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CORRECTIONS TO THE MEAN-FIELD APPROXIMATION
FOR LATTICE GAUGE THEORY

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ABSTRACT

Corrections to the mean-field approximation for lattice gauge theories are studied. For the pure gauge system we develop a method of computing these corrections which employs collective coordinate techniques and a convenient background gauge to deal with zero-frequency modes. It is shown that the corrections can be organized in a $1/d$ expansion, d being the dimensionality of space-time. The calculations are explicitly carried out for a theory with $U(1)$ symmetry, for definiteness. We further consider the gauge field in interaction with massless dynamical Susskind fermions. The problem of dynamical generation of fermion mass is discussed, as well as the shift in the deconfinement point due to fermion effects.

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1. - INTRODUCTION AND SUMMARY

Mean-field methods were applied to lattice gauge theories soon after the latter were born^{1),2)}. Wilson¹⁾ predicted that in general these theories undergo a first-order phase transition separating a strong-coupling disordered phase from a weak-coupling ordered one, where the lattice gauge field $U_{x,\mu}$ acquires a non-vanishing vacuum expectation value. However, this approach was abandoned for some time because it seems to contradict Elitzur's theorem³⁾, which states that local gauge invariance cannot be spontaneously broken (unless it is explicitly broken by a gauge-fixing term).

More recently, Drouffe⁴⁾ and Brezin and Drouffe⁵⁾ have emphasized the fact that the mean-field approximation corresponds to a saddle-point evaluation of the (properly rewritten) path integral of the theory. This approach has two major advantages: it opens the way to the calculation of corrections to the mean-field approximation, and it indicates how the mean-field approach can be reconciled with Elitzur's theorem. Indeed, it follows from gauge invariance that if some non-trivial saddle-point value of the gauge field is obtained, all its gauge orbits are also degenerate saddle-points: they contribute with equal weight to the path integral. This will obviously make the vacuum gauge invariant and give a vanishing vacuum expectation value to gauge-invariant operators such as $U_{x,\mu}$ itself.

The purpose of this paper is to discuss the corrections to the mean-field approximation. For continuous gauge groups, the existence of degenerate saddle-points in the non-confining phase implies the presence of zero-frequency modes which prevent one from making a naive loop expansion. These zero-frequency modes simply express the fact that fluctuations around the saddle-point which correspond to a gauge transformation have no restoring force. Previous authors^{6),7)} have dealt with this problem by first fixing the gauge in the original path integral (the axial gauge is the one which is most convenient for this procedure) and then looking for a saddle-point and computing corrections to it. Since gauge invariance is explicitly broken by gauge fixing, there are no degenerate saddle-points in the weak-coupling phase. However, this procedure of axial gauge fixing leads anyway to a mode of frequency of order $1/d$ and the $1/d$ nature of the resulting expansion is not evident.

We have adopted a different approach to this problem. The gauge is not fixed to start with, and we deal explicitly with the degenerate saddle-points by using collective-coordinate methods^{8),9)}. That is to say, perturbation theory is performed only in the genuine Gaussian fluctuations, and the collective co-ordinates arising from the gauge degrees of freedom are integrated exactly. This last

integration restores gauge invariance in the ordered phase. Genuine Gaussian fluctuations satisfy a background gauge condition, which is just an orthogonality condition to the gauge symmetry degrees of freedom⁹⁾. Corrections to the mean-field approximation can then be organized in a $1/d$ expansion, d being the dimensionality of space-time.

We have also studied the corrections to the free energy arising from the coupling of the gauge field to massless Susskind fermions¹⁰⁾, for the following reason: Susskind fermions decouple when $d \rightarrow \infty$ and consequently give contributions to the free energy which are of the same order as the first correction to the mean-field approximation. One can hope to acquire, in this way, some qualitative understanding of the effects of dynamical fermions in the phase diagram of lattice gauge theories.

This paper is organized in such a way that the two subjects discussed, namely a) corrections to the mean-field approximation in the pure gauge case and b) the effects of Susskind fermions and their dynamical mass generation in mean-field theory, can be read independently. In Section 2 we briefly review the mean-field approach to the pure gauge theory and also give a qualitative discussion of our way of incorporating dynamical fermions in the mean-field picture. Section 3 deals with our method of computing corrections to the mean-field result in the pure gauge case. Explicit calculations are carried out for simplicity for the $U(1)$ gauge group, but the generalization to any other continuous gauge group is straightforward. Numerical results for $U(1)$ are given in Section 4.

Section 5 is devoted to the inclusion of dynamical Susskind fermions in the mean-field picture. Our way of dealing with this problem is such that the known strong coupling results¹¹⁾⁻¹³⁾ are guaranteed from the start. The fermion corrections to the mean-field free energy are computed in the case of $U(1)$, and the dynamical mass generation is discussed. For $d \gg 1$, it occurs in and only in the confining phase. It seems, however, that at $d = 4$, the $q\bar{q}$ condensate may survive in spite of quarks being deconfined, but in any case it will eventually disappear as the coupling is further decreased. A firm conclusion on this point would require the calculation of higher order $1/d$ effects. We have also calculated, always for $U(1)$ theory, the shift of the deconfinement points due to fermion effects.

2. - STATEMENT OF THE MODEL AND THE MEAN-FIELD APPROXIMATION

In this paper we shall be dealing with lattice gauge fields coupled to massless Susskind fermions. Our conventions are as follows: we set the lattice spacing a equal to 1 and label sites by a d dimensional vector x whose components $(x_1, \dots, x_d)$ are integers. Links are labelled by the site x to which they are attached and a unit vector in the positive μ direction, to be denoted by $\hat{\mu}$. The partition function we consider is

$$Z = \int \prod_{x, \hat{\mu}} d\mu(U_{x, \hat{\mu}}) \prod_x d\chi_x d\bar{\chi}_x \exp[S_G + S_F] \quad (2.1)$$

where d_μ is the Haar measure over the gauge group, S_G the Wilson action

$$S_G = \beta \sum_{x, \hat{\mu} \neq \hat{\nu}} \text{Tr} [U_{x, \hat{\mu}} U_{x+\hat{\mu}, \hat{\nu}} U_{x+\hat{\nu}, \hat{\mu}}^+ U_{x, \hat{\nu}}^+ + \text{h.c.}] \quad (2.2)$$

and S_F the fermionic action corresponding to massless Susskind fermions

$$S_F = \sum_{x, \hat{\mu}} \frac{1}{2} \gamma_\mu(x) \{ \bar{\chi}(x+\hat{\mu}) U_{x, \hat{\mu}}^+ \chi(x) - \bar{\chi}(x) U_{x, \hat{\mu}} \chi(x+\hat{\mu}) \} \quad (2.3)$$

where

$$\gamma_\mu(x) = (-1)^{x_1 + x_2 + \dots + x_{\mu-1}} \quad (2.4)$$

Notice that $\chi(x)$ and $\bar{\chi}(x)$ are Grassmann variables which carry colour (and possibly flavour) but no spin indices. It is well known that because of the spectral multiplication problem, the naive Wilson action describes 2^d degenerate quark states. Because of the fact that in the Susskind action, (2.3), there is only one spin degree of freedom per site, this action describes only $2^{d/2}$ degenerate quark states. It turns out that this reduction in the number of degrees of freedom is exactly what one needs to get rid of unwanted multiplicity factors that would spoil the convergence of the loop expansion for fermions, and to check that, as is known to happen in the strong-coupling limit¹⁴⁾, fermion effects can be systematically computed in a $1/d$ expansion. This explains our choice of Susskind fermions.

Let us now turn to the mean-field approximation and forget for the moment the fermion action in Eq. (2.1). The starting point of the mean-field approach is to decouple the group variables $U_{x, \mu}$ in the Wilson action by replacing the

plaquette interaction by the interaction of $U_{x,\hat{\mu}}$ with a random external field $A_{x,\hat{\mu}}$ at each link. This is achieved [for $U(1)$ theory, generalization is obvious] by multiplying the integrand in Eq. (21) by

$$1 = \int \prod_{\ell} dV_{\ell} dV_{\ell}^* \delta(U_{\ell} - V_{\ell}) \delta(U_{\ell}^* - V_{\ell}^*) =$$

$$= \int \prod_{\ell} \frac{dV_{\ell} dV_{\ell}^* dA_{\ell} dA_{\ell}^*}{16\pi^2} \exp \frac{i}{2} \left\{ \sum_{\ell} A_{\ell}^* (U_{\ell} - V_{\ell}) + \text{c.c.} \right\} \quad (2.5)$$

and replacing U_{ℓ} by V_{ℓ} in the Wilson action. Here $\ell = (x, \hat{\mu})$ is a shorthand notation for a link. Since $U_{x,\hat{\mu}}$ variables are now decoupled, the integration over the Haar measure in (2.1) can be easily done. Defining

$$\omega(AA^*) = \log \int d\mu(U) \exp \frac{i}{2} \{ A^* U + A U^* \} \quad (2.6)$$

Eq. (2.1) in the absence of fermions becomes

$$Z = \int [dV dA] \exp \left\{ S_0[V] - \frac{i}{2} \sum_{\ell} (A_{\ell}^* V_{\ell} + A_{\ell} V_{\ell}^*) + \sum_{\ell} \omega(A_{\ell}^* A_{\ell}) \right\} \quad (2.7)$$

The mean-field approximation consists of the computation of this path integral by a saddle-point technique. This result is exact as $d \rightarrow \infty$, and we shall be interested in computing $1/d$ corrections to it. But let us first come back to the fermion problem and discuss the way of incorporating them into the mean-field picture. One obvious thing to do would be to integrate them explicitly from the start and define an effective gauge field action $S_{\text{eff}}[V]$ which includes the effect of virtual $q\bar{q}$ propagation. This is perfectly correct for heavy quarks of masses of order a^{-1} , because in such a case the effective action is still short range on the lattice scale and mean-field ideas can be meaningfully applied to it. But we have already emphasized that this is not the correct procedure for massless fermions¹⁵⁾ since the known strong coupling results^{11),13)} - fermions getting a dynamically generated mass - are not matched by this approach. We shall rather use an alternative approach put forward in Ref. 15), which consists of keeping the fermions in interaction with the compact gauge variable $U_{x,\hat{\mu}}$ rather than the mean field $V_{x,\hat{\mu}}$. This procedure matches the strong-coupling results and it reduces to including an extra term in the integrand of the right-hand side of Eq. (2.7)

$$e^{-F_f\{A\}} = e^{-\sum_{\ell} \omega(A_{\ell}^* A_{\ell})} \int [d\mu(U) d\chi d\bar{\chi}] \exp \left\{ S_F + \frac{i}{2} \sum_{\ell} (A_{\ell}^* U_{\ell} + A_{\ell} U_{\ell}^*) \right\} \quad (2.8)$$

Notice that the U_ℓ integrations are still decoupled and that the fermion bilinears just shift the external source A_ℓ on each link. It is the Grassmann integration that couples all the links of the lattice. In principle, the fermion contribution should be included in the search for a saddle-point in $\{V, A\}$. However, (cf. Section 5) this term is of order $1/d$ as compared with pure gauge terms, and therefore it comes on the same footing as the first corrections to the mean-field result. This is essentially a consequence of fermion fields being associated with sites while gauge variables U are living on links.

The one-link integral (2.6) is given by

$$\omega(A^*A) = \log I_0(\sqrt{-A^*A}) \quad (2.9)$$

where $I_0(x)$ is the modified Bessel function. The saddle-point equations read

$$\frac{\partial S_G}{\partial V_\ell} - \frac{i}{2} A_\ell^* = 0 \quad ; \quad \frac{\partial S_G}{\partial V_\ell^*} - \frac{i}{2} A_\ell = 0 \quad (2.10)$$

$$-\frac{i}{2} V_\ell + \frac{\partial \omega}{\partial A_\ell^*} = 0 \quad ; \quad -\frac{i}{2} V_\ell^* + \frac{\partial \omega}{\partial A_\ell} = 0 \quad (2.11)$$

In order to solve these equations one usually assumes a translationally-invariant solution. This is reasonable since these configurations should represent the ground state of the system. Let ρ be the saddle-point value of V_ℓ and (V_ℓ^*) and let $i\alpha$ be the corresponding value of A_ℓ and (A_ℓ^*) . We assume ρ and α to be real. The saddle-point equations now reduce to

$$\rho = \frac{d}{d\alpha} \log I_0(\alpha) = \frac{I_1(\alpha)}{I_0(\alpha)} \quad (2.12)$$

and

$$\alpha = 4\bar{\beta}\rho^3 \quad ; \quad \bar{\beta} = \beta(d-1) \quad (2.13)$$

The mean-field free energy per link is given by

$$\frac{1}{Nd} F_{MF} = -\bar{\beta}\rho^4 + \alpha\rho - \log I_0(\alpha) \quad (2.14)$$

The mean-field equations (2.12), (2.13) admit for all $\bar{\beta}$ the solution $\rho = \alpha = 0$. This solution represents the true minimum of F_{MF} in the strong-coupling region

(small $\bar{\beta}$). At $\bar{\beta} = 1.39$, a new solution appears which dominates over the trivial one when $\bar{\beta} = \bar{\beta}_c = 1.82$. At this value of the gauge coupling, the mean-field ρ jumps from zero to a value $\rho = 0.9$, already very close to 1. As $\bar{\beta}$ is further increased, ρ tends to 1. In the non-confining phase, the external field α is very large, since it starts from $\alpha \approx 5.32$ and tends to infinity as $\rho \rightarrow 1$. In fact, the non-confining phase is characterized by a small parameter $(1-\rho)$ related to a large parameter α . This seems to be a general feature of any gauge group. A very good estimate of the saddle-point solution is obtained by using an asymptotic form of the modified Bessel function $I_0(\alpha)$. In this way one obtains

$$\rho = 1 - \frac{1}{8\bar{\beta}} + O(\bar{\beta}^{-2}) \quad (2.15)$$

$$\alpha = \frac{1}{2(1-\rho)} + O((1-\rho)^{-2}) \quad (2.16)$$

Notice that these relations are identical to those obtained in Ref. 15) for $U(\infty)$

The mean-field action in Eq. (2.7) is always gauge invariant under simultaneous gauge transformations of $V_{x,\mu}$ and $A_{x,\mu}$. This means that in the weak-coupling non-confining phase there is an infinite number of degenerate saddle-points obtained by performing an arbitrary gauge transformation on the homogeneous solution (ρ, α) discussed above:

$$\tilde{V}_{x,\mu} = \exp i [\phi(x+\hat{\mu}) - \phi(x)] \cdot \rho \quad (2.17)$$

$$\tilde{A}_{x,\mu} = \exp i [\phi(x+\hat{\mu}) - \phi(x)] \cdot \alpha \quad (2.18)$$

This implies the existence of zero-frequency modes due to the absence of restoring forces for fluctuations related to the gauge symmetry. We shall discuss in the next section how the use of collective-coordinate methods allows us to integrate the zero-modes exactly when computing corrections to the mean-field approximation.

3. - BEYOND THE MEAN-FIELD APPROXIMATION: PURE GAUGE CORRECTIONS

In the mean-field approximation - at fixed $\bar{\beta}$ - the free energy per link is of order 1. We shall now discuss the calculation of $1/d$ corrections to the saddle-point result by restricting ourselves to the pure gauge case and temporarily ignoring the fermion contribution $F_f[A]$ to the free energy.

Let us first briefly discuss the corrections to the trivial saddle-point $\rho = \alpha = 0$. It is rather simple to understand why the saddle-point result is exact as $d \rightarrow \infty$ in spite of the fact that the number of integration variables grows as Nd . The mean-field value of the free energy vanishes at $\rho = \alpha = 0$, which means that in Eq. (2.7) we obtain $Z_{MF} = 1$. Let us now assume that we switch off the Wilson action $S_G[V]$ in Eq. (2.7): in this case the link integrations are decoupled but the result for Z is still 1 because the V integrations give δ functions in A and $\omega(A^*A)$ vanishes at the origin. This means that corrections to Z_{MF} come only from the expansion of the Wilson action: this is just the strong-coupling expansion which is also an expansion in $1/d$ simply because the gauge coupling has been rescaled in the form $\beta = \tilde{\beta}(d-1)$. The calculations are straightforward and we shall not discuss them any further.

We now turn to the calculation of the corrections around non-trivial values of ρ and α . Here we must deal with the N zero frequency modes which simply indicate the absence of restoring forces for quantum fluctuations which correspond to a gauge transformation of the classical solution. These zero modes prevent one from carrying out a naive perturbation expansion. The correct procedure to follow is to use collective coordinate methods, which consists of integrating exactly the symmetry degrees of freedom (this means, in our case, summing over all degenerate saddle-points) and developing a perturbation theory only in the genuine Gaussian fluctuations orthogonal to the zero frequency modes. Keeping in mind Eq. (2.7) for the partition function, our procedure will consist of first making the integration over the $A_{x,\mu}$ fluctuations which, as we shall see, are local integrations that can always be unambiguously made, and then integrating the $V_{x,\mu}$ fluctuations. The zero frequency modes occur in this last integration.

Let us first examine the zero frequency modes and the orthogonality conditions that define the genuine Gaussian fluctuations. We shift the integration variables in Eq. (2.7) according to

$$V_{x,\mu} = \rho + \sqrt{x_{,\mu}} \hat{\mu} = \rho + \lambda_{x,\mu} \hat{\mu} + i \xi_{x,\mu} \hat{\mu} \quad (3.1)$$

$$A_{x,\mu} = i\alpha + a_{x,\mu} \hat{\mu} = i\alpha + X_{x,\mu} \hat{\mu} + i Y_{x,\mu} \hat{\mu} \quad (3.2)$$

The classical solution $V_{x,\mu}^0 = \rho$ we are expanding around is transformed by an infinitesimal gauge transformation performed at the site s into

$$V_{x,\mu}'^0 = [1 + i\epsilon(s)(\delta_{x,s} - \delta_{x,s-\hat{\mu}})] \rho \quad (3.3)$$

which shows that the zero-frequency modes occur, as expected, among the imaginary fluctuations $\xi_{x,\hat{\mu}}$ in Eq. (3.1). It follows from Eq. (3.3) that the imaginary fluctuations which are orthogonal to the zero mode satisfy the conditions

$$\oint \sum_{\mu} (\xi_{x,\hat{\mu}} - \xi_{x-\hat{\mu},\hat{\mu}}) = 0 \quad ; \quad \forall x \quad (3.4)$$

which is nothing but the background gauge condition, since this expression can be rewritten as

$$\text{Im} \left\{ \sum_{\mu} (V_{x,\hat{\mu}} V_{x,\hat{\mu}}^* - V_{x-\hat{\mu},\hat{\mu}} V_{x-\hat{\mu},\hat{\mu}}^*) \right\} = 0 \quad ; \quad \forall x \quad (3.5)$$

with our choice of $V_{x,\hat{\mu}}^0 = \rho$ (and $\rho \neq 0$). We need to take the imaginary part of the usual expression because the $V_{x,\hat{\mu}}$ fields are not unitary.

Next we enforce the background gauge condition in the path integral (2.7) by the standard Faddeev-Popov¹⁶⁾ method, by multiplying it by 1 written in the form

$$1 = \Delta_{\text{PF}}[\rho, \sigma] \int_0^{2\pi} \left(\prod_x d\phi_x \right) \prod_x \delta \left[\sum_{\mu} \cos(\phi_x - \phi_{x+\hat{\mu}}) \xi_{x,\hat{\mu}} - \sum_{\mu} \cos(\phi_x - \phi_{x-\hat{\mu}}) \xi_{x-\hat{\mu},\hat{\mu}} + \sum_{\mu} \sin(\phi_x - \phi_{x+\hat{\mu}}) (\rho + \lambda_{x,\hat{\mu}}) + \sum_{\mu} \sin(\phi_x - \phi_{x-\hat{\mu}}) (\rho + \lambda_{x-\hat{\mu},\hat{\mu}}) \right] \quad (3.6)$$

where the argument of the δ function is just the gauge transform of the background gauge condition (3.4). The gauge invariance of the integrand of Eq. (2.7) and of the Faddeev-Popov determinant $\Delta_{\text{PF}}[\rho, v]$ allows us to factorize in the standard way the integration over the volume of the gauge group, and we obtain

$$\begin{aligned} Z = (2\pi)^N \cdot \exp[-F_{\text{MF}}] \cdot \int [d\sigma da] \Delta[\rho, v] \cdot \prod_x \delta \left[\sum_{\mu} (\xi_{x,\hat{\mu}} - \xi_{x-\hat{\mu},\hat{\mu}}) \right] \\ \cdot \exp \left\{ \bar{S}_G[\rho, v] - \frac{i}{2} \sum_{\ell} (a_{\ell}^* v_{\ell} + a_{\ell} v_{\ell}^*) + \sum_{\ell} \bar{\omega}(a_{\ell}, \alpha) \right\} \end{aligned} \quad (3.7)$$

where $\bar{S}_G[\rho, v]$ and $\bar{\omega}[\alpha, a]$ are the shifted Wilson action and one-link integral, respectively. The constant terms define F_{MF} and linear terms in $v_{x,\mu}$ or $a_{x,\mu}$ are of course absent. The gauge fixing procedure adopted here amounts to an exact integration of the symmetry degrees of freedom. The background gauge condition will allow us later on to integrate away the zero-frequency mode. The factor $(2\pi)^N$ in the partition function arises because the integration over the

volume of the gauge group factorizes, since we are computing a gauge-invariant quantity. In computing the expectation value of any non-gauge-invariant operator, the group integration will not factorize, and will average to zero the saddle-point result. This is the way in which gauge invariance is restored.

We next sketch the rest of the one-loop calculation and carefully collect all contributions to the one-loop correction to the free energy, which we call ΔF .

a) There is clearly an entropy factor arising from the volume of the gauge group

$$\Delta F_1 = - N \log(2\pi) \quad (3.8)$$

b) Let us next discuss the $a_{x,\mu}$ integration. The expansion of $\bar{\omega}(a_x, \alpha)$ is of the form

$$\begin{aligned} \bar{\omega}(a_x, \alpha) &= -\frac{1}{2} \left\{ \frac{I_0''(\alpha)}{I_0(\alpha)} - \left(\frac{I_0'(\alpha)}{I_0(\alpha)} \right)^2 \right\} X_{x,\mu}^2 - \frac{1}{2\alpha} \frac{I_0'(\alpha)}{I_0(\alpha)} Y_{x,\mu}^2 + \dots \\ &= -\frac{1}{2} \left(1 - \frac{\rho}{\alpha} - \rho^2 \right) X_{x,\mu}^2 - \frac{\rho}{2\alpha} Y_{x,\mu}^2 + \dots \end{aligned} \quad (3.9)$$

where the dots indicate higher order local interactions whose strength decreases like α^{-2n} for an n point vertex. It is possible to show, from the convexity properties of the $\bar{\omega}(a, \alpha)$ function, that the quadratic terms are negative-definite. Therefore, the Gaussian integration over the $a_{x\mu}$ fluctuations is perfectly well defined. In rewriting the expansion in terms of α and ρ we have used the saddle-point equations as well as identities among Bessel functions which yield $I_0''(\alpha)$ in terms of $I_0'(\alpha)$ and $I_0(\alpha)$. The Gaussian integration yields a quadratic term in $v_{x,\mu}$ which adds to $\bar{S}_G(\rho, v)$, and a constant term

$$\Delta F_2 = \frac{Nd}{2} \log \left[\frac{\rho}{\alpha} \left(1 - \frac{\rho}{\alpha} - \rho^2 \right) \right] \quad (3.10)$$

potentially dangerous since it is of order Nd . However, the leading term in d will cancel against similar terms arising from the $v_{x,\mu}$ integrations. This has to be so if the mean-field is to be exact as $d \rightarrow \infty$, and it is guaranteed to happen for reasons similar to the ones advanced in the discussion of the corrections to the trivial saddle-point $\rho = \alpha = 0$.

c) After the $a_{x,\mu}$ integration has been done, the quadratic piece of the action in $v_{x,\mu}$ takes the form, in momentum space:

$$S_G^{(2)} = - \int_{-\pi}^{\pi} \frac{d^d p}{(2\pi)^d} \left\{ \sum_{\mu,\nu} \lambda_{\mu}^*(p) \Lambda_{\mu\nu}(p) \lambda_{\nu}(p) + \sum_{\mu,\nu} \xi_{\mu}^*(p) \Omega_{\mu\nu}(p) \xi_{\nu}(p) \right\} \quad (3.11)$$

where

$$\Lambda_{\mu\nu}(p) = \left\{ \frac{\alpha}{2\rho(d-1)} \left[1 + 2 \cos p_{\mu} - \sum_{\lambda} \cos p_{\lambda} \right] + \frac{1}{2(1 - \frac{\rho}{\alpha} - \rho^2)} \right\} \delta_{\mu\nu} - \frac{\alpha}{4\rho(d-1)} (1 + e^{ip_{\mu}})(1 + e^{-ip_{\nu}}) \quad (3.12)$$

and

$$\Omega_{\mu\nu}(p) = \frac{\alpha}{2\rho(d-1)} \sum_{\lambda} (1 - \cos p_{\lambda}) \cdot \delta_{\mu\nu} - \frac{\alpha}{4\rho(d-1)} (1 - e^{ip_{\mu}})(1 - e^{-ip_{\nu}}) \quad (3.13)$$

The spectrum of the imaginary fluctuations is immediately obtained since $\Omega_{\mu\nu}(p)$ is the identity plus a rank-1 matrix. We then find $(d-1)$ degenerate eigenvalues given by

$$\omega_i(p) = \frac{\alpha}{2\rho(d-1)} \sum_{\lambda} (1 - \cos p_{\lambda}) \geq 0 \quad (3.14)$$

and the zero frequency mode $\omega^0(p) \equiv 0$, whose normalized eigenfunction is a d dimensional vector

$$\psi_{\mu}^{(0)}(p) = \frac{1 - e^{ip_{\mu}}}{\left[2 \sum_{\lambda} (1 - \cos p_{\lambda}) \right]^{1/2}} \quad (3.15)$$

The $(d-1)$ massless excitations given by Eq. (3.14) are simply the massless photons that must show up in the deconfined phase of the theory. The positive sign of their associated frequencies $\omega_i(p)$ ensures the stability of the saddle-point against imaginary fluctuations.

Let us now turn to the real fluctuations described by $\Lambda_{\mu\nu}(p)$. The diagonalization of this matrix is slightly less straightforward because of the presence of the $\cos p_{\mu}$ term in the diagonal part. This term is clearly of order $1/d$, and the eigenvalues of $\Lambda(p)$ can be computed using a $1/d$ expansion, but some care must be exercised because in the absence of this term Λ has $(d-1)$ degenerate eigenvalues. The $(d-1)$ approximately degenerate eigenvalues of Λ are given by

$$\omega_R(p) = \frac{\alpha}{2\rho(d-1)} \sum_{\lambda} (1 - \cos p_{\lambda}) + \frac{\alpha}{2\rho} \cdot \frac{2\rho - \alpha + \alpha p^2}{\alpha - \rho - \alpha p^2} + O(1/d) \quad (3.16)$$

and the non-degenerate one is:

$$\tilde{\omega}_R(p) = -\frac{\alpha}{\rho(d-1)} \sum_{\lambda} \cos p_{\lambda} + \frac{\alpha}{2\rho} \cdot \frac{2\rho - \alpha + \alpha p^2}{\alpha - \rho - \alpha p^2} + O(1/d) \quad (3.17)$$

It can easily be verified that these eigenfrequencies are positive definite. The saddle-point is then also stable against real fluctuations. Equations (3.16) and (3.17) represent heavy massive excitations since from the weak-coupling estimates (2.15) and (2.16) it follows that their mass behaves like $(1-\rho)^{-2}$ when $\rho \rightarrow 1$. They will ultimately decouple in this limit, thereby reproducing the correct weak-coupling spectrum.

The $(d-1)$ photons and the d massive excitations (3.16) and (3.18) give a one-loop contribution to the free energy

$$\Delta F_3 = \frac{N(d-1)}{2} \int \frac{d^d p}{(2\pi)^d} [\log \omega_L(p) + \log \omega_R(p)] + \frac{N}{2} \int \frac{d^d p}{(2\pi)^d} \log \tilde{\omega}_R(p) \quad (3.18)$$

The leading Nd term will cancel against the one coming from Eq. (3.10), and the full correction will be $O(1/d)$. We have checked that $O(1/d)$ terms in Eqs. (3.16) and (3.17) do not contribute to ΔF_3 , to leading order in $1/d$.

d) Let us now turn to the zero mode. If the imaginary fluctuations $\xi_{\mu}(p)$ are expanded in normalized eigenfunctions of $\Omega_{\mu\nu}(p)$

$$\xi_{\mu}(p) = \sum_{n=0}^{d-1} C_n(p) \Psi_{\mu}^{(n)}(p) \quad (3.19)$$

the Jacobian of the transformation $\xi_{\mu}(p) \rightarrow C_n(p)$ is of course 1. However, the background gauge condition (3.4) becomes, in momentum space

$$\prod_{\mu} \delta \left[\sum_{\mu} (\xi_{x,\hat{\mu}} - \xi_{x-\hat{\mu},\hat{\mu}}) \right] = \prod_p \frac{1}{[2 \sum_{\mu} (1 - \cos p_{\mu})]^{1/2}} \delta[C_0(p)] \quad (3.20)$$

The zero-frequency mode is then integrated away, but there is a Jacobian coming from the normalization of the zero-mode wave function. We then obtain a contribution to the free energy

$$\Delta F_4 = \frac{N}{2} \int \frac{d^d p}{(2\pi)^d} \log \left[\sum_{\mu} (2 - 2 \cos p_{\mu}) \right] + \frac{N}{2} \log(2\pi) \quad (3.21)$$

the second term arising because the C_0 integration is not Gaussian.

e) Finally, we must consider the Faddeev-Popov determinant, which is computed from Eq. (3.6) by standard methods

$$\Delta_{PF}[\rho, v] = \det C \quad (3.22)$$

where C is a matrix defined in site space

$$C_{x, x'} = \delta_{x, x'} \cdot \sum_{\mu} (2\rho + \lambda_{x, \mu} + \lambda_{x-\hat{\mu}, \mu}) - \\ - \delta_{x', x+\hat{\mu}} (\rho + \lambda_{x, \mu}) - \delta_{x', x-\hat{\mu}} (\rho + \lambda_{x-\hat{\mu}, \mu}) \quad (3.23)$$

This determinant can be computed by a perturbation expansion in the λ fluctuations. The zeroth order term gives a contribution to the free energy

$$\Delta F_5 = -N \log \rho - N \int \frac{d^d p}{(2\pi)^d} \log \left[\sum_{\mu} (2 - 2 \cos p_{\mu}) \right] \quad (3.24)$$

The perturbation expansion of the Faddeev-Popov determinant turns out to be an expansion in $1/d$. Therefore, the quantum fluctuations in $\det C$ do not contribute to the $1/d$ correction to the mean-field result. This was to be expected on general grounds: the "curvature" of the collective coordinate is felt only in higher orders of perturbation theory.

This completes our discussion of the one-loop corrections to the mean-field result. Collecting all contributions to the free energy given by Eqs. (3.8), (3.10), (3.18), (3.21) and (3.24), and retaining only the leading $1/d$ behaviour of all momentum space integrals, we obtain the following correction to the free energy per link:

$$\frac{1}{Nd} \Delta F = \frac{1}{d} F_1(\alpha, \rho) \quad (3.25)$$

$$F_1(\alpha, \rho) = -\frac{1}{2} \log(4\pi d \rho^2) - \frac{1}{2} \log \left[\frac{\alpha}{2\rho - \alpha + \alpha \rho^2} \right] + \\ + \frac{3}{8} + \frac{1}{2\rho} (\alpha - \rho - \alpha \rho^2) - \frac{1}{8\rho^2} (\alpha - \rho - \alpha \rho^2)^2 \quad (3.26)$$

The logarithmic terms in this expression include all the Jacobians in the problem as well as the contribution of the non-degenerate eigenvalue $\tilde{\omega}_R(p)$ of the real fluctuations. The constant term $(+3/8)$ is the contribution of the $(d-1)$ massless

photons, and the remainder is the contribution of the $(d-1)$ massive real excitations $\omega_R(p)$. From the weak-coupling estimates one can easily see that F_1 diverges as $\log \alpha$ in the weak-coupling limit, just as the mean-field result does. It must be emphasized that the one-loop result does not exhaust all the $1/d$ corrections. There are, for example, corrections coming from the cubic and quartic terms in the Wilson action, and which are of the same nature as the corrections computed in the confining phase. However, they are small because the propagator for the ξ fluctuations behaves as $(1-\rho)$ and the one for λ fluctuations behaves as $(1-\rho)^2$. A quick estimate shows that these $1/d$ corrections vanish at least as $(1-\rho)^2$ in the non-confining phase and can then be safely ignored. Furthermore, higher order terms in the expansion (3.9) of the one-link integral yield, after integration over $a_{x,\mu}$, new effective interactions among the $v_{x,\mu}$ fields. This is again a source of $1/d$ corrections to the mean-field result, but it can easily be checked that they fade away like high powers of $(1-\rho) \sim \alpha^{-1}$ in the weak-coupling limit. The one-loop corrections (3.26) are then the only prominent ones in the ordered phase, and they are the only ones that will be retained in the numerical analysis of the next section.

4. - THE MAGNITUDE OF CORRECTIONS IN THE PURE GAUGE CASE

The free energy $F(\bar{\beta}, \rho)$ given in Eq. (2.14) of Section 2 has a local minimum at $\rho = 0$ for all $\bar{\beta}$. At this trivial minimum the correction to free energy is easily found to be (we work here with the free energy per link and suppress factors of $(Nd)^{-1}$)

$$\Delta F' = - \frac{\bar{\beta}^2}{2d} + O(\alpha^{-2}) \quad (4.1)$$

For $\bar{\beta} \geq 1.39$, a non-trivial minimum appears. The corresponding correction to free energy has been computed in the preceding section, where it has been denoted by ΔF . When both these corrections are taken into account, the non-trivial minimum takes over the trivial one at a value of $\bar{\beta}$ different from $\bar{\beta}_c$, at $\bar{\beta} = \bar{\beta}'_c$, say. We shall now calculate the shift $\delta\bar{\beta}_c = \bar{\beta}'_c - \bar{\beta}_c$.

Denote by $\rho(\bar{\beta})$ the value of ρ at the non-trivial minimum. Since $F(\bar{\beta}, 0) = 0$, $\bar{\beta}'_c$ is defined by the equation

$$F[\bar{\beta}_c, \rho(\bar{\beta}_c)] = 0 \quad (4.2)$$

It is clear that $\bar{\beta}'_c$ is to be found from

$$F[\bar{\beta}'_c, \rho(\bar{\beta}'_c)] + \Delta F[\bar{\beta}'_c, \rho(\bar{\beta}'_c)] = \Delta F'[\bar{\beta}'_c] \quad (4.3)$$

$$\left. \frac{\partial F}{\partial \rho} \right|_{\rho(\bar{\beta}_c)} = 0 \quad ; \quad \left. \frac{\partial F}{\partial \bar{\beta}} \right|_{\rho} = -\rho^4 \quad (4.4)$$

and for large enough d one obtains:

$$\delta \bar{\beta}_c = - \left\{ \frac{1}{\rho^4} [\Delta F'(\bar{\beta}) - \Delta F(\bar{\beta}, \rho)] \right\}_{\bar{\beta}=\bar{\beta}'} + O(d^{-2}, (1-\rho)^2 d^{-1}) \quad (4.5)$$

With $\bar{\beta}_c = 1.82$ and $\rho(\bar{\beta}_c) = 0.9$ this yields

$$\bar{\beta}_c' \approx 1.82 - 0.76 \frac{\log d}{d} - \frac{0.04}{d} \quad (4.6)$$

At $d = 4$ we get $\bar{\beta}_c' = 1.55$. The corresponding value obtained in the Monte Carlo simulation¹⁷⁾ is 1.51 [notice that β of Ref. 17) differs from ours by a factor 2]. This excellent agreement seems, however, somewhat fortuitous since it is a priori not obvious that higher order corrections are that small.

5. - MEAN-FIELD THEORY IN THE PRESENCE OF DYNAMICAL FERMIONS

The purpose of this section is to discuss the effect of Susskind fermions in the mean-field equations for $U(1)$ lattice gauge theory, namely to compute the fermion contribution to the free energy $F_f[A]$ defined by Eq. (2.8). We shall closely follow the method used in Ref. 15), which consists of first making the $U_{x,\hat{\mu}}$ integration to obtain an effective fermion Lagrangian, and then doing the Grassman integration. It has recently been shown by Kluberg-Stern et al.¹⁴⁾ that in the strong-coupling limit [which corresponds to setting $A_{x,\hat{\mu}} \equiv 0$ in Eq. (2.8)] Susskind fermion effects can be systematically computed in a $1/d$ expansion. We shall find that this is still true in the presence of a fluctuating external field $A_{x,\hat{\mu}}$.

The gauge group integration gives just the known result for the one-link integral, with the external field modified by the presence of matter sources

$$A_{x,\hat{\mu}} \rightarrow A_{x,\hat{\mu}} + \eta_{\mu}(x) \bar{\chi}(x+\hat{\mu}) \chi(x) \quad (5.1)$$

$$A_{x,\hat{\mu}}^* \rightarrow A_{x,\hat{\mu}}^* - \eta_{\mu}(x) \bar{\chi}(x) \chi(x+\hat{\mu}) \quad (5.2)$$

and the effective Lagrangian is obtained by expanding the one-link integral in the Grassmann variables $\chi(x)$, $\bar{\chi}(x)$. If we neglect the quantum fluctuations in $A_{x,\hat{\mu}}$ and keep only its mean-field value, the effective fermionic action takes the form

$$S_{\text{eff}} = \sum_{x,\hat{\mu}} \frac{1}{2} A(\alpha) \gamma_{\mu}(x) [\bar{\chi}(x+\hat{\mu}) \chi(x) - \bar{\chi}(x) \chi(x+\hat{\mu})] + \quad (5.3)$$

$$+ \sum_{x,\hat{\mu}} \frac{1}{2} B(\alpha) \bar{\chi}(x) \chi(x) \bar{\chi}(x+\hat{\mu}) \chi(x+\hat{\mu})$$

where

$$A(\alpha) = \frac{I'_0(\alpha)}{I_0(\alpha)} = \rho \quad (5.4)$$

$$B(\alpha) = \frac{1}{2} \left[1 - \left(\frac{I'_0(\alpha)}{I_0(\alpha)} \right)^2 \right] = \frac{1}{2} (1 - \rho^2) \quad (5.5)$$

Use has been made of the saddle-point equation (2.12) to express $A(\alpha)$ and $B(\alpha)$ in terms of the mean field ρ . This is legitimate in the context of a $1/d$ expansion because the fermion-free energy is of order $1/d$ with respect to the gauge mean-field free energy. The effective action (5.3) represents Susskind fermions moving in a background mean-field ρ plus a hopping term for the composite field $(\bar{\chi}(x)\chi(x))$ which is the only one that survives in the confining phase $\rho = 0$, and which is then responsible for the dynamical mass generation. As we move towards the weak-coupling limit $\rho = 1$, the four-fermion interaction fades away and we recover the free-fermion Lagrangian. Therefore, fermions become massless at some value of ρ which remains to be determined.

Fluctuations in the $A_{x,\hat{\mu}}$ field can easily be included. However, we have explicitly verified that they do not contribute to the leading $1/d$ behaviour of $F_f(A_l)$. We shall therefore restrict ourselves to the discussion of the effective Lagrangian (5.3). Before doing the integration over the Fermi fields it is convenient to decouple the composite fields $\bar{\chi}(x)\chi(x)$ by using the identity¹⁴⁾

$$\exp \left\{ \frac{1}{2} \sum_{x,x'} \bar{\chi}(x) \chi(x) (V^{-1})_{xx'} \bar{\chi}(x') \chi(x') \right\} =$$

$$= (\det V)^{1/2} \int \prod_x \frac{d\lambda(x)}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \sum_{x,x'} \lambda(x) V(x-x') \lambda(x') + \sum_x \lambda(x) \bar{\chi}(x) \chi(x) \right\} \quad (5.6)$$

With the identification

$$(V^{-1})_{xx'} = \frac{1}{2} B [\delta_{x, x'+\hat{\mu}} + \delta_{x, x'-\hat{\mu}}] \quad (5.7)$$

the potential $V(x-x')$ is easily obtained

$$V(x-x') = \int_{-\pi}^{\pi} \frac{d^d p}{(2\pi)^d} e^{ip \cdot (x-x')} \frac{1}{B \sum_{\mu} \cos p_{\mu}} \quad (5.8)$$

The integration over the Grassmann variables can now be done. We define a matrix $\Lambda(x, x')$ by

$$\Lambda(x, x') = \lambda(x) \delta_{x, x'} + \frac{1}{2} \rho \eta_{\mu}(x) [\delta_{x', x+\hat{\mu}} - \delta_{x', x-\hat{\mu}}] \quad (5.9)$$

Then,

$$\exp\{-F_f[\rho]\} = [\det V]^{1/2} \int \prod_x \frac{d\lambda(x)}{\sqrt{2\pi}} \cdot \exp \left\{ \text{Tr} \log \Lambda - \frac{1}{2} \sum_{x, x'} \lambda(x) V(x-x') \lambda(x') \right\} \quad (5.10)$$

The integration over $\lambda(x)$ is finally done by the saddle-point method. For a traditionally invariant saddle-point λ_0 , the saddle-point equations read

$$(\Lambda_0^{-1})_{xx} - \lambda_0 \sum_{x'} V(x-x') = 0 \quad (5.11)$$

where Λ_0 is the Λ matrix (5.9) with $\lambda(x) = \lambda_0$. It follows from Eq. (5.8) that

$$\sum_{x'} V(x-x') = (Bd)^{-1} \quad (5.12)$$

The propagator Λ_0^{-1} for Susskind fermions can be computed by standard methods: the $\eta_{\mu}(x)$ factors can be turned into γ_{μ} matrices by unitary transformations. Then, the saddle-point equation (5.11) reads:

$$\frac{\lambda_0}{Bd} = \int_{-\pi/2}^{\pi/2} \frac{d^d p}{\pi^d} \frac{\lambda_0}{\lambda_0^2 + \rho^2 \sum_{\mu} \sin^2 p_{\mu}} \quad (5.13)$$

which is essentially the lattice version of the Nambu-Jona-Lasinio equation¹⁸⁾

Besides the trivial solution, $\lambda_0 = 0$, there may be a non-trivial one which signals the spontaneous breaking of chiral invariance. This non-trivial solution is known to exist in the strong-coupling limit, where $\rho = 0$. Indeed, it follows from Eq. (5.13) that in this case $\lambda_0 = \sqrt{B(0)d}$. Whether or not it continues to exist at weak coupling is a dynamical question that we shall discuss later on. The saddle-point value of the fermion free energy is given by:

$$F_f(\rho) = -\frac{N}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d^d p}{(2\pi)^d} \log \left[\lambda_0^2 + \rho^2 \sum_{\mu} \sin^2 p_{\mu} \right] + \frac{N}{2} \frac{\lambda_0^2}{Bd} - \frac{1}{2} \text{Tr} \log V(\rho) \quad (5.14)$$

It is of course understood that, when there are both a trivial and a non-trivial solution of Eq. (5.13) for λ_0 , only the one that minimizes $F_f(\rho)$ is to be retained in Eq. (5.14). Corrections to the saddle-point result (5.14) are of order $1/d$. This can be shown by the methods of Ref. 14). In fact, the only difference with the strong-coupling effective Lagrangian studied by the authors of Ref. 14) is the weakening of the strength of the four-fermion interaction as ρ departs from zero and the presence of a free quark propagation term. This last term does not invalidate the $1/d$ nature of the loop expansion for Susskind fermions.

To leading order in $1/d$ expansion momentum space, integrals are easily evaluated¹⁵⁾ and the self-consistent equation (5.13) for λ_0 solved. In this case the non-trivial solutions exists only if $B > (1/2)\rho^2$, which means $\rho^2 < 1/2$ and is given by

$$\lambda_0 = \left[\left(\frac{1}{2} - \rho^2 \right) d \right]^{1/2} \quad (5.15)$$

The free energy (5.14) can also be evaluated at large d . It turns out that the non-trivial solution (5.15) always dominates whenever it exists, so the fermion free energy is given by

$$\frac{1}{N} F_f(\rho) = \frac{1}{2} \theta(\rho^2 - \frac{1}{2}) \log \left[\frac{1 - \rho^2}{\rho^2} \right] + \theta(\frac{1}{2} - \rho^2) \frac{\frac{1}{2} - \rho^2}{1 - \rho^2} \quad (5.16)$$

According to our general prescription, this contribution to the free energy is to be understood as a function of α , and consequently it modifies the right-hand side of Eq. (2.12). One could have expected to obtain a non-vanishing mean field of order $1/d$ for any $\bar{\beta}$ in the strong-coupling region due to the screening effects of massless fermions. However, it can easily be verified that this is not the case: the fermion contribution does not change the saddle-point at $\rho = 0$ at strong coupling. Indeed, since fermions have acquired a dynamical mass, they

cannot screen an infinite string tension. The only effect of fermions is to produce a further shift of the critical $\bar{\beta}$ towards the strong-coupling region, so that the system deconfines earlier: the non-trivial minimum now takes over at $\bar{\beta} = \bar{\beta}_c''$, say. The magnitude of $\delta_f \bar{\beta}_c = \bar{\beta}_c'' - \bar{\beta}_c'$ is easily found, for large d , by using arguments analogous to those employed in Section 4.

For large d , $\bar{\beta}_c''$ cannot differ much from $\bar{\beta}_c = 1.82$ and the corresponding value of ρ must be in the neighbourhood of $\rho(\bar{\beta}_c) = 0.9$. For this value of ρ the condition for the occurrence of dynamical mass generation is not met and $\lambda_0 = 0$. A straightforward calculation yields

$$\delta_f \bar{\beta}_c = \frac{1}{2d} \left\{ \frac{1}{\rho^4} \left[\log \frac{1-\rho^2}{\rho^2} - 1 \right] \right\}_{\beta=\bar{\beta}_c} + o(1/d^2) \quad (5.17)$$

and with $\rho = 0.9$ one further gets

$$\delta_f \bar{\beta}_c = - \frac{1.87}{d} + o(d^{-2}) \quad (5.18)$$

Note that for $d = 4$ this effect is in fact large: the central $\bar{\beta}$ is displaced from 1.56 down to about 1.08, well below the values of $\bar{\beta}$ where the uncorrected free energy develops a non-trivial minimum. This of course means that non-leading contributions to the fermion free energy are becoming important at $d = 4$. Thus, we cannot present any definite conclusion concerning the coincidence (or lack of it) between the deconfinement point and the point where chiral invariance is restored. All we can say is that at $d = 4$ the fermion originated corrections to the mean field are fairly large and tend to make the system deconfine sooner when coming from the strong-coupling side, as expected from our intuitive image of screening. It is therefore not impossible that a $q\bar{q}$ condensate survives in spite of quarks being deconfined. Of course, this must be a transient phenomenon since dynamical mass generation is certainly impossible for weak enough couplings.

The estimate (5.18) corresponds to $2^{[d/2]}$ species of Dirac fermions. One can argue that in a theory with N_f flavours and N_c colours, a very rough estimate of the shift of the transition region due to fermion effects can be obtained from (5.18) by multiplying the right-hand side by $N_f/(2^{[d/2]} N_c)$.

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