Ref.TH.3319-CERN

THE GRAVITINO AND SPIN-ZERO BOSON MASSES

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ABSTRACT

We demonstrate that in theories with a local $N = 1$ supersymmetry spontaneously broken at the tree level the spin-zero boson masses are bounded below by the gravitino mass. This result is proved for an arbitrary number of chiral superfields. It suggests that in such a theory the mass of the gravitino should be less than $O(100)$ GeV and the supersymmetry breaking scale μ_s should be less than $\sqrt{m_W m_{\text{Planck}}} \approx 10^{10}$ GeV. We discuss possible ways of avoiding this apparent contradiction between having light scalars and a heavy gravitino.

Ref.TH.3319-CERN
3 June 1982

It seems that supersymmetry is an inescapable ingredient of the future unified theory. However, supersymmetry must be broken in the real world and one would like to know how, and at what energy scale. Presumably global supersymmetry is broken spontaneously, and if it is to have any relevance to the hierarchy problem¹⁾ some relic of supersymmetry must persist down to $O(100)$ GeV, in order to protect the masses of some Higgs scalar fields. This does not prevent the primordial scale of supersymmetry breaking from being considerably higher than $O(100)$ GeV, and it has been proposed²⁾⁻⁵⁾ that the spontaneous breaking of supersymmetry may start as high as the Planck mass. Gravitational interactions become important at this scale, and they can only be combined with local supersymmetry in an extended supergravity theory⁶⁾. It is attractive to conjecture that the required breakdown of global supersymmetry is triggered by the spontaneous breakdown of local supersymmetry, in which case the gravitino acquires a mass through the superHiggs effect⁷⁾. The most straightforward scenario of this type would have local $N = 1$ supersymmetry broken spontaneously at the tree level, and it is natural to ask whether this possibility is compatible with the desired¹⁾⁻⁵⁾ low-energy phenomenology.

In this paper we prove that the gravitino mass can be no larger than the spin-zero boson masses in a theory with local $N = 1$ supersymmetry broken spontaneously at the tree level. In such a theory the gravitino should therefore be lighter than $O(100)$ GeV. Our results reflect the fact that the interactions induced by local supersymmetry cannot be neglected in scenarios²⁾⁻⁵⁾ where global supersymmetry is broken at a scale $\geq \sqrt{m_W m_{\text{Planck}}} \approx 10^{10}$ to 10^{11} GeV. There may be some ways to evade this conclusion. Perhaps the upper bound on the gravitino mass could be relaxed in a theory with $N \geq 2$ local supersymmetry? Perhaps the bound can be violated by radiative corrections to our tree level results? Perhaps local supersymmetry is broken non-perturbatively? Perhaps one should not worry yet about the cosmological constant? A crucial ingredient in our proof is the requirement that the cosmological constant Λ vanishes. However, it has been suggested⁸⁾ that the effective cosmological constant at large distance scales may vanish even if it is very large at short distances around the Planck length. Perhaps we do not need to set $\Lambda = 0$ at the bare Lagrangian level?

It has long been known that the scalar potential is positive semi-definite in a globally supersymmetric theory, and the minimum is strictly positive if and only if global supersymmetry is spontaneously broken⁹⁾. These results do not apply to locally supersymmetric theories, whose scalar potentials contain both positive and negative pieces^{10),11)}. This property enables one to construct spontaneously broken $N = 1$ locally supersymmetry theories with a vanishing

cosmological constant¹¹⁾. Parametrizing the spin-zero fields in such a way that their kinetic energy term is canonical, the scalar potential in the minimal model with a single chiral superfield A is¹¹⁾⁻¹⁴⁾ (using Planck units)

$$V = \exp\left(\frac{1}{2}|A|^2\right) \left[2|f_A + \frac{1}{2}A^*f|^2 - 3|f|^2 \right] \quad (1)$$

where f is the usual chiral superpotential and $f_A \equiv \partial f / \partial A$. We will also use the notation $\tilde{f}_A \equiv f_A + \frac{1}{2}A^*f$, and use A indiscriminately to denote both chiral superfields and their spin-zero components, while the fermionic components will be denoted by ψ_A . The gravitino ψ has a bilinear coupling with $\tilde{f}_A \psi_A$ which give it a mass

$$m_\psi = \exp\left(\frac{1}{4}|A|^2\right) |f| \quad (2)$$

through the superHiggs effect⁷⁾. The cosmological constant vanishes when

$$2|\tilde{f}_A|^2 = 3|f|^2 \quad (3)$$

The order parameter for the breaking of local supersymmetry is $|f|$, whereas $|f_A|$ is that for global supersymmetry. The model (1) yields the formula¹¹⁾

$$m_A^2 + m_B^2 = 4m_\psi^2 \quad (4)$$

relating the masses of the two spin-zero mass eigenstates and of the gravitino (2). Thus the spin-zero masses are comparable with the gravitino mass, and an immediately distinguishable contribution to the spin-zero mass matrix comes from the term

$$\frac{1}{2} \exp\left(\frac{1}{2}|A|^2\right) |A|^2 |f|^2 \quad (5)$$

in the expression (1).

In real life we need more than one chiral superfield A_i ($i = 1, \dots, N$). The potential (1) now becomes¹²⁾⁻¹⁴⁾

$$V = \exp\left(\frac{1}{2} \sum_i |A_i|^2\right) \left[2 \sum_i |\tilde{f}_{A_i}|^2 - 3|f|^2 \right] \quad (1')$$

The combination $\Sigma_i \tilde{f}_{A_i} \psi_{A_i} / \Sigma_i |\tilde{f}_{A_i}|^2$ is the goldstino which will be eaten by the gravitino ψ to give it a mass

$$m_\chi = \exp\left(\frac{1}{4} \sum_i |A_i|^2\right) |f| \quad (2')$$

and the mass formula (4) becomes¹⁴⁾

$$\text{Supertrace}(m^2) = 2(N-1)m_\chi^2 \quad (4')$$

It is evident that the generalized potential (1') contains terms

$$\frac{1}{2} \exp\left(\frac{1}{2} \sum_i |A_i|^2\right) |A_i|^2 |f|^2 + \dots \quad (5')$$

which give contributions of order m_ψ (2') to the masses of all the spin-zero fields. This is rather worrisome if one wishes m_ψ to be much larger than some of the m_{A_i} which one would like to be $O(100 \text{ GeV})$ so as to be available for Weinberg-Salam symmetry breaking. Our worries are deepened by the sum rule (4'), which tells us that at least some of the spin-zero bosons must have masses $\geq m_\psi$. However, all is perhaps not yet lost. We might expect by analogy with Eq. (4) that the particular combinations of spin-zero fields which are supersymmetric partners of the goldstino will acquire supersymmetry breaking masses of order m_ψ . But we might also hope that other spin-zero components which are not associated with the goldstino might escape this fate. Perhaps all the supersymmetry breaking (4') could be shuffled into a harmless sector of the theory? We now exclude this possibility.

We choose a basis for the chiral superfields in which the goldstino is ψ_{A_1} and establish lower bounds on the masses of the scalars $A_2, \dots, A_N$. The spin-zero fields will in general have vacuum expectation values v_i ($i = 1, \dots, N$). The condition (3) for the vanishing of the cosmological constant Λ becomes

$$2|\tilde{f}_{A_1}|^2 + 2 \sum_{i=2, \dots, N} |\tilde{f}_{A_i}|^2 = 3|f|^2 \quad (6)$$

In our basis we have $\tilde{f}_{A_i} = 0$ ($i = 2, \dots, N$) and hence

$$2|\tilde{f}_{A_1}|^2 = 3|f|^2 \quad (7)$$

Let us write the scalar field terms in the superpotential $f(A_i)$ in the general power series expansion

$$f(A_i) = \sum_{\{p_i\} = 0}^{\infty} a_{p_1 \dots p_N} \prod_{i=1}^N \hat{A}_i^{p_i} \quad (8)$$

where $\hat{A}_i \equiv A_i - v_i$. We should impose on the theory the extremal conditions $\partial V / \partial A_i = 0$. Once this is done the $\hat{A}_i \hat{A}_j^*$ terms in the mass matrix take the form

$$\exp\left(\frac{1}{2} \sum_i |v_i|^2\right) \times \left\{ 2 \sum_i \left| \sum_j a_{ij} \hat{A}_j + \sum_i a_{ii} \hat{A}_i + \frac{1}{2} v_i^* \sum_i a_i \hat{A}_i \right|^2 \right. \\ \left. + \frac{1}{2} |a|^2 \sum_i |\hat{A}_i|^2 - \left| \sum_i a_i \hat{A}_i \right|^2 \right\} \quad (9)$$

where we have introduced the abbreviated notation

$$a \equiv a_{0 \dots 0}; a_i \equiv a_{0 \dots 0, p_i=1, 0 \dots 0}; a_{ij} \equiv a_{0 \dots 0, p_i=1, 0 \dots 0, p_j=1, 0 \dots 0} \quad (10)$$

In terms of these coefficients the conditions $\tilde{f}_{A_i} = 0$ ($i = 2, \dots, N$) become

$$a_i + \frac{1}{2} v_i^* a = 0 \quad (i=2, \dots, N) \quad (11)$$

The first term in Eq. (9) is positive semi-definite, while we can use the conditions (11) to rewrite the $i = 2, \dots, N$ parts of the last two terms as

$$\exp\left(\frac{1}{2} \sum_i |v_i|^2\right) \frac{1}{2} |a|^2 \sum_{i,j=2, \dots, N} (\delta_{ij} - \frac{1}{2} v_i v_j^*) \hat{A}_i^* \hat{A}_j \quad (12)$$

In a phenomenological theory¹⁾⁻⁵⁾ our spin-zero fields either have zero vacuum expectation values v_i (e.g. squarks, sleptons) or else $v_i \ll 1$ [e.g. the Weinberg-Salam Higgs fields, or even the Higgses appearing in grand unified theories which have vacuum expectation values $O(10^{-17}, 10^{-3})$, respectively]. Since at the extremum $f(v_i) = a$, we conclude that the contribution (5') does indeed provide an effective lower bound on the masses of the spin-zero fields. This applies to spin-zero fields which are not partners of the goldstino, and hence there is no way of shuffling the local supersymmetry breaking safely away from the real world. Although we have not included gauge interactions in our analysis, we do not believe that they will alter the essential conclusions. The

gravitino mass is still given by $|f|$ in such a theory, while the gauge interactions can only add to the spin-zero masses, making the inequality on m_ψ even stronger.

We conclude that the gravitino should have a mass $\lesssim 0(100)$ GeV in a theory with local $N = 1$ supersymmetry broken at the tree level. This is phenomenologically interesting and not necessarily disastrous, since the helicity $\pm 3/2$ gravitino states would presumably not be produced in present-day experiments, while the helicity $\pm 1/2$ states would have interactions analogous to those of conventional spin $1/2$ particles¹⁵⁾. However, the new terms in the potentials (1) and (1'), as compared to the conventional global supersymmetry potential $\sum_i |f_{A_i}|^2$, have certainly proven that they cannot be neglected in any scenario²⁾⁻⁵⁾ where global supersymmetry is broken at an energy scale $\mu_s \geq (m_W m_{\text{Planck}})^{1/2} = \sqrt{m_W}$ in Planck units. They could be neglected if

$$\mu_s^4 \equiv \sum_i |f_{A_i}|^2 \ll |f|^2 \quad \text{i.e.} \quad \mu_s \ll \sqrt{m_\psi} \quad (13)$$

and we have just seen that $m_\psi \lesssim 0(100)$ GeV in a local $N = 1$ supersymmetry theory broken at the tree level. In such a theory the supersymmetry breaking scale μ_s should be less than $(m_W m_{\text{Planck}})^{1/2} \approx 10^{10}$ to 10^{11} GeV.

What options are available if we want to have a heavier gravitino? As mentioned earlier in this paper, the gravitinos could perhaps be heavier in a local $N \geq 2$ theory, though we are not very sanguine. A more promising way out may be to break local supersymmetry not at the tree level, but rather by radiative corrections. In any case, radiative corrections may produce useful violations of the mass sum rules (4) and (4'). A more distant hope is that local (and perhaps global) supersymmetry may be broken non-perturbatively, so that the theoretical framework of this paper is irrelevant. Finally, we note that a crucial ingredient in our proof was the condition (6) for the vanishing of the cosmological constant. If one does not impose this condition, one is formally in De Sitter space¹¹⁾, and the interpretation of particle mass terms in the Lagrangian and potential is not at all clear. However, it has been suggested by Hawking⁸⁾ and others that the effective cosmological constant may vanish at large distances, even if the bare parameter in the Lagrangian which describes short-distance physics is in fact very large. We recall also that it is difficult¹²⁾ to reconcile the Fayet-Iliopoulos¹⁶⁾ mechanism of supersymmetry breaking with a zero cosmological constant. One might therefore be prepared to relax the condition (6) for the time being, or at least until the cosmological constant problem is clarified. However, phenomenological life might be more interesting if the gravitino mass were indeed $\lesssim 0(100)$ GeV as our result suggests.

Unless one of the above-mentioned escape routes can be exploited, it is difficult to see how scenarios with a supersymmetry breaking scale $\mu_s > \sqrt{m_W m_{\text{Planck}}}$ can be maintained. An even more striking conclusion is obtained if one combines our results with the cosmological observations of Pagels and Primack¹⁷⁾ and of Weinberg¹⁸⁾. They conclude that either $m_\psi < 1 \text{ keV}$ ($\mu_s < 2 \times 10^6 \text{ GeV}$) or else $m_\psi > 10^4 \text{ GeV}$ ($\mu_s > 10^{11} \text{ GeV}$). Our results may exclude the latter alternative except for a possible small window with $\mu_s \sim 10^{10}$ to 10^{11} GeV (?), at least in an attractive class of theories. Therefore the gravitino may need to be lighter than a keV.

ACKNOWLEDGEMENTS

We thank S. Ferrara for communicating the results of Ref. 14) to us before publication.

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