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ELECTRODYNAMICS IN HIGH-Z SYSTEMS

G. SOFF, U. MÜLLER, T. DE REUS, P. SCHLÜTER,
A. SCHÄFER, J. REINHARDT, B. MÜLLER, AND
W. GREINER

Invited lecture at the summer school "Atomic Physics
of Highly Ionized Atoms", Cargese, France, June 1982

JUNE 1982

Gesellschaft für Schwerionenforschung mbH
Planckstr. 1 · Postfach 110541 · D-6100 Darmstadt 11 · Germany

ELECTRON EXCITATION PROCESSES AND QUANTUM ELECTRODYNAMICS IN HIGH-Z SYSTEMS*

Gerhard Soff

Gesellschaft für Schwerionenforschung mbH
Planckstraße 1, Postfach 110541
D-6100 Darmstadt 11, West Germany
and

Udo Müller, Theo de Reus, Paul Schlüter, Andreas Schäfer,
Joachim Reinhardt, Berndt Müller, and Walter Greiner

Institut für Theoretische Physik der
Johann Wolfgang Goethe-Universität
Robert-Mayer-Straße 8-10, Postfach 111 932
D-6000 Frankfurt am Main, West Germany

1. INTRODUCTION

Collisions of very heavy ions with bombarding energies close to the Coulomb barrier offer us the possibility to perform a spectroscopy of electronic states in transient superheavy systems within a charge range of $100 \leq Z \leq 190$ ($_{92}\text{U} + _{98}\text{Cf}$). The pivotal question to answer is whether the binding energy of the strongest bound electronic state can reach or exceed twice the electron rest mass. An important task in this connection is the investigation of electron excitation processes in superheavy systems. Two possible excitation mechanisms are visualized in Fig. 1. As an example we consider the schematic level structure for a head-on collision of $\text{Pb} + \text{Cm}$ with a total charge $Z = 178$ at 5.9 MeV/u bombarding energy. Binding energies of the adiabatic $1s\sigma^-$ and $2s\sigma^-$ states¹⁾ are displayed versus the internuclear separation R . The turning point between incoming and outgoing trajectory is given by $R_{\min} = 17$ fm.

*This work was supported by the Bundesministerium für Forschung und Technologie (BMFT) and the Deutsche Forschungsgemeinschaft (DFG). One of us (G.S.) acknowledges the support of the DFG-Heisenberg Programm.

Fig. 1: Binding energies of the $1s\sigma$ and $2s\sigma$ state versus inter-nuclear separation for the Pb+Cm system. One-step and two-step electron excitation processes are indicated qualitatively. On the right hand side characteristic numbers describing the collision are given.

A typical one-step excitation process is the direct ionization from the $1s\sigma$ -state to the positive continuum. In addition the multi-step processes play an important role. Actually they form the dominant part of inner shell vacancy production probabilities. An example for a two-step process is depicted in Fig. 1, where a $2s\sigma$ -electron is ionized to the continuum. The remaining hole is filled by the $1s\sigma$ -electron which leaves a vacancy in the K-shell. Since initial and final state of both processes are indistinguishable, they have to be treated coherently. This is achieved by solving coupled channel equations for the electron occupation amplitudes of the quasimolecular adiabatic states.

Strong relativistic effects are best visible in the wavefunctions. Fig. 2 shows the radial density $r^2|\psi(r)|^2$ of the $1s$ -electron in the field of an extended nucleus. The Schrödinger hydrogenic $1s$ wavefunction is given by $\psi_{1s}(r) = 2(Z/a_B)^{3/2} e^{-rZ/a_B}$, where $a_B = \hbar_e/\alpha = 52918 \text{ fm}$ is the Bohr radius. The axes in fig. 2 have been scaled appropriately in order to make the shape of the

nonrelativistic density independent of the charge Z . Deviations therefore are solely due to relativistic effects. While the shape of $r^2|\psi(r)|^2$ does not alter much in the region of the ordinary periodic table, the wavefunction becomes severely contracted as $Z\alpha$ approaches or exceeds the value 1. Although a complete "collapse of the wavefunction" is averted, the scaled Bohr radius a_B/Z loses its importance if $Z\alpha > 1$ and the only relevant length scale is determined by the nuclear radius R . The inset in Fig. 2 gives the electron density at the origin, $|\psi(0)|^2$, as a function of Z . Apart from the Z^3 -dependence known from the nonrelativistic solution the density increases by additional three orders of magnitude going to $Z = 180$.

The figure also displays the electron density at the origin for the $2p_{1/2}$ -wavefunction. For a nonrelativistic p-wave, characterized by the sharp angular momentum $\ell = 1$, this value is exactly zero. The $2p_{1/2}$ Dirac bispinor, however, carries a mixture of $\ell = 0$ (upper component) and $\ell = 1$ (lower component orbital angular momentum). In atoms with $Z\alpha > 1$ the "small" and "large" components of the wavefunction become comparable in magnitude. As a consequence the distinction between $s_{1/2}$ - and $p_{1/2}$ -states is diminished. This explains the steep rise of the $2p_{1/2}$ density as well as the strong increase of its binding energy as a function of Z .

In fig. 3 we compare the radial density distribution of the K-shell electron in lead with that in the superheavy atom $Z=164$, where the maximum of $|\psi|^2 r^2$ is located at about $r \sim 20$ fm. The radial expectation value $\langle r \rangle_{1s}$ changes from 840 fm for $Z=82$ to 140 fm for $Z=164$.

The shrinkage of the $1s$ -wavefunction at shortest internuclear distances is a typical feature in collisions of very heavy ions. Sticking to our example $\text{Pb} + \text{Cm}$ at $E_{\text{lab}} = 5.9$ MeV/u we find a radial expectation value for the $1s$ orbital of about 120 fm at the distance of closest approach. Due to this extreme localization (within the Compton wavelength of an electron, $\lambda_e = 386$ fm) transfer of high momenta components to the strongly bound electrons resulting from the nuclear motion becomes possible. Therefore δ -electrons up to kinetic energies of 2 MeV can be observed experimentally³, whereas the classical allowed maximum energy transfer to an electron at rest is about 10 keV.

Fig. 2: The radial electron density of the $1s_{1/2}$ state in super-heavy atoms with nuclear charges $Z = 100, 135$, and 170 . For comparison the nonrelativistic Schrödinger wavefunction is drawn. The axes are scaled with powers of Z in order to make the nonrelativistic density independent of the charge. The inset shows the drastic increase of the electron density at the origin over its nonrelativistic value. The wavefunctions of the $1s_{1/2}$ and $2p_{1/2}$ states become very similar in the region $Z\alpha > 1$.

Fig. 3

After a short theoretical survey concerning electron excitation processes we discuss in chapter 3 as a special topic the bound state solution of the two-center Dirac equation. In the following we present our numerical results and compare them with some experimental data. The influence of strong collisional magnetic fields on spin polarization of electrons is treated in chapter 5. Section 6 and 7 deal with the selfenergy of electrons in critical fields and with some aspects of the Glashow-Weinberg-Salam interaction. After this we present a few new results concerning positron creation in deep inelastic nuclear collisions. Finally we discuss the possibility of internal electron-positron pair creation in supercritical compound systems.

In this chapter we summarize the essential equations where our calculations are based on. We emphasize that in superheavy quasi-molecules a full relativistic treatment of the electron motion is absolutely necessary since the combined charge $Z=Z_1+Z_2 \gg 1/\alpha$ and the electron binding energies become comparable with the electron rest mass or even exceed it. If we want to describe the dynamical evolution of an electron involving two colliding nuclei we thus have to solve the time-dependent two-centre Dirac equation.

$$i\partial/\partial t \phi_i(\vec{R}(t)) = H_{\text{TCD}}(\vec{R}(t)) \phi_i(\vec{R}(t)). \quad (2.1)$$

H_{TCD} is the relativistic two-centre Hamiltonian which depends sensitively on the internuclear separation $R(t)$. For this reason it is useful to expand the total wave function ϕ_i into Born-Oppenheimer states ϕ_n which are represented by the stationary molecular states. The expansion reads

$$\phi_i(t) = \sum_j a_{ij}(t) \phi_j(\vec{R}(t)) \exp(-i\chi_j(t)), \quad (2.2)$$

where the sum includes an integration over continuum states with positive and negative energies. The phase factors χ_j are chosen as

$$\chi_j(t) = \int^t dt' \langle \phi_j(\vec{R}(t')) | H_{\text{TCD}}(\vec{R}(t')) | \phi_j(\vec{R}(t')) \rangle. \quad (2.3)$$

Inserting the expansion of eq. (2.2) into eq. (2.1) and projecting with stationary eigenfunctions we obtain the following set of first-order coupled differential equations for the occupation amplitudes $a_{ij}(t)$

$$\dot{a}_{ij}(t) = \sum_k a_{ik}(t) \langle \phi_j | \partial/\partial t | \phi_k \rangle \exp\{+i(\chi_j - \chi_k)\}. \quad (2.4)$$

The time derivative operator can be expressed in terms of a radial and rotational coupling

$$\partial/\partial t \rightarrow \dot{R} \partial/\partial R - i\vec{\omega} \cdot \vec{J}. \quad (2.5)$$

Our calculations are restricted to radial couplings only.

The set $\phi_i(t)$, already containing the dynamical excitations, can be used as a basis for solving the many-particle problem. To do this, one can expand the total wave function in a basis of many-electron configurations which may be represented by Slater determinants of the single particle basis functions ϕ_i . The amplitude for exciting a final configuration starting from a given initial configuration turns out to be just the determinant of the corresponding single-particle amplitudes $a_{ij}(t)$. Therefore the number of physical particles p or holes q in a particular level can be

deduced completely from the single-particle amplitudes^{4,5)}

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$$N_p = \sum_{r>F} |a_{rp}|^2 \quad \text{for } p>F, \quad (2.6)$$

$$N_q = \sum_{r>F} |a_{rq}|^2 \quad \text{for } q<F. \quad (2.7)$$

F denotes the Fermi surface of occupied states. For the number of correlated particle-hole pairs $N_{p,q}$ one finds

$$N_{p,q} = N_p \cdot N_q + \left| \sum_{r<F} a_{rp}^* a_{rq} \right|^2. \quad (2.8)$$

This formula should be applied to analyze experiments like those presently performed for coincidence between electrons and 1s σ -vacancy formation^{3,6)}. Eq. (2.8) holds also for particle-particle or hole-hole correlations if the sign of the second term is inverted.

In this section we find solutions of the stationary Dirac equation for a two-center potential. We start with the representation of the Dirac equation in spherical coordinates:

$$\left[i\gamma_5 \sigma_r \left(\frac{\partial}{\partial r} + \frac{1}{r} - \frac{\beta}{r} \hat{K} \right) - \frac{Z_1 e^2}{|\vec{r} - \frac{1}{2} \vec{R}|} - \frac{Z_2 e^2}{|\vec{r} + \frac{1}{2} \vec{R}|} + \beta m_e \right] \phi_\mu(\vec{r}) = E \phi_\mu(\vec{r}). \quad (3.1)$$

The spin-orbit operator is defined through the relation

$$\hat{K} \equiv \beta(\vec{\sigma} \vec{\ell} + 1). \quad (3.2)$$

It is known that the two-center Dirac equation is more difficult to handle than its nonrelativistic counterpart, since it is not separable in any orthogonal coordinate system. The most accurate and flexible approach at present is based on a multipole expansion of the wave function^{7,8)}

$$\phi_\mu(\vec{r}) = \sum_{\kappa} \phi_{\mu\kappa}(\vec{r}) = \sum_{\kappa} \begin{pmatrix} g_{\kappa}(r) X_{\kappa}^{\mu} \\ i f_{\kappa}(r) X_{-\kappa}^{\mu} \end{pmatrix}. \quad (3.3)$$

$f_{\kappa}(r)$ and $g_{\kappa}(r)$ are the radial wave functions, and the spinor spherical harmonics are given by

$$X_{\kappa}^{\mu} = \sum_{m=\pm 1/2} (\ell, \frac{1}{2}, j, \mu-m, m) Y_{\ell}^{\mu-m}(\theta, \phi) X^m. \quad (3.4)$$

$-\kappa$ is the eigenvalue of $\hat{K}$ and connected to the angular momentum through

$$\kappa = \begin{cases} \ell & \text{for } j = \ell - \frac{1}{2} \\ -\ell - 1 & \text{for } j = \ell + \frac{1}{2} \end{cases}, \quad (3.5)$$

$j = |\kappa| - \frac{1}{2}$ being the total angular momentum. The magnetic quantum number μ is the projection of the total angular momentum on the axis connecting the two nuclei (z axis). After expanding the two center potential into multipoles

$$V(\vec{r}, \vec{R}) = \sum_{\ell=0}^{\infty} V_{\ell}(r, R) P_{\ell}(\cos \theta), \quad (3.6)$$

where for $r < R/2$,

$$V_{\ell}(r, R) = \frac{-2Z_1 e^2}{R} \left(\frac{2r}{R} \right)^{\ell} - \frac{2Z_2 e^2}{R} \left(\frac{-2r}{R} \right)^{\ell}, \quad (3.7)$$

and where for $r > R/2$,

$$V_{\ell}(r, R) = \frac{-Z_1 e^2}{r} \left(\frac{R}{2r} \right)^{\ell} - \frac{Z_2 e^2}{r} \left(\frac{-R}{2r} \right)^{\ell}, \quad (3.8)$$

the coupled radial equations read

$$\begin{aligned} \frac{d}{dr} g_{\kappa}(r) &= (E+m) f_{\kappa}(r) - \frac{\kappa+1}{r} g_{\kappa}(r) \\ &\quad - \sum_{\bar{\kappa}, \ell} f_{\bar{\kappa}}(r) V_{\ell}(r, R) A_{(-\kappa, \ell, -\bar{\kappa})}^{\mu}, \\ \frac{d}{dr} f_{\kappa}(r) &= \frac{\kappa-1}{r} f_{\kappa}(r) - (E-m) g_{\kappa}(r) \\ &\quad + \sum_{\bar{\kappa}, \ell} g_{\bar{\kappa}}(r) V_{\ell}(r, R) A_{(\kappa, \ell, \bar{\kappa})}^{\mu}. \end{aligned} \quad (3.9)$$

The coefficients $A_{(\kappa, \ell, \bar{\kappa})}^{\mu}$ are easily determined by angular momentum algebra

$$\begin{aligned} A_{(\kappa, \ell, \bar{\kappa})}^{\mu} &= \langle \chi_{\kappa}^{\mu} | P_{\ell} | \chi_{\bar{\kappa}}^{\mu} \rangle \\ &= \sum_{m=-1/2}^{+1/2} (-1)^{\mu-m} \left(\ell_{\kappa}, \frac{1}{2}, j_{\kappa}, \mu-m, m \right) \left(\ell_{\bar{\kappa}}, \frac{1}{2}, j_{\bar{\kappa}}, \mu-m, m \right) \\ &\quad \left[(2\ell_{\kappa}+1)(2\ell_{\bar{\kappa}}+1) \right]^{1/2} \begin{bmatrix} \ell_{\kappa} & \ell & \ell_{\bar{\kappa}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ell_{\kappa} & \ell & \ell_{\bar{\kappa}} \\ m-\mu & 0 & \mu-m \end{bmatrix}. \end{aligned} \quad (3.10)$$

The sum in Eq. (3.9) has been truncated at a sufficiently large angular momentum $j_{\max}$. The remaining $2(2j_{\max}+1)$ coupled differential equations can be solved by a five-point Adams integration code. The energy eigenvalues are determined by iteration, which is stopped when the change of the energy value is less than 10^{-14} . In this case the accuracy of the two-center wave function is better than $3 \cdot 10^{-3}$.

Fig. 4a

Fig. 4b and 4c

The adiabatic correlation diagram for the symmetric Pb-Pb system has been published elsewhere ¹⁾. In the following we discuss a few examples of relativistic correlation diagrams for asymmetric systems. Fig. 4 shows the dependence of the binding energies on the two-center distance R for the $^{82}\text{Pb} - ^{96}\text{Cm}$ system. For a more

Fig. 4d : Correlation diagram for the superheavy quasimolecule Pb-Cm. For two-center distances $R < 400$ fm the energies of the 7 lowest bound states are presented on a linear scale.

accurate analysis various representations of the correlation diagram are chosen. In fig. 4a we plot on a double logarithmic scale the 21 lowest σ - (solid lines) and π - (dashed lines) states between $R=16$ fm and $R = 3000$ fm. The dashed-dotted lines denote the energies for extended nuclei. The relativistic splitting between the states $2p_{3/2}\sigma - 2p_{1/2}\sigma$ is striking. It amounts to more than the electron rest mass. At the critical distance $R_{\text{cr}} = 24$ fm the $1s\sigma$ state dives into the negative energy continuum. The energy levels in rectangles I and II between 300 and 1000 fm are presented separately on a linear scale in figs. 4b and 4c. The sudden energy change of the strongest bound states near the distance of closest approach is most impressive if energies as a function of two-center distance R are drawn on a linear scale (Fig. 4d).

The strongest energy change is found for the $2p_{1/2}\sigma$ -state. Its binding energy increases by 56.4% from $R=100$ fm to $R=16$ fm. The variation of the $1s\sigma$ binding energy amounts to about the electron rest mass in a two-center distance interval of $\Delta R=84$ fm. The nuclear extension lowers the binding energy of the $1s\sigma$ -state by 10% and that of the $2p_{1/2}\sigma$ -state by 15%. The three following asymmetric correlation diagrams $^{92}\text{U}-^{96}\text{Cm}$, $^{82}\text{Pb}-^{98}\text{Cf}$, $^{92}\text{U}-^{98}\text{Cf}$ show an equivalent structure as the $^{82}\text{Pb}-^{96}\text{Cm}$ system.

Fig. 5a

Fig. 5b and 5c

Fig. 6a

Fig. 6b

Fig. 6c

Fig. 7a

Fig. 7b

Fig. 7c

4. RESULTS FOR THE VACANCY FORMATION AND δ -ELECTRON PRODUCTION IN SUPERHEAVY QUASIMOLECULES

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After expanding the two-centre potential into multipoles and restricting ourselves to the dominant monopole part V_0 we obtain two first-order coupled differential equations

$$\begin{aligned} d/dr u_1 &= -\kappa/r u_1 + (E + m - V_0) u_2, \\ d/dr u_2 &= - (E - m - V_0) u_1 + \kappa/r u_2, \end{aligned} \quad (4.1)$$

where $u_1 = r \cdot g(r)$ and $u_2 = r \cdot f(r)$. The eqs. (4.1) are solved numerically both for bound states and continuum states. With the obtained wavefunctions we solved the coupled channel equations (2.4) for the occupation amplitudes. Using the expressions (2.6), (2.7) and (2.8) we calculated the vacancy production probabilities and δ -electron emission rates. A large variety of results are already published in refs. 4,5,9 and 10. For corresponding experimental results we refer to refs. 3,6,11 - 14. In the following we thus restrict ourselves to the presentation of few selected examples for electron excitation calculations.

Binding energies as well as the radial expectation value $\langle r_{1s0} \rangle$ play an important, but contrary role when calculating K-hole formation and δ -electron production. Whereas a strong increase in the binding energy hinders, e.g., the ionization of an electron from the K-shell - thus producing a K-hole and, eventually, a δ -electron - the strong decrease of $\langle r_{1s0} \rangle$ leads to high Fourier frequencies in the pulse spectrum - thus stimulating K-shell ionization. Therefore one can predict a maximum for K-hole production rates, P_{1s0} , around $Z \sim 170$. Subsequently we investigate this effect more systematically for K-hole formation as well as for δ -electron production rates. For this purpose we evaluated K-vacancy production rates for various combined nuclear charges $Z=134$ up to $Z=184$. We present coupled channel calculations stressing the Z -dependence of $P_{1s0}(b)$ in Fig. 9. Part a shows K-hole production rates for charges $Z \leq 164$. We observe an increasing slope with rising binding energy. Vacancy formation increases especially for small impact parameters as a consequence of the sharp localized $1s0$ -state. The latter effect allows the transfer of high Fourier frequencies to the K-shell originating from the nuclear motion, thus leading to increased K-hole production. In Fig. 9b combined nuclear charges beyond $Z=164$ are considered. In this region the ionization probabilities decrease due to the extremely strong binding energy which amounts nearly twice the electron rest mass.

Fig. 9: Number of created $1s\sigma$ -vacancies versus impact parameter b . The different curves belong to different total charges Z . Asymmetric systems are assumed. Part (a) for $Z \leq 164$, part (b) for $Z \geq 164$.

Another observable quantity where the influence of $1s\sigma$ -binding energies and the radial expectation value $\langle r_{1s\sigma} \rangle$ can be demonstrated is the differential emission probability of δ -electrons as a function of the kinetic electron energy. In Fig. 10 δ -electron cross sections are displayed for systems with united charge $Z=134$ up to $Z=184$. The δ -electrons have to be measured in coincidence with $1s\sigma$ -vacancies. In all calculations we chose a Fermi level of $F=3s\sigma$ and a bombarding energy $E_{lab}=4.7$ MeV/u. Asymmetric systems are considered. In the region of united charge $Z < 164$ the δ -electron cross section rises with increasing Z . The high-kinetic part with $E_e > 500$ keV grows faster than the low-kinetic one. This is due to the stronger localization of the $1s\sigma$ -orbit which allows for high momenta transfer as described above. But if the united charge exceeds $Z \sim 170$ the influence of the rapidly increasing $1s\sigma$ -binding energy becomes dominant. Thus the emission cross section even diminishes which is shown for $Z=184$. Therefore it would be highly desirable to measure more systematically K-hole formation and δ -electron cross sections in dependence on the united charge Z in order to verify the interplay of strong binding and sharp localization in ionization processes.

Fig. 10: δ -electron distribution in coincidence with $1s\sigma$ -vacancies versus the kinetic electron energy for different combined charges Z .

For the J+Pb system with $Z_1+Z_2=135$ the δ -electron spectra are shown as a function of the kinetic electron energy in Fig. 11. The dependence is of exponential form and less steep for more central collisions. Part a shows a triple coincidence measurement (scattering angle, δ -electron energy, and K-hole formation) by W. Koenig, C. Kozhuharov et al.^{3,6)}. Good agreement is achieved for all cm-kinetic electron energies and for all impact parameters. In part b the double differential electron cross section $d^2\sigma/d\Omega dE$ is displayed. The triangle denote the total spectrum of emitted δ -electrons. The experimental data representing coincidence measurements with K-vacancies are in overall agreement with our theoretical calculations. Also for the lower Z-system Br+Pb fair agreement with the measured data of ref. 6 is achieved¹⁵⁾.

The δ -electron distribution of the system Pb+Sn in coincidence with $1s\sigma$ -vacancies is displayed in Fig. 12a. For comparison with experimental data of Kozhuharov et al.³⁾ we divided our results by 4π assuming a spherical symmetric angular distribution of the δ -ray emission. The latter assumption was verified experimentally³⁾. We obtain a theoretical cross section which slightly underestimates the measured data. The δ -electron distribution for the selected impact parameters $b=0,5,10$ and 20 fm is shown in fig. 12b for the same system.

Fig. 11a: Triple coincidence measurements^{3,6} (scattering angle, δ -electron energy, and K-hole formation) of δ -electrons versus $R_0=R_{\min}$ in the system J+Pb at $E_{\text{Lab}}=500$ MeV. Different curves belong to different kinetic electron energies.

Fig. 11b: Total δ -electron distribution (triangles) and δ -electrons in coincidence with K-vacancies for the same system versus kinetic energies are results of Refs. 3,6. The latter data are compared with absolute values of coupled channel calculations.

Fig. 14: Differential cross section for δ -electron emission with respect to the kinetic electron energy E for various superheavy quasimolecules.

As an example for $2p_{1/2}\sigma$ excitation we finally show in fig. 15 the number of created $2p_{1/2}\sigma$ vacancies per Xe+Pb collision. Different bombarding energies are considered. We have to stress that rotational coupling contributions and electron screening corrections are neglected. Obviously the impact parameter dependence of $P(b)$ for the $2p_{1/2}\sigma$ state is less steep than for the strongest bound $1s\sigma$ -state.

Fig. 13: a) Production probabilities for δ -electrons in coincidence with $1s$ -vacancies versus impact parameter b , considering a $I + \text{Pb}$ collision at $E_{\text{lab}} = 3.94 \text{ MeV/u}$. The different curves belong to different kinetic electron energies. b) The same for the $I + \text{U}$ system.

In fig. 14 we compare for various superheavy quasimolecules the differential cross section for δ -electron emission with respect to the kinetic electron energy E . The systems $\text{Br} + \text{Pb}$ and $\text{Br} + \text{U}$ at $E_{\text{lab}} = 6.5 \text{ MeV/u}$, $\text{I} + \text{Pb}$ and $\text{I} + \text{U}$ at $E_{\text{lab}} = 3.94 \text{ MeV/u}$ and $\text{Au} + \text{U}$ at $E_{\text{lab}} = 3.5 \text{ MeV/u}$ are considered. A coincidence measurement with $1s$ vacancies is assumed. In each system we find the typical experimental shape. However, the exponential decline depends strongly on the combined nuclear charge Z and the bombarding energy, which reflects the different momentum components in the initial bound state wavefunction.

Fig. 14: Differential cross section for δ -electron emission with respect to the kinetic electron energy E for various superheavy quasimolecules.

As an example for $2p_{1/2}\sigma$ excitation we finally show in fig. 15 the number of created $2p_{1/2}\sigma$ vacancies per Xe+Pb collision. Different bombarding energies are considered. We have to stress that rotational coupling contributions and electron screening corrections are neglected. Obviously the impact parameter dependence of $P(b)$ for the $2p_{1/2}\sigma$ state is less steep than for the strongest bound $1s\sigma$ -state.

Fig. 15: Number of created $2p_{1/2}\sigma$ -vacancies per collision versus classical impact parameter b for the Xe+Pb system. Various bombarding energies are considered.

5. SPIN POLARIZATION OF ELECTRONS BY STRONG COLLISIONAL MAGNETIC FIELDS

The strongest magnetic fields on a microscopic scale accessible to experimental observation are created^{7,16)} in heavy ion collisions with $(Z_1+Z_2)\alpha > 1$. The maximum magnetic field strength may reach values in the order of $\vec{B}_{\max} = 10^{15} \text{G}$.

In superheavy quasimolecules the $1s\sigma$ -electron moves almost adiabatically close to the nuclei and is highly localized. Hence it may serve as a test particle for the high- $\vec{B}$ limit of Quantum-electrodynamics. The strong collisional magnetic fields cause a spin polarization of electrons which will be discussed in this chapter.

In the following discussion we neglect electron screening and retardation effects which should have only small influence on polarization. We now have to solve the coupled channel equations taking into account the scalar and vector potentials in the transvers Coulomb gauge. For this purpose we split the relativistic Hamiltonian into two parts,

$$H = H_0 + H' \quad (5.1)$$

where H_0 is the two-center Dirac Hamiltonian. H' contains any interaction responsible for electron excitations which is not included in H_0 . In the case of magnetic interactions it is given by

$$H' = -\vec{\alpha} \cdot \vec{A} \quad (5.2)$$

with the vector potential $\vec{A}$ created by the current of both colliding ions. This leads to a set of first-order coupled differential equations for the occupation amplitudes $a_{ij}(t)$.

$$\dot{a}_{ij}(t) = -\sum_k a_{ik}(t) \{ \langle \phi_j | \partial/\partial t | \phi_k \rangle + i \langle \phi_j | H'(t) | \phi_k \rangle \} \exp(i\chi_{jk}) \quad (5.3)$$

with the phases

$$\chi_{ik} = \chi_i - \chi_k. \quad (5.4)$$

In contrast to eq. (2.4) there occurs an additional magnetic interaction term resulting from the vector potential $\vec{A}$ which reads in the Coulomb gauge

$$\begin{aligned}
\vec{A}_C(\vec{r}, t) &= - \sum_{i=1,2} \frac{z_i e^2 \vec{v}_i(t)}{2 |\vec{r} - \vec{R}_i(t)|} - \\
&\quad - \sum_{i=1,2} \frac{z_i e^2 \vec{v}_i(t) \cdot [\vec{r} - \vec{R}_i(t)] [\vec{r} - \vec{R}_i(t)]}{2 |\vec{r} - \vec{R}_i(t)|^3} \\
&= \vec{A}_C^{(1)} + \vec{A}_C^{(2)} .
\end{aligned} \tag{5.5}$$

The corresponding result in the Lorentz-gauge $\vec{A}_L(\vec{r}, t)$ becomes

$$\vec{A}_L(\vec{r}, t) = 2\vec{A}_C^{(1)}(\vec{r}, t) . \tag{5.6}$$

For simplicity we restrict our derivations to symmetric systems with

$$Z_1 = Z_2 = Z \tag{5.7}$$

and use nuclear trajectories prescribed as Rutherford hyperbolas in the x-z plane, where the z-axis connects both nuclei. The matrix elements resulting from the interaction term in eq. (5.3) are evaluated in the monopole approximation since exact continuum solutions of the two-centre Dirac equation are unknown up to now. Due to selection rules or triangle rules we find that all contributions to couplings between s-states with the same magnetic quantum number just cancel in the Coulomb gauge

$$\langle s' \pm \frac{1}{2} | H'_{\text{Coulomb}} | s \pm \frac{1}{2} \rangle = 0 . \tag{5.8}$$

This is different from the Lorentz gauge, where we obtain

$$\langle s' \pm \frac{1}{2} | H'_{\text{Lorentz}} | s \pm \frac{1}{2} \rangle = - \frac{Ze^2}{3} i v_{R0} \int_{r_>}^{\infty} r^2 dr \frac{r_<}{2} (fg' - f'g) . \tag{5.9}$$

However, also in this case there is no contribution to the Zeeman splitting: Due to the minus sign in the integrand of eq. (5.9) the diagonal matrix elements ($s'=s$) vanish. For the spin flip transitions we find in summary the simple result

$$\langle s' + \frac{1}{2} | H' | s + \frac{1}{2} \rangle = \mp \frac{Ze^2}{3} i v_\phi \int_0^\infty r^2 dr \frac{r}{2} (fg' + f'g) = \mp i A_{s'+, s-+} \quad (5.10)$$

which is the same in the Coulomb as well as in the Lorentz gauge. The only contributing rotational matrixelement in the monopole approximation leads to

$$\langle s+1/2 | -i\vec{\omega} \cdot \vec{j} | s+1/2 \rangle = \mp b v_\omega / 2R^2 \quad (5.11)$$

Up to now we only discussed the fate of a single electron influenced by the collision dynamics. In order to describe the many-electron problem the field operator $\hat{\psi}$ is expanded in terms of the complete adiabatic basis states ϕ_q . This leads us to expressions for the number operators of particles N_{q+}^y with a given spin projection along a marked axis y. The spin polarization of holes q along the y axis is defined by

$$\eta_q = (N_{q+}^y - N_{q-}^y) / (N_{q+}^y + N_{q-}^y) \quad (5.12)$$

and correspondingly for particles

$$\eta_p = (N_{p+}^y - N_{p-}^y) / (N_{p+}^y + N_{p-}^y) \quad (5.13)$$

Using symmetry, time-reversal-symmetry and completeness relations of the expansion coefficients $a_{i\pm j\pm}$ to a given spin projection we can rewrite the two preceding equations into

$$\eta_q = \frac{2\text{Im} \left(\sum_{r < F} a_{q-, r+}^* a_{q-, r-} \right)}{1 - \sum_{r < F} |a_{q-, r-}|^2 - \sum_{r < F} |a_{q-, r+}|^2} \quad (5.14)$$

$$\eta_p = \frac{2\text{Im} \left(\sum_{r < F} a_{r-, p+}^* a_{r-, p-} \right)}{\sum_{r < F} |a_{r-, p-}|^2 + \sum_{r < F} |a_{r-, p+}|^2} \quad (5.15)$$

The solutions for a modified set of coupled channel equations containing expressions for $\dot{a}_{i+j+}$, $\dot{a}_{i+j-}$, $\dot{a}_{i-j+}$ also take into account the rotational coupling eq. (5.11), the diagonal and off-diagonal magnetic interaction and as usual the radial coupling.

We only consider results for the collision Pb+Cm ($Z_1+Z_2=178$) hence being the heaviest system investigated with respect to $1s\sigma$ -ionization probabilities.

As a main result we found that the additional magnetic interaction does not change the total ionization probabilities but leads to spin polarization of electron states. The created polarization is preserved during the collision despite the importance of multi-¹⁶⁾ step excitation processes. As dominant effect the Zeeman splitting of the $1s\sigma$ -state gives rise to a stronger ionization of one of the usually degenerated spin states whereas the other becomes less ionized. However, the total sum remains almost exactly the same as obtained with the radial coupling only.

The spin polarization η_q of created $1s\sigma$ -vacancies has been calculated according to eq. (5.14). Its dependence on the impact parameter b is presented in Fig. 16. As can be seen, $\eta_q=0$ for head-on collisions since the rotational velocity v_ϕ is zero in this case. Its maximum is obtained for medium impact parameters between $b=30$ and 80 fm. For larger impact parameters (distances) the magnetic field is too small in order to influence ionization processes. This theoretical prediction still needs experimental verification. Ionization measurements of $1s\sigma$ -electrons and also δ -electron spectroscopy therefore may yield direct information about the behaviour of electrons in strong magnetic fields.

Fig. 16: Spin polarizations of $1s\sigma$ -vacancies versus impact parameter b . $E_{\text{lab}} = 3.6$ MeV/u and 5.9 MeV/u, respectively. $Z_1 + Z_2 = 178$.

6. THE SELF-ENERGY OF ELECTRONS IN CRITICAL FIELDS

The K-electron binding energy E_{1s} increases strongly as a function of the nuclear charge Z . For $Z = 150$, E_{1s} amounts to about the electron rest mass and hence one enters the truly relativistic domain. For $Z \geq 170$ the binding energy exceeds twice the electron rest mass and the K-shell electron gets imbedded as resonance in the negative energy continuum, which opens the possibility of spontaneous positron production¹⁷⁾.

The major motivation of our investigations was the question whether field theoretical corrections, such as vacuum-polarization and self-energy may prevent such an extraordinary strong binding. These processes are visualized by the Feynman-diagrams in fig. 17. The double lines indicate the exact propagators and wave functions in the Coulomb field of the nucleus. The dominant vacuum-polarization contribution is provided by the attractive Uehling potential.

Fig. 17: Feynman diagrams for the lowest order vacuum-polarization (a) and self-energy (b). The double lines indicate the exact propagators and wave functions in the Coulomb field of a nucleus.

Its influence on electronic binding energies for superheavy systems has been calculated by various authors¹⁸⁻²⁰⁾. For the critical nuclear charge Z_{cr} the Uehling potential leads to an energy shift $\Delta E_{VP}^{(n=1)} = -11.8 \text{ keV}$ ²⁰⁾, which decreases Z_{cr} by 1/3 of a unit. The remaining vacuum-polarization effects in lowest order of the fine-structure constant α but in all orders of $(Z\alpha)^n$ with $n \geq 1$ were evaluated by M. Gyulassy^{21,22)} and by Rinker and Wilets²³⁾.

These authors made use of the angular momentum decomposition of the electron propagator in spherically symmetric potentials that was developed by Wichmann and Kroll²⁴⁾. The obtained energy shift of $\Delta E_{VP}^{(n>1)} = +1.15 \text{ keV}$ ²¹⁾ is very small compared with the total K-shell binding energy of 1 MeV.

Electronic self-energy corrections for high-Z systems have been first studied in the pioneering work of Brown, Langer and Schaefer²⁵⁻²⁷⁾. In these theoretical investigations the traditional expansion²⁸⁾ of the Feynman diagrams in powers of the coupling constant ($Z\alpha$) of the external field was avoided. This method was further refined and successfully applied in computations of electron energy shifts in high-Z elements by Desiderio and Johnson²⁹⁾, who allowed for a realistic nuclear charge distribution as well as the electron-electron interaction in the Hartree-Fock approximation. The precise analysis of self-energy corrections by P. Mohr³⁰⁾ is based on the Coulomb potential for point-like nuclei. Due to the singular nature of the potential these calculations are restricted to nuclear charges below $Z = \alpha^{-1} \sim 137$. Cheng and Johnson³¹⁾ continued the calculations of ref. 29) up to $Z = 160$, where a repulsive energy shift for K-shell electrons of $\Delta E_{SE} = +7.3 \text{ keV}$ was found.

Recently Liesen et al.¹⁴⁾ measured the ionization probability $P(b)$ of the strongest bound electron states versus the classical impact parameter b in collisions of Pb and Cm with a combined charge $Z_1 + Z_2 = 178$. For almost central collisions deviations from an empirical scaling law for $P(b)$ were found. The authors speculated that a strong self-energy shift of the quasimolecular $1s\sigma$ -state could be responsible for the observed modification of the ionization probability.

In our calculations we employed the methods developed by Desiderio and Johnson²⁹⁾, which may be slightly simplified by restriction to K-shell electrons. We performed our calculations for hydrogen-like systems. The external potential energy $V(x)$ is determined by the nuclear charge distribution, for which a homogeneously charged sphere with a radius $R = 1.2 A^{1/3} \text{ fm}$ has been assumed. To check our computer code, we computed the self-energy contribution to the K-shell binding energy in mercury ($Z=80$). Assuming a fictitious nuclear mass number $A=1$ ($R=1.2 \text{ fm}$) we obtained $\Delta E_{SE} = 206.1 \text{ eV}$ which is about 0.7 eV smaller than the result of Cheng and Johnson³¹⁾ for a point-like nucleus. For the superheavy system $Z = 130$ we found $\Delta E = 2.537 \text{ keV}$ for a nucleus with $A=1$, as compared with the point-nucleus value (ref. 31) of $\Delta E_{SE} = 2.586 \text{ keV} \pm 0.156 \text{ keV}$. These numbers are drastically reduced if one takes into account a realistic nuclear size determined by $A = 2.5 Z$. This lowers the energy shift to $\Delta E_{SE} = 1.896 \text{ keV}$. The complementary result of Cheng and Johnson is $\Delta E_{SE}^{HF} = 1.844 \text{ keV} \pm 0.029 \text{ keV}$, where in addition, electron screening

effects within a mean field Hartree-Fock (HF) potential were taken into account. For $Z = 150$ the present calculation leads to $\Delta E_{SE} = 4.963$ keV and for $Z = 160$ to $\Delta E_{SE} = 7.759$ keV, respectively. The latter number differs by 393^{SE} eV from the corresponding HF-values of ref. 31), where the numerical error was estimated to ± 354 eV. Presumably this slight disagreement is caused by the considerable difference of a HF-potential from a Coulomb potential for finite size nuclei. Our calculation for $Z=169$ yielded $\Delta E_{SE} = 10.839$ keV. For the critical nuclear charge $Z = 170$ we adjusted the nuclear mass number and hence the nuclear radius such that the K-electron energy eigenvalue differed only by 10^{-3} eV $\sim 10^{-9} E_{1s}^b$ from the border line of the negative energy continuum. As the most important result we found an energy shift of $\Delta E_{SE} = 10.989$ keV, which still represents only a 1% correction to the total K-electron binding energy. Therefore it may safely be neglected in investigations of ionization probabilities¹⁴⁾ in superheavy quasimolecular systems. If one adds to this the vacuum-plarization calculations of ref. 20) and 21) for critical external potentials, the total energy shift due to radiative corrections of order α amounts only to 300 eV. This tiny effect is at present far outside of any measurable consequences. The various calculations for the self-energy correction of K-electrons in high-Z atoms are summarized in fig. 18³²⁾. On a logarithmic scale the energy shift is displayed versus the nuclear charge Z . For $Z \geq 70$ it is well described by an exponential increase.

We conclude that radiative corrections as vacuum-polarization or self-energy may not prevent the K-shell binding energy from exceeding $2mc^2$ in superheavy systems with $Z > Z_{cr} \sim 170$.

Fig. 18: The self-energy shift ΔE of K-shell electrons as a function of the nuclear charge Z . The calculations are performed to first order in α but to all orders in the coupling constant ($Z\alpha$) of the external field. The dots denote the numerical results of P. Mohr³⁰⁾ for 1s-electrons in the Coulomb field of point-like nuclei. The squares represent the values obtained by Cheng and Johnson for a Hartree-Fock potential and extended nuclei. The results of the present calculations for extended nuclei are indicated by crosses.

7. THE GLASHOW-SALAM-WEINBERG MODEL AND ELECTRON POLARIZATIONS IN HEAVY-ION COLLISIONS

In this section we roughly estimate the order of electron polarization phenomena in heavy-ion collision caused by the Glashow-Salam-Weinberg interaction. In this model the interaction Lagrangian for the coupling of a neutral Z-boson to leptons is given by

$$L_1 = - \frac{g}{4 \cos \theta_w} \bar{e}(x) \gamma_\mu (c_v + \gamma_5) e(x) Z^\mu(x) \quad (7.1)$$

with $c_v = 1 - 4 \sin^2 \theta_w$ and $\sin^2 \theta_w = 0.22$. θ_w denotes the Weinberg angle and g the coupling constant being related with the charge by $e = g \sin \theta_w$. $e(x)$ is the field operator of the electron and $Z^\mu(x)$ described the field of the neutral Z-boson in analogy to the A^μ -field in the case of the electromagnetic interaction. On the other hand the coupling to quarks is determined by³³⁾

$$L_2 = \frac{g}{4 \cos \theta_w} \{ \bar{u}(x) \gamma_\mu (c'_v + \gamma_5) u(x) + \bar{c}(x) \gamma_\mu (c'_v + \gamma_5) c(x) - \bar{d}' \gamma_\mu (c''_v + \gamma_5) d' - \bar{s}' \gamma_\mu (c''_v + \gamma_5) s' - \dots \} Z^\mu(x) \quad (7.2)$$

with

$$\begin{aligned} c'_v &= 1 - \frac{8}{3} \sin^2 \theta_w \\ c''_v &= 1 - \frac{4}{3} \sin^2 \theta_w \end{aligned} \quad (7.3)$$

$u(x)$ and $c(x)$ are the fields of the up- and charm-quarks, respectively.

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad (7.4)$$

is the Fermi coupling constant. The mass of the charged W-boson M_W and of the neutral vector boson M_Z are related through

$$M_Z \cos \theta_w = M_W \quad (7.5)$$

Furthermore we introduced the mixing

$$\begin{aligned} d' &= d \cos \theta_c + s \sin \theta_c \\ s' &= -d \sin \theta_c + s \cos \theta_c \end{aligned} \quad (7.6)$$

for the fields of the down- and strange-quarks. θ_c is the Cabbibo

angle. In order to simplify the hadronic current (7.2) we perform some approximations:

- (1) Only the up- and down-quarks are considered. Hence, the contribution of the "sea-quarks" is neglected. This immediately leads to

$$L_2 \rightarrow \frac{g}{4\cos\theta_w} \{ \bar{u}(x) \gamma_\mu (c'_v + \gamma_5) u(x) - \bar{d} \gamma_\mu (c''_v + \gamma_5) d \} Z^\mu(x) \quad (7.7)$$

- (2) We replace the quark currents by the nucleon currents

$$\begin{aligned} J_p^Z &= 2 J_u^Z + 1 J_d^Z \\ J_n^Z &= 1 J_u^Z + 2 J_d^Z \end{aligned} \quad (7.8)$$

Thus we neglect the internal structure of the nucleons except for their quark content. This represents an approximation since for the charged currents eq. (7.8) is valid only for the vector part whereas the axial vector part has to be renormalized by a factor of about 1.25. With

$$\begin{aligned} J_u^Z &= \bar{u} \gamma_\mu (c'_v + \gamma_5) u \\ J_d^Z &= - \bar{d} \gamma_\mu (c''_v + \gamma_5) d \end{aligned} \quad (7.9)$$

we get

$$\begin{aligned} J_p^Z &= \bar{p} \gamma_\mu (2 c'_v - c''_v + \gamma_5) p = \bar{p} \gamma_\mu (c_v + \gamma_5) p \\ J_n^Z &= \bar{n} \gamma_\mu (c'_v - 2c''_v - \gamma_5) n = - \bar{n} \gamma_\mu (1 + \gamma_5) n \end{aligned} \quad (7.10)$$

In consequence it follows from equation (7.7)

$$L_2 \rightarrow \frac{g}{4\cos\theta_w} \frac{1}{3} \{ \bar{p} \gamma_\mu (c_v + \gamma_5) p - \bar{n} \gamma_\mu (1 + \gamma_5) n \} Z^\mu(x) \quad (7.11)$$

- (3) We adopt the nonrelativistic limit for the protons and neutrons.

$$p = \begin{pmatrix} \phi_p \\ \chi_p \end{pmatrix} \sim \begin{pmatrix} \phi_p \\ 0 \end{pmatrix} \quad (7.12)$$

With this approximation it results

$$\bar{p}\gamma_0 p \sim \phi_p^+ \phi_p$$

$$\bar{p}\gamma_5 p \sim 0$$

$$\vec{p}\gamma p \sim 0$$

$$\vec{p}\gamma\gamma_5 p \sim \phi_p^+ \sigma_p \phi_p$$

(4) In non-polarized nuclear matter the term $\sum_N \phi_N^+ \sigma_N \phi_N$ is much smaller than $\sum_N \phi_N^+ \phi_N$. Furthermore, due to the specific value of the Weinberg angle we also neglect the contribution of the protons ($1 - 4 \sin^2 \theta_w \sim 0.1$). Then the expression (7.11) simplifies considerably

$$L_2 \rightarrow - \frac{g}{4 \cos \theta_w} \frac{1}{3} \phi_n^+(x) \phi_n(x) Z^0(x) \quad (7.12)$$

With $\rho_n(x) = \phi_{n,f}^+(x) \phi_{n,i}(x)$ the electron-nucleus interaction takes the form $\tilde{\rho}_n(\vec{x}) e^{i x_0 (E_f - E_i)}$. Due to the large mass of the Z-boson the corresponding propagator may be written as

$$\frac{g_{0\mu} - \frac{k_0 k_\mu}{M_Z^2}}{k^2 - M_Z^2} \sim \frac{g_{0\mu}}{M_Z^2} = D_Z(k)_{0\mu} \quad (7.13)$$

The interaction matrix element now may be transformed into

$$\begin{aligned} M &= - \int d^4x \int d^4y j_e^\mu(x) \int \frac{d^4k}{(2\pi)^4} D_Z(k)_{0\mu} e^{ik(x-y)} \frac{g}{4 \cos \theta_w} \rho_n(y) \\ &= - \frac{g}{4 \cos \theta_w} \frac{1}{M_Z^2} \int d^4x \int d^4y j_e^\mu(x) g_{0\mu} \rho_n(y) \delta^4(x-y) \\ &= - \frac{g}{4 \cos \theta_w} \frac{1}{M_Z^2} \int d^4x j_e^0(x) \rho_n(x) \end{aligned} \quad (7.14)$$

This integral contributes only inside the nuclear volume. The electron current is split off into helicity eigenstates e_R and e_L .

$$\begin{aligned}
j_e^0 &= - \frac{g}{4 \cos \theta_w} e_f^+ (c_v + \gamma_5) e_i \\
&= - \frac{g}{4 \cos \theta_w} \frac{1}{2} \left\{ e_f^+ \frac{1+\gamma_5}{2} e_i \frac{1+c_v}{2} - e_f^+ \frac{1-\gamma_5}{2} e_i \frac{1-c_v}{2} \right\} \\
&= - \frac{g}{8 \cos \theta_w} \left\{ e_{R,f}^+ e_i (1-2 \sin^2 \theta_w) - e_{L,f}^+ e_i 2 \sin^2 \theta_w \right\} \quad (7.15)
\end{aligned}$$

$$\text{with } e_{R,L}^+ = e^+ \frac{1 \mp \gamma_5}{2} \quad (7.16)$$

If we insert this into eq. (7.14) we obtain

$$M = \frac{G}{4\sqrt{2}} \int d^4 x \left\{ (1-2 \sin^2 \theta_w) e_{L,f}^+ e_i - 2 \sin^2 \theta_w e_{R,f}^+ e_i \right\} \rho_n(x) \quad (7.17)$$

Thus it results a polarization of electrons which are scattered by neutral currents. It is given by

$$\frac{W(e_L) - W(e_R)}{W(e_L) + W(e_R)} = \frac{(1-2 \sin^2 \theta_w)^2 - (2 \sin^2 \theta_w)^2}{(1+2 \sin^2 \theta_w)^2 - (2 \sin^2 \theta_w)^2} \sim 0.16 \quad (7.18)$$

Provided that the initial states are unpolarized the final polarization amounts to about 16%.

Finally we compare the matrix elements of the electromagnetic interaction with that resulting from the neutral currents

$$\begin{aligned}
M_n &= \frac{g^2}{(4 \cos \theta_w)^2 M_Z^2} \int d^4 x \frac{1}{2} \left\{ e_{L,f}^+ e_{L,i} (1-2 \sin^2 \theta_w) \right. \\
&\quad \left. - e_{R,f}^+ e_{R,i} 2 \sin^2 \theta_w \right\} \rho_n(x) \quad (7.19)
\end{aligned}$$

$$\begin{aligned}
M_{el-mag} &= e^2 \int d^4 x \int d^4 y \frac{1}{2} \left\{ e_{L,f}^+ e_{L,i} + e_{R,f}^+ e_{R,i} \right\} \cdot \\
&\quad \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} e^{i k (x-y)} \rho_p(y) \quad (7.20)
\end{aligned}$$

Now we assume

$$\rho_p(y) = \tilde{\rho}_p(\vec{y}) e^{i(E_f - E_i)y_0}$$

It follows

$$\begin{aligned}
 |M_{el-mag}| &= |e^2 \int d^4x \frac{1}{2} \{e_{L,f}^+ e_{L,i} + e_{R,f}^+ e_{R,i}\} \int \frac{d^4k}{(2\pi)^4} \int d^3y \\
 &\delta(k^0 - E_f + E_i) \cdot \frac{1}{k^2 + i\epsilon} e^{i[k_0 x_0 - \vec{k}(\vec{x}-\vec{y})]} \tilde{\rho}_p(\vec{y}) \Big| = \\
 &\left| e^2 \int d^4x \frac{1}{2} \{e_{L,f}^+ e_{L,i} + e_{R,f}^+ e_{R,i}\} \int \frac{d^3\vec{k}}{(2\pi)^4} \int d^3\vec{y} \frac{1}{(E_f - E_i)^2 - \vec{k}^2 + i\epsilon} \cdot \right. \\
 &\left. e^{-i\vec{k}(\vec{x}-\vec{y})} e^{i(E_f - E_i)x_0} \tilde{\rho}_p(\vec{y}) \right| = \\
 &\left| \frac{e^2}{8\pi^2} \int d^4x \frac{1}{2} \{e_{L,f}^+ e_{L,i} + e_{R,f}^+ e_{R,i}\} e^{i(E_f - E_i)x_0} \right. \\
 &\left. \int d^3\vec{y} \frac{1}{|\vec{y}-\vec{x}|} e^{i|E_f - E_i||\vec{x}-\vec{y}|} \tilde{\rho}_p(\vec{y}) \right| \quad (7.21)
 \end{aligned}$$

where we employed

$$\int \frac{d^3\vec{k}}{(2\pi)^4} \frac{e^{i\vec{k} \cdot (\vec{x}-\vec{y})}}{(E_f - E_i)^2 - \vec{k}^2 + i\epsilon} = -\frac{1}{8\pi^2} \frac{1}{|\vec{y}-\vec{x}|} e^{i|E_f - E_i||\vec{x}-\vec{y}|} \quad (7.22)$$

In general eq. (7.21) may not be further simplified. The comparison with eq. (7.19) clearly expresses the difference between the Coulomb - and the point-interaction. The weak interaction is proportional to the overlap of the electron density and nucleon density. A rough estimate is attainable from eq. (7.20)

$$\begin{aligned}
 \left| \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} e^{ik(x-y)} \right| &\geq \left| \int \frac{d^4k}{(2\pi)^4} \frac{1}{|k^2|_{\max}} e^{ik(x-y)} \right| = \\
 \frac{1}{|k^2|_{\max}} \delta^4(x-y) &\quad (7.23)
 \end{aligned}$$

The maximum momentum transfer which is observed experimentally^{3,6)} for electron excitation process in superheavy quasimolecules is about 2 MeV. As upper limit we assume $|k^2|_{\max} = (7 \text{ MeV})^2$. Then we find

$$\begin{aligned}
 \left| \frac{M_n}{M_{el-mag}} \right| &= \frac{g^2 (7\text{MeV})^2}{(4\cos\theta_w)^2 M_Z^2 e^2} \frac{N}{Z} \sim \frac{1}{(4\sin\theta_w \cos\theta_w)^2} \left(\frac{7\text{ MeV}}{70\text{GeV}} \right)^2 \frac{N}{Z} \\
 &\sim \frac{N}{3Z} 10^{-8} \sim 10^{-8} \quad (7.24)
 \end{aligned}$$

Thus the interaction via neutral currents is strongly suppressed and may be safely neglected in polarization measurements of electron excitation processes in superheavy quasimolecules. Still we have to emphasize that this does not exclude observable effects due to the neutral current interaction in superheavy systems. A corresponding measurement would be favourable whenever an electromagnetic transition is forbidden by angular momentum or parity selection rules.

8. POSITRON CREATION IN SUPERCRITICAL QUASIMOLECULES

At the nuclear charge value $Z \approx 173$ the $1s$ -state enters the negative energy continuum, $E_{1s} < -mc^2$; the binding energy has reached the threshold where spontaneous pair production becomes possible. At this point the spectrum of eigenstates of the Dirac equation is subject to a characteristic change: The $1s$ -state becomes a resonance in the lower energy continuum. In a Gedankenexperiment it is possible to increase the charge of a bare nucleus from $Z < Z_{cr} \approx 173$ to $Z > Z_{cr}$, the unoccupied $1s$ -state will then be filled under emission of two (due to spin-degeneracy) positrons. The new stable ground state of the system consists of the nucleus plus two electrons in the K-shell; it is called the charged vacuum^{34,35}). The experimental exploration of this new phenomenon would constitute an important test of the theory of Quantum Electrodynamics (QED) in the region of strong fields.

At present, however, the unique way of extending the periodic table of elements, where these atomic effects can be pursued, lies in heavy ion collisions. In such scattering experiments, however, the dynamics of the collision becomes extremely important. The time-scale must be sufficiently long to allow the electron (positron) to adjust to the variation of the combined Coulomb field of the two nuclei. Since the typical velocities required to bring the nuclei close together are about $v/c \approx .1$, the adiabatic picture is meaningful only for electrons in relativistic motion.

In this section we will concentrate on positron creation. In particular, it is our aim to give an adequate description for positron production in supercritical collision systems, where $Z_T + Z_P$ exceeds 173. In supercritical collisions the resonance property of the $1s$ -state must be handled with care. In ref. 17 a formalism is developed, which avoids those difficulties and moreover has heuristic value for the interpretation of the positron creation process. The method is based on the observation that the continuum wavefunction of the supercritical system at resonance energy $E_p = E_{res}$ is quite similar to the discrete $1s$ -state in the subcritical case except for an oscillating tail - small in amplitude - reaching out to infinity. This structure reflects the occurrence of a tunneling process through the gap separating the particle- and antiparticle solutions of the Dirac equation. Apart from the asymptotic behaviour the $1s$ -wavefunction retains much of its identity, e.g. the strongly localized charge distribution having the extension of the atomic K-shell. This fact can be used to develop a general method to treat resonance scattering. In this context Wang and Shakin³⁶) introduced a projection formalism for resonances in the nuclear continuum shell model: After having defined a normalizable quasibound wavefunction ϕ_R , a new continuum $\mathcal{V}_{Ep}$ is constructed which spans a subspace orthogonal to ϕ_R and replaces the old continuum $\mathcal{V}_{Ep}$. Here $\mathcal{V}_{Ep}$ satisfies the Dirac equation

$$(H_{\text{TCD}} - E_p) |\varphi_{Ep}\rangle = 0 \quad (8.1)$$

and contains the resonance; E_p denotes the energy of the positron. The modified continuum wavefunction $\tilde{\varphi}_{Ep}$ are eigenstates of a projected Hamiltonian

$$(P H P - E_p) |\tilde{\varphi}_{Ep}\rangle = 0, \quad (8.2)$$

with

$$P = 1 - Q := \int dE_p |\tilde{\varphi}_{Ep}\rangle \langle \tilde{\varphi}_{Ep}|, \quad (8.3)$$

$$Q = |\phi_R\rangle \langle \phi_R| + \sum_{\alpha} |\varphi_{\alpha}\rangle \langle \varphi_{\alpha}|,$$

where Q is an operator projecting on the resonance state ϕ_R and on all higher bound and electron continuum states φ_{α} . If ϕ_R was chosen judiciously the newly defined modified continuum $\tilde{\varphi}_{Ep}$ will no longer show resonance behaviour. Using eqs. (8.3) and the orthogonality relations, one obtains

$$(H - E_p) |\tilde{\varphi}_{Ep}\rangle = \langle \phi_R | H | \tilde{\varphi}_{Ep} \rangle |\phi_R\rangle. \quad (8.4)$$

The modified continuum states satisfy the original Dirac equation supplemented by an inhomogeneous term containing an integral over the solution $\tilde{\varphi}_{Ep}$.

If the states ϕ_R and $\tilde{\varphi}_{Ep}$ are used as part of the basis the $1s\sigma$ -state couples to the positron continuum by two separate coupling operators

$$\dot{R} \langle \tilde{\varphi}_{Ep} | \partial / \partial R | \phi_R \rangle + i/\hbar \langle \tilde{\varphi}_{Ep} | H | \phi_R \rangle. \quad (8.5)$$

The second matrixelement arises since ϕ_R and $\tilde{\varphi}_{Ep}$ are not exact eigenstates of the two-centre Hamiltonian H . It does not depend on the nuclear motion and leads, in the static limit $R(t) = \text{const}$, to an exponential decay of a hole prepared in ϕ_R with the width

$$\Gamma = 2\pi |\langle \tilde{\varphi}_{E_{\text{res}}} | H | \phi_R \rangle|^2. \quad (8.6)$$

The values of Γ deduced by this method agree well with the exact widths obtained by a phase shift analysis of the old continuum $\mathcal{V}_{Ep}$.

The developed formalism thus has led to the emergence of "induced" and "spontaneous" positron coupling, the latter resulting from the presence of an unstable state ϕ_R in the expansion basis. In practise, however, this does not result in a threshold behaviour at the border of the supercritical region. Firstly both coupling matrixelements enter via their Fourier transforms depending on the time development of the heavy ion collision. Their contributions have to be added coherently so that in a given collision there is no physical way to distinguish between them. Secondly in Coulomb collisions the rapid variation of the quasimolecular potential causes significant contributions from the dynamical coupling, whereas the period of time for which the internuclear distance $R(t)$ is less than R_{cr} is usually very short ($\sim 10^{-21}$ s) compared to the decay time of the $1s\sigma$ -resonance ($\sim 10^{-19}$ s).

Therefore, the predicted production rates and energy spectra of positrons continue smoothly from the subcritical to the supercritical region. It turns out that the emergence of a new coupling is accompanied by a reduction of the radial matrixelement $\langle \mathcal{V}_{Ep} | \partial/\partial R | \phi_R \rangle$ and both effects nearly cancel in collisions on Coulomb trajectories. Qualitative deviations of the positron production rate in supercritical collision systems are expected only under favourable conditions, i.e. in encounters with a prolonged interaction time. Since the "spontaneous" and "dynamical" couplings exhibit a different functional dependence on the nuclear motion (the "induced" coupling is proportional to the velocity $\dot{R}$ while the "spontaneous" part depends on the distance R), an increase in collision time can be expected to provide a clear signature for supercritical collisions. Therefore Rafelski et al.³⁷⁾ suggested the study of positron emission in heavy ion reaction at bombarding energies above the Coulomb barrier, where the formation of a dinuclear system or of a compound nucleus would eventually lead to a time delay within the bounds of the critical distance R_{cr} . During this delay time T the spontaneous decay of the $1s\sigma$ -resonance, by filling dynamically created K-shell holes under emission of positrons, might be strongly enhanced. For an extensive discussion concerning the influence of a nuclear time delay on positron spectra and δ -electron distributions we refer to refs. 38-40.

Since beams of very heavy ions at energies close to the Coulomb barrier or even above have become available at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, a number of experiments has been performed to study positron emission in highly charged collision systems. At the same time, experiments measuring K-hole and δ -electron production have yielded independent information on the excitation mechanism and have served to strengthen

confidence in the underlying theoretical framework, see refs. 3,6, 11-14.

Before we will reflect on experimental results for positron creation and their comparison with theory, we first want to discuss the main theoretical predictions. The first calculations had been performed in the framework of perturbation theory which describes much of the physics involved in the excitation processes. However, the growing importance of multi-step processes indicates that perturbative calculations are of limited validity. In the following, therefore, we will present the results of coupled channel calculations¹⁷⁾.

We have solved the system of differential equations using the monopole approximation including up to 8 bound states and ~ 15 states in the upper continuum, separately for the angular momentum channels $\kappa = +1$ and $\kappa = -1$.

Positron emission rates increase very fast with total nuclear charge, flattening somewhat for the highest Z-values. If parametrized by a power law $(Z_T + Z_P)^n$ the power takes values of 20 down to 13. Here a Fermi-level at $F = 3s\sigma$, $4p_{1/2}\sigma$ is assumed, i.e. the states above the $3s\sigma$ - and $4p_{1/2}\sigma$ -levels are empty, which should give an upper bound for positron production rates. If the distance of closest approach $R_{\min}$ is kept fixed instead of the impact velocity, n becomes still larger. In collisions of bare nuclei ($F=0$) positron production is increased by up to two orders of magnitude. Here also the Z -dependence is extremely steep ($n \sim 29$). Mainly responsible for this effect is the contribution of the $1s$ -state which in normal collisions ($F>0$) is suppressed due to the small K-vacancy probability. If the K-shell is empty it becomes the dominant final state for pair production. This clearly reflects the strong coupling between the $1s$ -state and the antiparticle continuum which it approaches and even enters in the supercritical region. In all cases investigated the channels $\kappa = -1$ and $\kappa = +1$ contribute about equally to the total result.

Now we turn to the discussion of experimental results and their comparison with the described predictions. But first we want to point out a major problem in analyzing the experimental data.

Already for bombarding energies well below the Coulomb barrier ($E \sim .8E_C$) the nuclei can be excited by Coulomb excitation, the emitted photons with energy above 1022 keV can undergo pair conversion. Although this process takes place long after the collision ($\sim 10^{-13}$ sec, while "atomic" positrons are emitted within $\sim 10^{-20}$ sec), it cannot be distinguished experimentally from the quasimolecular mechanism by ordinary methods, resulting in a background which is difficult to handle. For nuclei with a simple level-structure

(e.g., $^{208}\text{Pb}: 2^+, 3^-$) Coulomb excitation can be calculated. The resulting pair creation is then deduced from the theoretically known conversion coefficient. Otherwise one has to measure simultaneously the γ -spectrum and fold it with the conversion coefficient. Here one has to know - or to assume - the γ -ray multipolarity.

In collisions with lighter targets, where the combined charge of projectile and target nucleus, $Z_u = Z_p + Z_T$, is well below the critical charge so that no significant contribution of atomic positrons is expected, the γ -ray spectra have been measured. The data can be fitted nicely by pair conversion if E1 multipolarity is assumed. Beginning in the region $Z_p + Z_T \gtrsim 160$, however, all experiments have found an increase which could not be explained by nuclear conversion alone (Fig. 19). This was the first hint for "atomic" positron production in heavy-ion collisions and, up to now, all conclusions had to rely on the described procedure for background subtraction.

Fig. 19: In the left part the ratio between measured positron production rates and the expected pair conversion rates deduced from the nuclear γ -spectra is displayed as a function of the united charge. For $Z_u > 160$ an increase in positron emission is observed⁴¹, which cannot be explained by nuclear excitation processes. The right part shows theoretical predictions for the positron cross section, $\sigma_{e^+}(Z)$, in heavy ion collisions at $E=5.9 \text{ MeV/u}$. A strong increase is expected.

The first generation of experiments further established the dependence of positron excitation rates on the kinematic conditions as well as on the combined charge Z_u . Fig. 20 shows results of Kozhuharov et al.⁴²⁾ for three collision systems, Pb-Pb, U-Pb, and U-U, at 5.8 MeV/u bombarding energy, measured with an orange-type β -spectrometer. The positron production probability in an energy window $E(e^+) = 490 \pm 50$ keV is shown as a function of projectile cm-scattering angle θ_{cm} . The nuclear background is subtracted. Because target and projectile cannot be distinguished in the experiment, the theoretical values have been symmetrized with respect to forward and recoil scattering.

Fig. 20: For 5.8 MeV/u Pb-Pb, Pb-U, and U-U collisions positron production probability in an energy window $E_{e^+} = 490 \pm 50$ keV is displayed as a function of projectile scattering angle θ_{cm} . The experimental data are taken from Kozhuharov et al (ref. 42). Here, the nuclear background is subtracted.

The shape of the theoretical curves is in quite good agreement with the experimental data. Also the Z -dependent increase is well described, which spans an order of a magnitude while $\Delta Z/Z$ is only 12%. On the other hand the absolute magnitude of the theoretical values is generally too high.

Using another type of experimental set-up, a solenoidal spectrometer, Backe et al.⁴⁴⁾ obtained integrated and differential positron probabilities for various impact parameters for the heavy-ion collision systems Pb-Pb, Pb-U, U-U, and U-Cm. As in the experiment described above general agreement with theory is found. In the Pb-Pb case and, for smaller distances of closest approach, even in Pb-U and U-U collisions the data seem to agree also in absolute values, in contrast to the experiment discussed above. But in the heaviest accessible system U-Cm ($Z_U=188$) and for larger distances $R_{\min}$ theory again has a tendency to overestimate the measured data. The experimental slopes are somewhat steeper than predicted, while the Z -dependence is overestimated.

For given kinematic parameters $\{E, b\}$ the Z -dependence of the total positron probabilities is given by Z^{-17} , for lighter systems as well as for heavier ones⁴⁴⁾. From these data no qualitative signature for the "diving" of the $1s$ -state in U-U, U-Cm collisions can be extracted, in agreement with theoretical predictions.

More sensitive information can be obtained by the measurement of energy spectra of positrons detected in coincidence with the scattered ions. Their knowledge is most useful if one wants to find deviations hinting to the positron creation mechanism.

Fig. 21 shows the first published positron spectra of Backe et al.^{44,45)} for three collision systems, U-Pd, U-Pb, and U-U, at 5.9 MeV/u bombarding energy; the ions are detected in an angular window $\theta_{\text{lab}}=45^\circ \pm 10^\circ$. For U-Pd ($Z=138$) no atomic positrons are expected, the data can be fully accounted for by nuclear conversion (light curve). Extrapolating this procedure to the U-Pb system (light curve) the sum of background and calculated QED positron rates (full curve) is in excellent agreement with the observed emission rates. In the spectrum of the supercritical U-U system ($E_{1s} \approx 1200$ keV for $R_{\min} \approx 21$ fm) some deviations are seen.

To obtain theoretical predictions for positron production it is essential to include the "spontaneous" coupling for collisions where the $1s\sigma$ -state joins the lower continuum. If it is left out of the calculation the resulting positron spectra would be strongly altered: The "induced" radial coupling is changed at the same time as the spontaneous coupling becomes important. Both contributions add up coherently and cannot be observed separately¹⁷⁾. A promising strategy to get a clear qualitative signature for the

Fig. 21: Spectra of emitted positrons in 5.9 MeV/u collisions measured by Backe et al.^{44,45)} in coincidence with ions scattered in the angular window $\theta_{lab} = 45^\circ \pm 10^\circ$. The spectrum in the lightest system, U-Pd, is explained by nuclear pair conversion alone (light line). In the U-Pb and U-U systems the sum of nuclear and calculated atomic positron production rates (full lines) is displayed.

diving process will be to modify the time structure and to select heavy-ion collisions with prolonged nuclear contact time³⁷⁾. Such nuclear reactions are expected to occur at energies close to or above Coulomb barrier. The nuclear delay time T should provide a handle to distinguish supercritical systems.

Within the schematic trajectory model³⁹⁾ we have performed coupled channel calculations for the four heavy-ion collision systems Pb-Pb, Pb-U, U-U, and U-Cm, corresponding to $Z_{united} = 164, 174, 184, \text{ and } 188$, respectively. The contributions of s - and $p_{1/2}$ -partial waves ($\kappa = \pm 1$), which are nearly equal for Coulomb-trajectories¹⁷⁾, are taken into account.

Independently of assumptions on the incoming and outgoing

path, of dissipation of nuclear kinetic energy or angular momentum, and of the position of the Fermi level, all positron spectra exhibit the following features:

In subcritical collision systems ($Z_T + Z_P \leq 173$) a delay time causes modulations in the positron spectrum with a width $\Delta E = 2\pi\hbar/T$. In Fig. 22a positron spectra are displayed for a Pb-Pb collision, ($E_{lab} = 8.73$ MeV/u, $b = 7.11$ fm, $F = 3\sigma$, $4p_{1/2}\sigma$) with delay times $T = 0$ (pure Rutherford scattering), 3., 6., and $10 \cdot 10^{-21}$ sec. The modulations are due to interference effects in much the same way as predicted for the δ -electron spectra in deep inelastic heavy-ion collisions⁴⁰). Fig. 22b shows the corresponding spectra for a Pb-U collision ($E_{lab} = 8.97$ MeV/u, $b = 7.27$ fm, $F = 3$), a system, which is just at the border to supercriticality. The spectra of Figs. 22a, b are qualitatively similar, although the heavier system is characterized by the presence of constructive interference.

In addition to the oscillatory interference patterns an enhancement of positron production in time-delayed supercritical collisions is observed, where the binding energy of the lowest bound states exceeds the value $2mc^2$. For long delay times a distinct peak in the positron spectrum is found at the location of the supercritical bound state resonance (binding energy minus $2mc^2$) due to the spontaneous pair-creation mechanism. A detailed analysis of the spectra reveals that this peak emerges gradually as $Z_U = Z_T + Z_P$ exceeds Z_{cr} .

Positron spectra for the supercritical system U-U ($E_{lab} = 7.35$ MeV/u, $b = 3.72$ fm, $F = 3$) are shown in Fig. 22c. With increasing delay time the position of the maximum drifts slowly from the kinematic maximum to the "resonance energy", which depends on the combined charge, the separation of the two nuclei and on the nuclear charge distribution.

The integrated positron emission probability P_{e+} as a function of T for these three collision systems and for U-Cm ($E_{lab} = 7.5$ MeV/u, $b = 3.74$ fm, $F = 3$) is displayed in Fig. 22d, where the solid lines denote s-state-, the dashed lines $p_{1/2}$ -state contributions. The absolute values, $P_{e+}(T=0)$, should not be compared since the impact energies are chosen differently. However, the variation of P_{e+} with delay time T is of particular interest. In the investigated subcritical systems, $P_{e+}(T)$ remains roughly unchanged or is even reduced by up to a factor two due to destructive interference between incoming and outgoing path. In the supercritical systems the probability rises approximately in linear relation with T , which reflects the increasing contribution of the spontaneous decay. In the schematic model its slope depends on the separation of the two nuclei, R_{min} ; in general it depends also on the time-dependent charge distribution during nuclear contact. At small values of T interference

Fig. 22: Spectra of positrons created in subcritical (parts a,b) and supercritical (part c) heavy-ion collisions assuming grazing Coulomb trajectory (full lines) and nuclear reactions leading to delay times $T=3\cdot, 6\cdot, \text{ and } 10\cdot 10^{-21}$ sec, resp., using a schematic model for the trajectory. The

kinematic parameters $\{E, b\}$ chosen are displayed within the figures, dissipation of energy or angular momentum is not taken into account. A Fermi level $F=3s\sigma$, $4p_{1/2}\sigma$ is assumed. Whereas for the lighter collision systems modulations in the positron spectra are present, a distinct peak at the "resonance" energy $E_{1s\sigma}(R_{\min})$ builds up for systems with $Z_u > 173$. - Part d: Probability for the emission of positrons, integrated over kinetic positron energy, as a function of reaction time T . Full lines: contribution of s-partial waves, dashed lines: $p_{1/2}$ -partial waves' contribution. For the supercritical systems U-U and U-Cm $P_{e^+,s}$ increases strongly with T .

effects seem to prevail even in supercritical collisions. The results described so far were obtained within the schematic model for the nuclear motion. It facilitates a systematic study of the time delay effect and allows for an investigation of the conceptually interesting limit of large sticking times. To analyze a given experiment, however, the employed nuclear trajectories should be consistent with the elastic and inelastic heavy-ion scattering data. For a corresponding discussion we refer to ref. 38, 39.

However, for any chosen set of experimental parameters, the nuclear reaction time T may (and will) not be a sharp but distributed over a certain range. Then the positron spectrum is determined by

$$\int dT \frac{dP(T)}{dE_{e^+}} f(T)$$

with the time distribution $f(T)$. As an assumption we took a Gaussian centered at $\bar{T}$

$$f(T) = \frac{1}{\sqrt{2\pi\tau}} \exp \left(-\frac{(T-\bar{T})^2}{2\tau^2} \right) \quad (8.7)$$

The resulting positron spectra for the parameters $\bar{T} = 16 \cdot 10^{-21} \text{ s}$ and $\tau = 0$ or $\tau = 2 \cdot 10^{-21} \text{ s}$, are displayed in fig. 23 for a central U-U collision at $E_{\text{lab}} = 5.9 \text{ MeV/u}$. If we consider the p-states (fig. 23a) only we observe that the oscillations disappear already for $\tau = 2 \cdot 10^{-21} \text{ s}$, except for the first peak.

Also in the total spectrum (fig. 23) the oscillations are damped out for increasing τ . But most striking is the occurrence of the dominant first peak, which originates from the spontaneous part of the positron production mechanisms. Nevertheless we may conclude that the appearance of several oscillations in the lepton spectra can be expected only for sufficient sharp nuclear reaction times.

Fig. 23: a) Differential positron production probability versus kinetic positron energy E_{e^+} in a central U-U collision at $E_{\text{lab}} = 5.9$ MeV/u. It is assumed that the nuclear reaction time is distributed over a certain range which is determined by a Gaussian centered at $\bar{T}$. For the width τ we have chosen $\tau=0$ and $\tau=2 \cdot 10^{-21}$ s, respectively.
 b) The same as in a) but including also the contribution of the s-states. Most striking is the appearance of the first pronounced peak which originates from spontaneous positron production.

The superheavy compound system may not be a static object. In the framework of a simple model assumption we investigated⁴⁶⁾ the influence of the internal dynamics of the dinuclear system on the energy spectra of the emitted positrons. The internuclear separation $R(t)$ is supposed to oscillate around the distance of closest approach R_0 of the Coulomb trajectory for a fixed duration T

$$R = R_0 (1 - \alpha_0 \sin(2\pi\nu t)) \quad (8.8)$$

In fig. 24 dP/dE is plotted for a central U-U collision at $E_{\text{lab}} = 6.2$ MeV/u.

Fig. 24: Positron spectra calculated under the assumption that the distance between the nuclear centres oscillates around the distance of closest approach R_0 of the Coulomb trajectory (see eq. (8.8)). a) Nuclear reaction times $T = 0, 12, 36, 60, 84, 108 \cdot 10^{-21} \text{ s}$ are considered. The longest duration T corresponds to the most pronounced "spontaneous peak", etc. Note the logarithmic scale.

Fig. 24b: The same as in a) for the reaction time $T=36 \cdot 10^{-21} \text{ s}$ on a linear scale.

In order to demonstrate the qualitative effect we have restricted the calculations to couplings between the s-states only. The parameters in the expression (8.8) are fixed by $\alpha_0 = 0.25$ and $\nu = 0.125 \cdot 10^{21} \text{ s}^{-1}$. Various sticking periods between $T=0$ and $T=108 \cdot 10^{-21} \text{ s}$ are considered in fig. 24 a). Fig. 24 b) shows on a linear scale the computed positron spectrum for $T=36 \cdot 10^{-21} \text{ s}$. As in fig. 23 we find the dominant "spontaneous peak". But in addition a second pronounced peak appears at about $E_{e+} = 800 \text{ keV}$. This reflects the fact, that part of the oscillation energy of the dinuclear system is transferred to the emitted lepton. The presented model calculation thus may be understood as a classical picture for the conversion process of internal pair creation in the supercritical quasimolecule.

We now turn again to the influence of a nuclear time delay in deep-inelastic heavy ion reactions on δ -electron spectra. To determine the excitation amplitudes in delayed collisions, we make here the simplest possible ansatz for the nuclear motion. The branches of two Rutherford hyperboles having the same distance of closest approach are joined by a section without any radial motion. Hence the effect of the reaction enters only via an additional phase⁴⁰⁾ in the set of coupled differential equations for the occupation amplitudes $a_{ij}(t)$. We calculated δ -electron spectra in the central heavy-ion collision $U+U$, where the nuclei stick together for about 10^{-21} to 10^{-20} s . Fig. 25 shows the spectra of emitted δ -electrons choosing a delay time $T=0$ (fig. 25a) and $T=10^{-20} \text{ s}$ (fig. 25b). A coincidence with created $1s\sigma$ -vacancies is assumed. Only the excitation of s-state electrons is considered. Since the p-states also contribute considerably the present data may not be compared quantitatively with experimental data. The spectra in fig. 25 are calculated according to eq. (2.8). The dashed lines show the separate contributions of both terms in eq. (2.8). The oscillatory pattern in fig. 25b is caused by the real correlation between the δ -electron and the $1s\sigma$ -vacancy. The term $N_q \cdot N_q$ which describes accidental coincidences produces always a smooth distribution.

Fig. 25: Differential emission probability of δ -electrons in dependence on the kinetic electron energy for the system U+U at $E_{\text{LAB}} = 6.2 \text{ MeV/u}$ and $b=0$. A coincidence with $1s\sigma$ -vacancies is assumed. Only the s-states are taken into account. The dashed-dotted line shows the contribution of the accidental coincidences $N_p \cdot N_q$ of eq. (2.8).
a) No nuclear sticking time b) $T = 10^{-20} \text{ s}$.

In collisions of very heavy ions such as U+U with $E_{LAB} > 3$ MeV/u both nuclei are strongly Coulomb excited. For bombarding energies at about the Coulomb barrier transfer reactions or even deep inelastic nuclear reactions can take place which lead to additional excitations of the nuclei. This internal excitation energy may be irradiated by a photon or may be transferred to a bound electron or to an electron of the negative energy continuum, which leads to ionization and electron-positron pair creation, respectively. The latter process requires nuclear transition energies ω larger than twice the electron rest mass m . Nuclear EO-transitions are characterized by the absence of single photon emissions, because a photon must carry at least one unit of angular momentum. Therefore EO-transitions predominantly occur by bound state electron conversion or by internal electron-positron pair creation.

In this section we investigate the internal electron-positron pair creation accompanying the decay of excited nuclei. We mainly focus our attention to the calculation of the pair conversion coefficient (PCC)

$$\beta = P_{e^+, e^-} / P_\gamma. \quad (9.1)$$

β is defined as the ratio of the pair production probability compared with that of photon emission for a specific nuclear transition with energy ω . Since the energy of the electron and the positron takes continuous values we may express β also as integral of the positron spectrum $d\beta/dE$ with respect to the positron energy E ,

$$\beta(\omega) = \int_1^{\omega-1} \frac{d\beta}{dE}(E) dE. \quad (9.2)$$

$d\beta/dE$ is called differential pair conversion coefficient (DPCC). The lower bound of the integral is determined by the rest mass of the electron, which corresponds to vanishing kinetic energy, while the upper bound is given by the nuclear transition energy ω minus m_e . We will use the units $\hbar=m_e=c=1$. The investigation of β , which is typically of the order of 10^{-4} - 10^{-3} , instead of P_{e^+, e^-} has several theoretical and experimental advantages. In contrast to P_{e^+, e^-} the pair conversion coefficient, β , to a good approximation is independent of the nuclear wave function. Therefore only the knowledge of the electron and positron wave function for a given nuclear charge distribution is necessary for the evaluation of β . Experimentally it is easier to measure the ratio of two counting rates. The theory of internal pair creation is strongly connected with conversion of bound state electrons. In the latter process the excitation energy of a nucleus is transferred to a bound state electron which becomes excited to the positive energy

continuum. Here the ratio of the probabilities of inner-shell vacancy formation and photon emission is defined as conversion coefficient,

$$\alpha = P_e/P_\gamma. \quad (9.3)$$

In particular this mechanism is important for low energy nuclear transitions. A major motivation for the investigation of internal pair creation results from the test of quantum electrodynamics in the presence of strong external fields. A large contribution to the total positron production rate in supercritical collisions stems from internal pair conversion following nuclear Coulomb excitation. For the experimental separation of both processes it is therefore necessary to know precisely the conversion coefficient β . To subtract the internal conversion process one has to measure the nuclear photon spectrum and multiply it with β . Our theoretical investigations are especially motivated by the recently observed structures^{47, 48)} in the energy distribution of emitted positrons which have been produced in collisions of very heavy ions like U-Th, U-U, and U-Cm. These systems are supercritical and in principle may lead to spontaneous positron production. A conventional explanation for this peculiar structure would be positron production via EO-conversion processes. Contrary to EO-processes nuclear transitions of multipolarity E1 or E2 should also be observable in the emitted photon spectra provided that proper Doppler shift corrections are performed.

The basic processes under investigation are depicted schematically in Fig. 26. The nucleus which undergoes EO-transition is labeled by its initial and final state angular momenta $J_i, J_f=J_i$ and eigenenergies $E_i, E_f=E_i-\omega$. Process a) describes the electron-positron pair creation. An electron of the negative energy continuum ($\epsilon=-E < mc^2$) with Dirac quantum number κ is lifted to the positive energy continuum. The final state energy obviously amounts to $E' = \epsilon + \hbar\omega$ whereas the angular momentum quantum number remains unchanged. Since neither the initial electron state energy nor the final state energy is fixed one expects a continuous energy distribution for the emitted positrons. Process b) indicates the conversion of a K-shell electron ($n=1, l=0, j=\frac{1}{2}, \kappa=-1$) with energy eigenvalue $E_{1s_{1/2}}$. Thus bound states with definite energies are involved. Energy conservation then simply causes monoenergetic lepton emission for a fixed nuclear transition energy $\hbar\omega$. Process c) symbolizes monoenergetic positron production. Here an electron of the negative energy continuum is excited to a bound state, e.g. to the $1s_{1/2}$ -state. This represents a rather rare process since it requires a vacant bound state with a large overlap with the nuclear interior. If we investigate a realistic heavy-ion collision of ^{92}U on ^{92}U with a bombarding energy close to the Coulomb barrier, the number of created K-vacancies per collision with impact para-

Fig. 26 : Schematic representation of electron conversion processes accompanying nuclear E0-transition from a state $\{E_i, J_i\}$ to a state $\{E_f = E_i - \hbar\omega, J_f = J_i\}$. a) Electron-positron pair production leading to a continuous energy distribution of positrons and electrons. b) Conversion of K-shell electrons - a monoenergetic electron-production mechanism. c) Monoenergetic positron-production - a negligible process.

meter $b \lesssim 20$ fm is typically of the order of 10%¹⁷⁾. The transition time (X-ray decay) of an electron from a higher shell to the K-shell is⁴⁹⁾ about 10^{-17} sec. This has to be compared with nuclear transition times of about 10^{-13} sec, after which conversion takes place. Hence the K-vacancy will be decayed before it could eventually be filled again due to pair conversion of a nuclear state.

Therefore the monoenergetic positron formation is suppressed in contrast to ordinary K-shell conversion by at least 7 orders of magnitude, because one can conclude⁵⁰⁾ that the E0-conversion of a K-shell electron has a probability larger by about 3 orders of magnitude than the monoenergetic e^+ -production where the electron occupies the vacant K-shell. This ratio even increases by one order of magnitude if one considers higher bound states in relation to the positron production process. Thus process c) can be neglected completely.

The ratio η of the two conversion probabilities P_{e^+,e^-} for electron-positron pair creation and P_{e^-} for the ionization of bound state electrons

$$\eta = \frac{P_{e^+,e^-}}{P_{e^-}} \quad (9.4)$$

must be investigated more closely. The differential ratio with respect to the positron energy E obviously is

$$d\eta/dE = \frac{dP_{e^+,e^-}}{dE} / P_{e^-} \quad (9.5)$$

and consequently

$$\eta = \frac{\hbar\omega - mc^2}{mc^2} \int_2 \frac{d\eta}{dE} dE. \quad (9.6)$$

The final expressions for the conversion probabilities are to lowest order in r

$$\frac{dP_{e^+,e^-}}{dE} \propto |M|^2 \sum_{\kappa=\pm 1} |C_{e^+,e^-}|^2, \quad (9.7)$$

$$P_{e^-} \propto |M|^2 \sum'_m |C_{e^-}|^2, \quad (9.8)$$

where M denotes the nuclear EO matrix element

$$M = \int_0^\infty dV_n \rho_n(\vec{r}_n) r_n^2, \quad (9.9)$$

and C is a normalization factor

$$C = \lim_{r \rightarrow 0} \frac{ff' + gg'}{r^{2j-1}}. \quad (9.10)$$

Here f, f' and g, g' are the radial Dirac wavefunctions belonging to the initial and final state of the electron, respectively. Σ' in eq. (9.8) denotes the sum over all occupied states (E_m, j_m) excluding the summation over magnetic substates; in eq. (9.7) only $j=1/2$ ($\kappa=\pm 1$) - states are taken into account.

Thus the ratio eq. (9.5) now reads

$$\frac{d\eta}{dE} = \frac{\sum_{\kappa=\pm 1} |C_{e^+,e^-}|^2}{\sum'_m |C_{e^-}|^2}, \quad (9.11)$$

where the nuclear matrixelement drops out. Due to Eq. (9.10) the ratio η of pair conversion to bound state electron conversion is completely determined by the density of the electron wavefunctions at the nuclear origin.

It is also possible to derive an analytical expression for the ratio $d\eta/dE$. For this purpose we utilized electron wavefunctions that are solutions of the Dirac equation with a point-nucleus potential. It results

$$\frac{d\eta}{dE} = \frac{(EE' - \gamma^2) e^{\pi(B' - B - \bar{B})}}{\pi(Z\alpha)^2(\omega + 2\gamma)\Gamma(2\gamma + 1)} \cdot \left(\frac{pp'}{Z\alpha\bar{p}} \right)^{2\gamma - 1} \left| \frac{\Gamma(\gamma + iB)\Gamma(\gamma + iB')}{\Gamma(\gamma + i\bar{B})} \right|^2. \quad (9.12)$$

The variables belonging to the final state electron are always indicated by a prime.

Here we used the abbreviations

$$\begin{aligned} E' &= \omega - E \\ \bar{E} &= \gamma + \omega \\ \gamma &= [\kappa^2 - (Z\alpha)^2]^{\frac{1}{2}} \\ B &= \frac{Z\alpha E}{p}, \quad B' = \frac{Z\alpha E'}{p'}, \\ p &= [E^2 - 1]^{\frac{1}{2}}, \quad p' = [E'^2 - 1]^{\frac{1}{2}}, \end{aligned} \quad (9.13)$$

According to eqs. (9.10) and (9.11) we computed the differential conversion ratio $d\eta/dE$ with respect to the positron energy E for EO-transitions. As bound state only the atomic K-shell has been taken into account. The conversion probability of higher bound states is at least one order of magnitude smaller. For the nucleus ${}_{92}\text{U}$ the energy distributions of emitted positrons is depicted in Fig. 27. Nuclear transition energies of 1323 keV, 1423 keV, 1523 keV, and 1623 keV are considered, which correspond to maximum positron energies of $E_{\text{max}}^{\text{kin}} = 300 \text{ keV}$, 400 keV, 500 keV, and 600 keV, respectively.

In Fig. 28 we compare our numerical results for $d\eta/dE$ (solid lines) using Dirac wavefunction for finite size nuclei with the analytical formula (9.12) (dashed lines). Good agreement is achieved for low and medium Z nuclei. Even for $Z = 92$ the agreement is still remarkable. In conclusion we can state that nuclear EO-transitions with $\omega > 2m \sim 1 \text{ MeV}$ may be identified by measuring the positron to K-shell vacancy production rate. The differential

Fig.27 : Differential conversion probability ratio $d\eta/dE$ with respect to the kinetic positron energy E for nuclear $E0$ -transitions in U. Nuclear transition energies $\omega = 1323$ keV, 1423 keV, 1523 keV, and 1623 keV are considered, corresponding to maximum kinetic positron energies of $E_{\max} = 300$ keV, 400 keV, 500 keV, and 600 keV.

ratio $\frac{d\eta}{dE}$ as well as η are typical quantities for $E0$ -transitions.

For the purpose of nuclear spectroscopy formula (9.12) can be applied as a good approximation.

The numerical integration of (9.11) yields the dimensionless ratio η . For $Z = 92$ η is presented in Fig.29 as function of the nuclear transition energy.

For $\omega = 1.423$ keV we find that the probability for K-shell conversion is 556 times larger than the e^+ , e^- pair creation probability. In the considered range of transition energies η varies from 10^{-4} to $2 \cdot 10^{-2}$. The dashed line represents results of R. Thomas⁵¹⁾. Both calculations are in fair agreement.

Fig. 28 : Differential conversion probability ratio $d\eta/dE$ as function of the kinetic positron energy E . The nuclei $Z = 50$ and $Z = 92$ are considered. The nuclear transition energy was chosen to be $\omega = 1\,623$ keV. The solid line represents a numerical calculation utilizing Dirac wavefunctions for finite size nuclei. The dashed line follows from the analytical expression.

For comparison of the spectral shape of $d\eta/dE$ in fig. 27 we show in fig. 30 the equivalent differential conversion coefficient $d\beta/dE$ for nuclear E1 and E2 transitions, which are defined in eq. (9.1) and (9.2). Obviously the ratio β determines the number of created electron-positron pairs divided by the number of created photons. For details concerning the theoretical formalism and for the numerical numbers we refer to refs. 52-54. Similar positron distributions as in fig. 27 are found.

Fig. 29: The ratio η versus the nuclear transition energy ω for $Z = 92$. The dashed line is obtained by R. Thomas⁵¹⁾. Both calculations are in fair agreement.

Fig. 30 : Differential conversion coefficient $d\beta/dE$ with respect to the kinetic positron energy E for nuclear E1- and E2- transitions in ^{92}U . The same transition energies as in fig. 27 .

For completeness we show in fig.31 the K-shell conversion coefficient α_K (eq. 9.3) as function of the nuclear transition energy ω . Nuclear transitions of multipolarities E1, E2 and M1 in ^{92}U are considered. The data are taken from the table 55. Thus for $\omega > 600$ keV the ionization of the N-shell is suppressed typically by a factor of 100 compared with the emission of a single photon.

Fig. 31: The conversion coefficient $\alpha_K = P_{e_K^-} / P_\gamma$ as function of the nuclear transition energy ω . Nuclear transitions of multipolarity E1, E2 and M1 in ^{92}U are considered. The data are taken from the table 55.

Now we can discuss the probability that the recent observed structures in positron spectra originates from nuclear EO-transitions. One convincing argument against this interpretation is related to the shape of the e^+ -energy distribution. According to Fig. 27 the halfwidth of the spectra should be at least $\Delta E \geq 150$ keV. However, the observed structure is much narrower. The second argument is connected with the energy distribution of the emitted δ -electrons. The measured differential positron production probability dP_{e^+}/dE with respect to the kinetic positron energy at $E \sim 400$ keV for U-U collisions with a bombarding energy of

$E_{\text{lab}} = 5.9 \text{ MeV/u}$ and ion scattering angles of $\theta_{\text{ion}}^{\text{lab}} = 45^\circ \pm 10^\circ$ amounts to $dP_{e^+}/dE \approx 10^{-7}/\text{keV}$ ⁴⁷⁾.

Assuming that these positrons are of pure nuclear origin one can easily deduce the probability P_N for nuclear excitation

$$P_N \approx \frac{dP_{e^+}}{dE} / \frac{d\eta}{dE} \approx 10^{-2} . \quad (9.14)$$

In principle the competing K-shell conversions should lead to a sharp line in the electron spectra at $E_{e^-} \approx 1300 \text{ keV}$.

However, the experimental energy resolution for these high energy electrons is presently not better than $\Delta E_{e^-} = 25 \text{ keV}$, which therefore yields a differential e^- -production probability of

$$dP_{e^-}/dE_{e^-} \approx P_N/\Delta E_{e^-} = 4 \cdot 10^{-4}/\text{keV} . \quad (9.15)$$

This is still about an order of magnitude larger than the expected δ -electron energy of $E_{e^-} \approx 1300 \text{ keV}$, which results from dynamical electron excitations during the collision. This factor can be increased by an order of magnitude if one measures the emitted electrons in coincidence with the decaying K-vacancy (e.g., K_α -radiation). Hence we can conclude: If the observed structure in a positron spectra is caused by nuclear EO-transitions one should also observe a distinct peak in the δ -electron distribution. Furthermore we emphasize that sharp peaks in the energy distribution of emitted positrons with a halfwidth of less than 100 keV may not originate from nuclear EO-conversion processes.

We now turn to the discussion of electron-positron pair ⁵⁶⁾ conversion in supercritical compound systems. We consider a collision of very heavy ions such as $^{92}\text{U} + ^{92}\text{U}$ with a combined charge $Z=Z_1+Z_2=188$. According to the ideas of Reinhardt et al. ³⁹⁾ the projectile and target may come into nuclear contact even for bombarding energies slightly below the nuclear Coulomb barrier. ¹⁹⁾ These authors furthermore assume that for a period of $T \sim 4 \cdot 10^{-19} \text{ s}$ a superheavy compound system is produced. In the superheavy system $Z=184$ the electron K-shell is imbedded as resonance in the negative energy continuum. A vacant K-shell thus may lead to spontaneous positron production ¹⁷⁾. Reinhardt et al. predicted a pronounced peak in the positron spectra due to the spontaneous part of the positron production mechanism thereby indicating the energetical location of the $1s_{1/2}$ -resonance in the negative energy continuum.

Motivated by these theoretical investigations we considered the question whether positrons may be created due to conversion processes during the sticking period T . In this case pair

conversion is not distinguishable from ordinary K-shell conversion. This situation is visualized in fig. 32. The energy spectrum of the Dirac equation is shown with the upper continuum and lower continuum, which includes the supercritical K-shell denoted by its energetical location E_{res} .

Fig. 32 : Conversion processes indicated by arrows in supercritical systems. The energy spectrum of the Dirac equation is shown with the upper continuum and lower continuum, which includes the supercritical K-shell denoted by the energetical location E_{res} . The curves in the lower and upper continuum display the spectral shape of the positron and electron distribution, respectively. The two arrows on the right hand side denote the conversion of bound state electrons in high lying states.

A supercritical nucleus $Z=184$ which undergoes a transition with $w > 2$ during the sticking period T may transfer this excitation energy to one of the electrons in the negative energy continuum. The remaining hole is defined as positron. But also the K-shell electron can be lifted to the upper continuum. If T is longer than the spontaneous decay width of the K-shell resonance the K-vacancy will be filled again leading to spontaneous positron emission. The conversion processes are indicated by the arrows in fig.32. The curves in the lower and upper continuum display the spectral shape of the positron and electron distribution, respectively. In the positron spectra a sharp peak appears at $E = E_{res}$. The two arrows on the right hand side denote the conversion of bound state electrons in high lying states. The conversion coefficients β and the differential distribution $d\beta/dE$ are calculated in precisely the same manner as described extensively in ref. 53. For the nuclear charge distribution a homogeneously charged sphere with a radius $R_n = 10.88$ fm has been assumed. In fig. 33 we show the

Fig. 33 : Differential ratio $d\beta/dE$ for the electron-positron pair creation compared with the photon emission probability as function of the positron energy E . Nuclear transitions with $w=4$ and of multipolarity E1, E2 and M1 in a super-critical nucleus $Z=184$ with a radius $R_n = 10.88$ fm are considered. The dashed curves signify the various portions of the electron angular momentum states.

differential ratio $d\beta/dE$ for the electron-positron pair creation compared with the photon emission probability as function of the positron energy E . Nuclear transitions with $w=4$ and of multipolarity E1, E2 and M1 are considered. Striking is the appearance of the pronounced peak at $E = E_{res}$. The dashed curves signify the various portions of the electron angular momentum states. The contributions of the states with $\kappa = -1$ and the states with $\kappa \neq -1$ are given separately. As expected the resonance shows up only in the ($\kappa = 1$)-part..

In fig. 34 we can compare $d\beta/dE$ for $w=4$ and $w=6$. Again the multiplicities E1, E2 and M1 are considered. In addition the maximum values for $d\beta/dE$ for the multiplicities E3, M2 and M3 are given. Obviously lepton emission predominantly occurs for nuclear transitions with magnetic multiplicities.

In the following table we present for the multiplicities E1, E2, M1 the total conversion coefficient β following from a numerical integration of $d\beta/dE$. The second line always gives percentage contribution of the $\kappa = -1$ electron angular momentum state, which mainly correspond to K-shell conversion.

Fig. 34 : The same as in fig. 33 for $w=4$ and $w=6$. In addition the maximum values for $d\beta/dE$ for various multipolarities are given.

ω	E1	E2	M1
4	1.74E-2 95.4%	3.26E-1 97.1%	4.03 96.2%
6	2.17E-2 92.1%	3.21E-1 96.2%	1.11 94.2%
8	2.04E-2 88.7%	2.28E-1 95.3%	4.43E-1 92.0%

Thus especially for magnetic transitions the total probability of lepton production may exceed the photon emission probability even for $w > 2$. However, one has to emphasize that the contribution of the discussed conversion processes to the positron yield in a realistic heavy ion collision depends strongly on the specific nuclear structure especially for the compound system. The observed positron production probability per collision is about 10^{-5} . This requires transition times shorter than 10^{-14} s in the compound nucleus in order that internal pair conversion can contribute considerably. Furthermore we should stress that in contrast to the measured structure in the positron spectra just one peak can be expected due to the discussed pair production mechanism.

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We acknowledge very fruitful discussions with P. Armbruster, H. Backe, H. Bokemeyer, F. Bosch, J.S. Greenberg, W. Koenig, C. Kozhuharov, D. Liesen, P. Mokler, and P. Senger concerning their experiments. We are grateful to W. Betz for making his calculations on the two-centre Dirac equation available to us.