavours. This is a question of cosmological significance.

A further restriction on the program of measuring as many independent parameters as possible is imposed by the inability to separately measure the individual quark couplings. We shall see below that, at least at present, it is only possible to measure certain combinations of the various couplings.

Sakurai [5] has presented a complementary representation of the fundamental couplings in the form of his "tetragon":

Each link between any two apexes in the figure corresponds to a particular reaction $f_1\bar{f}_2 \rightarrow f_2\bar{f}_2$ or its line reversed equivalent. Sakurai thus thinks in terms of the product of any two fundamental fermion couplings. Moreover, if the neutral current is mediated by a series of gauge bosons rather than a single Z^0 then the product of the two couplings becomes a sum over the products, i.e. the couplings no longer factorise. The various processes represented by the above figure are as follows:

1. $\nu e \rightarrow \nu e$ and $\bar{e}\bar{e} \rightarrow \nu\bar{\nu}$
2. $\nu q \rightarrow \nu q$ and $\bar{q}\bar{q} \rightarrow \nu\bar{\nu}$
3. $e q \rightarrow e q$ and $\bar{q}\bar{q} \rightarrow e\bar{e}$
4. $\bar{e}\bar{e} \rightarrow \mu\bar{\mu}$ and $\bar{q}\bar{q} \rightarrow \mu\bar{\mu}$
5. $\bar{q}\bar{q} \rightarrow \mu\bar{\mu}$ and $\bar{\mu}\bar{\mu} \rightarrow \nu\bar{\nu}$
6. $\bar{\mu}\bar{\mu} \rightarrow \nu\bar{\nu}$

With 6 links and V,A couplings, there are 12 different numbers that can be measured. If the neutral current is mediated by a single gauge boson, then one can apply factorisation as follows:

Taking all factorisation relations into account, there remain 7 independent quantities to describe the couplings. The two schemes are thus equivalent, these 7 quantities also reduce to the single $\sin^2\theta_w$ in the standard model.

To conclude this section we can remark that it is the aim of experiments at PETRA, PEP and eventually LEP to measure the processes:

$$\begin{aligned} e^+e^- &\rightarrow e^+e^- \\ &\mu^+\mu^- \\ &\tau^+\tau^- \\ &\nu\bar{\nu}(\gamma) \\ &q\bar{q} \rightarrow \text{hadrons} \end{aligned}$$

in order to determine as many of the neutral current fermion couplings as possible. The parameters thus determined can be compared with the prediction of the standard (or any other) model.

The way in which the vector and axial vector couplings appear in the total cross sections, the angular distributions and the polarisation dependences will be discussed in the next section.

3. Electroweak effects in $e^+e^- \rightarrow f\bar{f}$

I shall only consider fermion antifermion production via the s channel processes: $e^+e^- \rightarrow \gamma, Z^0 \rightarrow f\bar{f}$.

The case where $f = e$ (Bhabha scattering) can also proceed via t channel γ and Z exchange and this makes the formalism more complicated [6].

a) As a first step consider this process with the exchange of a γ only. This demonstrates the symmetry properties of the electromagnetic interaction which are broken by the weak interaction.

From the interaction $\mathcal{L}_{EM} = -\frac{e^2 Q_f^2}{s} \bar{\psi}_f \gamma_\mu \psi_f (\bar{\psi}_e \gamma^\mu \psi_e)$

it is instructive to separate out the separate helicity components [7] (in the limit of massless leptons):

$$\bar{e}_L e_R \rightarrow f_L \bar{f}_R \quad \frac{4s}{a^2} \cdot \frac{d\sigma}{d\Omega} = Q_f^2 (1+\cos\theta)^2 \quad (2a)$$

$$\bar{e}_L e_R \rightarrow f_R \bar{f}_L \quad = Q_f^2 (1-\cos\theta)^2 \quad (2b)$$

$$\bar{e}_R e_L \rightarrow f_L \bar{f}_R \quad = Q_f^2 (1-\cos\theta)^2 \quad (2c)$$

$$\bar{e}_R e_L \rightarrow f_R \bar{f}_L \quad = Q_f^2 (1+\cos\theta)^2 \quad (2d)$$

where θ is the angle between the e^- and f^- . The subscripts R and L refer to right and left handed electrons or fermions. The single γ exchange (spin 1) means that the initial (and final) pair of fermions have opposite helicity. Moreover, as required by the fact that the electromagnetic interaction conserves parity, the substitution: $\bar{e}_R \rightarrow \bar{e}_L$ etc, ($\equiv$ parity transformation) leaves each separate helicity amplitude unaltered. Note also the form of the θ dependence. Due to angular momentum conservation, the substates $e_L \rightarrow f_R$ and $e_R \rightarrow f_L$ cannot occur at 0° , hence the $1-\cos\theta$. Similarly, the substates $e_L \rightarrow f_L$, which cannot take place at 180° , contain the factor $1+\cos\theta$.

For unpolarised $e^\pm$ beams, the angular distribution is obtained by averaging over all four initial helicity states; the terms in $\cos^2\theta$ all cancel and one has the well known result

$$\frac{4s}{a^2} \cdot \frac{d\sigma}{d\Omega} = Q_f^2 (1+\cos^2\theta)$$

which for the case of the $\mu^+\mu^-$ final state ($Q_f = -1$) gives

$$\frac{4s}{a^2} \cdot \frac{d\sigma}{d\Omega} = (1+\cos^2\theta) \quad (3)$$

and integrating over all production angles, $\sigma = \frac{87}{s}$ nb.

b) Now add the weak neutral current and define the weak couplings of right and left handed electrons to be R and L respectively. For the final state fermion the notation is R_f and L_f . The vector and axial vector couplings are related to the R and L as follows.

$$a = L-R \quad a_f = L_f - R_f \quad v = L+R \quad v_f = L_f + R_f \quad (4)$$

The manifestation of parity violation in the weak interaction is that in general $L \neq R$ and $L_f \neq R_f$.

The four helicity states are now modified thus

$$e_L^+ e_R^- \rightarrow f_L^+ \bar{f}_R^- \quad \frac{4s}{\alpha^2} \frac{d\sigma}{d\Omega} = (1+\cos\theta)^2 |Q_f + LL_f g|^2 \quad (5a)$$

$$e_L^+ e_R^- \rightarrow f_R^+ \bar{f}_L^- \quad (1-\cos\theta)^2 |Q_f + LR_f g|^2 \quad (5b)$$

$$e_R^+ e_L^- \rightarrow f_L^+ \bar{f}_R^- \quad (1-\cos\theta)^2 |Q_f + RL_f g|^2 \quad (5c)$$

$$e_R^+ e_L^- \rightarrow f_R^+ \bar{f}_L^- \quad (1+\cos\theta)^2 |Q_f + RR_f g|^2 \quad (5d)$$

The weak amplitude g is essentially the Fermi coupling constant times a few factors and an energy/propagator term

$$g = \frac{\sqrt{2}G_F}{4\pi\alpha} \cdot \frac{s \cdot m_Z^2}{(s - m_Z^2 + i m_Z \Gamma_Z)}$$

In equations (5a-d) we have in fact explicitly assumed factorisation by writing the product LL_f etc. To be more general, one can write instead $\epsilon_{LL} = \Sigma LL_f$ etc although the arguments that follow do not depend on this assumption.

Note that the individual helicity states are no longer invariant under parity transformation (unless $L=R$ and $L_f=R_f$). However, as we shall now show it is necessary to observe the individual amplitudes (or asymmetric

combinations of them) if the parity violating nature of the interaction is to be revealed.

3.1 Angular Distributions and Total Cross Section.

If the beams are unpolarised and the final state polarisation is not measured, then the angular distribution is obtained by adding the four helicity components and dividing by 4.

Then

$$\begin{aligned} \frac{4s}{\alpha^2} \cdot \frac{d\sigma}{d\Omega} = & \lambda(1+\cos\theta)^2 [(Q_f + LL_f g)^2 + (Q_f + RR_f g)^2] \\ & + \lambda(1-\cos\theta)^2 [(Q_f + LR_f g)^2 + (Q_f + RL_f g)^2] \end{aligned} \quad (6)$$

Already it can be seen that the parity asymmetry has been removed by combining essentially states of opposite parity. A parity transformation $L \rightarrow R$ etc leaves $\frac{d\sigma}{d\Omega}$ unaltered and so the angular distribution and anything derived from it by integrating over regions of θ (eg total cross section, forward-backward angular asymmetry) is intrinsically a parity conserving quantity.

Working out equation 6 and substituting $a = L-R$ etc, leads to

$$\begin{aligned} \frac{4s}{\alpha^2} \frac{d\sigma}{d\Omega}(\bar{f}f) = & (1+\cos^2\theta)(Q_f^2 - \frac{1}{2}Q_f v v_f g + \frac{1}{8}(v^2 + a^2)(v_f^2 + a_f^2)g^2) \\ & - 2\cos\theta(\frac{1}{2}Q_f a a_f g - \frac{1}{4}v a v_f a_f g^2) \end{aligned} \quad (7)$$

The interference terms (proportional to g) have a $v v_f$ term associated with $(1+\cos^2\theta)$ and an $a a_f$ term associated with $\cos\theta$. The vector and axial components can therefore be isolated by either integrating over $\cos\theta$ ($\cos\theta$ term drops out) or forming the forward-backward asymmetry $(1+\cos^2\theta$ term drops out).

Thus

$$\frac{3s}{4\pi\alpha^2} \cdot \sigma(\bar{f}f) = R_f = Q_f^2 - \frac{1}{2}Q_f v v_f g + \frac{1}{8}(v^2 + a^2)(v_f^2 + a_f^2)g^2 \quad (8)$$

whereas the forward-backward asymmetry = $A_f = \frac{\int_0^1 \frac{d\sigma}{dQ} - \int_{-1}^0 \frac{d\sigma}{dQ}}{\int_{-1}^1 \frac{d\sigma}{dQ}}$

is given by

$$A_f = \frac{3}{4} \cdot \frac{-\frac{1}{2}Q_f a_f g + \frac{1}{2}v_f a_f g^2}{(Q_f^2 - \frac{1}{2}Q_f v_f g + \frac{1}{2}(v_f^2 + a_f^2)g^2)} \quad (9)$$

At PETRA energies, where the weak squared terms can be neglected, the following approximate but transparent formulae are obtained

$$R_f = Q_f^2 - \frac{1}{2}Q_f v_f g$$

$$A_f = -\frac{3}{8}Q_f a_f g$$

ie total cross sections measure vector couplings and asymmetries measure axial couplings.

At higher energies, above the interference region where the weak squared term dominates, both the total cross section and the F-B asymmetry measure quadratic combinations of the vector and axial couplings.

To illustrate the dependence of R_f and A_f on the variables of interest, s and $\sin^2\theta_w$, some examples are shown in figs 1 and 2. Fig 1 shows the variation of R_μ with $\sin^2\theta_w$ for various energies (s) in the PETRA energy range. If $\sin^2\theta_w$ is close to $\frac{1}{2}$ as indicated by v_e , v_μ and eq experiments, then the change in R_μ due to weak effects is so small that it cannot be measured. On the other hand, measurable effects would occur if $\sin^2\theta_w$ differs appreciably from $\frac{1}{2}$, ie even if v_e or v_μ are close to zero, they can still be observed. The muon asymmetry A_μ does not depend too strongly on $\sin^2\theta_w$ (at PETRA energies) since it enters only as a small perturbation to the denominator in eq 9, through the propagator term ($m_Z^2 = 37.3^2/(\sin^2\theta_w \cos^2\theta_w)$) or through the weak squared term. Due to the a_μ dependence, and with $a = a_\mu = -1$, the effect on the asymmetry can be substantial and observable at PETRA energies, as shown in fig 2. The

Figure 1. The variation of R_μ with $\sin^2\theta_w$ for two values of s .

Figure 2. The variation of A_μ with s for three values of $\sin^2\theta_w$.

asymmetry A_μ will therefore measure the axial couplings via an observable non-zero effect but due to its lack of dependence on $\sin^2\theta_w$ it does not offer a crucial test of the standard model.

In the case of $q\bar{q}$ final states, R_f corresponds to a contribution from a particular quark flavour to R , the total hadronic cross-section in units of the point like muon QED cross section. Then $R = 3 \sum_f R_f$ where the factor 3 arises from colour and the sum is carried out over all possible quarks eg d, u, s, c and b at present PETRA energies. The $\sin^2\theta_w$ dependence of R is shown in fig 3 for several values of s. The ratio of the interference term to the γ term is proportional to $v\bar{v}_f/q_f$ which is a factor 25 or 6 larger for d or u quarks than for muons (setting $\sin^2\theta_w = 0.23$) and therefore more readily measurable in hadron final states than $\mu^+\mu^-$. For values away from $\frac{1}{2}$, R changes substantially from 3.87 and can reach extreme values. A careful measurement of R can therefore measure $\sin^2\theta_w$ quite accurately. Note that a measurement of R at a single value of s would lead to two possible solutions for $\sin^2\theta_w$. The two solutions have different s dependences however (see the crossing regions in fig 3) and a measurement of R over a wide range of s can lead to a single solution. The way in which the propagator term modifies the total cross sections is shown in fig 4 which shows R_μ and R_{hadrons} as a function of s.

Turning to the quark-antiquark forward-backward asymmetry (the hadronic equivalent of the muon asymmetry) a few interesting points arise. Firstly, it can be seen in eqn 9 that the asymmetry varies like $\frac{a_f}{Q_f}$, ie it is bigger for fractionally charged quarks than for muons. For d, s and b quarks the asymmetry could be as large as 30% at the highest PETRA energies. Unfortunately, we now encounter a fundamental obstacle; particles and antiparticles are distinguished by their charge. Thus a quark with positive charge would be either $Q_f = \frac{2}{3}$ quark or a $Q_f = \frac{1}{3}$ antiquark and since the asymmetry depends on a_f/Q_f , which is always positive in the standard model, the asymmetries cancel in the summation over quark flavours. Quantitatively, this works as follows, summing over

Figure 3. The variation of R_{hadrons} with $\sin^2\theta_w$ for three values of s.

Figure 4. The variation of R_{hadrons} and R_μ as a function of s for $\sin^2\theta_w = 0.23$.

all $Q_f = \frac{2}{3}$ quarks and $Q_f = \frac{1}{3}$ antiquarks, the combined asymmetry becomes

$$A_{\text{tot}}(q\bar{q} \text{ or } \bar{q}q) = \frac{\sum |Q_f| a_f}{\sum Q_f^2} \cdot A_\mu$$

$$= \frac{(-\frac{1}{3} + \frac{2}{3} - \frac{1}{3} + \frac{2}{3} - \frac{1}{3} - \frac{1}{3}) A_\mu}{(\frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9})} = \frac{3}{11} A_\mu$$

So the resulting angular asymmetry would only be 3/11ths of the muon asymmetry and is not very promising at PETRA.

Possible methods of determining quark charges are as follows:

(a) A determination of the momentum weighted charges in a hadron jet can in principle be related to the parent quark charge - on a statistical basis [8]. For example, $\sum_1^N Q_i P_i^Y$ has a distribution with a positive mean value for $Q_f = \frac{2}{3}$ quarks and a negative mean value for $Q_f = -\frac{1}{3}$ quarks. The summation is carried out over the N particles in a jet where the i th particle has charge Q_i and momentum P_i . The exponent Y is chosen empirically to maximise the separation between positively and negatively charged quarks. In practice, the measure is not a 100% reliable estimator since the widths of the two measured distributions are so large [9] due to experimental effects that they overlap almost completely. More work is necessary to show whether or not this technique will prove fruitful either at PETRA or LEP energies.

(b) The semi-leptonic decays of quarks also offer a clue to the charge of the parent quark

eg
 $b \rightarrow \mu^- \nu c$
 $c \rightarrow \mu^+ \nu s$ etc

where we can now see that a μ^- is an indicator for a b or $\bar{c}$ etc. So the asymmetry of hadron jets containing a μ^- (or μ^+ with $\theta_{\text{jet}} \rightarrow 180^\circ - \theta_{\text{jet}}$) will suffer from the same cancellation effects mentioned above. In this particular case however, one can do better. Firstly, the different lifetimes of successive species of quarks means that only those muons need be selected which originate close to the main hadron vertex (direct muons), the spatial resolution being of the order of a mm. This essentially excludes all decays except those from b and c .

Then

$$A_{c+b} = \frac{\frac{2}{3} - \frac{1}{3}}{\frac{4}{9} + \frac{1}{9}} A_\mu = \frac{3}{5} A_\mu$$

still only a few percent but now possibly measurable and more important, providing a measurement of a_c and a_b . However, these measurements have to be made against a background from meson decays which reduce the statistical accuracy. A significant improvement can be made by measuring the transverse momentum of the muons relative to the jet axis. Muons from heavy b (or B meson) decay are likely to have a larger transverse momentum than those from c decay and this could be used to try and separate b and c jets. Even so, the $p_T(\mu)$ distributions from b and c decay overlap substantially and it is unlikely that the full 30% effect could be seen.

3.2 Parity

We saw that total cross-sections or forward-backward asymmetries for leptons and quarks are all parity conserving quantities - if the initial beams are unpolarised. This leads to certain advantages or disadvantages depending on one's point of view:

1) Since the electromagnetic and strong interactions conserve parity, they too can contribute to cross sections and angular asymmetries via single and multiple γ or gluon exchange or emission. Higher order QED processes otherwise known as radiative corrections must therefore be calculated carefully since they affect all the measured quantities by a

few percent. QCD can also affect the value of R, eg $R \rightarrow R(1+\alpha_s/\pi+\dots)$ and the quark asymmetry.

2) Because the parity conserving part of the neutral current is not measured in e^+q or ν scattering, e^+e^- can be viewed as providing fresh insight into the structure of the weak current - provided QED and QCD higher order contributions are understood. This can be an advantage.

3) But nevertheless, the classic, traditional signature for weak effects is parity violation. The observation of a parity violating quantity in e^+e^- could not be due to QED or QCD according to our present understanding and would be a clear unambiguous indication of a weak effect - just as a non zero polarisation asymmetry in e^+q scattering means just that. The next section deals with possible ways of detecting parity violating effects in e^+e^- , now and in the future.

3.3 Polarisation effects

Referring to equations 5a-d, the way in which the parity violating properties of the interaction can be revealed is to measure or define the spin state of the initial or final state fermions. For example, if the beams were polarised - say e^-_R only - then the cross section would be given by 5c+5d which is no longer invariant under a parity operation (L $\leftrightarrow$ R etc). We can now derive from equations 5a-d various helicity dependent cross sections.

a) Beam polarisation asymmetry.

By analogy with the polarisation asymmetry in e^+q scattering, the difference between the cross sections for e^-_R and e^-_L scattering has the property that electromagnetic components disappear leaving a completely parity violating interference (or pure weak) term.

ie

$$\frac{4s}{\alpha^2} \left[\frac{d\sigma}{dQ^2}(e^-_R) - \frac{d\sigma}{dQ^2}(e^-_L) \right] = \frac{1}{2}(1+\cos\theta)^2 [(Q_f + R R_f g)^2 - (Q_f + L L_f g)^2] + \frac{1}{2}(1-\cos\theta)^2 [(Q_f + R L_f g)^2 - (Q_f + L R_f g)^2]$$

after some algebra, this leads to

$$\frac{4s}{\alpha^2} \left[\frac{d\sigma}{dQ^2}(e^-_R) - \frac{d\sigma}{dQ^2}(e^-_L) \right] = \frac{1}{2}(1+\cos^2\theta) [2Q_f a_f g - \frac{1}{2} v_f a_f^2 g^2] + 2\cos\theta [Q_f v_f a_f g - \frac{1}{2} (v_f^2 + a_f^2) v_f a_f g^2]$$

or, integrating over $\cos\theta$ (cos θ term vanishes) and summing over d,u,s,c,b quarks:

$$R(e^-_R) - R(e^-_L) = \sum_f [Q_f a_f g - \frac{1}{2} v_f a_f^2 g^2]$$

It actually makes more sense to evaluate $(R(e^-_R) - R(e^-_L))/R$ because both QCD effects and experimental normalisation uncertainties cancel out. Note $2R = R(e^-_R) + R(e^-_L)$, and then

$$R_A = \frac{R(e^-_R) - R(e^-_L)}{R} = \frac{\sum_f [Q_f a_f g - \frac{1}{2} v_f a_f^2 g^2]}{\sum_f [Q_f^2 - \frac{1}{2} Q_f v_f g + \frac{1}{2} (v_f^2 + a_f^2) (v_f^2 + a_f^2) g^2]} \quad (10)$$

$$\approx \frac{\sum_f Q_f a_f g}{\sum_f Q_f^2} \text{ in the interference region}$$

In this case, the interference term is proportional to v_f (not v_e) and therefore is substantial, even for $\sin^2\theta_W$ close to $\frac{1}{2}$. The behaviour of this quantity (eq 10) as a function of s ($\sin^2\theta_W = 0.23$) is shown in fig 5. It would be quite easily measurable at PETRA energies although, alas, the beams are not polarised in this way, nor will they be at LEP at first. The prospects at the planned SLAC "single pass collider" are more promising because it will be possible to polarise the electron beam (but not positrons) and this is enough, for, as we saw above, a right handed electron will only pick out left handed positrons with which to interact.

The flux of polarised positrons is essentially one half the total positron beam flux.

Equation 10 above also shows that the beam polarisation asymmetry for producing $\mu^+\mu^-$ will be close to zero due to the presence of ν_f in the ν interference term and ν in the pure weak term.

b) Final state polarisation

The polarisation of final state fermions can be non zero, even for unpolarised e^+e^- beams. Again from equations 5a-d, the polarisation is given by

$$P_f = \frac{\frac{d\sigma_f(f)}{d\Omega_R} - \frac{d\sigma_f(f_L)}{d\Omega_L}}{\frac{d\sigma_f(f)}{d\Omega_R} + \frac{d\sigma_f(f_L)}{d\Omega_L}}$$

$$\text{where } \frac{4s}{\alpha^2} \frac{d\sigma}{d\Omega}(f_R) = \frac{1}{2} [(1+\cos\theta)^2 [Q_f + RR_f g]^2 + (1-\cos\theta)^2 [Q_f + LR_f g]^2]$$

$$\frac{4s}{\alpha^2} \frac{d\sigma}{d\Omega}(f_L) = \frac{1}{2} [(1+\cos\theta)^2 [Q_f + LL_f g]^2 + (1-\cos\theta)^2 [Q_f + RL_f g]^2]$$

Then

$$P_f(\cos\theta) = \frac{(1+\cos^2\theta)(2Q_f v_f g - \frac{1}{2}(v^2+a^2)v_f a g^2) + 2\cos\theta(2Q_f a v_f g - \frac{1}{2}v a(v_f^2+a_f^2)g^2)}{(1+\cos^2\theta)(4Q_f^2-2Q_f v v_f g + \frac{1}{2}(v^2+a^2)(v_f^2+a_f^2)g^2) - 2\cos\theta(2Q_f a a_f g - v a v_f a_f g^2)} \quad (11)$$

Once again, QCD effects should cancel out when determining P_f . Weak terms in the denominator can effectively be ignored at PETRA energies. Furthermore, the $(1+\cos^2\theta)$ term in the numerator contains $v(=0)$ so this can be neglected in favour of the term proportional to $\cos\theta$. So we obtain the approximate formula

$$P_f(\cos\theta) = \frac{a v_f g}{Q_f} \frac{\cos\theta}{1+\cos^2\theta}$$

Figure 5. The beam polarisation asymmetry R_A as a function of s for three values of $\sin^2\theta_w$.

Figure 6. The angular variation of the quark and lepton polarisation for two values of s . In this figure $\sin^2\theta_w = 0.23$.

Note that the final state polarisation measures the same combination of weak couplings (ie av_f) as the beam polarisation asymmetry. But if the beams can't be polarised then it would be the only source of this information. In fact, the observation of non zero polarisation of final state particles in e^+e^- annihilation would be the first manifestation of the parity violating effects of the weak interaction in e^+e^- since it could not arise from the electromagnetic or strong interaction. Note that from equation (11) the polarisation of final state μ or τ is likely to be small due to v_f whereas the quark polarisation can be large. Using the numbers in table 2, one can see that the combination $\frac{av_f}{Q_f}$ is four times larger for d, s and b quarks than for u and c quarks. The observation of the s quark polarisation (via Λ decay) appears promising. It is worth noting that the angular dependence $\frac{\cos\theta}{(1+\cos^2\theta)}$ means that the average

polarisation is close to zero! This can be seen in fig 6 where the various fermion polarisations are plotted as a function of $\cos\theta$ for different s values. Although the polarisation for s quarks reaches large values, it changes sign and an experiment with limited statistics, averaging over production angle would not see much of an effect. Therefore, let us define the average polarisation for forward and backward produced fermions ie P_F and P_B and form two orthogonal combinations

$$\frac{1}{2}(P_F - P_B) : = P_A, \text{ the forward-backward polarisation asymmetry}$$

$$\frac{1}{2}(P_F + P_B) : = \langle P \rangle, \text{ average polarisation.}$$

It then turns out that P_A is a good measure in the interference region (see fig 7) whereas $\langle P \rangle$ is close to zero in the interference region, but becomes large as the weak squared term takes over (fig 8).

The classic methods of measuring the polarisation involve the decay angular distribution of the polarised particle

Figure 7. The variation of P_A with s for quarks and leptons. In this figure $\sin^2\theta_w = 0.23$.

Figure 8. The variation of $\langle P \rangle$ with s for quarks and leptons. In this figure $\sin^2\theta_w = 0.23$

eg

Ranft and Ranft for example [10] have calculated via a cascade Monte Carlo process that up to 50% of the initial polarisation will survive into so called "leading" Λ^0 where "leading" means $x_{\Lambda} > 0.1$ (where $x_{\Lambda} = p_{\Lambda}/E_{\text{beam}}$).

In principle, another way of measuring the polarisation is via the so-called leptonic decays of heavy quarks.

for example

$$\text{where } \bar{P}(\theta) = 1 + \alpha P_b \cos \theta$$

but here, the probable mechanism is that the b quark forms a meson B^0 which then decays weakly. The polarisation information is then lost in the spinless meson. One can make the nice observation that in general, the polarisation is induced by the weak neutral current and analysed by

eg
a)

$$\text{where } \bar{P}(\theta) = 1 + P_{\mu} \cos \theta$$

or b)

$$\text{where now } \bar{P}(\theta) = 1 + \alpha P_{\Lambda} \cos \theta \text{ with } \alpha = 0.64$$

Method a) would be suitable for τ decay, but not for μ decay due to the lifetime. Method b) can be used for Λ^0 produced in e^+e^- collisions but the big question is whether or not the quark polarisation survives the fragmentation process and is transmitted to the Λ^0 . This will depend crucially on the fragmentation process for baryon production. At the very least, one can measure the decay angular distribution of Λ^0 and compare it with the expression

$$\bar{P}(\theta) = 1 + 0.64 \alpha_f^2 \Lambda \cos \theta$$

where in addition to the traditional polarisation parameter $\alpha = 0.64$, there is an additional factor: the fragmentation depolarising parameter α_f .

The process of baryon production in connection with polarising effects has been considered by several authors [10]. Usually, to produce a baryon, a diquark is needed from the vacuum

the weak charged current. In any case, it is likely that polarisation measurements will take on increasing importance in e^+e^- processes as both the energy and the statistics are increased.

To conclude this section, it is worth tabulating the various observables and listing the particular weak couplings or combination of couplings that each observable measures. This is shown in table 4.

Table 4

	Interference region	
	PEP, PETRA + SLC →	"Resonance" region LEP
$\sigma(\mu^+\mu^-)$	vv_μ	$(v^2+a^2)(v_\mu^2+a_\mu^2)$
$A(\mu^+\mu^-)$	aa_μ	$vav_\mu a_\mu$
$R(e^+e^- \rightarrow \text{hadrons})$	$v\sum_q v_q$	$(v^2+a^2)\sum_q (v_q^2+v_q^2)$
$A(q\bar{q})$	$a\sum_q a_q$	$va\sum_q a_q$
$\sigma_R - \sigma_L$	$a\sum_q v_q$	$va\sum_q (v^2+a^2)$
$\text{Pol}(\Lambda)$	av_s	$va(v_s^2+a_s^2)$

4. Radiative corrections

Since many of the observables used to determine the strength of electroweak effects in e^+e^- annihilations are intrinsically parity conserving quantities, QED and QCD processes (to all orders) can also contribute and they can cause effects similar to those produced by Z^0 exchange. For example, higher order QED graphs change the total cross section for $\mu^+\mu^-$ and produce a forward-backward angular asymmetry. These effects are of the order of a few percent at PETRA energies ($1+\alpha$). Similarly, QCD effects (gluon emission) change the total cross section for $q\bar{q}$ and produce a forward-backward quark angular asymmetry. These effects are also of the order of a few percent ($1+\alpha_s/\pi$).

In order to extract the electroweak information as reliably as possible,

it is necessary to understand these radiative corrections, to calculate them carefully or to measure them if possible.

Higher order QED processes are handled by using the calculations of Berends and Kleiss [4] and all groups working at PETRA and PEP apply these (or similar) corrections to their data.

One of the PETRA experiments, namely JADE, has carried out a careful comparison of the Berends and Kleiss calculations with measurements of the processes $e^+e^- \rightarrow \gamma \mu^+\mu^-$ and $e^+e^- \rightarrow \gamma + (\text{hadrons} = q\bar{q})$ where the γ is radiated from one of the initial states $e^\pm$ and has an angle and energy which allows it to be detected by the apparatus. A comparison of the momentum spectrum of the γ and the $\mu^\pm$ angular distribution with the curves of Berends and Kleiss is shown in fig 9 where good agreement can be seen. This provides confidence in using the calculations in regions where the γ is not detected (eg small angles and low momentum). To quantify the situation, the QED contribution to the F-B asymmetry in $\mu^+\mu^-$ is 1.3% when integrated over the acceptance of the JADE detector. This can be compared with an expected asymmetry of electroweak origin of about 9% at present energies. Fig 9 suggests that the error on the 1.3% correction is probably quite small. In fact, radiative corrections to lepton final states are less of a problem in present detectors with " 4π " γ detection (eg JADE Mark J) since radiative effects are largely observed and the final correction is small.

The problem of QCD corrections has not been investigated quite so well. Calculations have been made by Jersak et al [11] who calculated R and the quark F-B asymmetry including graphs up to order α_s .

They also included finite quark mass effects which for example cause an axial suppression near threshold since a $q\bar{q}$ pair at rest cannot be in a

Figure 9. The differential momentum and angular distribution for the radiative process $e^+e^- \rightarrow \mu^+\mu^- \gamma$.

Figure 10. The effect of a low mass Z^0 on the $\mu^+\mu^-$ p-p asymmetry.

$j^P = 1^+$ state. In general, the correction turns out to be larger than $(1 + \alpha_s/\pi)$. In the standard model, the value of R gets modified by up to 20% at the peak of the Z^0 and the result suggests that more investigation, including higher order terms needs to be carried out if reliable information is to be extracted from LEP data in the Z^0 region.

5. Extensions to the standard model

There is no shortage in the literature of models with a "richer" gauge boson structure than the standard model. We know from history that a proliferation of "elementary" particles tends to imply an underlying substructure. Before the discovery of N^* and Δ baryons and the ρ , ω , ϕ etc mesons, the nuclear force was thought to be mediated by pions.

The eventual plurality of baryons and mesons and the subsequent experiments at large Q^2 at SLAC in the late 1960s put an end to this picture and we now have a substructure of quarks with their associated gauge particles - gluons. The neutral weak interaction of leptons and quarks can be drawn thus:

It is debatable whether there is a proliferation of leptons and quarks, the discovery of another heavy lepton would tend to suggest that there is! Similarly the existence of a complicated Z structure would also support this idea. The answer lies in spectroscopy and high Q^2 experiments.

In the meantime, the data from present experiments at PETRA and PEP can be used to exclude, restrict or eventually confirm these extended gauge models all of which incidentally have to coincide with the standard model

at low Q^2 or to differ from the standard model by having extra parity conserving pieces only (since the parity violating part has been well tested already).

The various models achieve the richer gauge structure by adding an extra symmetry group to the standard model $SU(2) \times U(1)$.

eg

$$\begin{array}{l}
 \text{Barger et al [12]} \quad \begin{array}{c} SU(2) \times U(1) \times SU(2)' \\ \left\{ \begin{array}{c} W_1^\pm \\ Z_1 \end{array} \right\} \quad \begin{array}{c} W_2^\pm \\ Y \\ Z_2 \end{array} \end{array} \\
 \text{gauge particles} \\
 \text{Elias, Pati \& Salam [13]} \quad \begin{array}{c} SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \\ \left\{ \begin{array}{c} W_L^\pm \\ Z_L \end{array} \right\} \quad \begin{array}{c} W_R^\pm \\ Z_R \\ Y \end{array} \end{array} \\
 \text{de Groot et al [14]} \quad \begin{array}{c} SU(2) \times U(1) \times \tilde{U}(1) \\ \left\{ \begin{array}{c} W^\pm \\ Z_1 \end{array} \right\} \quad \begin{array}{c} Y \\ Z_2 \end{array} \end{array}
 \end{array}$$

In addition, a series of papers by Schildknecht et al [15] has covered the properties of more general gauge group extensions to the standard model, namely $SU(2) \times U(1) \times G$.

The extra symmetry group corresponds to the insertion of an additional piece into the interaction Lagrangian

$$\mathcal{L} = \frac{\sqrt{2}G_F}{4} \{ [j_3 - \sin^2 \theta_w j_{em}]^2 + c j_{em}^2 \} \quad (12)$$

ie

where $C=0$ corresponds to the standard model. Since C is associated with j_{em} it is a parity conserving quantity, therefore not detectable in the electron deuteron polarisation asymmetry measurements at SLAC. Nor would it be observable in ν scattering experiments since the neutrino does not couple to the photon.

The extra symmetry group chosen together with the assumed structure of the Higgs sector determines the analytic form for C . For example, expressions for C for two of the above models are as follows:

$$\text{Barger et al [12]} \quad C = \sin^4 \theta_w \frac{(m_2^2 - m_1^2)(m_2^2 - m_1^2)}{m_1^2 m_2^2}$$

$$\text{and de Groot et al [14]} \quad C = \cos^4 \theta_w \frac{(m_2^2 - m_1^2)(m_2^2 - m_1^2)}{m_1^2 m_2^2}$$

The strength of the extra piece of the Lagrangian is proportional to $\sin^4 \theta_w$ (≈ 0.04) in one case and $\cos^4 \theta_w$ (≈ 0.5) in the other; ie there is a factor of 10 difference between the two and they can be used as extreme cases of these types of models. Moreover, since C is a positive definite quantity, the form of C above leads to the condition $m_1 < m_2$ (standard) $< m_2$ although more recent models [15] with a different Higgs structure and different m_1, m_2 dependence for C have $m_1, m_2 > m_2$. C is also non-zero for models where the leptons and quarks are composite particles. Nevertheless, some of these models predict a gauge boson with a mass less than that of the standard model. The masses are not predicted in these models and in the limiting case m_1 or $m_2 = m_Z$ the other Z decouples and the model becomes identical to the standard model.

The quantity C also corresponds to a modification to the vector coupling (since C is associated with the vector electromagnetic current) thus $v^2 + (-1+4\sin^2 \theta_w)^2 + 16C$. So if v^2 varied away from its low Q^2 value of approximately zero as data at higher Q^2 (near the Z^0 boson mass) is obtained, this would indicate the presence of the C term.

C can also be expressed in terms of the sum rule [17]

$$16C = \frac{\int ds}{s} \sigma(e^+e^- \rightarrow \gamma, Z_1, \dots, Z_N + \text{all}) - \frac{\int ds}{s} \sigma(e^+e^- \rightarrow \gamma, Z(\text{standard}) + \text{all})$$

ie it represents the possible deviation from the standard model due to an arbitrary series of gauge bosons $Z_1 \dots Z_N$.

Other deviations from the standard model have been proposed by Professor Sakurai [18] and will be discussed in more detail in his lectures. This γ -Z mixing (non gauge invariant) model retains the v and a couplings with their values determined at low Q^2 but the relation between m_Z and $\sin^2 \theta_w$ ie $m_Z^2 = \frac{37.32}{\sin^2 \theta_w \cos^2 \theta_w}$ is discarded and m_Z becomes a parameter to be determined by experiment. The effect of varying m_Z on the $\mu^+\mu^-$ asymmetry (eg propagator effect) can be seen in fig 10 which shows that a measurement of A_μ at PETRA energies can already say plenty about the value of m_Z in the context of this model.

6. Interpretation of e^+e^- data

This section is intended to review what has been learnt about the electroweak interaction from e^+e^- annihilation measurements. We shall be interested in the values of the lepton and quark vector and axial couplings v_f and a_f and a comparison with the predictions of the standard model. The value of $\sin^2 \theta_w$ determined from lepton and hadron final states will also be considered. Finally the restrictions on the mass of the Z^0 will be presented.

The data come mainly from five experiments which have been running on the e^+e^- storage ring PETRA. Experimental physicists these days tend to work under a cloak of acronymy; at PETRA for example the experiments are called CELLO, JADE, MARK-J, PLUTO and TASSO. Meanwhile the competing e^+e^- machine PEP at Stanford has also started to produce results and the combined data from about 7 different experiments has helped considerably to overcome the problem of the low cross sections. For the present, no data are available on polarisation (parity violating) effects nor has anyone yet observed quark charge asymmetry effects analogous to the muon asymmetry.

6.1 Lepton sector coupling constants v, a

In order to determine the lepton coupling constants, data from three processes have been analysed ie

In all the analyses, the results have been presented as v^2 and a^2 although strictly, the measured quantities are v_e^2 , v_μ^2 , v_τ^2 etc in the three processes. Assuming e - μ - τ universality allows data from the three processes to be combined in a single analysis. There is a justification for this assumption. Fig 11 shows the measured total cross sections for $\mu^+\mu^-$ and $\tau^+\tau^-$ production which is sensitive to $v_\mu v_\mu$ and $v_\tau v_\tau$. The data support μ - τ universality in that both v_μ and v_τ appear to be close to zero. However, the possibility: $v_e = 0$ and v_μ and v_τ non zero and not equal to each other is not excluded. To test this latter possibility, one should measure the τ polarisation (via the $\mu\nu v_\tau$ decay mode). From equation 11, this measures a_τ and the zeroing effect of v_e can be avoided.

Further evidence for μ - τ universality comes from the shape of the differential cross section - which is sensitive to $a_\mu a_\tau$. The measured $\frac{d\sigma}{dQ}$ are shown in fig 12; they do not contradict each other. Most experiments quote a result for the forward-backward asymmetry since apparatus biases largely cancel out. The various experiments tabulate as follows:

Figure 11. The total cross sections for $\mu^+\mu^-$ and $\tau^+\tau^-$ production as a function of $W = \sqrt{s}$.

Figure 12. The angular distributions for $\mu^+\mu^-$ and $\tau^+\tau^-$ production at $s = 1150 \text{ GeV}^2$.

	A_μ (measured)	A_μ (expected)
CELLO	$-6.4 \pm 6.4\%$	-9.1%
JADE	$-12.7 \pm 2.7 \pm 1\%$	-9.1%
MARK-J	$-8.4 \pm 2.9 \pm 1.1\%$	-9.5%
PLUTO	$+7 \pm 10\%$	-5.8%
TASSO	$-15.1 \pm 3.2\%$	-9.2%
NAC	$-0.9 \pm 5.7\%$	-5.0%
MARK II	$-4.0 \pm 3.5\%$	-5.0%
		Average $-9.4 \pm 1.5\%$
		Expected -8.1%

	A_τ (measured)	A_τ (expected)
CELLO	$-10.3 \pm 5.2\%$	-9.2%
JADE	$-4.7 \pm 4.2 \pm 1.2\%$	-7.5%
MARK J	$-7.0 \pm 7.2 \pm 2.1\%$	-9.5%
TASSO	$-0.4 \pm 6.6\%$	-9.1%
		Average $-5.8 \pm 2.8\%$
		Expected -8.5%

(where two errors are quoted, the second is systematic)

The τ and μ results are in fairly good agreement and together provide a clear, unambiguous demonstration of the existence of neutral current effects in e^+e^- reactions. The reason why the expectation of the standard model is different for τ and μ is because the theory has been integrated over apparatus angular acceptance and this is slightly smaller in some experiments for τ than for μ .

There is always the possibility that the apparatus has introduced an asymmetry via instrumental biases. To investigate this, JADE tested for instrumental effects by measuring a large sample of cosmic muons and found $A_{\text{cosmic}} = -0.7 \pm 1.1\%$, close to zero as expected. The $e^+e^- \rightarrow \mu^+\mu^-$ asymmetry has also been measured in the same experiments at low s where weak effects are expected to be negligible. No evidence for an experimental bias can be found.

Combining data for the various final states, a fit can be made to the measured angular distributions to find v^2 and a^2 . Since data is continually being collected, these values are progressively being improved; the status quoted here corresponds to the spring 1982

conferences [19]. The results for the various experiments can be listed as follows:

	v^2	a^2
CELLO	-0.16 ± 0.32	1.12 ± 0.48
JADE	0.40 ± 0.28	1.24 ± 0.32
MARK-J	0.12 ± 0.24	0.88 ± 0.32
TASSO	-0.44 ± 0.52	2.12 ± 0.40
MAC		0.16 ± 0.88
MARK II	0.20 ± 0.40	0.96 ± 0.64

They can be combined to obtain a result for the lepton couplings from all e^+e^- experiments:

$$\begin{aligned} v^2 &= 0.10 \pm 0.14 & \text{and} & & a^2 &= 1.22 \pm 0.17 \\ \text{or} & (g_v^2 = 0.026 \pm 0.035 & \text{and} & & g_a^2 &= 0.305 \pm 0.043) \end{aligned}$$

remembering they have been derived assuming $e-\mu-\tau$ universality and factorisation. A further assumption in the v^2, a^2 analyses was to absorb propagator effects into the coupling constants, i.e. the results are actually for $\frac{v^2 a^2}{m_Z^2 - s}$ and $\frac{a^2 m_Z^2}{m_Z^2 - s}$. If $m_Z \approx 90$ GeV then the above numbers would have to be divided by a factor 1.16.

A comparison can now be made with the couplings determined in νe elastic scattering, also a purely leptonic process with no hadronic structure involved. Elastic νe scattering measures the product of the neutrino and electronic couplings (see Sakurai tetragon). It is customary however, to assume $V-A$ coupling for the neutrinos and to use the data to determine g_a and g_v for the electron - see fig 13. The cross sections depend on terms like $(g_a + g_v)^2$ and $(g_a - g_v)^2$; hence the ellipses from the measurement of a single number σ . All three elliptical regions coincide only in the small region. $(g_v, g_a) \approx (0, -\frac{1}{2})$ or $\approx (-\frac{1}{2}, 0)$ leaving a dominant axial or dominant vector ambiguity from νe reactions alone. A comparison with the results from e^+e^- can be made in the $v^2 - a^2$ plane (remember $v = 2g_v$ and $a = 2g_a$) as shown in fig 14. This is a comparison between all νe and all

Figure 13. The measurement of g_a and g_v in νe elastic scattering experiments.

Figure 14. A comparison of lepton weak couplings determined in e^+e^- and νe experiments.

e^+e^- data under the assumptions:

- 1) v couplings are assumed to be V-A
- 2) factorisation
- 3) $e-\mu-\tau$ universality.

The comparison removes the v ambiguity, the dominant axial solution alone survives and furthermore, both sets of purely leptonic processes are in good agreement with each other. It is worth noting that e^+e^- measurements are already competitive in determining the axial coupling but lag behind a little on the determination of the vector coupling.

The results for v^2 and a^2 overlap the line corresponding to the standard model ($a^2 = 1$, $v^2 = (-1+4\sin^2\theta_w)^2$) and it is thus reasonable to proceed to the next stage which is to determine $\sin^2\theta_w$. This is done in the fitting procedure by fixing $a^2 = 1$, substituting $v^2 = (-1+4\sin^2\theta_w)^2$ and including the propagator dependence $m_Z^2 = 37.3^2/(\sin^2\theta_w \cos^2\theta_w)$. Typical results as shown below are obtained

CELLO	0.25 ± 0.12
JADE	0.25 ± 0.12
MARK J	0.25 ± 0.11
TASSO	0.29 ± 0.09 ~ 0.11

The fact that the various results are all close to 0.25, even though the quoted errors would allow a large scatter is due to the fact that $\sin^2\theta_w$ is essentially determined via v^2 which happens to be close to zero. Then, as can be seen in fig 15, a measurement of v^2 which is negative will yield a single value for $\sin^2\theta_w$ of 0.25. If the measurement gives a slightly positive value for v^2 then there are two solutions for $\sin^2\theta_w$ symmetric about 0.25 which overlap and merge to give again 0.25. (Propagator effects can shift the value slightly off 0.25). Due to the quadratic dependence of the χ^2 surface, a value of v^2 which is negative will lead to a value of $\sin^2\theta_w$ with smaller errors than if v^2 were slightly positive. To obtain an average value for $\sin^2\theta_w$, it is safer to use the average value for v^2 from the various experiments. Assuming a

parabolic error for the combined value and fixing $a^2 = 1$, then from all experiments on e^+e^-

$$\sin^2\theta_w = 0.25 \pm 0.06$$

6.2 Quark sector coupling constants v_f, a_f

In this case, we have only one process

$$e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$$

because a summation is carried out over all the quarks.

The cross section values used in the analysis by JADE are shown in fig 30 and they cover the range from $s = 144 \text{ GeV}^2$ to $s = 1350 \text{ GeV}^2$; a variation of almost a factor 10 in s . The s range is just as important as the accuracy of the measurement of R since different values of $\sin^2\theta_w$ correspond to different s dependences.

A model independent analysis can be carried out as follows: From equation (8), summing over d, u, s, c and b quarks and multiplying by a factor 3 for colour,

$$R = 3\sum_f Q_f^2 + \frac{3g_V}{2} + \frac{3g_A^2}{16}$$

$$\text{where } V = v\sum_f Q_f$$

$$W = (v^2 + a^2)\sum_f (v_f^2 + a_f^2)$$

$$\text{and now } g = -\sqrt{2}G_F/4\pi\alpha$$

Neglecting propagator effects, the above equation can be fitted to the data to determine V and W and the measurement then corresponds to a determination of these particular combinations of couplings. Including QCD effects following the prescription of Jersak et al [11], the result for V and W shown in fig 16 is obtained. Fig 16 also shows the prediction for the V - W locus from the standard model. The l.s.d. ellipse overlaps part of the standard model and hence the measured V and W agree with the calculated value for certain values of $\sin^2\theta_w$. The next stage is to determine the value of $\sin^2\theta_w$. This is done by fitting the full

Figure 15. The dependence of v^2 on $\sin^2 \theta_w$ showing how $\sin^2 \theta_w = 0.25$ arises and errors which depend on v^2 as well as the error on v^2 .

Figure 16. A model independent fit to determine the V and W combinations of the quark weak couplings.

expression for R , substituting the expressions for v_f and a_f from table 2 and including propagator effects as before. The effect of the overall normalisation uncertainty in the R values (assumed to be independent of s) is taken into account by including the normalisation as a fit parameter with measured value 1.00 ± 0.07 . The result can be expressed as an l.s.d. contour in the $\sin^2 \theta_w$ - normalisation plane - see fig 17. The fitted normalisation of 0.986 is well within the 7% uncertainty.

The fitted value for $\sin^2 \theta_w$ is

$$\sin^2 \theta_w = 0.24 \pm 0.05 \quad (d, u, s, c \text{ and } b \text{ quarks})$$

where the error contains both statistical and systematic effects. It is worth emphasising the fact that e^+e^- processes produce all quarks democratically and provide a unique opportunity to measure the couplings of s , c and b quarks.

A further source of error is in the uncertainty in α_s . Varying α_s by $\pm 30\%$ (ie $\alpha_s = 0.18 \pm 0.05$) produced a change of ± 0.03 in $\sin^2 \theta_w$.

An important conclusion is that $\sin^2 \theta_w$ has the same value for a democratic mixture of all known quarks as it does for leptons (νe and e^+e^- results) or for light quarks (νq and eq results). Moreover, the e^+e^- measurements have been made at much larger values of Q^2 than previously.

In the analysis, one can "freeze" out the d and u quarks by fixing $\sin^2 \theta_w$ for d and u quarks at the world average of 0.23 and determine the mixing angle for s , c and b quarks alone. The result in this case is

$$\sin^2 \theta_w = 0.25 \pm 0.12 \quad (s, c \text{ and } b \text{ quarks})$$

It seems that $\sin^2 \theta_w$ has just that value which makes the lepton coupling to the weak neutral current almost pure axial. This value is close to the "special" value of $\frac{1}{2}$ displayed in table 3. Whether this is an approximate coincidence or is significant, remains to be seen.

6.3 What is the mass of the Z^0 ?

There are several ways to draw conclusions about the Z^0 mass from e^+e^- data. These can be listed as follows:

1. Search for peak in energy scan data.
2. Comparison of data with multi Z (low Z mass models).
3. Deviation from linear s (propagator) dependence (Sakurai model).
4. Have faith in the standard model, then $m_Z^2 = \frac{37.32}{\sin^2 \theta_W (1 - \sin^2 \theta_W)}$

6.3.1 Energy scan

Much of the data at PETRA has been taken in small steps of beam energy, scanning for narrow peaks. The results of this scan above 30 GeV are shown in fig 18 where no significant peaks are observed. Since a Z^0 with conventional couplings would produce an R value of several thousand, this can essentially be ruled out in the energy range covered.

6.3.2 Multi gauge boson models.

The measured cross sections for lepton and hadrons final states can be compared with the predictions of the extended gauge models discussed above. In the case of the two boson models of Barger et al and de Groot et al which each predict two bosons with masses m_1 and m_2 the data is compared with theory for all possible m_1 and m_2 values. Regions of m_1, m_2 space can be excluded if the fitted χ^2 is unacceptable (eg $\chi^2 > \chi_{min}^2 + 5$) leaving a restricted region of allowed m_1 and m_2 values. Fig 19 shows the result for these two models. Disregarding the small pathological regions under the toes of the curves (where the bosons barely couple) one can say

$$\begin{array}{lcl} m_Z > 50 \text{ GeV} & \text{Barger et al} & \text{95\% CL limits} \\ m_Z > 90 \text{ GeV} & \text{de Groot et al} & \end{array}$$

In terms of the C parameter, this corresponds to $C \lesssim 0.03$ 95% CL

Figure 17. A two parameter fit to determine the overall normalisation of the JADE data and a value of $\sin^2 \theta_W$.

Figure 18. A measurement of R in fine steps of energy. No evidence is seen for a peak whereas a Z^0 with conventional couplings would give an R value of several thousand.

Figure 19. The regions of m_1 and m_2 space excluded by e^+e^- data. The shaded areas correspond to values of m_1 and m_2 still allowed by the data.

Figure 20. The q^2 dependence of $\sin^2 \theta_w$ determined in various processes.

referring to equation 12, the coefficient of the extra piece of j_{em}^2 is slightly smaller than the coefficient of j_{em}^2 in the standard model ($\sin^2 \theta_w$). Alternatively, this can be expressed as the sum rule

$$\frac{\int ds}{\int ds} \frac{\sigma(e^+e^- \rightarrow \gamma, Z, \dots, N, \text{all})}{\sigma(e^+e^- \rightarrow \gamma, Z(\text{standard}), \text{all})} - \frac{\int ds}{\int ds} \frac{\sigma(e^+e^- \rightarrow \gamma, Z(\text{standard}), \text{all})}{\sigma(e^+e^- \rightarrow \gamma, Z(\text{standard}), \text{all})} < 0.43$$

ie multigauge bosons can still contribute as much to this sum rule as the standard model alone.

6.3.3 Comparison with Sakurai model

If the complete set of JADE data on lepton and hadron final states is compared with the Sakurai model where the v and a couplings are all fixed at the low q^2 values but the m_Z is a free parameter, then $m_Z > 54$ GeV with 95% confidence. This procedure is an extended version of the method whereby the mass is deduced via propagator effects in the asymmetry A_μ .

6.3.4 m_Z in the standard model

The values of $\sin^2 \theta_w$ determined from $e^+e^- \rightarrow$ lepton pair and hadrons can be inserted in the relation $m_Z^2 = \frac{37.32}{\sin^2 \theta_w \cos^2 \theta_w}$ then:

$e^+e^- \rightarrow l^+l^-$	$77 < m_Z < 116$ GeV	95% CL
$e^+e^- \rightarrow q\bar{q}$	$78 < m_Z < 112$ GeV	95% CL

But note that this result depends heavily on the mass formula of the standard model (ie Higgs structure) and is the most model dependent result. The results of these considerations on m_Z can be summarized as follows

asymmetry which would be a factor 1.25 larger at the current highest energies than for point coupling. If the mass of the Z^0 turns out to be very large then the $\mu^+\mu^-$ asymmetry will continue to rise linearly through the PETRA/PEP energy range and will thereafter rise less rapidly than linearly, eventually falling to zero due to the weak squared term, i.e.

If the Z^0 is discovered then attention can be turned to study the detailed properties of the Z^0 (and W via $e^+e^- \rightarrow W^+W^-$), the Higgs or techniparticle structure, a program which will probably be carried out at the SLC and LEP in a few years time.

In the meantime however, measurements of polarisation effects and quark charge asymmetries are likely to provide new and interesting information.

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Method	Limit on m_Z
Direct scan	$> 36.8 \text{ GeV}$
Extended models	
$SU(2) \times U(1) \times SU(2)'$	$> 50 \text{ GeV}$
$SU(2) \times U(1) \times \tilde{U}(1)$	$> 80 \text{ GeV}$
Sakurai model	$> 54 \text{ GeV}$
Standard model	$116 > m_Z > 77 \text{ GeV}$ all lepton data
	$112 > m_Z > 78 \text{ GeV}$ Jade hadron data

Conclusions

The data from experiments at PETRA and PEP now demonstrate the existence of neutral current effects in e^+e^- processes for the first time. Traditionally, this would have been termed a violation of QED in an electromagnetic process. Information is also available on the neutral weak couplings of leptons and quarks which complements the results from neutrino and electron scattering. In the lepton sector the information derived from νe and e^+e^- processes are of comparable quality.

The important conclusion from e^+e^- data is that the range of validity of the standard model has been extended to include μ and τ leptons and the second and third generation of quarks. Furthermore, the Q^2 values at which measurements have been taken are about an order of magnitude greater than in other processes. The situation is summarised in fig 20, (borrowed from Perkins [20] with the e^+e^- data updated) which shows the Q^2 dependence of $\sin^2\theta_w$.

Although the data are consistent with the standard model, one should not lose sight of the fact that they are also consistent with the old fashioned four fermion point coupling with axial and vector couplings which happen to agree with the values predicted by the standard model. No propagator effects, which would indicate a finite range of the weak force, have yet been detected. But if the standard model is really correct, then there is a Z^0 with a mass of about 90 GeV. Therefore one would expect that the $\bar{p}p$ collider should see it soon, or that the experiments at PETRA and PEP should see propagator effects in the $\mu^+\mu^-$

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