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THE MASSES OF HEAVY FLAVOURED HADRONS

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A B S T R A C T

We propose a model-independent way to obtain upper bounds on the masses of beautiful hadrons and remark that these upper bounds are in fact estimates. We also propose simple interpolation formulae for hyperfine splittings.

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You have heard, in other talks, discussions on the production mechanisms of hadrons with visible heavy flavour. Here, I just want to present a method to estimate what will be the masses of these objects, especially the beautiful mesons and baryons. However, naturally, once the $t\bar{t}$ system is discovered the method will also apply to calculate masses of objects with visible top.

For us a baryon with heavy flavour Q will be (qqQ) , and a meson will be $(\bar{q}Q)$. q will be an ordinary quark u or d . Q will be t, b, c and even s (which we dare to treat as heavy because of the success of the description of the $s\bar{s}$ and $c\bar{c}$ states in this way). One can think of other systems like (qQQ) (the Ξ is of this type !) or (QQQ) like the Ω^- , but the production of these things, when $Q = c$ or b seems very unlikely and will not be observed before a very long time presumably.

The most crude prediction of masses one can make is based on the crude additive quark model. For instance, if we take $m_s = \frac{1}{2}M\phi = 0.51$ GeV we predict $m_{Q^-} = 1.53$ instead of the experimental number 1.672. So if you do not want an accuracy better than about 150 MeV, this is good enough (the spin effects, however, cannot be disregarded ! The mass differences $\rho-\pi$, $\Delta-N$ show this very clearly).

The next, much more sophisticated thing you can do is to manufacture a potential between quarks or between quark and antiquark, which fits existing hadrons and try to use this potential to predict the masses of beautiful or truthful hadrons. One can constraint the fit further by demanding that in the three-quark system only two-body forces are important and that $V_{qq'} = \frac{1}{2}V_{q\bar{q}'}$, a property which is

strictly true for the one-gluon exchange part of the force and which has the merit of giving results insensitive to the particular choice of quark masses made. Specifically, if we neglect the change in the kinetic energy we can make the replacements

$$m'_{q_i} = m_{q_i} + \Delta$$

$$V'_{q_i \bar{q}_j} = V_{q_i \bar{q}_j} - 2\Delta$$

$$V'_{q_i q_j} = V_{q_i q_j} - \Delta,$$

and these changes preserve the above-mentioned relation between qq and $q\bar{q}$ forces. As an example, Richard ¹⁾ using a potential which fits the $b\bar{b}$, $c\bar{c}$, $c\bar{s}$ and $s\bar{s}$ systems ²⁾ has obtained $M_{Q^-} = 1665$ (experiment: 1672). Bhadury et al. ³⁾ and Stanley and Robson ⁴⁾ have made over-all fits of all mesons and baryons with a universal potential and make predictions like, for instance, $m_{B^*} = 5.344$, $m_B = 5.308$, $m_{\Lambda_b} = 5.574$, $m_{\Sigma_b} = 5.046$, $m_{\Sigma_b^*} = 5.862$ MeV. Their approach is sometimes criticized because of the large number of parameters in their potentials. This seems unavoidable if one wants to treat spin effects in a non-perturbative way and take into account annihilation diagrams to explain the impurity of the quark content of mesons [such problems do not occur if one restricts oneself to heavy quarks as was done in Ref. 1)].

Anyway there is a need for a less model-dependent method of prediction of the masses of beautiful and truthful hadrons. What we want to point out is that if one believes in flavour-independence of the forces and if one believes that there is some effective, but unknown, Hamiltonian for the quark-antiquark, and three quark systems containing one heavy quark, one can make some general predictions.

Because of flavour-independence the heavy quark mass is the only characteristic of the heavy quark. It will appear in the kinetic energy as $p^2/2M_Q$, in the spin-spin forces as $(1/m_Q)f(r_i, \vec{\sigma}_i)$. [For instance, in the case of a contact term we have just

$$c_{q_i} \frac{1}{m_Q m_{q_i}} (\vec{\sigma}_Q \cdot \vec{\sigma}_{q_i}) \delta(\vec{r}_Q - \vec{r}_{q_i})$$

neglecting terms of the order of $(1/m_Q)^2$ which are under control.] The same is true for spin-orbit terms, which anyway are unimportant in the ground states of hadrons.

In short, the Hamiltonian of the $Q\bar{q}$ or Qqq system is essentially of the form

$$H = \frac{1}{m_Q} H_1 + H_2$$

Notice that this does not say that light quark motion is not relativistic ! Anyway when we see this form we can use an elementary theorem which is

If $H = A + \lambda B$, the ground state energy of the system $E(\lambda)$ is a concave function of λ , i.e.,

$$\frac{d^2 E}{d\lambda^2} \leq 0.$$

So the binding energy of the Qqq system (with given external quantum numbers like spin, isospin, etc.) will be a concave function of $1/m_Q$. In other words, if

$$M(Q\bar{q}) \text{ or } M(Qqq) = A + Bm_Q + \frac{C}{m_Q} + \frac{D}{m_Q^2}$$

the theorem says that D is negative.

It is perhaps slightly daring to treat strange quarks as heavy quarks but the success in the treatment of the $s\bar{s}$ and $c\bar{s}$ systems seems to justify it ²⁾. Then the concavity property allows us to get, for instance, an upper bound on $M(B^*)$ from $M(D^*)$ and $M(K^*)$. The only extra ingredient needed is a set of quark masses. Though quark mass differences change very little from one good fit to another good fit, there is some freedom in the choice of the absolute value of the masses. We have found, however, that taking two sets of masses gives very little change in the predictions. If we take the set we proposed ²⁾ to fit simultaneously $b\bar{b}$, $c\bar{c}$, $c\bar{s}$, $s\bar{s}$:

$$m_b = 5.174, \quad m_c = 1.8, \quad m_s = 0.518 \text{ GeV},$$

we obtain from the known masses of K , K^* , D , D^* , Λ , Σ , Λ_c , Σ_c :

$$m_B \leq 5.263 \text{ GeV}, \quad m_{B^*} \leq 5.340 \text{ GeV},$$

$$m_{\Lambda_b} \leq 5.629 \text{ GeV}, \quad m_{\Sigma_b} \leq 5.826 \text{ GeV}.$$

These inequalities are interesting because they are seen to be very constraining in the meson case. Indeed, we get

$$m_B + m_{B^*} < 10.603 \text{ GeV.}$$

According to CUSB ⁵⁾ the T''' , with a mass of 10.580 GeV, is lighter than $m_B + m_{B^*}$ because they see no signal from the decay

$$m_{B^*} \rightarrow m_B + \gamma,$$

at the T''' energy.

If this is true this means that the upper bound we get is less than 25 MeV away from the experimental value.

We are thus led to ask ourselves the question whether formulae of the type

$$M = A + m_Q + B/m_Q$$

do not represent reasonable interpolation formulae.

This can be checked on potential models both for the two-body system and for the three-body system. Our limited "numerical experiments" indicate that by doing this one never makes a mistake of more than 50 MeV.

In other words we expect

$$5.213 \leq m_B \leq 5.263 \text{ GeV, } 5.290 < m_{B^*} < 5.340 \text{ GeV,}$$

$$5.581 \leq m_{\Lambda_b} \leq 5.629 \text{ GeV, } 5.776 \leq m_{\Sigma_b} \leq 5.826 \text{ GeV.}$$

To end up with still less rigorous considerations, let us suggest to predict vector-pseudoscalar mass differences by formulae of the type

$$M(Q\bar{q})_{1-} - M(Q\bar{q})_{0-} = A/m_Q + B/m_Q^2.$$

Taking into account deviations from this formula, we predict

$$50 < M_{B^*} - M_B < 57 \text{ MeV.}$$

Similarly, numerical experiments indicate that the $\Sigma_Q - \Lambda_Q$ mass difference, where

$$\Sigma_Q = \left[(qq)_{J=1} \begin{matrix} Q \\ \end{matrix} \right]_{J=\frac{1}{2}}$$

and

$$\Lambda_Q = \left[(qq)_{J=0} \begin{matrix} Q \\ \end{matrix} \right]_{J=\frac{1}{2}}$$

is well represented by

$$\Sigma_Q - \Lambda_Q = A + B/m_Q.$$

In this way, from $\Sigma - \Lambda = 77 \text{ MeV}$ and $\Sigma_c - \Lambda_c = 166 \text{ MeV}$, one predicts

$$\Sigma_b - \Lambda_b = 197 \pm 10 \text{ MeV.}$$

On the other hand, we expect that the mass difference $\Sigma_Q^* - \Sigma_Q$, where

$$\Sigma_Q^* = \left[(qq)_{J=1} \begin{matrix} Q \\ \end{matrix} \right]_{J=\frac{3}{2}},$$

vanishes for $m_Q \rightarrow \infty$ because the hyperfine interaction of the heavy quark with the light quarks vanishes, so that $A = 0$.

From $\Sigma^* - \Sigma = 190 \text{ MeV}$, we expect

$$\Sigma_c^* - \Sigma_c = 55 \text{ MeV} \quad \text{and} \quad \Sigma_b^* - \Sigma_b = 19 \text{ MeV.}$$

Notice that since Σ_b is allowed to decay strongly via $\Sigma_{c,b} \rightarrow \Lambda_b + \pi$, with an appreciable phase space available, the widths of Σ_b and Σ_b^* exceed their splitting, and we expect, as pointed out by Lipkin⁶⁾, interesting interference phenomena in the angular distribution of the decay products as a function of energy.

REFERENCES

- 1) J.M. Richard - Phys.Letters 100B (1981) 515.
- 2) A. Martin - Phys.Letters 100B (1981) 511.
- 3) A. Bhadury et al. - Phys.Rev.Letters 44 (1980) 3180.
- 4) D.P. Stanley and D. Robson - Phys.Rev. D21 (1980) 3180; Phys. Rev.Letters 45 (1980) 235.
- 5) J. Lea-Franzini and P. Franzini - Physics Reports 81 (1982) 240.
- 6) H.J. Lipkin - Private communication.