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ON THE WIDTH OF Σ HYPERNUCLEI AND Σ^- ATOMS

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A B S T R A C T

The self-energy of a Σ particle in symmetric nuclear matter is investigated by using a boson exchange model for the ΣN interaction. The imaginary part of the self-energy is shown to be strongly density dependent and has saturation properties around $\rho \approx 0.7 \rho_0$. This peculiar feature allows our results to be in good agreement with the Σ atom data while simultaneously providing a width of about 5 MeV or smaller for all nuclei and different states.

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With the first experiments on Σ hypernuclei showing a narrow width ($\Gamma < 8$ MeV for ${}^9_{\Sigma}\text{Be}$)¹⁾, much lower than the semi-classical estimates based on the mean free path for the reaction $\Sigma N \rightarrow \Lambda N$ ²⁾, a great amount of theoretical work has been stimulated in order to understand this peculiar feature²⁾⁻⁷⁾. More recent experiments in ${}^6_{\Sigma}\text{H}$ ⁸⁾ and ${}^{12}_{\Sigma}\text{C}$, ${}^{16}_{\Sigma}\text{O}$ ⁹⁾, where the widths of the observed states are systematically around 4-5 MeV, have come to confirm these first findings. On the other hand, the phenomenological studies of Σ^- atoms¹⁰⁾ suggest an optical Σ nucleus potential given by :

$$V(r) = -U(r) - iW(r)$$

with

$$U(r) = (28 \pm 3) \rho(r) / \rho_0 \text{ [MeV]}$$

and

$$W(r) = (15 \pm 2) \rho(r) / \rho_0 \text{ [MeV]}_{(1)}$$

where $\rho(r)$ is the nuclear density and $\rho_0 = 0.17 \text{ fm}^{-3}$ the normal nuclear matter density. The width of a particular bound nuclear state is then given by the expression

$$\Gamma = \int |\phi(r)|^2 2W(\rho(r)) d^3r \quad (2)$$

where the nuclear wave function $\phi(r)$ is normalized to unity. In view of the values of W in (1), it is easy to understand the difficulties in getting small numbers for the width of the nuclear states. Gal et al.³⁾ have made an extensive study of different hypernuclei by using the potential in Eq. (1), and the calculated widths are always bigger than 20 MeV for the 1s states, with values around 10 MeV for the 1p state of light nuclei like ${}^{12}\text{C}$ or ${}^{16}\text{O}$.

The present trend of widths of about 5 MeV or smaller, for the different nuclei analyzed so far, cannot be understood in terms of the potential (1), which is adjusted to reproduce the levels and widths of the Σ^- atomic states. Some hints have been directed at a different density dependence than the one assumed in Eq. (1)^{5),6)}, or to the energy dependence^{5),6)} or non-localities³⁾ of the optical potential.

A direct way to evaluate this optical potential in nuclear matter consists in determining the Green's function for a Σ particle that propagates in the medium, since the Σ self-energy gives us directly the optical potential. The Σ self-energy is an explicit function of the Σ energy and momentum as well as of the nuclear density, hence careful study of this magnitude can

provide an answer to these questions and eventually solve the apparent contradiction between the atomic and nuclear data. This is the purpose of the present work.

The Σ decays much faster inside the nucleus than in free space because in its interaction with the nucleons there is a channel open for the $\Sigma N \rightarrow \Lambda N$ reaction. This can be brought diagrammatically into a Σ self-energy many-body diagram shown in Fig. 1a, where, for the moment, the wavy line symbolically stands for the $\Sigma N \rightarrow \Lambda N$ interaction. The imaginary part of the self-energy comes from having the intermediate states, the Λ , the nucleon-particle and the nucleon-hole on their mass shell. This can easily be done since there are about 80 MeV ($M_\Sigma - M_\Lambda$) available to throw the Λ in the continuum and produce particle-hole excitations in the medium.

The next step is to make a model for the $\Sigma N \rightarrow \Lambda N$ interaction. For this purpose we have used the boson exchange model of the Nijmegen group ¹¹⁾. This model is particularly simple in the present case since the Σ has isospin $T = 1$ and Λ has $T = 0$. Thus the interaction must be mediated by a $T = 1$ meson, in particular the π and ρ mesons. The presence of a long-range force due to one-pion exchange introduces some peculiar features into the problem. Indeed, if we erase the straight baryon line in Fig. 1a, we are left with the diagram that describes the renormalization of a pion in a nuclear medium, but just at the lowest order. The full response of the medium to the pion field requires instead the iterated series that comes from Fig. 1b after having removed the straight baryon line ¹²⁾⁻¹⁶⁾. A recent review on these pionic modes of excitation in nuclear matter and finite nuclei can be seen in Ref. 17).

In Fig. 1b, the double lines stand for $\Delta(\frac{3}{2}, \frac{3}{2})$ isobars, which play an important rôle in the pion propagation, although in this case the Δ -h excitation would not contribute directly to the Σ width. The mechanism of Fig. 1b, with the spin-isospin dependent part of the interaction that we need here, conveniently adapted for the case of the nucleon self-energy, plays an essential rôle in adequately describing the energy dependence of the nucleon effective mass ^{18), 19)}. There, in addition to the bare π and ρ exchange, a correlation function is introduced that modulates the π and ρ exchange, giving rise to an effective interaction between nucleons of the following type :

$$\tilde{V}(\vec{q}) = \left[\tilde{V}_{ij}^{\pi}(\vec{q}) + \tilde{V}_{ij}^{\rho}(\vec{q}) \right] \sigma_i^{(1)} \sigma_j^{(2)} \vec{\tau}_i^{(1)} \vec{\tau}_j^{(2)} \quad (3)$$

where

$$\begin{aligned}\tilde{V}_{ij}^{\pi}(\vec{q}) &= \frac{f^2}{\mu^2} \left\{ q_i q_j D_0^{\pi}(q) F_{\pi}^2(q^2) - (q_i q_j + \frac{1}{3} q^2 \delta_{ij}) \tilde{D}_0^{\pi}(q) \tilde{F}_{\pi}^2(q^2) \right\} \\ \tilde{V}_{ij}^{\rho}(\vec{q}) &= \frac{f^2}{\mu^2} C_{\rho} \left\{ \left[\frac{2}{3} \vec{q}^2 \delta_{ij} - (q_i q_j - \frac{1}{3} \vec{q}^2 \delta_{ij}) \right] D_0^{\rho}(q) F_{\rho}^2(q^2) \right. \\ &\quad \left. - \left[\frac{2}{3} (\vec{q}^2 + q^2) \delta_{ij} - (q_i q_j - \frac{1}{3} \vec{q}^2 \delta_{ij}) \right] \tilde{D}_0^{\rho}(q) \tilde{F}_{\rho}^2(q^2) \right\}\end{aligned}\quad (4)$$

q_c is a characteristic correlation parameter, which we take to be equal to the ω meson mass, C_{ρ} is the ratio

$$C_{\rho} = \frac{f_{\rho}^2}{m_{\rho}^2} : \frac{f^2}{\mu^2} \quad (5)$$

and $D_0^{\pi, \rho}(q)$, $F_{\pi, \rho}(q^2)$ are the pion (ρ meson) propagators and form factors, respectively

$$D_0^i(q) = \frac{1}{q^2 - \vec{q}^2 - m_i^2 + i\epsilon} \quad (6)$$

$$F_i(q^2) = \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q^2} \right) \quad (7)$$

The form factors have been chosen of the monopole type with values of Λ_i suited to the description of pion absorption²⁰⁾, np and $\bar{p}p$ charge-exchange scattering²¹⁾, pion-nucleus scattering¹⁵⁾, etc. The values of the coupling constants and form factor parameters have been chosen as in Refs. 15) and 18) with $C_{\rho} = 2$, which implies a spin-spin correlation parameter $g' \approx 0.6$.

In the present case we also need the effective interaction between $\pi N - \pi N$, which we have taken from Ref. 11), and put in the form of (3) and (4) by assuming a similar correlation function. The values of the coupling constants are taken as :

$$\begin{aligned}\frac{f_{\pi\pi\pi}^2}{4\pi\mu^2} &= \frac{f^2}{4\pi\mu^2} \cdot \frac{2}{9} \\ \frac{f_{\pi\pi\rho}^2}{4\pi m_{\rho}^2} &= \frac{f_{\rho}^2}{4\pi m_{\rho}^2} \cdot \frac{1}{2}\end{aligned}\quad (8)$$

Consequently, the NN effective interaction can be cast in a form

$$\tilde{V}_{ij}^{NN}(\vec{q}) = \tilde{V}_{ij}^{\pi}(\vec{q}) + \tilde{V}_{ij}^{\rho}(\vec{q}) \equiv a \delta_{ij} + b \vec{q} \cdot \vec{q}_j \quad (9)$$

where a and b can be immediately deduced from (4). Analogously we will have

$$\tilde{V}_{ij}^{YN}(\vec{q}) = a' \delta_{ij} + b' \vec{q} \cdot \vec{q}_j \quad (10)$$

where Y stands for the $\Sigma\Lambda$ vertex of the strange particles. Hence the induced interaction of Fig. 1b will be written in terms of a geometric series

$$\tilde{V}_{ie}^{YN} \mathcal{U} \tilde{V}_{ej}^{YN} + \tilde{V}_{ie}^{YN} \mathcal{U} \tilde{V}_{em}^{NN} \mathcal{U} \tilde{V}_{mj}^{YN} + \dots \quad (11)$$

where $\mathcal{U}$ is the Lindhard function accounting for p-h and Λ -h excitation

$$\mathcal{U}(q) = \mathcal{U}_N(q) + \mathcal{U}_\Delta(q) \quad (12)$$

By carrying out the sum in (11) and the spin, isospin sums, we can write an expression for the Σ self-energy

$$\Sigma^*(K) = i \delta_{ss'} \int dt \epsilon' \int \frac{d^4 q}{(2\pi)^4} \frac{1}{k^0 - q^0 - \epsilon(\vec{k} - \vec{q}) + i\epsilon} M(q^0, \vec{q}) \quad (13)$$

where

$$M(q^0, \vec{q}) = \frac{\mathcal{U}}{1 - \mathcal{U}_a - \mathcal{U}_b \vec{q}^2} \left\{ 3a'^2 + 2a'b' \vec{q}^2 + b'^2 \vec{q}^4 - \frac{2a'^2 b \vec{q}^2 \mathcal{U}}{1 - \mathcal{U}_a} \right\} \quad (14)$$

In (13), s, s' and t, t' are the spin and isospin of the initial and final Σ states, $\epsilon(\vec{k} - \vec{q})$ the kinetic energy of the intermediate Λ particle and $k^0, \vec{k}$ the energy and trimomentum of the Σ particle (k^0 contains the additional 80 MeV of the mass difference between the Σ and the Λ).

In order to carry out the q^0 integration, special care has to be taken of the analytical structure of the integrand. There is a cut in the real q^0 axis with the function continuous in the upper half plane for $\text{Re } q^0 > 0$

and in the lower half plane for $\text{Re } q^0 < 0$, plus the pole of the Λ propagator at $q^0 = k^0 - \epsilon(\vec{k}-\vec{q}) + i\epsilon$. The special analytical structure suggests a Wick rotation to evaluate the q^0 integral ¹⁸⁾ with the result

$$\begin{aligned} \Sigma^*(K) = & -\delta_{ss'}\delta_{tt'} \frac{1}{(2\pi)^2} \int_0^\infty q^2 dq \times \\ & \times \left[\int_0^\infty \frac{dx}{2\pi} \frac{m_\Lambda}{Kq} \ln \frac{(K^0 - (K-q)^2/2m_\Lambda)^2 + x^2}{(K^0 - (K+q)^2/2m_\Lambda)^2 + x^2} M(q^0, \vec{q}) \right]_{q^0=i\epsilon} \\ & - \int_{-1}^1 d\cos\phi \theta(K^0 - \epsilon(\vec{K}-\vec{q})) M(q^0, \vec{q}) \Big|_{q^0=K^0 - \epsilon(\vec{K}-\vec{q})} \end{aligned} \quad (15)$$

where ϕ is the angle between $\vec{k}$ and $\vec{q}$. In order to calculate the lowest order, shown in Fig. 1a, one can use the same formula by simply setting $a = b = 0$ in Eq. (14).

Equation (15) shows the $k^0, \vec{k}$ dependence of the self-energy, while the density dependence is implicit in the Lindhard function $\mathcal{U}$.

We have carried out the integrations in (15) by keeping in mind three different physical situations :

- a) the hypernuclear p state for a nucleus like ^{12}C or ^{16}O ,
- b) the hypernuclear s state for the same nuclei, and
- c) the atomic Σ state.

In case a), with the p state basically with zero energy, we take $k^0 = 80$ MeV by setting the origin of energies in this baryon line at the mass of the Λ . A typical momentum for the p shell, $k = 180$ MeV, is taken in this case. In case b), we assume a binding of 15 MeV to the Σ state and thus $k^0 = 65$ MeV, while $k \approx 140$ MeV. Finally, in case c), $k^0 = 80$ MeV and k is set equal to zero.

The results can be seen in Figs. 2 and 3 for the lowest order and the full induced interaction. Both are calculated for case a). The real part of Σ^* is negative, which corresponds to attraction, and the imaginary part, also negative, is related to the width by means of Eq. (2). The real part is almost a linear function of the density while the imaginary part shows a stronger divergence from a linear function in ρ . This reveals the effect of the exclusion principle in the intermediate nucleon states, a feature also found in Refs. 3), 5) and 6). Here the exclusion principle is taken into account in an exact way by means of the Lindhard function.

When introducing the full induced interaction of Fig. 1b, there are some drastic changes. The real part shows a small reduction, for densities around ρ_0 , with respect to the lowest order results. But the effects on the imaginary part are really dramatic. At densities lower than $\rho = 0.2 \rho_0$, it starts deviating from a linear function and around $\rho = 0.7 \rho_0$ it shows saturation up to $\rho = \rho_0$ and beyond. At $\rho = \rho_0$, there is a reduction factor of nearly 3 with respect to the lowest order results and about a factor 4 from the linear extrapolation from low densities in this same calculation. In order to interpret this strong reduction factor, one has to look at Eq. (15), where only the second term contributes to the imaginary part of Σ^* , the first one providing only a real background. The most important change from the lowest order to the full calculation is the appearance of the quantity $1 - U_a - U_b \vec{q}^2$ in the denominator of $M(q^0, \vec{q})$ in (14). For the regime of $q^0, \vec{q}$ relevant to the problem, the real part of the Lindhard function is negative and this factor, playing the rôle of a medium polarization for the meson fields, effectively reduces the strength of the interaction.

With respect to the p state, the results for Γ in Σ atoms are very similar, but in the plateau the value of Γ is about 7.5 MeV instead of 6 MeV for the p state. For the s state, similar results are obtained with a value in the plateau of 5.4 MeV, showing a slight reduction due to the smaller phase space because of the binding. By assuming the Σ wave functions for the s and p states in $^{16}_\Sigma O$ and $^{12}_\Sigma C$ to be similar to those of the corresponding nucleons, we can compute the width of the hypernuclear states by means of (2), where one is making implicit use of the local density approximation.

The results are as follows :

$$^{16}O : \quad \begin{array}{ll} s\text{-state} & \Gamma = 4.97 \text{ MeV} \\ p\text{-state} & \Gamma = 4.65 \text{ MeV} \end{array}$$

$$^{12}C \quad \begin{array}{ll} s\text{-state} & \Gamma = 4.90 \text{ MeV} \\ p\text{-state} & \Gamma = 4.48 \text{ MeV} \end{array}$$

(16)

The saturation properties of $\Gamma(\rho)$ as a function of ρ , with Γ approximately constant in a wide range of densities, has the immediate consequence that the widths are very much similar for all nuclei and different nuclear states.

For nuclei lighter than those quoted in (16), we would get smaller widths, and for larger nuclei, values around 5 MeV. This is clearly in good agreement with the present experimental data, but more of these data are necessary in order to clarify the theoretical panorama. Indeed, in Ref. 2), Γ for the s states has the range ~22-27 MeV, while for the p states Γ goes from ~10 MeV for $^{12}_{\Sigma}\text{C}$ up to ~20 MeV for $^{40}_{\Sigma}\text{Ca}$. In Ref. 3), Γ for the p states of nuclei lighter than $^{16}_{\Sigma}\text{O}$ is smaller than 5 MeV, while for the s states it ranges from 2 MeV for $^5_{\Sigma}\text{He}$ up to 12 MeV for $^{16}_{\Sigma}\text{O}$. The p states calculated in these references have a tendency to be slightly unbound and this helps in generating smaller widths. As shown before, for bound p states, as well as for the more bound s states, the predictions here are always around 5 MeV or smaller.

At first sight there seems to be a contradiction between our results and the empirical estimates of Eq. (1) for the Σ atoms. Indeed, at $\rho = \rho_0$ we get $\Gamma = 7.5$ MeV, while Eq. (1) would give $\Gamma = 30 \pm 4$ MeV, a factor four larger. This comparison is nevertheless inadequate since the atomic wave function only sees the surface of the nucleus where the nuclear density is smaller. For atomic states with $\ell \geq 3$, as in Ref. 10), the atomic wave function goes like $r^{\ell+1}$ at small distances, and the overlap of the nuclear and atomic densities has a peak far outside the nucleus, in a region where the nuclear density is around $\rho \approx 0.1 \rho_0$ or less. It is clear then that by studying the Σ atoms one is only probing the region of small densities around $\rho \approx 0.1 \rho_0$. In this region our results give a nearly linear dependence on the density. Hence the atomic data are only sensitive to the slope of the curve in the origin. If we extrapolate our curve linearly from the origin, we get, for the imaginary part, $\Gamma^{\text{ext}}(\rho) \equiv 2W^{\text{ext}}(\rho) = 24 \rho/\rho_0$ [MeV], only 10% below the allowed region from the empirical estimates. Due to uncertainties in the coupling constants themselves, we cannot claim a precision of less than 10%. Therefore, we should consider the results here in good agreement with the empirical data for Σ atoms. The calculations have also been performed for SU(6) couplings rather than the standard ones. The changes are of the order of 10%. Other possible sources of imaginary parts coming from exchange terms apart from those considered here, like $\Sigma N \rightarrow N \Lambda$, have also been investigated, but they are negligible since they involve the exchange of kaons which have a smaller coupling to the baryons in this case.

To summarize, we have shown that highly non-linear effects (in the nuclear density) that could be traced back to the meson propagation inside nuclear matter, are responsible for a density dependence of Γ that shows saturation around $\rho \approx 0.7 \rho_0$. Such a dependence was essential here to provide simultaneously a width of around 5 MeV, or less, for all nuclei and different states, while still being in

agreement with the empirical data from Σ^- atoms. Although we have seen that the only region of interest for the atoms is the one at very low densities, we can suggest new parametrizations of the imaginary part showing saturation, as we have found in this work. A good parametrization to our results can be obtained with the following formula

$$\begin{aligned} \Gamma(\rho) &= \Gamma(0.7\rho_0) \frac{\arctan(3.7\rho/\rho_0)}{\arctan(3.7 \times 0.7)} , & \rho \leq 0.7\rho_0 \\ \Gamma(\rho) &= \Gamma(0.7\rho_0) , & \rho > 0.7\rho_0 \end{aligned}$$

with $\Gamma(0.7\rho_0) = 7.5 \text{ MeV}$

(17)

while for the p state hypernuclear case a similar good fit can be obtained by

$$\begin{aligned} \Gamma(\rho) &= \Gamma(0.7\rho_0) \frac{\arctan(5\rho/\rho_0)}{\arctan(5 \times 0.7)} , & \rho \leq 0.7\rho_0 \\ \Gamma(\rho) &= \Gamma(0.7\rho_0) , & \rho > 0.7\rho_0 \end{aligned}$$

with $\Gamma(0.7\rho_0) = 6.03 \text{ MeV}$

(18)

Although we have not attempted to construct a complete model for the real part of the Σ potential, it is nonetheless interesting to quote some features of the real part obtained here. For Σ atoms it is a very good approximation to assume the real part linear in ρ , especially when only the part of small ρ where the linear approximation is excellent, is effective in this case. For Σ hypernuclei we have instead found a better approximation by means of the formula

$$Re \Sigma(\rho) = Re \Sigma(\rho_0) \left(\frac{\rho}{\rho_0} \right)^{1 - 0.25 \rho/\rho_0}$$

with $Re \Sigma(\rho_0) = -12.86 \text{ MeV}$

(19)

It is clear that some other pieces which contribute to the real part are missing, among them the Hartree terms which would give a dependence linear in ρ .

Another interesting consequence of this work is that if one goes to a sufficiently heavy Σ^- atom, where the overlap of the Σ^- with the nucleus is larger, one might then explore a region of larger ρ in the nucleus. Then the widths should become smaller than those predicted with an optical potential, with the imaginary part proportional to ρ , which fits the lighter nuclei. It would be very interesting to contrast these predictions with experiment. The fact that such an effect has been observed in heavy pionic atoms²²⁾ should stimulate the measurements of these heavy Σ^- atoms.

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(a)

(b)

- Figure 1 -

- Figure 2 -

- Figure 3 -