A MICROSCOPIC APPROACH TO SIGMA-HYPERNUCLEI

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1. Introduction

A great deal of interest has been generated recently by the observed long lifetimes of Σ -hypernuclear levels. Unlike the Λ -hyperon which is stable in nuclear matter, the Σ decays via the strong interaction to Λ , releasing roughly 80 MeV in the conversion. Because of this sizeable energy release it had not generally been expected that Σ -hypernuclei would live long enough to be identified as such. It was very exciting then when in 1979 two groups reported finding structure with widths less than 8 MeV^{1,2}, apparently corresponding to unbound p-states formed in the strangeness-exchange reaction ${}^9\text{Be}(K^-, \pi^-){}_{\Sigma}^9\text{Be}$.

These findings have led to considerable experimental and theoretical effort. On the experimental side, at CERN and Brookhaven a variety of light hypernuclei have now been studied^{3,4}. The original ${}_{\Sigma}^9\text{Be}$ result has been confirmed, and the measurements have been extended to include ${}_{\Sigma}^6\text{Li}$, and ${}_{\Sigma}^{12}\text{C}$. It has even been suggested that in ${}_{\Sigma}^6\text{Li}$ the p-level may be as narrow as 3 MeV³. Unfortunately, the experimental situation for s-state hypernuclei is not nearly as clear, with the only evidence for narrow states to date being in the ${}_{\Sigma}^{12}\text{C}^1$, and possibly ${}_{\Sigma}^9\text{Be}^5$ systems.

Theoretically, a number of mechanisms have been proposed for suppressing $\Sigma N \rightarrow \Lambda N$ conversion in the nucleus. Gal and Dover⁶ have emphasized the dominance of the $I = 1/2$ 3S_1 interaction in the conversion process—the implication being that in special cases where the $I = 3/2$ 1S_0 interaction dominates it is possible to have rather long-lived states. Their conclusions, of course, apply mainly to p-state Σ -hypernuclei. Kisslinger⁷ has suggested that short-range correlations can dramatically reduce the widths, although this explanation has been

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questioned by Gal³⁾ after considering the tensor contribution to $\Sigma N \rightarrow \Lambda N$ conversion from π and ρ exchange. A particularly intriguing idea has been presented by Gal and Toker⁸⁾. Rather than seeking a mechanism to suppress conversion, they have pointed out that in special circumstances a very strongly absorptive potential can produce narrow bound states embedded in the continuum.

These proposals are only applicable to the p-levels. In s-state these models predict very broad widths in agreement with classical estimates^{6,9)}. A calculation which does predict narrow s-states (and is closest in philosophy to our approach) is the recent work of Stepien-Ruddha and Wycech¹⁰⁾. Stressing the importance of Pauli effects and nucleon binding, they constructed a simple, local Σ -nucleus potential consistent with Σ^- atomic data which produced narrow s-states in Σ hypernuclei.

Our suggestion is that narrow states in both S and P waves arise when explicit and careful attention is given to the nuclear many-body effects of Pauli blocking and nucleon binding. To demonstrate this we wish to calculate the Σ single-particle potential as accurately as possible using the ΣN scattering information as input. To describe the coupled ΣN - ΛN systems we construct a separable potential model. This model is presented in detail in sect. 2 with the resulting fits to the two-body scattering data. In sect. 3 the single-particle potential for the Σ is developed with special attention to the Pauli exclusion principle, and nucleon binding. In sect. 4 the Schrödinger equation is solved self-consistently for the (complex) eigenvalues in $\ell=0,1$ hypernuclear states. It is calculated that, because of two very

distinct mechanisms, both s and p states are expected to be long-lived. In sect. 5 we highlight the findings of our study.

2. ΣN Scattering

Fundamental to our aim of understanding the long lifetimes of Σ hypernuclear states is the description of the underlying ΣN interactions. The strong coupling to ΛN in the isospin $I=1/2$ channel complicates this description considerably and, despite the scarcity of ΣN data, there are clear experimental indications that P-wave contributions are important. Even at energies as low as 5 MeV in the c.m. frame the $\Sigma^- p \rightarrow \Sigma^- p$ and $\Sigma^- p \rightarrow \Lambda n$ differential cross sections show a very marked forward/backward asymmetry^{1,12)}, and a non-zero value for the Λ polarization has been reported¹¹⁾. It can also be expected that, because of the large final-state momentum, the $\Sigma^- p(^3S_1) \rightarrow \Lambda n(^3D_1)$ transition is a significant proportion of the conversion cross section.

Our program is to fit all of the low-energy ΣN scattering data within a separable potential model. As intimated above, a convincing description of the coupled ΣN - ΛN systems probably requires P and D-waves to be considered. Since with separable interactions the scattering amplitude can be written in a closed form, it is straightforward to include these higher partial waves. In addition, we ultimately wish to relate these results to ΣN interactions within the nucleus and, in this case, the relative simplicity of the separable model will allow us to perform fairly sophisticated nuclear calculations with a minimum of purely technical complications.

The ΣN cross sections are not well determined and, therefore, as in all model calculations, we need some reasonable premise for

constraining the parameters. The most extensive treatments of ΣN to date are the OBE calculations of Nagels *et al.*^{13,14,15} and their results should provide solid guidance for our fit. Their basic philosophy is that NN data can be described well by an OBE picture and that, with the assistance of SU(3) and SU(6) to fix the relative strengths of coupling constants, the model can be extended to the ΣN interactions. All of their calculations include the exchange of nonets of pseudo-scalar and vector mesons, and uncorrelated two-pion exchange. Recent works also consider the contributions from a nonet of heavy, scalar mesons. The only phenomenology entering their models is hard-core repulsion, and one of the main differences between models is the assumptions made regarding the core radii in different channels. These radii plus $F/(F+D)$ ratios for meson couplings are treated as free parameters for fitting all the ΣN and ΛN data.

The resulting fits are impressive, with a χ^2 /data-point less than one for ΣN in all models. Although in details the model predictions differ, in general conclusions they do not. The results of their best ΣN fit [model D_1^{14}], which are relevant to our present purposes, are summarized below.

- (i) isospin 3/2: (a) 1S_0 : attractive.
- (b) 3S_1 : weak and repulsive.
- (ii) isospin 1/2: (a) 1S_0 : repulsive.
- (b) 3S_1 : attractive, with the elastic $\Sigma^+ p$ cross section dominated by $^3S_1 \rightarrow ^3S_1$ transitions, and the $\Sigma^+ p \rightarrow \Lambda n$ conversion dominated by $^3S_1 \rightarrow ^3S_1$ and $^3S_1 \rightarrow ^3D_1$.
- (c) 3P_1 : attractive, with the

forward/backward asymmetry in the cross section due mainly to $^3S_1 - ^3P_1$ constructive interference.

In fact, this model also suggests that the $I=3/2$ 1P_1 interaction is important to the $\Sigma^+ p$ description, but to include this term in our model would require at least one extra parameter. Since our primary concern is the coupled ΣN - ΛN systems such an additional freedom does not seem warranted. These results will be returned to shortly after specifying our potential model. For the moment we only remark that our initial suspicion that $\lambda \neq 0$ ΣN - ΛN interactions are significant is supported by Nagels *et al.*

We assume that the channel-coupling potentials are all rank-one separable. For N coupled channels, the partial t -matrix describing scattering from state $\alpha \rightarrow \beta$ is in general

$$t_{\beta\alpha} = |v_{\beta} > \lambda^{\beta\alpha} < v_{\alpha}| + |v_{\beta} > \sum_{\gamma=1}^N \lambda^{\beta\gamma} < v_{\gamma} | G_0^{\gamma} t_{\gamma\alpha}, \quad (2.1)$$

where $\lambda^{\beta\alpha} = -1$ (+1) for attractive (repulsive) interactions, and G_0^{γ} is the γ -channel Green's function, defined by

$$G_0^{\gamma}(E) = (E + \Sigma m_{\gamma} - (m_{\Sigma} + m_N) - K^2/2\mu_{\gamma} + i\epsilon)^{-1}, \quad (2.2)$$

with Σm_{γ} the sum of the masses of the particles in that channel, and μ_{γ} their reduced mass. Following the procedure of Londergan *et al.*¹⁶, we impose the fairly mild restriction that all constants λ are the same sign within any set of coupled channels. In this case the solution of (2.1) takes the particularly simple form

$$t_{\beta\alpha} = \frac{|v_{\beta} > \lambda^{\beta\alpha} < v_{\alpha}|}{1 - \sum_{\gamma=1}^N \lambda^{\gamma\gamma} < v_{\gamma} | G_0^{\gamma} | v_{\gamma} >}, \quad (2.3)$$

where the summation extends over the diagonal elements of all coupled

channels. The form factors $|v_Y\rangle$ are chosen to be

$$\begin{aligned} \langle p | v_Y \rangle &= \bar{v}_Y (\beta_0^2 + p^2)^{-1} \quad \text{for } l = 0, \\ &= \bar{v}_Y p \beta_1^{-1} (\beta_1^2 + p^2)^{-1} \quad \text{for } l = 1, \\ &= \bar{v}_Y p^2 (\beta_2^2 + p^2)^{-2} \quad \text{for } l = 2, \end{aligned} \quad (2.4)$$

and where the β_l are the inverse ranges, and $\bar{v}_Y$ the strengths of the interactions. With the normalization of the t-matrix such that the elastic scattering amplitude $f_{\alpha\alpha}$ is $-\pi i u_{\alpha\alpha}$, the overlap integrals $\langle v_Y | G_0^Y | v_Y \rangle$ in (2.3) are found to be

$$\begin{aligned} \int_0^\infty \frac{dk k^2 v_Y^2(k^2)}{E + \Delta m - k^2/2u_Y + i\epsilon} &= \frac{-\pi i u_Y \bar{v}_Y^2}{(\beta^2 + k_Y^2)^2} \left[\frac{1}{2\beta} (\beta^2 - k_Y^2) + i k_Y \right], \quad l = 0, \\ &= \frac{-\pi i u_Y \bar{v}_Y^2}{\beta^2 (\beta^2 + k_Y^2)^2} \left[\beta/2 (\beta^2 + 3k_Y^2) + i k_Y^3 \right], \quad l = 1, \\ &= \frac{-\pi i u_Y \bar{v}_Y^2}{(\beta^2 + k_Y^2)^4} \left[\frac{1}{16\beta} (\beta^6 + 5\beta^4 k_Y^2 + 15\beta^2 k_Y^4 - 5k_Y^6) + i k_Y^5 \right], \quad l = 2, \end{aligned} \quad (2.5)$$

where k_Y is the γ -channel momentum.

This completes our general definition of the t-matrix and our problem now is to minimize the number of parameters in the model without sacrificing any of its essential features. The difficulty is especially clear in the light of our earlier discussion where, regardless of the chronic shortage of ΣN data, it was apparent that the description of the $I = 1/2$ channel required P and D waves to be considered.

Consequently we are obliged to make some rather severe assumptions about the interactions to arrive at a non-trivial fit. First, the range β is arbitrarily chosen to be the same in all channels. Clearly, in a more realistic calculation the ΛN potential is expected to be of shorter range than the ΣN interaction. For instance, the longest range

ΣN potential is generated by one pion exchange, whereas in the elastic ΛN interaction two pions must be exchanged. On the other hand, each pion in $\Lambda N \rightarrow \Lambda N$ scattering is emitted with roughly 80 MeV of energy corresponding to the Σ - Λ mass difference. To some extent this compensates for the shorter range of two pion exchange. Second, in S state the 1S_0 strengths are related to 3S_1 in each isospin channel by the weighting $|\sigma_\Sigma \cdot \sigma_N|$ with the signs the same as $(\sigma_\Sigma \cdot \sigma_N)(\sigma_\Sigma \cdot \sigma_N)$ in agreement with the OBE calculation. We also include the $I = 1/2$ 3P_J waves assuming the potential strengths to be attractive, and equal in all total J states. Finally, an attractive 3D_1 interaction is included in the ΛN channel (but not in ΣN because of the large suppression of D-wave at low energies).

Assuming the $I = 1/2$ strengths to be the same in both ΣN and ΛN channels, this gives a total of five free parameters; a single range β^{-1} , and one strength parameter in each of $I = 1/2$ 3S_1 , 3P_J , 3D_1 , and $I = 3/2$ 3S_1 channels. These are adjusted to fit all existing $\Sigma^+ p$ data, which comprises:

- (i) elastic $\Sigma^+ p$ differential, and total cross sections¹²⁾,
- (ii) conversion $\Sigma^+ p \rightarrow \Lambda n$ differential, and total cross sections¹¹⁾,
- and (iii) charge-exchange $\Sigma^+ p \rightarrow \Sigma^0 n$ total cross section¹¹⁾. Because of the limited angular region accessible experimentally, the total cross sections quoted are the differential cross sections integrated over the angular interval and then multiplied by a factor appropriate for an isotropic distribution. That is,

$$\sigma_{\Sigma N} = \frac{4\pi}{\cos\theta_{\text{MAX}} - \cos\theta_{\text{MIN}}} \int_{\cos\theta_{\text{MIN}}}^{\cos\theta_{\text{MAX}}} \frac{d\sigma(\theta)}{d\Omega} d\cos\theta. \quad (2.6)$$

Since with this definition the total cross section contains Coulomb contributions, and neglects P-wave in extrapolating from the narrow angular interval, we follow the experimental procedure in comparing calculated cross sections with the data.

The results of our fit are shown in figs. 1 and 2 and it is found that, despite the heavy constraints placed on the parameters, the fit is overdetermined, with a $\chi^2/\text{data-point}$ of 0.75 for the 45 points. The best fit coupling strengths are listed in table 1, and in table 2 the scattering lengths and effective ranges in the elastic $\bar{\Sigma}^0 p$ channel are given.

Considering the poor quality of ΣN data it is clear that the fits to the cross sections will not be sensitive to even large changes in the parameters. However, there is an accurately known datum, free of P-wave contributions, that is sensitive to the ratio of 3S_1 to 1S_0 amplitudes. This is the capture ratio at rest, defined by¹³⁾

$$R = \sum_{S=0}^1 \frac{(2s+1)}{4} \frac{a_{2s+1}(\bar{\Sigma}^0 p \rightarrow \Sigma^0 n)}{a_{2s+1}(\bar{\Sigma}^0 p \rightarrow \Lambda n) + a_{2s+1}(\bar{\Sigma}^0 p \rightarrow \Sigma^0 n)}, \quad (2.7)$$

where $2s+1=1,3$ for singlet, triplet respectively. The calculated value of R is 0.485, in excellent agreement with the measured value $0.47 \pm 0.01^{13)}$. With some confidence we can now look at the predictions of the model in more detail.

Most importantly it is found that the inclusion of the $I=1/2$ P and D-wave terms is essential to the description of $\Sigma N \rightarrow \Lambda N$ conversion. The P-waves comprise less than 2% of the elastic $\bar{\Sigma}^0 p$ cross section at 160 MeV/c lab momentum, but yield nearly 16% of the conversion cross section. The $I=1/2$ $^3S_1 \rightarrow ^3D_1$ transition is even more important,

contributing 25% of the conversion cross section. In agreement with Nagels *et al.*, we find that the $I=1/2$ $^3S_1 \rightarrow ^3S_1$ amplitude dominates the $\bar{\Sigma}^0 p$ reactions and is responsible for 75% of the elastic cross section and over 50% of the conversion reaction.

The $I=1/2$ 3S_1 amplitude is of particular interest. In the reaction $K^0 d \rightarrow \pi^+ \Lambda p$ a strong enhancement of the Λp invariant mass distribution has been established near the $\bar{\Sigma}^0 n$ threshold¹⁷⁾. It has often been suggested that this enhancement cannot be explained by threshold effects alone and that the cusp arises from either a nearby resonance pole or an unstable ΣN bound state in the 3S_1 channel¹⁸⁾. The status of this pole has been addressed in several model calculations. The OBE calculations of Nagels *et al.*, do not support a bound state, but their results are fairly sensitive to the parameters in this respect. In some of their models no resonances are found^{13,15)} while others predict a resonance above the $\bar{\Sigma}^0 p$ threshold on the second energy sheet¹⁴⁾. Tokar and Gal¹⁹⁾ have studied the $K^0 d \rightarrow \Lambda \pi^+ p$ reaction in a Faddeev calculation. Using only S-wave separable interactions, they found that the data could be reproduced with or without a ΣN bound state, but that the best reproduction of the shoulder in the Λp distribution favoured a resonance pole rather than a bound state. This model (A) placed the pole on the second Riemann sheet at a momentum $k = 0.162 - i0.163 \text{ fm}^{-1}$.

The results of our work support Tokar and Gal's model A, producing a pole above the $\bar{\Sigma}^0 p$ threshold at $k = 0.163 - i0.082 \text{ fm}^{-1}$. The closeness of this pole to the physical sheet creates very strong energy dependence in the scattering amplitude near threshold. This is demonstrated in fig. 3 where, with zero coupling to the 3D_1 ΛN channel, the

variation of the off-shell amplitude $\pi\pi t(\kappa'=\kappa=0;E)$ near threshold for different 3S_1 coupling strengths is shown.

This strong energy dependence of the (dominant) 3S_1 amplitude will be of particular importance to our study of Σ hypernuclear states.

Clearly the large enhancement of the $\Sigma^+p \rightarrow \Lambda n$ conversion near threshold will tend to decrease the lifetime of the Σ -hyperon in the nuclear medium.

3. The Σ -nucleus potential

Very little is known about the nature of the Σ -nucleus interaction. Until recently the high energies and low intensities inherent in the formation reaction $KN \rightarrow \Sigma\pi$ had made the experiments too difficult.

One of the few indications we do have comes from Batty's analysis of Σ^- atoms²⁰). In that work the atomic energy shifts were reproduced with a phenomenological potential of the factored (tp) form

$$V_{\Sigma}(r) = - \frac{4\pi}{2\mu_{\Sigma A}} \left(1 + \frac{m_{\Sigma}}{m_N} \right) A \bar{a}_{\Sigma p}(r), \quad (3.1)$$

with the medium-corrected ΣN scattering length $\bar{a}_{\Sigma}$ treated as a (complex) parameter. As pointed out by many authors though^{6,9}), the value of $\bar{a}_{\Sigma}$ consistent with the atomic data leads to predictions for the widths of hypernuclear levels which are two to three times broader than have been observed experimentally. [For example, in s-state $^{12}_{\Sigma^0}\text{C}$, experimentally $\Gamma \sim 10 \text{ MeV}^1$), whereas using (3.1) $\Gamma \sim 22 \text{ MeV}^6$]. Further, this effective scattering length bears little resemblance to the free value, with the absorptive component differing by as much as an order of magnitude from some ΣN analyses. (e.g. $\sim 0.19i \text{ fm}$ as compared with the $1.22i \text{ fm}$ of our work).

It is clear from the above results that nuclear many-body effects

must play a critical role in the connection between free ΣN , atomic ΣN , and nuclear ΣN interaction descriptions. Amongst these effects, Pauli blocking of the final state nucleon in $\Sigma N \rightarrow \Lambda N$ conversion and the binding of the nucleons are expected to be most significant.

In the atomic situation, the Σ is bound with essentially zero energy in a Coulomb orbital of high angular momentum and, therefore, absorption occurs primarily on the loosely bound valence nucleons at the nuclear surface. This is a region of low density and correspondingly low Fermi momentum (within a local density approximation). Pauli exclusion should not have a large effect on the absorptive strength of the potential in this circumstance.

A Σ bound in the ground state of a hypernucleus, on the other hand, is confined almost exclusively to the nuclear interior and absorption occurs mainly on the deeply bound s-state nucleons. In this high density central region both nucleon binding and Pauli effects make important modifications to Σ absorption.

To appreciate the size of correction that nucleon binding may introduce, consider a Σ bound with (say) 5 MeV which is absorbed on an s-state nucleon bound with 35 MeV. The total energy of the pair is $\sim 40 \text{ MeV}$ and upon decay the ΛN pair emerges with $\sim 40 \text{ MeV}$. Phase space considerations alone require that the amplitude for this conversion be reduced by $\sim 30\%$ relative to the atomic case where the ΣN pair have roughly zero energy.

In addition, for the ΣN decaying at rest the final state nucleon has $285 \text{ MeV}/c$ momentum which is comparable to the Fermi momentum of nuclear matter. This implies that a large proportion of

decays are forbidden because the final state nucleon is produced with sub-sea momentum. (i.e. The naive picture presented above predicts ~50% suppression for $\kappa_{\text{FERMI}} = 285 \text{ MeV/c.}$)

These simple estimates have been essentially confirmed by Dabrowski and Rozynek⁽²¹⁾. Using an OBE potential of the Nijmegen group⁽¹⁴⁾ in Brueckner reaction matrix theory, they demonstrated that in nuclear matter Pauli exclusion suppressed ΣN decay by 50% from classical estimates.

On the basis of these arguments Pauli blocking and nucleon binding are essential to understanding the lifetimes of Σ states and a complete microscopic description of the hypernucleus must account for these features.

With this advisory in mind, we turn now to the problem of constructing the Σ -nucleus potential. The coupled equations describing the Σ + A-nucleon system (assuming only pair-wise interactions) are

$$\begin{aligned} \left(K_{\Sigma} + \sum_i K_i + \sum_i v_{\Sigma i}^i + \frac{1}{2} \sum_{i \neq j} u_{ij} - E \right) |\Psi(1 \dots A; \Sigma) \rangle &= - \sum_i v_{\Sigma i}^i |\Psi(1 \dots A; \Lambda) \rangle, \\ \left(K_{\Lambda} + \sum_i K_i + \sum_i v_{\Lambda i}^i + \frac{1}{2} \sum_{i \neq j} u_{ij} - (E + \Delta m_{\Sigma\Lambda}) \right) |\Psi(1 \dots A; \Lambda) \rangle &= - \sum_i v_{\Lambda i}^i |\Psi(1 \dots A; \Sigma) \rangle. \end{aligned} \quad (3.2)$$

The K_i are single-particle kinetic energy operators, $v_{\gamma\gamma'}$ and u_{ij} are two-body YN and NN potentials, the v are the hyperon + A-nucleon wavefunctions, and $\Delta m_{\Sigma\Lambda}$ is the Σ - Λ mass difference.

The diverse schemes for approximating solutions to many-body equations like (3.2) form the foundation of the whole field of nuclear structure physics. These techniques stem from the observation that, as a good first approximation, the nucleus comprises particles which move independently—each bound in an average potential generated by all the other particles. The success of the independent-particle model in predicting many features of nuclei suggests that the most important corrections are due to two-body correlations.

Specifically, we wish to extract the interaction of a single YN pair from each of the $(A+1)$ -body equations (3.2). The sum over YN potentials in each channel can be written in terms of the Σ -nucleon-A pair, Λ -nucleon-A pair, single-particle, and residual interactions as

$$\sum_i \left(v_{\Sigma i}^i |\Psi(\Sigma) \rangle + v_{\Sigma\Lambda}^i |\Psi(\Lambda) \rangle \right) = v_{\Sigma\Sigma}^A |\Psi(\Sigma) \rangle + v_{\Sigma\Lambda}^A |\Psi(\Lambda) \rangle + v_{\Sigma} |\Psi(\Sigma) \rangle + \left[\sum_{i \neq A} \left(v_{\Sigma i}^i |\Psi(\Sigma) \rangle + v_{\Sigma\Lambda}^i |\Psi(\Lambda) \rangle \right) - v_{\Sigma} |\Psi(\Sigma) \rangle \right], \quad (3.3a)$$

$$\sum_i \left(v_{\Lambda i}^i |\Psi(\Lambda) \rangle + v_{\Lambda\Sigma}^i |\Psi(\Sigma) \rangle \right) = v_{\Lambda\Lambda}^A |\Psi(\Lambda) \rangle + v_{\Lambda\Sigma}^A |\Psi(\Sigma) \rangle + v_{\Lambda} |\Psi(\Lambda) \rangle + \left[\sum_{i \neq A} \left(v_{\Lambda i}^i |\Psi(\Lambda) \rangle + v_{\Lambda\Sigma}^i |\Psi(\Sigma) \rangle \right) - v_{\Lambda} |\Psi(\Lambda) \rangle \right]. \quad (3.3b)$$

Similarly, for nucleon A we may write

$$\frac{1}{2} \sum_{i \neq j} u_{ij} = \frac{1}{2} \sum_{i \neq j \neq A} u_{ij} + U_A + \left(\sum_{j \neq A} u_{Aj} - U_A \right). \quad (3.3c)$$

Neglecting the residual interactions, the hyperon single-particle potential operators are defined formally using (3.2) as

$$\begin{aligned} v_{\Sigma} |\Psi(\Sigma) \rangle &= \sum_{i \neq A} \left(v_{\Sigma i}^i + v_{\Sigma\Lambda}^i \frac{1}{(E + \Delta m_{\Sigma\Lambda} - K_{\Lambda} - \sum_{i \neq j \neq A} u_{ij} - V_{\Sigma} - v_{\Lambda\Lambda})} \right) \times \\ &\quad \times \sum_j \left(v_{\Lambda\Sigma}^j \right) |\Psi(\Sigma) \rangle, \\ v_{\Lambda} |\Psi(\Lambda) \rangle &= \sum_{i \neq A} \left(v_{\Lambda i}^i + v_{\Lambda\Sigma}^i \frac{1}{(E - K_{\Sigma} - \sum_{i \neq j \neq A} u_{ij} - V_{\Lambda} - v_{\Sigma\Sigma})} \right) \times \\ &\quad \times \sum_j \left(v_{\Sigma\Lambda}^j \right) |\Psi(\Lambda) \rangle, \end{aligned} \quad (3.3d)$$

and where, for economy of space, in the equations (3.3) the $(A+1)$ -body wavefunctions $|\Psi(1 \dots A; \gamma) \rangle$ have been abbreviated to $|\Psi(\gamma) \rangle$. Because residual interactions have been dropped in the propagators of (3.3), matrix elements of V_{γ} between independent-particle states will restrict the sum over j to $j=i$.

The $(A+1)$ -body equations themselves now become

$$\begin{aligned}
& (K_{\Sigma} + K_A + K_{A-1} + v_{\Sigma\Sigma}^A + v_{\Sigma}^A + U_A + \frac{1}{2} \sum_{i \neq j \neq A} u_{ij} - E) |\psi(\Sigma)\rangle = -v_{\Sigma\Lambda}^A |\psi(\Lambda)\rangle, \\
& (K_{\Lambda} + K_A + K_{A-1} + v_{\Lambda\Lambda}^A + v_{\Lambda}^A + U_A + \frac{1}{2} \sum_{i \neq j \neq A} u_{ij} - (E + \Delta m_{\Sigma\Lambda})) |\psi(\Lambda)\rangle = -v_{\Lambda\Sigma}^A |\psi(\Sigma)\rangle. \quad (3.4)
\end{aligned}$$

The ΣK_1 has been divided explicitly into the nucleon-A and (A-1)-core kinetic energy operators to emphasize the three-body nature of the interaction. That the equations (3.4) are indeed three-body equations, and that the recoil of the core is necessary in guaranteeing elastic unitarity has been stressed by many authors^{22,23}.

The ΣA pair can now be isolated from (3.4). Defining the YA pair wavefunctions $|\psi_{\sigma\alpha}(\Sigma A)\rangle$, and $|\psi_{\lambda\alpha}(\Lambda A)\rangle$ as the overlap integrals $\langle \phi(A-1) | \psi(\Sigma) \rangle$ and $\langle \phi(A-1) | \psi(\Lambda) \rangle$, with $\langle \phi(A-1) |$ the (A-1)-core wavefunction, Eqs. (3.4) are multiplied to the left by $\langle \phi(A-1) |$. Further, separating K_{A-1} into centre-of-mass and internal components, it is found that, since only the average of $K_{A-1}^{INT} + 1/2 \sum_{i \neq j \neq A} u_{ij}$ is required, this internal Hamiltonian can be replaced by $E-E_{\sigma\alpha}$, where $E_{\sigma\alpha}$ is the ΣA pair energy. Finally, writing the Y and nucleon kinetic energy operators in relative, and centre-of-mass terms they can be combined with the core kinetic energy to give

$$\begin{aligned}
K_Y + K_A + K_{A-1}^{CM} &= K_{YA} + P^2/2\mu_Y, \\
\mu_Y &= m_{A-1}(m_N + m_Y)/(m_A + m_Y), \quad (3.5)
\end{aligned}$$

and where P is the YA pair total momentum as measured in the Y-nucleus rest frame. Assembling all of the above results we arrive at the coupled YA pair equations in relative co-ordinates

$$\begin{aligned}
& (K_{\Sigma A} + P^2/2\mu_{\Sigma\Sigma} + v_{\Sigma\Sigma}^A + v_{\Sigma}^A + U_A - E_{\sigma\alpha}) |\psi_{\sigma\alpha}(\Sigma A)\rangle = -v_{\Sigma\Lambda} |\psi_{\lambda\alpha}(\Lambda A)\rangle, \\
& (K_{\Lambda A} + P^2/2\mu_{\Lambda\Lambda} + v_{\Lambda\Lambda}^A + v_{\Lambda}^A + U_A - (E_{\sigma\alpha} + \Delta m_{\Sigma\Lambda})) |\psi_{\lambda\alpha}(\Lambda A)\rangle = -v_{\Lambda\Sigma} |\psi_{\sigma\alpha}(\Sigma A)\rangle. \quad (3.6)
\end{aligned}$$

As pointed out earlier, Pauli exclusion is expected to play an important role in the Σ lifetime. These effects can be included explicitly by modifying the two-body potentials $v_{YY'}$ to the form $Q_{\alpha} v_{YY'}$, where Q_{α} is a projection operator for hyperon and nucleon states defined as

$$Q_{\alpha} \equiv 1_Y \otimes (|\alpha\rangle\langle\alpha| + \sum_{Y>K_F} |\gamma\rangle\langle\gamma|). \quad (3.7)$$

Here the $|\xi\rangle$ are independent-particle states for the nucleons, and the second term in (3.7) projects only onto unoccupied states above the Fermi sea. The presence of 1_Y for projection on hyperon states just reflects the fact that no Pauli restrictions apply to the hyperon in the nucleus. Finally, if the potential V_{Σ} has been chosen by a variational procedure to yield the best single-particle energies ϵ , then the difference between $E_{\sigma\alpha} |\psi_{\sigma\alpha}\rangle - \langle\alpha| V_{\Sigma\Sigma} |\psi_{\sigma\alpha}\rangle |\sigma\alpha\rangle - \langle\alpha| V_{\Sigma\Lambda} |\psi_{\lambda\alpha}\rangle |\sigma\alpha\rangle$ and $(\epsilon_{\sigma} + \epsilon_{\alpha}) |\psi_{\sigma\alpha}\rangle$ is a second-order correction, and therefore (3.6) becomes

$$(K_{\Sigma A} + P^2/2\mu_{\Sigma\Sigma} + Q v_{\Sigma\Sigma}^A + v_{\Sigma}^A + U_A - (\epsilon_{\sigma} + \epsilon_{\alpha})) |\psi_{\sigma\alpha}(\Sigma A)\rangle = -Q v_{\Sigma\Lambda} |\psi_{\lambda\alpha}(\Lambda A)\rangle, \quad (3.8a)$$

$$(K_{\Lambda A} + P^2/2\mu_{\Lambda\Lambda} + Q v_{\Lambda\Lambda}^A + v_{\Lambda}^A + U_A - (\epsilon_{\sigma} + \epsilon_{\alpha} + \Delta m_{\Sigma\Lambda})) |\psi_{\lambda\alpha}(\Lambda A)\rangle = -Q v_{\Lambda\Sigma} |\psi_{\sigma\alpha}(\Sigma A)\rangle, \quad (3.8b)$$

where Q projects only onto nucleon states above the Fermi sea. In this form it is convenient to eliminate the ΛA wavefunction from (3.8a) using (3.8b) to give the ΣA pair equation

$$(K_{\Sigma A} + P^2/2\mu_{\Sigma\Sigma} + Q v_{\Sigma A}^{eff} + v_{\Sigma}^A + U_A) |\psi_{\sigma\alpha}(\Sigma A)\rangle = (\epsilon_{\sigma} + \epsilon_{\alpha}) |\psi_{\sigma\alpha}(\Sigma A)\rangle, \quad (3.8c)$$

where the effective ΣA (complex) potential $v_{\Sigma A}^{eff}$ is defined as

$$v_{\Sigma A}^{eff} \equiv v_{\Sigma\Sigma} + v_{\Sigma\Lambda} \frac{1}{(\epsilon_{\sigma} + \epsilon_{\alpha} + \Delta m_{\Sigma\Lambda} - K_{\Lambda A} - P^2/2\mu_{\Lambda\Lambda} - U_{\Lambda} - Q v_{\Lambda\Lambda})} Q v_{\Lambda\Sigma}. \quad (3.8d)$$

We are now in a position to calculate the Σ single-particle potential. Clearly the total potential felt by the Σ interacting with the nucleus is just the sum of all the two-body interactions. That is, V_{Σ}

can be defined by $\langle \sigma | V_{\Sigma} | \sigma \rangle = \sum_{\alpha} \langle \sigma \alpha | v_{\Sigma A}^{\text{eff}} \psi_{\sigma \alpha}(\Sigma A) \rangle$. This form is not very helpful since the pair wavefunctions $\psi_{\sigma \alpha}$ are not known. This can be remedied though by rearranging (3.8). Asymptotically $|\psi_{\sigma \alpha} \rangle$ must reduce to the model wavefunctions $|\sigma \alpha \rangle$ as $v_{\Sigma A}^{\text{eff}} \rightarrow 0$. Therefore $|\psi_{\sigma \alpha} \rangle$ can be expressed in the integral relation

$$|\psi_{\sigma \alpha}(\Sigma A) \rangle = |\sigma \alpha \rangle + \frac{1}{(e_{\sigma} + e_{\alpha} - K_{\Sigma A} - p^2/2\mu_{\Sigma} - V_{\Sigma} - U_A)} Q v_{\Sigma A}^{\text{eff}} |\psi_{\sigma \alpha}(\Sigma A) \rangle. \quad (3.9)$$

If we now introduce the Brueckner reaction matrix τ_{α} with the property that $\tau_{\alpha} |\sigma \alpha \rangle = v_{\Sigma A}^{\text{eff}} \psi_{\sigma \alpha}$ then, using (3.9), τ_{α} is

$$\tau_{\alpha} = v_{\Sigma A}^{\text{eff}} + v_{\Sigma A}^{\text{eff}} (e_{\sigma} + e_{\alpha} - K_{\Sigma A} - p^2/2\mu_{\Sigma} - V_{\Sigma} - U_A)^{-1} Q \tau_{\alpha}. \quad (3.10)$$

Here τ_{α} is the effective ΣN interaction, in the sense that τ_{α} acting on the model wavefunctions is equivalent to $v_{\Sigma A}^{\text{eff}}$ acting on the pair wavefunction. Specifically, introducing the Σ states $|k \rangle, |k' \rangle$ the total Σ potential becomes

$$\langle k' | V_{\Sigma} | k \rangle = \sum_{\alpha} \int d\vec{p} \phi_{\alpha}^*(\vec{p}-\vec{q}) \langle k', \vec{p}-\vec{q} | \tau_{\alpha}(\omega) | k, \vec{p} \rangle \phi_{\alpha}(\vec{p}), \quad (3.11)$$

where q is the three-momentum transfer, ϕ_{α} is the nucleon single-particle wavefunction, and ω is the complicated energy variable defined by (3.10).

In practice V_{Σ} is difficult to calculate, particularly since ω also depends on V_{Σ} . The best approach is the Brueckner-Hartree iterative method in which τ is calculated initially using a reasonable spectrum of single-particle energies and the single-particle potentials in ω are set equal to zero. V_{Σ} is then determined by (3.11) and a new value for the Σ energy is fixed by the Σ -nucleus Schrödinger equation. This cycle is repeated until a self-consistent solution is achieved.

A remark worth making at this point is that a complete, self-

consistent solution would also include the perturbative effect that the Σ has on the nucleon states arising from the contribution of the ΣN potential to the nucleon single-particle potential. Practically, this influence can be safely ignored since first, it is an $O(1/A)$ correction and, second, it is known experimentally that the Σ is only weakly bound (if at all) so that the ΣN potential must be weak relative to NN interactions.

The approach used in this work is not as ambitious as the program outlined above. Instead we wish to approximate (3.11) to a more manageable form while retaining its most important features. Essentially our aim is to apply the Brueckner-Hartree method only to first order. That is, the single-particle potentials entering τ are to be set equal to zero and then the momentum-space Schrödinger equation self-consistently solved for the Σ binding energy. Higher-order corrections to V_{Σ} would involve the Σ and N potentials but in this work the primary interest is the lifetime of the hypernuclear levels and, therefore, with the energy available in asymptotic states. The energy dependence defined above should be suitable for this purpose.

In partial compensation for this area of ambiguity we invoke closure approximation to replace the nucleon energies e_{α} by an average value $-B$ which in practice is treated as a parameter. Our final simplification is to replace τ by its spin- and isospin-averaged value $\bar{\tau}$, which is exact to $O(1/A)$. Our final expression for the Σ^0 single-particle potential is then

$$V_{\Sigma}(k', k; E_{\Sigma}) = \int d\vec{p} \bar{\tau}(k', k; \omega) \sum_{\alpha} \phi_{\alpha}^*(\vec{p}-\vec{q}) \phi_{\alpha}(\vec{p}), \quad (3.12)$$

where $\underline{k}, \underline{k}'$ are the initial and final state Σ -nucleus relative momenta; $\underline{k}, \underline{k}'$ are the initial and final state ΣN momenta in the ΣN rest frame; and ω is the three-body energy

$$\omega = \epsilon_{\Sigma} - B - (\underline{k} + \underline{p})^2 / 2\mu_{\Sigma} . \quad (3.13)$$

Despite our simplifications, the many-body effects embedded in τ still make the integrand of (3.11) a very complicated function. It can be seen from the integral relation (3.10) that τ resembles the free ΣN t -matrix except that the free propagator has been replaced by one appropriate for a Σ interacting with a bound nucleon. This analogy can be pushed farther. If partial-wave mixing in τ is ignored [which is equivalent to using the angle-averaged Pauli operator^{24,25}] it can be shown that to describe scattering from the state α to β , τ satisfies

$$\begin{aligned} \tau_{\beta\alpha}(\omega) &= |v_{\beta}\rangle \lambda^{\beta\alpha} \left(1 - \sum_{\gamma} \lambda^{\gamma\gamma} \langle v_{\gamma} | Q^0 G_{\gamma}(\omega) | v_{\gamma} \rangle^{-1} \right) \langle v_{\alpha} | , \\ &\equiv |v_{\beta}\rangle D_{\beta\alpha}(\omega) \langle v_{\alpha} | , \end{aligned} \quad (3.14)$$

where Q^0 is the angle-averaged Pauli operator to be discussed later, the $|v_{\gamma}\rangle$ are the separable potentials of sect. 2, and the summation over γ includes all coupled channels.

The main difficulty presented in integrating (3.12) is the angular dependence of the energy variable ω , since all other factors normally have a closed, analytic form. A number of approximation schemes have been devised to circumvent this difficulty. In the simplest, fully-factored form $\bar{\tau}$ is removed from the integrand and evaluated at some average value of the nucleon momentum. A much better approach is to leave $\bar{\tau}$ within the integral but replace all angles in $\bar{\tau}$ by their average value²³). Although the latter is much better than the former, the angular dependence is ignored nonetheless. The obvious successes of

such models in describing scattering situations may be attributable to incident energies which are large in comparison with nucleon energies, and a $\bar{\tau}$ which is a slowly varying function of energy.

A priori, neither of the above features can be expected to hold in Σ hypernuclei. The Σ and N energies are comparable in the hypernucleus. Perhaps more importantly, all separable potential predictions of ΣN scattering lead to strong energy dependence in the (dominant) 3S_1 $I=1/2$ channel near threshold [c.f. sect. 2, also ref. 19].

Consequently, we make no further approximations in evaluating (3.12). That is, we evaluate the full, three-dimensional integral exactly.

The potential (3.12) can be put in a more convenient form for angular integration by changing variables from the nucleon momentum $\underline{p}$ to the ΣN total momentum $\underline{P} = \underline{k} + \underline{p}$. In this way ω becomes a scalar function of $\underline{P}$ and all angular dependence resides in the simpler $\bar{\tau}$ form factors and nuclear wavefunction. With this substitution V_{Σ} becomes

$$V_{\Sigma}(\underline{k}', \underline{k}; E_{\Sigma}) = \int d\underline{P} \bar{\tau}(\underline{k}' - \epsilon \underline{P}, \underline{k} - \epsilon \underline{P}; \omega(P^2)) F(\underline{P} - \underline{k}'; \underline{P} - \underline{k}) ,$$

with

$$\epsilon \equiv m_{\Sigma} / (m_{\Sigma} + m_N) , \text{ and } F(\underline{P} - \underline{k}'; \underline{P} - \underline{k}) \equiv \sum_{\alpha} \phi_{\alpha}^{*}(\underline{P} - \underline{k}') \phi_{\alpha}(\underline{P} - \underline{k}) . \quad (3.15)$$

With harmonic oscillator single-particle wavefunctions F may be expanded in partial waves as

$$\begin{aligned} F(\underline{P} - \underline{k}'; \underline{P} - \underline{k}) &= f_0 \left(1 + a^2 \nu (p^2 - \underline{p} \cdot (\underline{k} + \underline{k}') + \underline{k} \cdot \underline{k}') \right) e^{-a^2(p^2 + k^2 + k'^2) - \underline{p} \cdot (\underline{k} + \underline{k}')} , \\ &= f_0 e^{-a^2(p^2 + k^2 + k'^2)} \sum_{\ell\ell'} (2\ell+1) (2\ell'+1) \left(\rho_{\ell\ell'}^0 + \rho_{\ell\ell'}^1 \hat{\underline{p}} \cdot \hat{\underline{k}} \right) \\ &\quad P_{\ell}(\hat{\underline{p}} \cdot \hat{\underline{k}}) P_{\ell'}(\hat{\underline{p}} \cdot \hat{\underline{k}}') , \end{aligned} \quad (3.16)$$

with $f_0 = 4a^3/\pi^{3/2}$, $\nu = (A-4)/6$, and a^2 chosen to fit the r.m.s. matter radius. The functions $\rho_{\ell\ell'}^0$, and $\rho_{\ell\ell'}^1$, are given by

$$\begin{aligned} \rho_{\ell\ell'}^0 &= (1 - \nu(\ell + \ell') + a^2 \nu p^2) i_{\ell}(x) i_{\ell'}(x') - \nu(x i_{\ell+1}(x) i_{\ell'}(x') + x' i_{\ell}(x) i_{\ell'+1}(x')), \\ \rho_{\ell\ell'}^1 &= a^2 \nu k k' i_{\ell}(x) i_{\ell'}(x'), \end{aligned} \quad (3.17)$$

with $x = a^2 p k$, $x' = a^2 p k'$, and the i_{ℓ} are modified spherical bessel functions.

Similarly, the two-body $\bar{\tau}$ can be expanded as a function of the 3 angles $\hat{k} \cdot \hat{k}'$, $\hat{p} \cdot \hat{k}$, and $\hat{p} \cdot \hat{k}'$. The normalization of $\bar{\tau}$ is chosen to be the same as used for the two-body t-matrix. The spin, and isospin-averaged $\bar{\tau}$ is then

$$\bar{\tau}(\kappa', \kappa; \omega) = 1/4\pi \sum_{\text{ILSJ}} \frac{(2J+1)(2I+1)}{24} \tau_{\text{LSJ}}^I(\kappa', \kappa; \omega) P_L(\hat{k} \cdot \hat{k}'),$$

$$\tau_{\text{LSJ}}^I(\kappa', \kappa; \omega) = v_L(\kappa') D_{\text{LSJ}}^I(\omega) v_L(\kappa) \kappa^L \kappa'^L. \quad (3.18)$$

In our case the v_L are the Yamaguchi form factors in both S and P waves.

Note that (3.18) defines the expansion of $\bar{\tau}$ in the ΣN centre-of-mass

whereas the integral (3.15) requires $\bar{\tau}$ in the Σ -nucleus frame. Using

$$\tilde{\kappa} = \tilde{\kappa} - e\tilde{p}, \text{ and } \tilde{\kappa}' = \tilde{\kappa}' - e\tilde{p} \text{ it has been shown that in general}^{26,27})$$

$$\begin{aligned} \kappa^L \kappa'^L P_L(\hat{\kappa} \cdot \hat{\kappa}') &= (4\pi)^2 \sum_{a\alpha} \sum_{b\beta} \sum_{m\bar{m}} \left(\frac{(2L+1)}{(2a)} \right) \left(\frac{(2L+1)}{(2b)} \right) \frac{1}{(2a+1)(2b+1)} \left(-e\tilde{p} \right)^{a+b} \kappa^L \kappa'^L \\ &\quad \begin{pmatrix} a & L-a & L \\ \alpha & m-\alpha & m \end{pmatrix} \begin{pmatrix} b & L-b & L \\ \beta & m-\beta & m \end{pmatrix} Y_a^{\alpha}(\hat{p}) Y_b^{\beta}(\hat{p}) Y_{L-a}^{m-\alpha}(\hat{\kappa}) Y_{L-b}^{m-\beta}(\hat{\kappa}'). \end{aligned} \quad (3.19)$$

The two-body form factors can be expanded in the form

$$\begin{aligned} v(\kappa) &= (\beta^2 + (\tilde{\kappa} - e\tilde{p})^2)^{-1} = \frac{1}{2ePk} \sum_n (2n+1) Q_n \left(\frac{\beta^2 + \kappa^2 + e^2 p^2}{2ePk} \right) P_n(\hat{p} \cdot \hat{\kappa}), \\ &\equiv \sum_n v_n(\kappa; p) P_n(\hat{p} \cdot \hat{\kappa}), \end{aligned} \quad (3.20)$$

where the Q_n are Legendre polynomials of the second kind.

With equations (3.16)-(3.20) inserted into (3.15) the angular integrals can be performed. This is a fairly straightforward exercise

producing the general result

$$V_{\Sigma}(\kappa', \kappa; E_{\Sigma}) = 1/4\pi \sum (2\ell+1) V_{\ell}(\kappa', \kappa; E_{\Sigma}) P_{\ell}(\hat{k} \cdot \hat{k}'),$$

where

$$\begin{aligned} V_{\ell}(\kappa', \kappa; E_{\Sigma}) &= 4\pi f_0 e^{-a^2/2} (k^2 + k'^2) \int dp p^2 e^{-a^2 p^2} \sum_{\ell'=0}^{\ell} \sum_{\ell''=0}^{\ell'} (2\ell'+1) (2\ell''+1) \\ &\quad \rho_{\ell\ell''}^{\ell'} \sum_{\text{ILSJ}} \frac{(2J+1)}{4} \frac{(2I+1)}{6} D_{\text{LSJ}}^I(\omega) \sum_{ab} (-e\tilde{p})^{a+b} \kappa^L \kappa'^L \\ &\quad \sum_{\bar{m}\bar{m}'} v_n(\kappa; p) v_{\bar{m}}(\kappa'; p) \left(\frac{(2L)!}{(2a)! (2(L-a))!} \frac{(2L)!}{(2b)! (2(L-b))!} \right)^{1/2} (2L+1) \\ &\quad \sum_{L' L'' L''' L^{(4)}} \begin{pmatrix} 2L'+1 \\ L' \end{pmatrix} \begin{pmatrix} 2L''+1 \\ L'' \end{pmatrix} \begin{pmatrix} 2L''' +1 \\ L''' \end{pmatrix} \begin{pmatrix} 2L^{(4)}+1 \\ L^{(4)} \end{pmatrix} \begin{pmatrix} \ell & \ell' & L' \end{pmatrix}^2 \begin{pmatrix} n & \ell'' & L'' \end{pmatrix}^2 \\ &\quad \begin{pmatrix} m & \ell''' & L''' \end{pmatrix}^2 \begin{pmatrix} L' & L'' & L-a \end{pmatrix} \begin{pmatrix} L' & L''' & L-b \end{pmatrix} \begin{pmatrix} b & L'' & L^{(4)} \end{pmatrix} \begin{pmatrix} a & L''' & L^{(4)} \end{pmatrix} \\ &\quad \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \\ &\quad \left\{ \begin{matrix} L-a & L' & L'' \\ L & L-b & b \\ a & L''' & L^{(4)} \end{matrix} \right\}. \end{aligned} \quad (3.22)$$

This is not a very convenient looking expression for V_{ℓ} . However, with only S and P waves included in $\bar{\tau}$ it simplifies considerably, because at least two of the coefficients in the $9J$ symbol are always zero, thereby reducing it to a $3J$.

The advantage of the above form is that we have found it possible to reduce the Brueckner potential to an exact one-dimensional integral without relying on untested approximations affecting the energy-dependence.

4. Technical details and results

With the development of a reliable description of the Σ -nucleus interaction we can turn now to the search for Σ^0 bound states and resonances. These states appear as poles in the elastic S-matrix and at this point it is worth digressing to examine the movement of these

poles for a two-channel interaction. The positions of the various S -matrix singularities are shown in fig. 4.

In the limit of zero coupling to the ΛN channel the bound state lies on the positive imaginary axis and the unitarity condition $S^*(k) = S(-k^*)$ ensures that the resonant poles are symmetric about this axis (on the unphysical energy sheet). We also notice that, because $S(k)S(-k) = 1$, a pole of S on the second sheet corresponds to a zero of S on the physical sheet.

In the usual scattering situation the third quadrant pole is too far from the physical energy sheet to influence the cross section. The fourth quadrant pole on the other hand can lie very close to the physical momentum axis and is then responsible for the characteristic resonant bump in the amplitude.

For non-zero coupling to the ΛN channel all the S -matrix poles are perturbed to the left and all zeros to the right as shown. The bound state moves into the second quadrant and acquires a small real component which leads to the absorptive width of the state. The fourth quadrant pole moves away from the real axis causing the resonant width to increase with increasing absorption, as expected. The third quadrant pole however, shifts closer to the negative real axis. This pole still does not affect the cross section but, as indicated earlier, this is accompanied by a zero of the S -matrix approaching the physical real axis. The presence of the zero nearby on the physical sheet is manifested by a large reduction in the magnitude of the S -matrix at this energy.

There is a particularly interesting limit of the above case. If

the potential is sufficiently absorptive, it is possible for the third quadrant pole to cross the real axis into the physical region. We then have the unique situation that the strong absorption of the potential is responsible for creating a quasi-bound state. Of specific importance here is that, since this pole can lie arbitrarily close to the real axis, it is possible in principle for the state to have a vanishingly small width despite the fact that the effective Σ -nucleus potential is strongly absorptive.

We have labelled this situation a bound state although it is recognized to be an unusual one. The Σ wavefunction for this energy decays exponentially at large distances and the scattering phase shift has the characteristic bound state signature $\delta(0) - \delta(\infty) = \pi$. Conversely, the real component of the 'binding' energy is positive; embedded in the Σ continuum states. We also stress again that the pole responsible for this state is not the usual bound state one, but rather has moved from a nominally inaccessible region of the unphysical energy sheet.

The observation that a strongly absorptive potential can lead to narrow states is not new. Gal and Toker⁸) have discussed this possibility in detail, and found that with their one-channel phenomenological model narrow Σ p-states arose naturally from this mechanism. It should be noted though that their potential predicts very broad s-states (e.g. $\Gamma \sim 23$ MeV in $^{12}_0\text{C}$) because of the strong absorption.

So far we have not mentioned the ΛN channel momentum. Because we search for the self-consistent eigenvalues $E_\Sigma = \epsilon - i\Gamma/2$ of the Schrödinger equation, $\bar{\tau}(\omega)$ [eq. (3.14)] must be evaluated at complex energies. If the coupling between the ΣN and ΛN channels is again imagined as being increased from zero, the Σ bound state pole moves into the third

quadrant of the physical energy sheet. The corresponding ΔN channel energy however is perturbed from the positive real axis into the fourth quadrant of the unphysical energy sheet. Similarly for a Σ resonant energy the ΔN energy is on the unphysical sheet.

It is realized then that the correct evaluation of the $\bar{\tau}$ overlap integral $\langle v | Q^0 G(\omega) | v \rangle$ in the ΔN channel requires the analytic continuation of the Pauli operator Q^0 to the second sheet. This is not a trivial problem since in the usual Fermi gas approximation Q^0 is only a piece-wise continuous function of nucleon momentum. This point was not considered in the work of ref. 10) because there was no attempt to compute the Σ -nucleus potential self-consistently.

In nuclear matter approximation Q is defined by the zero temperature limit of the Fermi-Dirac distribution:

$$Q \equiv 1 - |0\rangle\langle 0| = 1 - \lim_{\theta \rightarrow \infty} \left(1 + \exp \left[\theta (k_N^2 - k_F^2) \right] \right)^{-1}, \quad (4.1)$$

where k_N , k_F are the nucleon, and Fermi momenta. Eventually the limit of θ approaching infinity will be taken but for now a finite value allows Q to be defined for complex momenta.

If partial-wave mixing in τ via Q is ignored then only the angle-averaged value of Q is required. With k_γ, p the relative, and total ΔN momenta this means

$$Q^0(k_\gamma; p; k_F) \equiv 1/2 \int d(\hat{p} \cdot \hat{k}_\gamma) Q(|k_\gamma - n_p|),$$

$$= 1 - \lim_{\theta \rightarrow \infty} \frac{1/4\pi p k_\gamma \theta \ln \left\{ \frac{1 + \exp[\theta(k_F^2 - (k_\gamma - n_p)^2)]}{1 + \exp[\theta(k_F^2 - (k_\gamma + n_p)^2)]} \right\}}{\ln \left\{ \frac{1 + \exp[\theta(k_F^2 - (k_\gamma - n_p)^2)]}{1 + \exp[\theta(k_F^2 - (k_\gamma + n_p)^2)]} \right\}}, \quad (4.2)$$

with $n \equiv n_N/(m_N + m_\pi)$. In the limit that $\theta \rightarrow \infty$ it is found that for complex momentum $k_\gamma = \kappa + i\gamma$ Q^0 behaves as

$$Q^0(k_\gamma; p; k_F) = 0 \quad \text{for } (k+n_p)^2, (k-n_p)^2 < k_F^2 + \gamma^2,$$

$$= 1 \quad \text{for } (k+n_p)^2, (k-n_p)^2 > k_F^2 + \gamma^2,$$

$$= \frac{k_F^2 - (k_\gamma - n_p)^2}{4\pi p k_\gamma} \quad \text{for } (k+n_p)^2 < k_F^2 + \gamma^2 < (k-n_p)^2,$$

$$= \frac{(k_\gamma + n_p)^2 - k_F^2}{4\pi p k_\gamma} \quad \text{for } (k+n_p)^2 > k_F^2 + \gamma^2 > (k-n_p)^2. \quad (4.3)$$

In the limit that $\gamma \rightarrow 0$ the above result of course reduces to the usual definition of the angle-averaged Pauli operator. With this technical detail clarified we return to the predictions of the Σ^0 single-particle potential.

Although no attempt is made to fit the model to the available Σ^0 -atomic data, the consistency with Batt's phenomenological result can be checked in some appropriate limit. At zero momentum transfer the volume integral of the model potential is proportional to Batt's value $\bar{a}_B$.

The theoretical value is given by the expression

$$\bar{a}_{th} = -\pi \frac{\hbar}{\mu_{\Sigma A}} \int d^3p F(p, p) \sum_{I, L, S, J} \frac{(2J+1)}{4} \frac{(2J+1)}{6} \tau_{LSJ}^I(\omega),$$

$$\text{with} \quad \tau_{LSJ}^I(\omega) = \frac{(e p)^{2L}}{(g^2 + e^2 p^2)^2} D_{LSJ}^I(-B - p^2/2\mu). \quad (4.4)$$

Notice that even at zero incident momentum (in the Σ -nucleus frame) $\bar{a}_{th}$ receives contributions from $L=1$ and higher ΣN partial waves.

For ^{13}C $\bar{a}_{th}$ has been examined in some detail. With the nucleon binding parameter B chosen (rather arbitrarily) to be 10 MeV and a Fermi momentum of 260 MeV/c it is found that $\bar{a}_{th} = 0.346 + i0.197$ fm in comparison with $\bar{a}_B = (0.35 \pm 0.04) + i(0.19 \pm 0.03)$ fm. The agreement of these results is remarkable and furthermore it is relatively insensitive to the choice of parameters.

The variation of $\bar{a}_{th}$ with k_F and B is shown in detail in fig. 5,

where the elliptical area defines the values allowed from Batty's analysis. With a Fermi momentum of 260 MeV/c and any value of $0 \leq B \leq 20$ MeV $\bar{a}_B$ is consistent with $\bar{a}_B$. Conversely, with $B = 10$ MeV any κ_F in the range $250 \leq \kappa_F \leq 275$ MeV/c is consistent (i.e. an average nuclear density of ~ 75 -100% of nuclear matter).

The above predictions are a very satisfying reflection on the model and the bound state calculations can proceed with some confidence. For the light hypernuclei considered in this work we have fixed the nucleon binding B at 10 MeV for simplicity. Because in s -state the Σ is largely confined to the nuclear volume, fairly high Fermi momenta have been chosen, ranging from 245 MeV/c in ${}^5_{\Sigma^0}\text{He}$ to 260 MeV/c in ${}^{16}_{\Sigma^0}\text{O}$ (or, average densities from 70-80% of nuclear matter). The self-consistent eigenvalues $\epsilon - i\Gamma/2$ for s -state hypernuclei are presented in table 3.

Several conclusions can be drawn from these results. Most importantly it is found that fairly narrow bound states result when Pauli exclusion and nucleon binding are incorporated in a microscopic calculation. We also notice that for $A \leq 9$ the model predicts that $\epsilon > 0$. These are bound states though and not resonances. That is, they arise from an S -matrix pole in the second quadrant of the momentum plane.

Although the earlier prediction of $\bar{a}$ was encouraging we also wish to test the sensitivity of these bound states (and the widths in particular) to small changes in the parameters.

Again for ${}^3_{\Sigma^0}\text{C}$, the variation has been examined in detail. With values of B and κ_F which approximately reproduce the extreme values permitted for $\bar{a}_B$ the binding has been calculated and these results are presented in table 4. The real binding $|\epsilon|$ varies between 2 and 5 MeV, but Γ seems fairly insensitive, changing only from 9-11 MeV.

Since Pauli exclusion and nucleon binding effects are major influences in producing these narrow states we have tried to isolate their contributions as much as possible. The binding has been calculated with, first, B fixed and κ_F varied between 225-300 MeV/c and, second, κ_F fixed and B varied from 0-30 MeV. These results are displayed in fig. 6. It can be seen that the inclusion of Pauli effects is essential to the correct description of the bound state, but, any reasonable choice of κ_F leads to a small width. The variation of Γ with large values of B is dramatic. The width is almost insensitive to changes in B from 0-20 MeV but decreases sharply for $B \geq 20$ MeV.

This strong dependence of Γ on nucleon binding for large B should not be surprising. To take an extreme example, if the Σ and nucleon binding energies were greater than the Σ - Λ mass difference then the $\Sigma N \rightarrow \Lambda N$ conversion would never be energetically favourable and the decay width of the state would necessarily be zero. For more realistic cases it is expected, from phase space considerations alone, that the width must decrease rapidly for large reductions in the ΛN channel energy.

As mentioned earlier, our determination of ϵ is expected to be less reliable than of Γ , mainly because of the neglect of single-particle potentials in the energy variable of $\bar{\tau}(\omega)$. On the other hand, the position and width calculated for the ${}^{12}_{\Sigma^0}\text{C}$ state agrees rather well with the experimental result of Bertini¹⁾. Also, the weakly bound state in ${}^9_{\Sigma^0}\text{Be}$ is consistent with the findings of Brückner *et al.*²⁾.

Experimentally, the clearest evidence of narrow hypernuclear states comes, not from the ground states so far discussed, but rather, from the recoillessly produced $1p$ levels. To date, the positions and widths of these levels have not been fixed accurately but some indications are

available. By comparing the Σ missing-mass spectrum with the corresponding well-known Λ spectrum, Brückner *et al.*⁽²⁾ have concluded that in ${}_{\Sigma}^9\text{Be}$ the state is unbound by ~ 9 MeV and have placed an upper limit on the width of 8 MeV. Bertini *et al.*⁽⁵⁾ have found a narrow p-level in ${}_{\Sigma}^{12}\text{C}$ at ~ 5 MeV excitation, and it has even been suggested recently that in ${}_{\Sigma}^6\text{Li}$ the width may be as small as 3 MeV⁽³⁾. It must be emphasized that these are only tantalizing indications. The data are not yet precise enough to pinpoint the positions to better than a few MeV accuracy.

Pauli exclusion and nucleon binding effects are not expected to influence P-wave as much as in the ground state calculations. The centrifugal barrier obliges the Σ to interact primarily in the low density region of the valence nucleons so that, for consistency, both the Fermi momentum k_F and B should be reduced from their S-wave values. In all the light nuclei we consider, the nucleon binding has been raised to zero MeV and also the Fermi momenta have been reduced substantially, ranging from 75 MeV/c in ${}_{\Sigma}^5\text{He}$ to 175 MeV/c in ${}_{\Sigma}^{16}\text{O}$.

Our method of solution is straightforward. The Schrödinger equation is again solved self-consistently and we search for poles in the S-matrix. Anticipating our results somewhat, we limit our attention to the pole lying in either the second or third quadrant of the momentum plane. The positions of these poles are given in table 5. Also, the elastic phase shifts δ and absorption coefficients η are plotted in fig. 7.

These results are rich in information and several important observations can be made. First, it is noticed that for $A \geq 9$ the model predicts the 1p level to be bound. This is evident from both the pole positions as given in the table and also the π phase change between $k = 0$ and ∞ . That these states do in fact correspond to the resonance pole rather than

the usual bound state pole is quickly confirmed by following their motion for increasing Fermi momentum.

Increasing the Fermi momentum effectively decreases absorption by decreasing coupling to the ΛN channel. Absorption depletes the Σ wavefunction at short distances, thereby acting as a short-range repulsion. By increasing k_F a bound state pole then should move towards the positive imaginary momentum axis. This is exactly the behaviour already encountered in S-wave. A resonance pole, in light of our earlier discussion, will move in the opposite direction, crossing the negative real axis into the unphysical third quadrant.

Choosing ${}_{\Sigma}^8\text{Be}$ as representative, we increase k_F from 125 to 200 MeV/c. It is found that the state becomes unbound, with the pole moving into the third momentum quadrant at $E_p = 5.23 + i2.19$ MeV, clearly demonstrating these poles to be of the resonant variety.

The second feature we notice are the remarkably long lifetime of these states, with $\Gamma \sim 4$ for ${}_{\Sigma}^{16}\text{O}$ and as small as ~ 0.5 for ${}_{\Sigma}^8\text{Be}$. This is truly paradoxical: it is the very fact that the potential is so strongly absorptive that the levels are long-lived.

The case of ${}_{\Sigma}^7\text{Li}$ deserves special attention because, although the pole is very near to the real axis, the state is not quite bound. However, since the original choice of k_F and B was to some extent arbitrary, the question naturally arises whether for any reasonable choice of parameters the pole does cross the real axis to become a bound state. We find in fact that it does not. By reducing the Fermi momentum to zero and increasing the nucleon energy to +20 MeV (i.e. $B = -20$ MeV) it is possible to decrease the width of the level to 0.06 MeV from 0.52 MeV, but the pole remains in the third quadrant.

Larger changes in κ_F and B than the above would not be warranted on any physical grounds, but the results of the model calculation should not be construed of course as meaning that the ${}^7_0\text{Li}$ p-state is necessarily unbound. Clearly a very small change in the overall strength of the coupling to the ΛN channel would be sufficient to produce binding in Li , without qualitatively affecting the p-state results for the other nuclei.

In brief summary then, we find that when nucleon binding and Pauli exclusion effects are included in a microscopic calculation narrow Σ hypernuclear levels result. Not only do we find remarkably long-lived resonance poles in p-state in close agreement with the experimental energies, but the model also predicts the ground states of light hypernuclei to be long-lived.

5. Summary and conclusion

Our aim in this work has been to calculate the widths of light Σ hypernuclear states as accurately as possible. To this end we constructed a separable potential model to describe ΣN scattering where it was found that, in addition to the usual S-wave interactions, the ${}^3S_1(\Sigma N) \rightarrow {}^3D_1(\Lambda N)$ transition was essential to the description of the conversion reaction. With these interactions as fundamental input, the Σ -nucleus single-particle potential was developed with careful attention to corrections arising from Pauli exclusion and nucleon binding effects. After a minimum number of approximations we were able to evaluate analytically the angular integrals in the defining relation for this potential, and to show that in the appropriate limit it reduced to the phenomenological result obtained from the analysis of Σ^- atoms.

Most importantly, the self-consistent solutions of the momentum-space Schrödinger equation predicted narrow Σ states in both S and P waves, although the mechanisms responsible were quite different. In s-state, Pauli exclusion and nucleon binding effects produced long-lived states with widths ranging from 1.8 MeV in ${}^5_0\text{He}$ to 12.5 MeV in ${}^{16}_0\text{O}$. In p-state on the other hand, the Pauli principle had little influence on the potential and the consequent strong absorption produced narrow states in the Σ continuum with widths from 0.5 MeV in ${}^9_0\text{Be}$ to 4.0 MeV in ${}^{16}_0\text{O}$.

In view of the two distinct causes of these narrow states it seems important that measurements of p-state Σ -hypernuclear levels should first be extended to very light nuclei where they are predicted to be exceptionally long-lived. Secondly one would like reliable data for the s-states.

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Table 1. Best-fit coupling strengths $\bar{v}_{\beta\lambda}^{\lambda\beta\sigma\nu_\alpha}$ (MeV/fm). Strengths are identical for ΣN and ΔN channels, except for 3D_1 which is ignored for ΣN . $\beta = 1.0563 \text{ fm}^{-1}$ in all channels.

I = 1/2					I = 3/2	
1S_0	3S_1	3P_J	3D_1		1S_0	3S_1
1S_0	164.46	0	0	0	-38.314	0
3S_1	0	-54.822	0	-37.581	0	12.771
3P_J	0	0	-14.357	0		
3D_1	0	-37.581	0	-25.762		

Table 2. Scattering lengths a and effective ranges r in the elastic Σ^+p channel.

I = 1/2				I = 3/2	
a	f_m	r		a	r
1S_0	-1.448 + i0.374	0.515 - i0.595		4.338	3.682
3S_1	1.027 + i4.757	3.010 - i0.717		-0.358	-7.232
3P_J	1.045 + i0.461				

Table 3. $1s$ binding in light Σ^0 hypernuclei.

Nucleus	κ_F (MeV/c)	B (MeV)	$\epsilon - i\Gamma/2$ (MeV)	Γ (MeV)
^5_0He	245	10	+2.49 - i0.88	1.75
^7_0Li	250	10	+0.81 - i0.99	1.98
^9_0Be	260	10	-0.46 - i1.95	3.89
$^{12}_0\text{C}$	260	10	-2.06 - i4.91	9.81
$^{13}_0\text{C}$	260	10	-2.59 - i5.31	10.62
$^{16}_0\text{O}$	260	10	-4.22 - i6.26	12.52

Table 4. Variation of $1s$ binding in $^{13}_0\text{C}$ with κ_F and B.

κ_F (MeV/c)	B (MeV)	$\bar{a}_{th}$ fm	$\epsilon - i\Gamma/2$ (MeV)	Γ (MeV)
250	22.5	0.39 + i0.19	-4.78 - i4.95	9.90
250	12.5	0.35 + i0.22	-2.22 - i5.63	11.26
260	10.0	0.35 + i0.19	-2.59 - i5.31	10.62
275	7.5	0.35 + i0.16	-3.00 - i4.65	9.30
275	-7.5	0.31 + i0.19	-2.00 - i4.84	9.68

Table 5. Position of 1p poles in light Σ^0 hypernuclei. Those states with $\gamma > 0$ are 'bound'.

Nucleus	K_F (MeV/c)	B (MeV)	$E_p = e - i\Gamma/2$ (MeV)	$E_p^{1/2} = K_F^{1/2} + i\gamma$ (MeV) ^{1/2}	Γ
$\Sigma^0_5\text{He}$	75	0	17.48 + i0.94	-4.18 - i0.112	-1.88
$\Sigma^0_7\text{Li}$	100	0	11.13 + i0.26	-3.34 - i0.039	-0.52
$\Sigma^0_8\text{Be}$	125	0	6.16 - i0.27	-2.48 + i0.054	0.54
$\Sigma^0_{10}\text{C}$	150	0	4.58 - i1.75	-2.18 + i0.402	3.50
$\Sigma^0_{160}\text{O}$	175	0	3.82 - i2.02	-2.02 + i0.501	4.04

1. The ΣN differential cross-sections at 160 MeV/c incident lab momentum.

Figure Captions

- a) Elastic $\Sigma^- p$ reaction, b) $\Sigma^- p \rightarrow \Lambda n$ conversion, and c) elastic $\Sigma^+ p$ reaction. The elastic data are taken from ref.^[12] and the conversion data from ref.^[11].
2. The low-energy ΣN total cross-sections. a) Elastic $\Sigma^+ p$ reaction, b) $\Sigma^- p \rightarrow \Lambda n$ conversion, c) elastic $\Sigma^- p$ reaction, and d) $\Sigma^- p \rightarrow \Sigma^0 n$ charge-exchange. The elastic data are taken from ref.^[12] and both the conversion and charge-exchange cross-sections from ref.^[11].

3. The energy dependence of the off-shell 3S_1 amplitude in the isospin 1/2 channel near the ΣN threshold for different 3S_1 coupling strengths as indicated (in units of MeV/fm). a) The real component of the amplitude f_R , and b) the imaginary component f_I . The normalization is chosen such that at zero energy $f_R + if_I$ is the $I=1/2$ 3S_1 scattering length in the $\Sigma^- p$ channel.
4. Movement of the S-matrix singularities in the ΣN momentum plane as coupling to the ΛN channel is increased from zero. For strongly absorptive interactions the pole in the third quadrant can move into the second quadrant to become a bound state pole.
5. Variation of the potential volume integral $\bar{a}_{ch}$ with Fermi momentum (K_F in MeV/c) and nucleon binding energy (B in MeV) in $^{13}_{\Sigma^0}\text{C}$. The elliptical region defines the values allowed by Baty's analysis of Σ^- atoms^[20].
6. Dependence of the s-state position (e) and width (r) on Fermi momentum (K_F in MeV/c) and nucleon binding energy (B in MeV) in $^{13}_{\Sigma^0}\text{C}$. Any reasonable choice of K_F and B leads to a fairly narrow state ($9 \leq r \leq 12$ MeV).
7. P-wave phase shifts δ and absorption coefficients η for low-energy elastic Σ -nucleus scattering. For the light nuclei indicated in figs. a) and b) no bound state exists as seen by the zero phase shift at zero energy. The 180° phase shift at zero energy for the nuclei with $A \geq 8$ shown in figs. c)-e) is evidence of a bound state. These states lie in the continuum, approximately where η is a minimum, and arise from the strong absorption of the potential.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2a

Fig. 2b

Fig. 2c

Fig. 2d

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5

Fig. 6

Fig. 7a

Fig. 7b

Fig. 7c

Fig. 7d

Fig. 7e