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LIMITS ON MASSES AND COUPLINGS OF EXCITED
ELECTRONS AND MUONS

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A B S T R A C T

We compute the effect of excited leptons on the electron and muon anomalous magnetic moments. We compare the limits given by $g-2$ measurements with those obtained recently in e^+e^- collisions and with future possibilities offered by high energy projects.

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There has recently been renewed interest in the search for excited leptons due to both experimental and theoretical developments. On the one hand, high energy e^+e^- collisions at PETRA and PEP recently gave new limits ¹⁾ on masses and couplings of possible e^* and μ^* objects; estimations of the possibilities offered by future machines like LEP or SLC have also been given ^{2),3)}. On the other hand, several theoretical models ⁴⁾ suppose that leptons, quarks and even gauge bosons are composite objects made of subconstituents; in these models the existence of excited leptons appears rather natural. The fundamental unknown quantity is the magnitude of the energy scale Λ of the interactions responsible for the binding of these subconstituents. This quantity certainly controls both the level spacing between low-lying and excited states and the extension of the composite objects. It has already been noticed that $(g-2)$ measurements give strong limitations on a possible substructure of electrons and muons. Brodsky and Drell ⁵⁾ expressed this constraint in terms of the ratio m/M of composite over constituent masses. In the present note we make explicit the constraint on masses and couplings of excited e^* and μ^* due to $(g-2)$ measurements. To our knowledge, this case has not been treated. In the standard reviews ⁶⁾ one only finds the much weaker effect of heavy leptons on the photon propagator or the effect of heavy bosons coupled in vector or axial form to the leptons. Here we have to deal with derivative tensor couplings which require special treatment. We consider both tensor and pseudo-tensor couplings and the experimental limits given by $g-2$ and electric dipolar moment measurements ⁷⁾⁻⁹⁾. We then compare these limits with those presently given by e^+e^- collisions and we quickly prospect the possibilities offered by high energy projects, LEP and SLC, with e^+e^- and γe collisions.

For a real photon, the general form of the $\ell'\ell\gamma$ coupling is given by

$$L = \frac{e}{2M} \bar{\psi}_{\ell'} \sigma^{\mu\nu} F_{\mu\nu} (a - ib\gamma^5) \psi_{\ell} + h.c.$$

It generalizes the one written by Low ¹⁰⁾: C invariance would require a and b to be real, CP invariance would require a to be real and b to be purely imaginary. Having factorized $1/M$ where M is the mass of the excited lepton ℓ' , the coupling constants a and b are dimensionless. Because of its derivative form, this coupling would not lead to a renormalizable perturbation theory; it can only be considered as an effective coupling and for a virtual photon it must be complemented by form factors. For precise numerical applications, we shall use a dipolar form $\Lambda^4/(q^2 + \Lambda^2)^2$, q^2 being the virtual photon mass squared.

With this coupling the contribution of the excited lepton ℓ' to the anomalous moment of the lepton ℓ at order α is given by the seven diagrams of Fig. 1. One can easily check that gauge invariance (charge conservation) is ensured separately for each of the diagrams 1b to 1g; diagram 1a is gauge invariant when added to the self-energy corrections to the lepton ℓ in the $\gamma\ell\ell$ vertex.

We have computed these diagrams by standard methods. Their contributions to the anomalous magnetic moment and electric dipole moment of the lepton ℓ defined by

$$L = e \bar{\psi}_\ell \left(\cancel{A} + \frac{\chi}{4m} \sigma^{\mu\nu} F_{\mu\nu} - i \frac{\chi'}{4m} \sigma^{\mu\nu} F_{\mu\nu} \gamma^5 \right) \psi_\ell$$

are given by

$$\chi = \frac{\alpha}{\pi} \left\{ (|a|^2 - |b|^2) \frac{m}{M} \left[-2 I'_0 + \frac{9}{2} I'_1 - J'_0 + 3(K'_0 + H'_0) + O\left(\frac{m}{M}\right) \right] \right. \\ \left. + (|a|^2 + |b|^2) \frac{m^2}{M^2} \left[2M^2 I_2 + I'_2 + \frac{9}{2} I'_1 + 3(-J'_1 + \frac{1}{2} J'_0 + M^2 K_1 + K'_0 + H'_0) + O\left(\frac{m}{M}\right) \right] \right\}$$

$$\chi' = \frac{\alpha}{\pi} \text{Re}ab^* \frac{m}{M} \left[-4 I'_0 + 9 I'_1 - 2 J'_0 + 6(K'_0 + H'_0) + O\left(\frac{m}{M}\right) \right]$$

using integrals given in the Table.

It appears that 90% of the dominant contributions come from diagrams 1d-1g, representing ℓ - ℓ' mixing effects. We can precisely write the ℓ - ℓ' mixing coupling as $(g - ig'\gamma^5)$ with

$$g = \frac{3a\alpha}{4\pi M^2} [m M K_1 + K'_0 + H'_0]$$

$$g' = \frac{3b\alpha}{4\pi M^2} [-m M K_1 + K'_0 + H'_0] .$$

The Feynman integrals have simple expressions at the limit $M \simeq \Lambda \gg m$ in which we are interested (the discussion of the case $\Lambda \gg M \gg m$ would be similar to the one given below for the effect of flavour-changing transitions like $\mu \rightarrow e\gamma$); one gets

$$\chi \simeq \frac{4\alpha}{\pi} (|a|^2 - |b|^2) \frac{m}{M} + \frac{9\alpha}{2\pi} (|a|^2 + |b|^2) \frac{m^2}{M^2}$$

$$\chi' \simeq \frac{8\alpha}{\pi} \text{Re}ab^* \frac{m}{M} .$$

A priori, an excited lepton ℓ' could have both a and b couplings. If a and b are relatively real (C invariance) an electric dipole moment χ' (CP violating) will appear. But there are rather stringent experimental limits on χ' , especially in the case of the electron ⁹⁾:

$$d_e = \frac{\chi'_e}{2m_e} = (0.7 \pm 2.2) \times 10^{-24} \text{ cm}.$$

This means

$$\frac{Reab^*}{M_{e*}} \lesssim \frac{1}{6 \times 10^7 \text{ GeV}}.$$

which seems to practically exclude this case.

For the muon, one has ⁷⁾:

$$d_\mu = \frac{\chi'_\mu}{2m_\mu} = (3.7 \pm 3.4) \times 10^{-19} \text{ cm}.$$

i.e., a less stringent constraint

$$\frac{Reab^*}{M_{\mu*}} \lesssim \frac{1}{500 \text{ GeV}}.$$

If the excited lepton can only have an a or b sizeable coupling, this would lead to an anomalous magnetic moment at order m/M given by $\chi \simeq \frac{4\alpha}{\pi}(|a|^2 - |b|^2) \times \frac{m}{M}$.

There is also the possibility of having two (almost) degenerate excited states with opposite parities (one coupled with a and the other one with b) such that the total contribution to χ vanishes at order m/M , leaving only the contribution $\frac{9\alpha}{2\pi}(|a|^2 + |b|^2)(m^2/M^2)$.

This is an illustration of the discussion given by Brodsky and Drell ⁵⁾: without chiral symmetry the contribution appears at order m/M ; with chiral symmetry it appears only at order $(m/M)^2$. In the second case, one would have to explain the absence of states of opposite parity approximately degenerate with the low-lying electron and muon.

We now give the constraints due to the (g-2) measurements:

$$\delta\chi_e \lesssim 2 \times 10^{-10}$$

From Ref. 8)

$$\delta\chi_\mu \lesssim 8.5 \times 10^{-9}$$

From Ref. 7).

With the first hypothesis (a or b negligible) one gets :

$$\frac{|a|^2 - |b|^2}{M_{e^*}} \lesssim \frac{1}{22 \text{ TeV}}$$

$$\frac{|a|^2 - |b|^2}{M_{\mu^*}} \lesssim \frac{1}{110 \text{ TeV}}$$

With the second hypothesis ($|a| = |b|$ or two degenerate states) one gets :

$$\frac{|a|^2 + |b|^2}{M_{e^*}^2} \lesssim \left(\frac{1}{3.8 \text{ GeV}} \right)^2$$

$$\frac{|a|^2 + |b|^2}{M_{\mu^*}^2} \lesssim \left(\frac{1}{110 \text{ GeV}} \right)^2$$

In Figs. 2 and 3 we compare these constraints with the ones obtained for e^+e^- annihilation ¹⁾ :

- i) $e^+e^- \rightarrow L^+L^-$ excludes stable heavy leptons up to $M_L = 14 \text{ GeV}$;
- ii) $e^+e^- \rightarrow \mu^{+*}\mu^{-*} \rightarrow \mu^+\mu^-\gamma\gamma$ excludes μ^* up to $M_{\mu^*} = 10 \text{ GeV}$;
- iii) $e^+e^- \rightarrow \mu^*\mu \rightarrow \mu^+\mu^-\gamma$ excludes μ^* up to $M_{\mu^*} = 30 \text{ GeV}$ with couplings such that

$$\frac{|a|^2 + |b|^2}{M_{\mu^*}^2} < \left(\frac{1}{330 \text{ GeV}} \right)^2$$

[Notice that in cases ii) and iii) one supposes no appreciable form factor effect associated with the virtual photon coupling]

- iv) $e^+e^- \rightarrow \gamma\gamma$ excludes e^* exchange with mass and couplings such that :

$$\frac{|a|^2 + |b|^2}{M_{e^*}^4} < \left(\frac{1}{58 \text{ GeV}} \right)^2$$

Without chiral symmetry it appears that the (g-2) constraint is much more stringent than the other ones, especially in the muon case. With chiral symmetry, the constraints are comparable. But even in this case the g-2 constraint is still very stringent for high mass states and it is useful to keep it in mind when discussing expectations for high energy projects.

With e^+e^- collisions in the range $\sqrt{s} = 100$ to 200 GeV , one can expect to push the limits i) to iv) up to $M = 100$ to 200 GeV with a comparable sensitivity to the couplings.

We also want to notice the possibilities offered by real $\gamma e^\pm$ collisions at the SLC ^{2),3)} [search for resonance effects $\gamma e \rightarrow e^* \rightarrow (f)$] and quasi-real $\gamma e^\pm$ collisions at LEP ³⁾ ($e^+e^- \rightarrow e^\pm e^{*\mp}$ by γ exchange).

The absence of departure from QED in Compton scattering ($\gamma e \rightarrow \gamma e$) will give limits similar to those given by $e^+e^- \rightarrow \gamma\gamma$ for high mass states ($M_{e^*} \gg \sqrt{s}$) :

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}^{\text{QED}} - \frac{\alpha^2(|a|^2 + |b|^2)}{s M_{e^*}^4} (s^2 + u^2) .$$

For $\sqrt{s} = 100$ GeV a departure less than 5% at large angles would mean

$$\frac{|a|^2 + |b|^2}{M_{e^*}^4} \lesssim \left(\frac{1}{200 \text{ GeV}} \right)^4 .$$

The absence of resonant effects like $\gamma e \rightarrow e^*$ with any kind of final state $e^* \rightarrow (f)$ will give a much more stringent constraint on the couplings for $M_{e^*} = \sqrt{s} < 100$ to 200 GeV. One can demand that the totally integrated spectrum :

$$\bar{\sigma} = \int \sigma d\sqrt{s} = \frac{4\pi^2}{M_{e^*}^2} \Gamma_{e^* \rightarrow e\gamma} = \frac{2\pi^2\alpha}{M_{e^*}} (|a|^2 + |b|^2)$$

be less than some background value $\sigma_0 \Delta$; σ_0 can be taken as the value of the Compton cross-section at large angle (i.e., 50 pb at $\sqrt{s} = 100$ GeV); Δ is the energy resolution of the beams. With $\Delta = 1$ GeV one gets :

$$\frac{|a|^2 + |b|^2}{M_{e^*}} < \frac{1}{10^6 \text{ GeV}}$$

which would be a very good limit as one can see in Fig. 3.

As a final remark, it may be asked whether these $g-2$ measurements can constrain flavour-changing transitions like $\mu \rightarrow e\gamma$. In the above calculation one can replace e^* or μ^* by e, μ or τ and get constraints on the couplings a or b , i.e., on $\Gamma_{f \rightarrow l'\gamma}$, for example on $\Gamma_{\mu \rightarrow e\gamma} = (\alpha m_\mu / 2)(|a|^2 + |b|^2)$.

With the same discussion as above on the chiral properties of a and b couplings, one will get

$$\delta\chi \simeq \left(\frac{m}{M}\right)^n \left(\frac{\Lambda^2}{M^2}\right) \frac{\Gamma_{\ell \rightarrow \ell' \gamma}}{M} \quad (n=1 \text{ or } 2)$$

where Λ is the scale of the interactions responsible for the flavour changing transitions. We have also to suppose that no special cancellation will occur with other diagrams involving different fermionic and bosonic ($W^\pm, Z, \dots$) internal lines.

Experimental limits on $\delta\chi_e$ and $\delta\chi_\mu$ will then constrain $\Lambda^2 \Gamma_{\ell \rightarrow \ell' \gamma}$. In the case of $\mu \rightarrow e\gamma$, knowing already from experiment ¹¹⁾ that $\Gamma_{\mu \rightarrow e\gamma} < 0.6 \times 10^{-28}$ GeV, the constraint will be ineffectual unless $\Lambda > 10^9$ GeV. In the case of $\tau \rightarrow \mu\gamma$ (and similarly for $\tau \rightarrow e\gamma$) one gets

$$\Lambda^2 \Gamma_{\tau \rightarrow \mu\gamma} < 3 \times 10^{-7} \text{ GeV}^3$$

With $\Lambda \simeq 100$ GeV one obtains $\Gamma_{\tau \rightarrow \mu\gamma} < 3 \times 10^{-11}$ GeV ($4.5 \times 10^{13} \text{ sec}^{-1}$) which is not far from the experimental limits ¹¹⁾ $\Gamma_\tau > 4 \times 10^{11} \text{ sec}^{-1}$, $B_{\tau \rightarrow e\gamma + \mu\gamma} < 12\%$.

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TABLE : Feynman integrals

$$I_2 = \int \prod_1^6 dy_i \frac{-24\Lambda^8}{(D_1)^5} y_1(y_1+y_2)$$

$$I'_{\frac{1}{2}0} = \int \prod_1^6 dy_i \frac{12\Lambda^8}{(D_1)^4} \begin{pmatrix} 1 \\ y_1 \\ y_1(y_1+y_2) \end{pmatrix} \quad D_1 \equiv m^2(y_1+y_2)^2 + (M^2-m^2)(y_1+y_2) + \Lambda^2(y_3+y_4+y_5+y_6)$$

$$J'_{\frac{1}{2}0} = \int \prod_1^4 dy_i \frac{2\Lambda^4}{(D_2)^2} \begin{pmatrix} 1 \\ \frac{1}{2}(y_1+y_2) \end{pmatrix} \quad D_2 \equiv m^2(y_1+y_2)^2 + (M^2-m^2)y_1 + \Lambda^2(y_3+y_4)$$

$$K_1 = \int \prod_1^3 dy_i \frac{\Lambda^4}{(D_3)^2} \cdot \frac{y_1}{m^2-M^2} \quad D_3 \equiv m^2y_1 + (M^2-m^2)y_1 + \Lambda^2(y_2+y_3)$$

$$K'_0 = \int \prod_1^3 dy_i \frac{-2\Lambda^4}{D_3} \cdot \frac{1}{m^2-M^2}$$

$$H'_0 = \int \prod_1^3 dy_i \frac{-2\Lambda^4}{D_4} \cdot \frac{1}{m^2-M^2} \quad D_4 = m^2y_1^2 + \Lambda^2(y_2+y_3)$$

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FIGURE CAPTIONS

Figure 1 : Diagrams contributing to $g_{\mu}-2$ at order α .

Figure 2 : Limits on μ^* mass and couplings

- (A) is excluded by $e^+e^- \rightarrow \mu^*\mu^*$
- (B) is excluded by $e^+e^- \rightarrow \mu^*\mu$
- (C) is excluded by $g_{\mu}-2$ at order m/M
- (D) is excluded by $g_{\mu}-2$ at order m^2/M^2

Figure 3 : Limits on e^* mass and couplings

- (A) is excluded by $e^+e^- \rightarrow \gamma\gamma$
- (B) is excluded by g_e-2 at order m/M
- (C) is excluded by g_e-2 at order m^2/M^2

//// : limit expected from the absence of the e^* effect in future $\nu e^{\pm}$ collisions.

Fig. 1

Fig. 2

Fig. 3