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SPIN-ZERO LEPTONS AND THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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A B S T R A C T

The anomalous magnetic moment of the muon $(g-2)_\mu$ imposes constraints on the masses and mixing of spin-zero leptons (sleptons). We develop the predictions of models of spontaneous supersymmetry breaking for the slepton mass matrix, and show that they are comfortably consistent with the $(g-2)_\mu$ constraints.

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During the present resurgence of interest in supersymmetry broken at low energies ¹⁾ new significance is attached to the classical phenomenological playgrounds of gauge theories such as the anomalous magnetic moments of the electron and muon ²⁾, flavour-changing neutral interactions ³⁾⁻⁵⁾, parity ⁶⁾ and CP violation ^{7),8)} in the strong interactions. The three latter phenomena make life rather difficult ^{3),7)} for the most general form of soft supersymmetry breaking, whereas simple models ⁹⁾⁻¹¹⁾ of spontaneously broken supersymmetry naturally ^{3),4),7)} respect the $\Delta F \neq 0$, P and CP violation constraints. As for the anomalous magnetic moments of the leptons, it has long been known that they vanish in an exactly supersymmetric theory ¹²⁾, and Fayet ²⁾ showed that in his model of supersymmetry breaking $(g-2)_\mu$ would be compatible with experiment if the spin-zero muon (smuon) masses were heavier than 15 GeV. Direct experimental searches ¹³⁾ now exclude the existence of lighter smuons. Fayet's analysis ²⁾ was in the context of a model with a very light photino $\tilde{\gamma}$ (see Fig. 1a), and Grifols and Méndez ¹⁴⁾ have recently made the interesting observation that his analysis is significantly altered for massive gauginos (see Figs. 1b,c). They show that there are potentially non-trivial constraints on the smuon masses in models of broken supersymmetry.

In this paper we analyze $(g-2)_\mu$ more deeply, taking into account the possibility of a non-trivial mass-matrix mixing between the spin-zero partners of the left- and right-handed muons. Since $(g-2)_\mu$ is a left-right transition operator there is a GIM ¹⁵⁾-like cancellation between the smuon mass eigenstates in Fig. 1c which provides a potential suppression mechanism. We analyze recent models ^{10),11)} of spontaneous supersymmetry breaking originating in the D and F sectors, respectively. We show that in the former case $(g-2)_\mu$ is suppressed by near degeneracy between the smuon mass eigenstates, while in the latter case $(g-2)_\mu$ is suppressed by small mixing angles between the left- and right-handed smuons. We close with some remarks about $(g-2)_e$ and about parity violation in the strong interactions.

When they examined Figs. 1a,b and c, Grifols and Méndez ¹⁴⁾ realized that there was a fundamental difference between the (almost ?) massless $\tilde{\gamma}$ diagram of Fig. 1a and the $\tilde{W}$ diagram of Fig. 1b as compared to the massive $\tilde{B}$ or $\tilde{Z}$ diagram of Fig. 1c. The first two diagrams are proportional to m_μ^2 , while the last one is apparently proportional to m_μ with the gaugino mass term replacing the other factor of m_μ . Figure 1a was shown by Fayet ²⁾ to make an acceptably small contribution to $(g-2)_\mu$:

$$\Delta(g-2)_\mu = -\frac{\alpha}{6\pi} m_\mu^2 \left(\frac{1}{m_{\tilde{S}_\mu}^2} + \frac{1}{m_{\tilde{t}_\mu}^2} \right) \quad (1)$$

where we have used Fayet's notation for the left- and right-handed smuons. Equation (1) is within the experimental range ¹⁶⁾ of $(-4.0, +5.2) \times 10^{-8}$ if $m_{S_\mu}, m_{t_\mu} > 15 \text{ GeV}$ ²⁾. Grifols and Méndez ¹⁴⁾ showed that Fig. 1b was acceptably small for all values of the sneutrino mass if $m_{\tilde{W}^\pm} > O(10) \text{ GeV}$. PETRA and PEP require $m_{\tilde{W}^\pm} > O(17) \text{ GeV}$, since measurements of the total cross-section and forward-backward asymmetry ¹⁷⁾ for $e^+e^- \rightarrow \tau^+\tau^-$ determine that the weak neutral couplings of the τ differ from those of a $\tilde{W}$. A priori the Majorana gaugino mass in Fig. 1c gives a contribution to $(g-2)_\mu$ which is larger than the other diagrams by a factor of order $(m_{\tilde{B}}^2 \text{ or } m_{\tilde{Z}}^2/m_\mu^2)$. There are also contributions which are not linear in the Majorana mass term, but they yield $\Delta(g-2)_\mu \sim m_\mu^2$ and are hence as harmless as Figs. 1a and 1b. The leading term from Fig. 1c involves a left-right transition, and we therefore introduce a parametrization of the smuon eigenstates :

$$\left. \begin{aligned} S\mu_1 &\equiv S_\mu \cos \alpha + t_\mu \sin \alpha \\ S\mu_2 &\equiv -S_\mu \sin \alpha + t_\mu \cos \alpha \end{aligned} \right\} \quad (2)$$

in terms of which Fig. 1c yields

$$\Delta(g-2)_\mu = -ab \frac{\cos \alpha \sin \alpha}{8\pi^2} \frac{m_\mu}{m_{\tilde{G}}} \times \left[\frac{1}{1-\eta_1} + \frac{2\eta_1}{(1-\eta_1)^2} + \frac{2\eta_1}{(1-\eta_1)^3} \log \eta_1 - (\eta_1 \leftrightarrow \eta_2) \right] \quad (3)$$

where $m_{\tilde{G}}$ is the mass of the appropriate gaugino, $\eta_i \equiv (m_{S\mu_i}^2/m_{\tilde{G}}^2)$ and "a" and "b" are the chiral gaugino couplings :

$$\mathcal{L} = a S_\mu \bar{\mu}_L \tilde{G} + b t_\mu \bar{\mu}_R \tilde{G} + (\text{herm. conj.})$$

(In one ¹⁰⁾ of the explicit models we consider later the neutral gaugino mass eigenstates are the $\tilde{\gamma}$ and the $\tilde{Z}$, neglecting gravitational effects, whereas in the other model ¹¹⁾ the $\tilde{B}$ and $\tilde{W}$ are mass eigenstates.)

Formula (3) exhibits a GIM-like cancellation between the different smuon exchanges. In the limit where $|m_{S\mu_1}^2 - m_{S\mu_2}^2| \ll m_{\tilde{G}}^2$ it becomes

$$\Delta(g-2)_\mu = -ab \frac{\cos \alpha \sin \alpha}{8\pi^2} \frac{m_\mu (m_{S\mu_1}^2 - m_{S\mu_2}^2)}{m_{\tilde{G}}^3} F(\eta) \quad (4)$$

where $\eta = (m_{S\mu_1}^2 + m_{S\mu_2}^2)/2m_{\tilde{G}}^2$ and

$$F(\eta) = \left[\frac{3}{(1-\eta)^2} + \frac{2+4\eta+2\log \eta}{(1-\eta)^3} + \frac{6\eta}{(1-\eta)^4} \log \eta \right]$$

Equation (4) falls within the experimentally allowed range provided

$$-4.0 \times 10^{-8} < -ab \frac{\cos \alpha \sin \alpha}{8\pi^2} \frac{m_\mu (m_{\tilde{S}\mu_1}^2 - m_{\tilde{S}\mu_2}^2)}{m_{\tilde{Z}}^2} F(\eta) < +5.2 \times 10^{-8} \quad (5)$$

We now analyze models of spontaneous supersymmetry breaking to see whether they are compatible with the constraint (5).

Spontaneous supersymmetry breaking may either be driven by the gauge (D) terms in the scalar field potential, or else by Yukawa (F) terms¹⁾. The first of these alternatives was discussed by Fayet²⁾, and a recent version of this class of models is due to Barbieri, Ferrara and one of the present authors (D.V.N.)¹⁰⁾. As is generally the case in D models, the axial coupling of the new $\tilde{U}(1)$ gauge boson in the BFN model¹⁰⁾ generates equal masses for the left- and right-handed sleptons, there is no mixing between the staus, selectrons and smuons, and the smuon mass matrix is of the form

$$\begin{pmatrix} \tilde{S}_\mu^* & t_\mu^* \end{pmatrix} \begin{pmatrix} D & \epsilon \\ \epsilon & D \end{pmatrix} \begin{pmatrix} \tilde{S}_\mu \\ t_\mu \end{pmatrix} \quad (6)$$

where ϵ is a possible small left-right mixing term. Because of the degeneracy between the diagonal terms in the matrix (6), even a small value of ϵ guarantees maximal mixing ($\cos \alpha \sin \alpha = \frac{1}{2}$). The splitting between the mass eigenstate smuons is 2ϵ , so condition (5) on the magnitude of the contribution to $(g-2)_\mu$ from the $\tilde{Z}$ loop in Fig. 1c becomes $[ab = -2\pi\alpha_e(1-\tan^2\theta_W)]$:

$$-9.4 \times 10^{-5} < \frac{\epsilon m_\mu}{m_{\tilde{Z}}^2} F(\eta) < +1.2 \times 10^{-4} \quad (7)$$

The BFN model¹⁰⁾ has no contribution to ϵ at the tree level, but it can arise in one-loop order, thanks, for example, to Fig. 2. Accordingly, we estimate

$$\epsilon = O\left(\frac{\alpha}{\pi}\right) m_\mu v \quad (8)$$

where $v = O(100 \text{ GeV})$. [Hence in this model one necessarily has $|m_{\tilde{S}\mu_1}^2 - m_{\tilde{S}\mu_2}^2|/m_{\tilde{Z}}^2 \ll \ll 1$, so Eq. (4) is a good approximation to the full formula (3).] Since the function $F(\eta) \leq O(1)$, the constraint (7) is satisfied for any plausible value of $m_{\tilde{Z}}$.

Turning now to F models, in these the sfermion mass matrices are generated by radiative corrections, and take the general forms 9),11)

$$m_{sf}^2 = \left(\sum_{i=1,2,3} \alpha_i^n C_i \right) \mu^2 \quad (9)$$

where the α_i are SU(3), SU(2) and U(1) coupling strengths, the sum runs over all gauge interactions coupling to a given fermion helicity, $n = 2$ or 3 , and the C_i are products of Clebsch-Gordan coefficients.

For example, in the original EIR model 11)

$$\begin{aligned} m_{sq_{2/3L}}^2 &\approx \left(\frac{4}{3} \alpha_3^2 + \frac{3}{4} \alpha_2^3 + \frac{25}{324} \alpha_1^3 \right) \mu^2 \\ m_{sq_{-1/3L}}^2 &\approx \left(\frac{4}{3} \alpha_3^2 + \frac{3}{4} \alpha_2^3 + \frac{25}{324} \alpha_1^3 \right) \mu^2 \\ m_{sq_{2/3R}}^2 &\approx \left(\frac{4}{3} \alpha_3^2 + \frac{100}{81} \alpha_1^3 \right) \mu^2 \\ m_{sq_{-1/3R}}^2 &\approx \left(\frac{4}{3} \alpha_3^2 + \frac{25}{81} \alpha_1^3 \right) \mu^2 \\ m_{sl_L}^2 &\approx \left(\frac{3}{4} \alpha_2^3 + \frac{25}{36} \alpha_1^3 \right) \mu^2 \\ m_{s2}^2 &\approx \left(\frac{3}{4} \alpha_2^3 + \frac{25}{36} \alpha_1^3 \right) \mu^2 \\ m_{sl_R}^2 &\approx \left(\frac{25}{9} \alpha_1^3 \right) \mu^2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} + O(m_q^2)$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} + O(m_l^2) \quad (10)$$

but there are also contributions with $n = 2$ from Eq. (9). Again there is no st , se , su mixing and from Eq. (9) we get

$$(S_\mu^*, t_\mu^*) \begin{pmatrix} D_L & \epsilon \\ \epsilon & D_R \end{pmatrix} \begin{pmatrix} S_\mu \\ t_\mu \end{pmatrix} \quad (11)$$

with $D_L \geq O(m_W) = O(100 \text{ GeV})$ and generally $\gg D_R$, as can be seen explicitly in Eq. (10). In the EIR model 11) the off-diagonal parameter ϵ in Eq. (11) can get tree graph contributions, though these may be very small. The leptons get their masses from an SU(5) 5-plet of Higgses H_2 which may have superpotential couplings to a $\bar{5}$ -plet of Higgses H_1 via a mass parameter δ , singlet fields X_i and $\underline{24}$ -plets Σ_j :

$$P = g_\mu \mu_L \bar{\mu}_R H_2 + H_1 (\delta + \lambda_i X_i + \lambda'_j \Sigma_j) H_2 + \dots \quad (12)$$

From the $F_{H_2} = \partial P / \partial H_2$ term in the scalar potential we extract

$$g_\mu [S_\mu t_\mu^*] [H_1 (\delta + \lambda_i X_i + \lambda'_j \Sigma_j)]^* + (\text{herm.})_{\text{conj.}} \quad (13)$$

We expect the second square parenthesis in Eq. (13) to be $O(v=O(100 \text{ GeV}))^2$ at most - it may well be much smaller - and therefore conclude that in the EIR class of models ¹¹⁾

$$\epsilon = m_\mu O(v) \quad (14)$$

since $m_\mu = g_\mu \langle 0 | H_2 | 0 \rangle = g_\mu O(v)$. If the second square parenthesis in Eq. (13) is much smaller than $O(v^2)$, then one falls back on the previously estimated contribution (8) to ϵ . Returning now to the matrix (11) we see that $|\epsilon| \ll |D_L - D_R|$ so that the mixing angle α of Eq. (2) is small :

$$\alpha \approx \frac{\epsilon}{D_L - D_R} \quad (15)$$

whereas $m_{S\mu_1} - m_{S\mu_2} \approx D_L - D_R$ so that the condition (5) reduces to $[ab = 2\pi\alpha_e / \cos^2\theta_W \text{ for } \tilde{B} \text{ exchange}] :$

$$-7.1 \times 10^{-9} < \frac{\epsilon m_\mu}{m_{\tilde{B}}^3} F(\eta) < +5.4 \times 10^{-5} \quad (16)$$

The value of the off-diagonal parameter ϵ may be larger in the EIR class ¹¹⁾ of models than in the BFN model ¹⁰⁾ (8), but there is still no danger of a conflict with the $(g-2)_\mu$ constraint (16).

We conclude this note with some comments related to the above analysis of $(g-2)_\mu$. The first observation is that since coincidentally the experimental bound ¹⁶⁾ on $\Delta(g-2)_e$ is $O(m_e/m_\mu) \times \Delta(g-2)_\mu$, in the Grifols-Méndez ¹⁴⁾ analysis the anomalous magnetic moment of the electron would have appeared to give bounds on supersymmetric particle masses comparable with those emanating from the

anomalous magnetic moment of the muon, whereas in our analysis it gives a much weaker constraint because of the extra lepton mass factor (8,14) appearing in ϵ . The final comment concerns parity violation in the strong interactions⁶⁾. This does not arise from the BFN mass matrix (6) because the mass eigenstates are pure scalar and pseudoscalar states. Parity violation can in principle arise from the EIR mass matrix (11), but it almost cancels in the processes considered by Suzuki⁶⁾ since the left u and d squark $(\text{mass})^2$ are identical within small corrections $O(m_u^2 - m_d^2)$, as are the right u and d squark $(\text{mass})^2$. Therefore this model is also comfortably consistent with the experimental constraints on nuclear parity violation.

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See also the discussion in the last paper of ref. 11).

FIGURE CAPTIONS

Figure 1 One-loop diagrams contributing to $(g-2)_\mu$: (a) essentially massless photino ($\tilde{\gamma}$) exchange, (b) $\tilde{W}$ and sneutrino (ν) exchange, and (c) $\tilde{B}$ or $\tilde{Z}$ exchange.

Figure 2 A one-loop contribution to the off-diagonal $s_{\mu_L} \overline{s_{\mu_R}}$ mass mixing which is proportional to m_μ .

FIG.1

FIG.2