Ref.TH.3294-CERN

DYNAMICALLY BROKEN SUPERGRAVITY AND THE HIERARCHY PROBLEM

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ABSTRACT

We propose a scenario for grand unified models based on local supersymmetry. We give arguments that condensates of strongly interacting gauge theories might break local supersymmetry. The gravitino mass induces mass splittings in the low energy theory and allows us to understand a hierarchy of $M_p = 10^{19}$ GeV to $M_w = 10^2$ GeV naturally.

Ref.TH.3294-CERN

6 May 1982

In strongly interacting globally supersymmetric ¹⁾ gauge theories condensate formation seems inevitable ^{2),3)}. This process leads to a breakdown of chiral symmetries but there are strong arguments that global supersymmetry remains unbroken ²⁾⁻⁵⁾. The reason for this amazing stability of global supersymmetry is probably linked to the rôle of the vacuum energy as an order parameter, which becomes transparent in the formulation of the index ^{4),5)}.

The situation changes completely if these theories are coupled to gravity, i.e., where supersymmetry is realized locally ⁶⁾. The coupling to supergravity can give negative contributions ^{7),8)} to the potential and the interpretation of the vacuum energy as an order parameter is lost. On the other hand we do not expect gravity to change qualitatively the dynamics of a strongly interacting gauge theory especially when the scale of this interaction is small compared to the Planck mass $M = 10^{19}$ GeV. We thus might expect condensation effects to appear as well.

Supersymmetry has to be broken in a realistic model. In the framework of global supersymmetry this would lead to a non-zero vacuum energy. Coupled to gravity this might result in an unacceptably large cosmological constant. In local supersymmetry we know, however, that supersymmetry can be broken while preserving $E_{\text{vacuum}} = 0$ ^{7),8)}. It is thus highly suggestive that recently proposed models ⁹⁾ such as "superGUTs" should in some way be incorporated in a locally supersymmetric scenario.

In this paper we make an attempt in this direction, coupling a gauge theory to $N = 1$ supergravity. We find strong evidence that condensates form in the regime of strong gauge coupling and that (contrary to the case of global supersymmetry) they might induce a breakdown of local Poincaré supersymmetry. As a result the super-Higgs effect ⁸⁾ occurs and the gravitino receives a mass. If we denote by μ the scale where the gauge interaction becomes strong, the gravitino mass in this scenario will typically be of the order

$$m_\psi \sim \mu^3 / M^2 \quad (1)$$

where M is the Planck mass. Coupled to a supersymmetric $SU(3) \times SU(2) \times U(1)$ model [or a grand unified $SU(5)$ model] this gravitino mass would be fed down to the supersymmetric partner of gauge bosons (e.g., the gluinos) through the four-fermion coupling in supergravity ¹⁰⁾. The masses of these gauge fermions will be of order

$$m \sim \frac{m_\psi}{M^2} \mu^2 \sim \mu^5 / M^4 \quad (2)$$

These masses, if for example of order 10 TeV, could lead to an acceptable low energy theory of strong and electroweak interactions ¹¹⁾ including a generation of a breakdown of $SU(2) \times U(1)$ at $M_W = 100$ GeV. With $\mu = 10^{16}$ GeV we would then naturally understand the huge ratio M_p/M_W . One might even speculate that μ coincides with the grand unification scale in an $SU(5)$ supersymmetric grand unified theory. A strongly interacting sugar-colour (sugar for supergravity) at 10^{16} GeV would thus naturally solve the puzzles of mass scales and hierarchies in physics.

We consider pure $SU(N)$ supersymmetric Yang-Mills coupled to $N = 1$ supergravity.

We will not write down the action here but will immediately go to a formulation at the effective Lagrangian level. Such a formulation has been given for pure globally supersymmetric Yang-Mills theories by the authors of Ref. 3). The Lagrangian can be represented as

$$\mathcal{L} = \int d^4\theta \left[(SS^*)^{1/3} + \frac{(SS^*)}{\mu^4} \right] + \int d^2\theta \left[S \log S/\mu^3 - S + h.c. \right] \quad (3)$$

with $S = W^\alpha W_\alpha$ and $W_\alpha = \bar{D}^2 e^{-V} D_\alpha e^V$. S is a chiral supermultiplet with components

$$S = (\lambda\lambda, \lambda \sigma_{\mu\nu} F^{\mu\nu} + \dots, F_{\mu\nu} F^{\mu\nu} + \dots) \quad (4)$$

where λ denotes the gauge fermion of $SU(N)$ and $F^{\mu\nu}$ is the field strength tensor. In this formulation it has been shown that $\lambda\lambda$ condensation occurs ³⁾ without breaking global supersymmetry. We have confidence in this description because of its success when applied to QCD ³⁾.

We now proceed and couple the chiral superfield S with action (3) to supergravity. We will do this following the pioneering work of Ref. 8) and we adopt the same notation as chosen there. The coupling is determined by one function $G(z, z^*)$ where $z = (\lambda\lambda)$:

$$G(z, z^*) = 3 \log(-\frac{\Phi}{3}) - \log\left(\frac{g_1^2}{M^6}\right) \quad (5)$$

where, deduced from (3)

$$\phi = -1 + \frac{a}{M^2 \mu^4} (zz^*) + \frac{b}{M^2} (zz^*)^{1/3} \quad (6)$$

and the superpotential is

$$g(z) = c \left(z \log^2 z / \mu^3 - z \right) \quad (7)$$

We have explicitly displayed the mass scales μ and M and we expect the coefficients a , b and c to be of order one. The potential is given by ⁸⁾

$$V = -\exp(-G) \left[3 + \frac{|G_{12}|^2}{G_{12} z^*} \right] \quad (8)$$

We want to remark here that by this choice of (6) and (7) we have not yet assumed that condensates form. It will be a question of the minimum of the potential whether $\langle z \rangle \neq 0$ or not. In order for the model to be acceptable some restrictions on ϕ and G have to be fulfilled. Reality of the vierbein requires $\phi < 0$ and for the kinetic term $M^2 G_{zz^*} |\partial_{\mu z}|^2$ to have the right sign we need $G_{zz^*} < 0$. With a cut-off at M these requirements are fulfilled for a large range of parameters $a < 0$, $b > 0$ and c . The potential (8) is a complicated function of z and z^* . To simplify the discussion we will make one adjustment of parameters :

$$b = -9a \quad (9)$$

This is an unnatural adjustment. It is equivalent (as will be seen later) to an adjustment of the cosmological constant to be zero. Since we are not aware of any theory that naturally has a vanishing cosmological constant we just remark that this problem remains. One might speculate that the dynamics provides this equality (9) if $N = 1$ supergravity is embedded in extended supergravities.

A first inspection of the potential shows that $V(z=0) = \infty$ which already indicates that $\langle z \rangle \neq 0$, i.e., $\lambda\lambda$ condensation will occur. We write

$$V(z, z^*) = f(z, z^*) B(z, z^*) \quad (10)$$

where

$$f = \frac{\exp(-G)}{G_{122}^* \phi |g|^2} \quad (11)$$

and

$$B = 3 \phi_{12} g (g_{12})^* + 3 (\phi_{12})^* g_{12} g^* - \\ - g \phi_{122}^* |g|^2 - |g_{12}|^2 \phi \quad (12)$$

f is strictly positive (remember that $\phi, G_{122}^* < 0$). A lengthy calculation shows that $B|_z = 0$ for $z_0 = \mu^3$. As a result of our choice (9), $B(z_0) = 0$. Combined with $f > 0$ these results show that for $\langle \lambda \lambda \rangle = \mu^3$ the potential has a minimum with energy zero. μ is the scale where the gauge interactions became strong and this scale still governs the scale of condensation and is not disturbed by gravitational effects. The absolute minimum of the potential is situated at $G_0|_z = 0$ and has negative energy. This does, however, not imply that the vacuum at $z_0 = \mu^3$ with $E = 0$ is unstable ¹²⁾. We thus might assume, following the arguments of Ref. 12) that the stable vacuum with $E = 0$ was chosen in the history of the early Universe ¹³⁾

The vacuum energy is zero, but this does not imply exact local supersymmetry. A more appropriate order parameter here is the gravitino mass. This is true since $E_{\text{vacuum}} = 0$ ^{7), 14)}, otherwise one has to be careful with the interpretation of a gravitino mass term. The absence of a cosmological term and the appearance of Poincaré (not de-Sitter) supersymmetry in the global limit is most elegantly expressed by ¹⁴⁾

$$\langle u \rangle = -\frac{g}{2} \frac{g_0^*}{\phi_0} \left[1 + \frac{3 |\phi_{0/2}|^2}{\phi_0^2 G_{0/22}^*} \right] = 0 \quad (13)$$

which can be verified in our case. Therefore the gravitino mass term

$$m_{\psi}^2 = M^2 \exp(-G_0) \quad (14)$$

is physical. The super-Higgs effect took place and the fermionic partner $\lambda \sigma^{\mu\nu} F_{\mu\nu}$ of $\lambda \lambda$ is absorbed by the gravitino. This can be independently observed by an inspection of the supersymmetry transformation ⁸⁾ on $\chi = \lambda \sigma^{\mu\nu} F_{\mu\nu}$

$$\delta \chi = 2 \exp(-G_0/2) \frac{G_{0/2}^*}{\sqrt{-2 G_{0/22}^*}} \epsilon + \dots \quad (15)$$

where the term on the right-hand side is related to the auxiliary field of the chiral supermultiplet. Combining (5)-(7) with (14) for $z_0 = \mu^3$ we arrive at a gravitino mass of the order of magnitude given in (1), since $|g_0|^2 \sim \mu^6$ and ϕ_0 is of order one. The masses of the scalars z, z^* can only be obtained after a redefinition of the kinetic terms which we will not perform here, since it is of no importance to the calculation of the gravitino mass ¹⁴⁾.

An exciting aspect of this scenario appears when this model is coupled to additional gauge interactions, e.g., QCD. The QCD coupling constant we assume to be weak compared to the sugar-colour coupling, such that the condensation process is not disturbed. The four-fermion coupling of the gluinos (g) to the gravitino ^{10),15)}

$$\frac{1}{M^2} \bar{g} g \bar{\psi}_\mu \psi^\mu \quad (16)$$

will induce a gluino mass term ¹⁵⁾

$$m_g \approx \frac{m_\psi}{M^2} \int_0^\Lambda \frac{d^4 p}{p^2 - m_\psi^2} \quad (17)$$

where Λ is a cut-off. A possible choice of this cut-off would be M , but since the gravitino mass has its origin in a condensation process at μ , $\Lambda = \mu$ seems to be a more appropriate choice. This leads to

$$m_g \approx \mu^5 / M^4 \quad (18)$$

Gluino masses of order 10 TeV would thus be induced if the sugar-colour scale is 10^{16} GeV.

We could now imagine a model based on $SU(N)_{SC} \times SU(5)$ coupled to supergravity. Sugar-colour becomes strong at 10^{16} GeV whereas $SU(5)$ has the usual grand unified coupling. Sugar-colour condensation might break supergravity as well as $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. [Of course, in this case we need sugar-colour multiplets which, in addition, have $SU(5)$ quantum numbers.] The gravitino receives a mass $m_\psi \approx 10^{10}$ GeV. This mass will be fed down to the fermionic partners of the $SU(3) \times SU(2) \times U(1)$ gauge bosons. The mass splittings of 10 TeV between the gauge bosons and the gauginos will induce a mass for the scalar partners of quarks and leptons at the two-loop level of order

$$\left(\frac{\alpha}{\pi}\right) m_g \sim (100 - 1000) \text{ GeV} \quad (19)$$

where α is one of the $SU(3) \times SU(2) \times U(1)$ couplings. Higgs bosons receive a negative mass-square due to the Yukawa interactions¹¹⁾ and $SU(2) \times U(1)$ is broken to $U(1)$ at a scale of 100 GeV. In this scenario the mass scales are understood in a natural way from the Planck scale, without unnatural adjustments of parameters and omissions of coupling constants. The input parameters are the sugar-colour and $SU(5)$ couplings at the Planck scale. A word of caution should be added here. We have argued that condensates might break local supersymmetry but not proved it. There was an unnatural adjustment of the cosmological constant and we have assumed that the vacuum with $E = 0$ was chosen in the history of the early Universe, and these questions have to be clarified. Optimistically one might hope these questions will be resolved by embedding $N = 1$ supergravity in extended supergravities.

Independently of the toy model presented in this paper, the general scenario of broken supersymmetry at the grand unification scale μ and a gravitino mass suppressed by powers of M is very appealing. It provides us with a solution of the hierarchy problem of understanding M_W in terms of μ and M .

ACKNOWLEDGEMENTS

I would like to thank S. Ferrara, L. Girardello, G. Veneziano and S. Yankielowicz for valuable discussions.

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