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OFF SHELL ONE-LOOP DIVERGENCES IN GAUGED $\phi(N)$
SUPERGRAVITIES

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ABSTRACT

We present the expression for the off shell one-loop infinities in gauged $O(N)$ supergravities thus explaining the agreement of two previous calculations of the gauge coupling β -function. The off shell gravitino contribution in the Weyl tensor squared divergence is found to have negative sign leading to the one-loop off shell finiteness of the $N=8$ and $N=1$, $d=10 \rightarrow 4$ supergravities in the gravitational 2^+ -sector.

1. INTRODUCTION

Ungauged (gauged) supergravities are known to be one-loop finite (renormalizable) on the mass shell (see e.g. [1]). Their off shell infinities are generally believed to be of no interest being dependent on the choice of the gauge. For example, the off shell effective action lack supersymmetry if the standart generally covariant background gauges (i.e. $\delta_\mu \psi = \bar{\psi}^{(\mu)}$, $\delta_\mu (h_{\nu\rho} - \frac{1}{2} g_{\nu\rho} h) = \eta_{\nu\rho}^{(\mu)}$, $\delta_\mu A_\rho = F_\rho^{(\mu)}$) are used (cf. [2]): these gauges cannot be considered as part of one supergauge and (ii) separately do not respect the background supersymmetry (e.g. do not contain the background gravitino). However, one may formally use these gauges (keeping in mind the possibility to maintain the off shell background supersymmetry in the proper superfield approach [3]) under the assumptions that there exist the off shell supersymmetrical extensions of the $W = R_{\mu\nu}^2 - \frac{1}{2} R^2$ and R^2 invariants for all $N \leq 8$ (cf. 4) and that all non-invariant counter-terms vanish on the field equations.

Our motivation for studying the off shell one-loop counter-terms in gauged $O(N)$ supergravities [5,6] was mainly to provide understanding of the agreement of two different calculations [7,8] of the gauge coupling β -function (and also to gain some indirect information on the off shell structure of extended theories). The analysis of ref. [7] was done on the mass shell in the gravitational sector ($\beta(g)$ was established using the relation $\Lambda = -12 g^2/\kappa^2$). This is justified in view of the known on shell finiteness in the limit $g \rightarrow 0$. In a more heuristic calculation of ref. [8] done in the gauge

field sector 8 was obtained by summing the $F_{\mu\nu}^2$ -infinities from all the fields in the multiplet. Several questions can be raised concerning this latter approach: (a) configurations $A_\mu \neq 0$, $g_{\mu\nu} = g_{\mu\nu}$, $\psi, \dots = 0$ lie off the mass shell when $\Lambda \neq 0$, where there are possible F^2 and RF^2 -infinities (the latter may contribute in β after going on shell);

(b) the formula

$$\beta_0(s) = -4 \left(\frac{1}{12} - s^2 \right) (-1)^{2s} C \quad (1)$$

for the contributions of massless spin s fields in the $F_{\mu\nu}^2$ -infinities (which can be easily proved for $s = 0, 1/2, 1$ [9]) was only assumed in [8] to be correct for $s = 3/2$;

(c) it should be proved that one can simply sum independent "minimal" contributions $\beta_0(s)$ (possible mixings of the gravitino "mass" term and non-minimal couplings may also contribute in β). It was pointed out recently [10] that the one-loop RF^2 -infinities are absent due to the chiral-dual invariance valid in the limit $g \rightarrow 0$. Note also ^{that} Eq.(1) was obtained in [1] for all s using the formalism of arbitrary spin second order operators. However, this derivation is not fully convincing being not based on the concrete gravitino lagrangian. In this paper we present the complete treatment of the problem of establishing the one-loop counter-terms in gauged supergravities starting with the known expressions for the lagrangians [5,6]. We prove that the validity of (1) for $s = 3/2$ is the result of non-trivial mixing between the "mass" $\bar{\psi}\psi$, "non-minimal" $\bar{\psi}\psi$ and also "minimal" terms for gravitino. We also calculate the off shell gravi-

tational infinities due to gravitino and find that its contribution in the W -infinities β_2 is negative while all lower spins are known to give positive ones in the standart background gauges ^{1/}. This opens the possibility to get $\beta_2 = 0$ in an appropriate model. Interestingly enough, we found that β_2 is really zero in $N = 8$ supergravity and also in the theory with the spectrum, arising after the dimensional reduction of the $N = 1, d = 10$ supergravity [12] (e.g. in the $SU(4)$ supergravity plus six $N = 4$ vector multiplets). In spite of the dependance of β_2 on the choice of the gauge these zero results for two most interesting supergravity theories (closely connected with the supersymmetric string model [13] and having zero trace anomalies [14]) seem not to be accidental.

2. $\alpha(2)$ SUPERGRAVITY.

In order to establish the gravitino contribution to the one-loop off shell infinities which depend only on the gravitational and gauge field backgrounds it is sufficient to consider the $\alpha(2)$ -model [5] with the relevant part of the lagrangian given by

$$\begin{aligned} \mathcal{L}_I = & -\frac{1}{\kappa^2} (R - 2\Lambda) + \frac{1}{4g^2} F_{\mu\nu}^2 + \\ & + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu^i \gamma_5 \gamma_\rho \partial_\sigma \psi_\nu^i + m \bar{\psi}_\mu^i \epsilon_{\mu\nu} \psi_\nu^i + \\ & + \frac{1}{2} m^{-1} \bar{\psi}_\mu^i F_{\mu\nu}^{ij} \psi_\nu^j \end{aligned} \quad (2)$$

1) Note that the often mentioned statement about $\beta_2 > 0$ for any spin cannot be actually proved in the background gravitino gauge. Let us remind that $\beta_2 < 0$ is valid also for the conformal supergravity gravitino [11].

2) We use the euclidean notations and also $R_{\mu\nu} = \partial_\lambda \Gamma_{\mu\nu}^\lambda - \dots$
(continued on p. 4)

where $K^2 = 16\pi G$, $m = 2g/k$, $\Lambda = 3m^2$, $\zeta\delta = 1,2$

and

$$F_{\mu\nu}^{\zeta\delta} = E^{\zeta\delta} F_{\mu\nu} , \quad F_{\mu\nu}^{\pm} = F_{\mu\nu} \pm \delta_5 F_{\mu\nu}^* \quad (3)$$

$$(D_{\mu})_{\lambda\rho}^{\zeta\delta} = g_{\lambda\rho} (\delta^{\zeta\delta} \partial_{\mu} + E^{\zeta\delta} A_{\mu} + \frac{1}{2} \epsilon^{\alpha\beta} \omega_{\mu}^{\alpha\beta} \epsilon_{\alpha\beta}^{\zeta\delta} \delta^{\zeta\delta}) + \delta^{\zeta\delta} F_{\mu}^{\lambda\rho}$$

The complete $O(2)$ theory is invariant under the supersymmetry transformations, including

$$\delta\psi_{\mu}^{\zeta\delta} = D_{\mu} \epsilon^{\zeta\delta} + \frac{1}{2} m \delta_{\mu}^{\zeta\delta} \epsilon^{\zeta\delta} - \frac{1}{2} m^{-1} F^{\zeta\delta} \epsilon_{\mu}^{\zeta\delta} \delta^{\zeta\delta} + \dots \quad (4)$$

Choosing the standart background gauge $\delta_{\mu}^{\zeta\delta} \psi_{\mu}^{\zeta\delta} = \delta^{\zeta\delta} \psi_{\mu}^{\zeta\delta}$

we get the ghost operator in the form

$$\Delta_{gh}^{\zeta\delta} = \hat{D}^{\zeta\delta} + 2m \delta^{\zeta\delta} . \quad (5)$$

"Averaging over gauges" with the help of the operator Δ_{gh} [15] (which is here convenient to take equal to (5)) results in following gravitino contribution in the one-loop partition function

$$\bar{Z}_{\psi}^{(1)} = (\det \Delta_{1/2})^{1/2} (\det \Delta_{gh})^{-1} (\det \Delta_{\alpha\nu})^{-1/2} = (\det \Delta_{1/2})^{1/2} (\det \Delta_{gh})^{-1/2} \quad (6)$$

$$\Delta_{1/2}^{\zeta\delta} = \sum_{\mu\rho} (D_{\mu})_{\lambda\rho}^{\zeta\delta} - m \delta^{\zeta\delta} g_{\mu\nu} + m^{-1} F^{\zeta\delta} F_{\mu\nu}^{\zeta\delta} \quad (7)$$

k2 contr.] $\delta_5 = \frac{1}{4!} \epsilon_{\mu\nu\lambda\rho} \partial^{\mu} \gamma^{\nu} \delta^{\lambda} \partial^{\rho}$, $\epsilon_{\mu\nu}^{\zeta\delta} = \frac{1}{2} \delta_{\mu}^{\zeta\delta} \delta_{\nu}^{\zeta\delta}$

$$F_{\mu\nu}^* = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\lambda\rho} , \quad F \cdot \epsilon \equiv F_{\mu\nu} \epsilon_{\mu\nu}^{\zeta\delta} \text{ etc.}$$

where $\Sigma_{\mu\rho}^{\lambda} = \frac{1}{2} \delta_{\rho}^{\lambda} \delta^{\mu}_{\mu}$, $\{\Sigma^{\lambda}, \Sigma^{\rho}\}_{\mu\nu} = 2g^{\lambda\rho} g_{\mu\nu}$.
 Our background field method quantization of gravitino though similar to the approaches of refs. [16] is different in that we do not assume the background fields to satisfy the classical field equations. ³⁾ This leads to gauge dependent terms in the effective action because the gravitino dependent part of (2) is invariant under (4) only on the mass shell.

To obtain the divergences of (6) we shall use the procedure of "squaring" of first order operators: one is to multiply

$\Delta = \Sigma^{\rho} \partial_{\rho} + Y$ by some auxiliary operator $\tilde{\Delta} = \Sigma^{\rho} \partial_{\rho} + \tilde{Y}$ such that there is no linear in ∂_{ρ} terms in the product, i.e. $\Sigma^{\rho} \tilde{Y} + Y \Sigma^{\rho} = 0$. Then one can use the well-known algorithm [18] for the second order operators (see also [19])

$$\Delta = -\partial^2(B) + X$$

$$(\log \det \Delta)_{\infty} = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left\{ \bar{b}_0 L^4 - \frac{1}{2} \bar{b}_2 L^2 - \bar{b}_4 \log \frac{L^2}{\mu^2} \right\}, \quad (8)$$

where $L \rightarrow \infty$, $\bar{b}_0 = \text{tr } 1$, $\bar{b}_2 = \text{tr}(\frac{R}{6} 1 - X)$, $\bar{b}_4 = \text{tr}(\frac{1}{2} X^2 \dots)$.

Taking into account that all divergent diagrams are proportional to $\text{tr}(\Sigma Y)^p$ we conclude that $(\det \Delta)_{\infty}$ and $(\det \tilde{\Delta})_{\infty}$ coincide, i.e. $\bar{b}_p(\Delta) = \frac{1}{2} \bar{b}_p(\Delta \cdot \tilde{\Delta})$, $p=0, 2, 4$.

In the case of (7) we get

$$\tilde{\Delta}_{\lambda/2 \mu\nu}^{ij} = \Sigma_{\mu\rho}^{\lambda} (\partial_{\lambda})_{\rho\nu}^{ij} + m \delta_{\mu\nu}^{ij} g_{\lambda\nu} - m^{-1} (F^{ij} \partial) g_{\mu\nu}, \quad (9)$$

³⁾ Note that the four-ghost vertices necessary in the complete treatment [17] can be ignored in one-loop approximation.

$$\begin{aligned} \Delta'_{\frac{1}{2}\mu\nu}{}^{\dot{\nu}} = & -(\Delta_{\frac{1}{2}} \cdot \tilde{\Delta}_{\frac{1}{2}})_{\mu\nu}{}^{\dot{\nu}} = -\partial^2_{\mu\nu}{}^{\dot{\nu}} + \delta^{\dot{\nu}}_{\mu\nu} m^2 + \\ & + \delta^{\dot{\nu}}_{\mu\nu} (R_{\mu\nu} - \epsilon_{\mu\rho} R^{\rho}_{\nu} + 2 R_{\rho[\mu} \epsilon_{\nu]\rho} - \frac{1}{2} \epsilon_{\mu\nu} R) - \\ & - 2 F_{\mu\nu}{}^{\dot{\nu}} - \epsilon \cdot F^{\dot{\nu}}_{\mu\nu} + m^{-2} F^{\mu\kappa}_{\nu} (\epsilon \cdot F^{\dot{\nu}}_{\kappa}) + \\ & + m^{-1} \sum_{\mu\nu} \partial_{\mu} (\epsilon \cdot F^{\dot{\nu}}_{\nu}) \end{aligned} \quad (10)$$

According to (6) $\bar{\epsilon}_r{}^{(\dot{\nu})} = -\frac{1}{2} [\bar{\epsilon}_r (\Delta'_{\frac{1}{2}}) - 3 \bar{\epsilon}_r (\Delta'_{gh})]$, where

$$\Delta'_{gh}{}^{\dot{\nu}} = -(\Delta_{gh} \cdot \tilde{\Delta}_{gh})^{\dot{\nu}} = -\partial^2{}^{\dot{\nu}} + (\frac{R}{\gamma} + 4m^2) \delta^{\dot{\nu}} - \epsilon \cdot F^{\dot{\nu}} \quad (11)$$

To present the results we shall use the following notations

(cf. [19])

$$\bar{\epsilon}_0 = \gamma, \quad \bar{\epsilon}_2 = \rho_1 R + \rho_2 \Lambda + \rho_0 \frac{\kappa^2}{g^2} F_{\mu\nu}^2, \quad (12)$$

$$\begin{aligned} \bar{\epsilon}_v = & \beta_1 R \bar{R}^* + \beta_2 W + \frac{1}{3} \beta_3 R^2 + \beta_4 R \Lambda + \beta_5 \Lambda^2 + \\ & + \beta_6 \partial^2 R + \gamma_1 (\kappa^2/g^2) R_{\mu\nu} T_{\mu\nu} + \gamma_2 (\kappa^2/g^4) T_{\mu\nu}^2 + \\ & + \gamma_3 (\kappa^2/g^2) (\partial_{\mu} F_{\nu})^2 + \beta_0 F_{\mu\nu}^2, \end{aligned} \quad (13)$$

$$T_{\mu\nu} = F_{\mu\lambda} F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F^2.$$

Then γ, ρ_i, β_n are given by the gravitino values in Table 1 multiplied by two (there are two gravitinos in $O(2)$ theory) while

$$\rho_0 = 0, \quad \gamma_1 = 0, \quad \gamma_2 = -\frac{1}{2}, \quad \gamma_3 = 0. \quad (14)$$

Our off shell results for $S = 3/2$ coincide on shell

($R_{\mu\nu} = \Lambda g_{\mu\nu} + \dots$) with the previously known for the trace anomaly [16] and the Λ -term renormalization [7]: $(\bar{\epsilon}_v)_{\text{on shell}} = \beta_1 R \bar{R}^* + \epsilon_{\frac{1}{2}} \Lambda^2$, $\epsilon_{\frac{1}{2}} = \frac{137}{45}$. We also automatically obtained the same value of the gravitino contribution to the gauge

field β -function as predicted by (1) (β_0 values of Table 1 are to be multiplied by the corresponding Casimir operators, e.g. $C = 2$ for $N = 2$ gravitinos). It should be stressed that the part $\Delta_{\mu\nu}^{ij} = -2^2 \delta_{\mu\nu}^{ij} - 2F_{\mu\nu}^{ij} - 6F_{\mu\nu}^{ij} g_{\mu\nu}$ of (10) leading to this value appears due to a non-trivial mixing of the "mass" and non-minimal terms in the product (10) (i.e. there are infinite diagrams with two " $m^{-1}F$ " and "two m " vertices) as well as due to the "minimal" interaction $(-\xi^{\mu\nu}\partial_\mu)^2 \delta_{\mu\nu}^{ij} = -2^2 \delta_{\mu\nu}^{ij} - F_{\mu\nu}^{ij}$.

The total result for the infinities in the $\mathcal{O}(2)$ theory can be established by summing the gravitino contribution with that of the Einstein-Maxwell theory given by (12) and (13) where γ , ρ_i , β_n are obtained by adding spin 2 and spin 1 values of Table 1, while

$$\rho_0 = 0, \quad \delta_1 = -\frac{1}{3}, \quad \delta_2 = \frac{13}{12}, \quad \delta_3 = \frac{1}{3}. \quad (15)$$

Let us remark that the $\Lambda \neq 0$ spin 2 values of Table 1 (see also [19]) are the off shell extension of the result of [20] $(\bar{p}_\nu)_{\text{on shell}} = \frac{1}{2}\Lambda^2 + \dots, \bar{q}_\mu = -\frac{58}{3}$. The δ_i 's in (15) were first presented in [21]. The interesting new result is the absence of the $F_{\mu\nu}^2 \cdot L^2$ infinities in the Einstein-Maxwell theory ($\rho_0 = 0$) being the outcome of a non-trivial cancellation (it can probably be understood using the duality invariance considerations and is in direct correspondence with the absence of the $F_{\mu\nu}^2 \log L^2$ infinities in the system of higher derivative gravity and the gauge field [19]). The final expression for the one-loop off-

shell infinities in the gauged $O(2)$ supergravity can be written in the form

$$\bar{\mathcal{L}}_0 = 0, \quad \bar{\mathcal{L}}_2 = \alpha_1 (R - 4\Lambda) + \alpha_2 \Lambda + 4\kappa^2 \beta_0 \left[-\frac{1}{\kappa^2} (R - 2\Lambda) + \frac{1}{4g^2} F_{\mu\nu}^2 \right], \quad (16)$$

$$\begin{aligned} \bar{\mathcal{L}}_4 = & \beta_1 R^* R^* + \beta_2 \left[(R_{\mu\nu} - \Lambda g_{\mu\nu} - \frac{\kappa^2}{2g^2} T_{\mu\nu})^2 - \frac{1}{3} (R - 4\Lambda)^2 \right] + \\ & + \frac{1}{3} \beta_3 (R - 4\Lambda)^2 + \beta_4 \partial^2 R + \delta_3 (\kappa^2/g^2) (\partial_\mu F_{\mu\nu})^2 + \\ & + \alpha_2 (g^2/\kappa^2) (R - 4\Lambda) + \alpha_2 (\kappa^2/g^2) T_{\mu\nu} (R_{\mu\nu} - \Lambda g_{\mu\nu} - \frac{\kappa^2}{2g^2} T_{\mu\nu}) + \\ & + 4g^2 \beta_0 \left[-\frac{1}{\kappa^2} (R - 2\Lambda) + \frac{1}{4g^2} F_{\mu\nu}^2 \right], \end{aligned} \quad (17)$$

where all coefficients are given in Table 2. We separated terms linear in the field equation (which would be absent if the gauges used were parts of one supergauge) and terms which can be extended to superinvariants. One could also include the $(\partial F)^2$ -terms in W and R^2 -extensions (cf. [22]) but this does not lead to complete elimination of the δ_3 -term. An apparent puzzle is $\alpha_1 \neq 0$ in (16) probably indicating a breakdown of the shell supersymmetry in the $O(2)$ theory by a regularization, taking into account quadratic divergences (it turns, however, that $\alpha_1 = 0$ in all $N \geq 3$ theories).

As an obvious consequence of (17) we conclude that (i) the theory is renormalisable on the mass shell

$$R_{\mu\nu} = \Lambda g_{\mu\nu} + \frac{\kappa^2}{2g^2} T_{\mu\nu}, \quad \partial_\mu F_{\mu\nu} = 0 \quad (18)$$

(ii) RF^2 -infinities are absent in the standard gauges, and (iii) the on shell renormalisations of Λ and g^2 are given

by the same coefficient β_0 . This clearly explains the agreement of the results of refs. [7] and [8] in the case of the $O(2)$ theory.

3. $N > 2$ GAUGED SUPERGRAVITIES

Let us begin with the $O(3)$ theory [5] with the lagrangian containing (along with the analog of (2)) also the spinor field terms

$$\frac{1}{2} \bar{\chi} \hat{D} \chi - \frac{1}{\sqrt{2}} m^{-1} \bar{\psi}_i (F^i{}_{\alpha\beta}) \gamma_{\alpha\beta} \chi \quad (19)$$

(here $i, j = 1, 2, 3$, $F^i{}_{\mu\nu} = \epsilon^{ijk} F_{\mu\nu}^k$). One can prove that the contribution of χ in the $F_{\mu\nu}^2$ -infinities is exhausted by the "minimal" one, i.e. that there is no "mixing" of the gravitino "mass" term and the non-minimal term in (19) (though the diagram with two " $m^{-1} F^i{}_{\alpha\beta}$ " and two " χ " vertices is formally divergent, its contribution is zero). This turns to be valid for all $N \geq 3$ theories. Starting with $N \geq 5$ there is a new non-minimal term $\bar{\chi} (G F) \chi$, which again provides only additional $T_{\mu\nu} R_{\mu\nu}$, $T_{\mu\nu}^2$, $(\partial_\mu F_{\mu\nu})^2$ -infinities in view of the absence of spinor mass terms in gauged supergravities [6]. The mass $m_0^2 = \frac{2}{3} \Lambda$ is present for the scalar fields [6] and it does "mix" with non-minimal $\phi^2 F_{\mu\nu}^2$ -terms but the $F_{\mu\nu}^2$ -infinities going from scalars and pseudoscalars mutually cancel (this cancellation occurs also for the $F_{\mu\nu}^2 L^2$ and $(\partial F)^2 \log L^2$ infinities). As a result, we can justify the possibility (implicitly assumed in [8]) to obtain the total gauge field

β -function by simply summing the contributions of gravitinos and the "minimal" values of β_0 for spins 1, 1/2 and 0, as given in Table 1.

The off shell infinities for the theories with $N \geq 3$ are given by (16), (17) (with $F_{\mu\nu} \rightarrow F_{\mu\nu}^a$, $a = 1, \dots, N$) and Table 2. Let us first comment on derivation of these results. All gravitational divergences are established using Table 1 (where spinors are assumed to be Majorana and scalars have the kinetic term $\frac{1}{2} \phi (-\partial^2 + \zeta R + m_0^2) \phi$) where we assumed that $m_0^2 = \frac{2}{3} \Lambda$, $\zeta = 0$ as apparently takes place in all $N \geq 4$ gauged supergravities.

To work out the spinor and scalar fields contributions to δ_1 and δ_2 in (13) one may extend the on shell diagram calculations of ref. [23] off the mass shell. ⁴⁾ The results coincide on shell with those of [23] (for $\Lambda = 0$). The evaluation of δ_3 presents a separate problem; explicit diagram calculations give the following results: the non-minimal $\bar{\psi} F \chi$ - terms contribution is $\frac{1}{6} (N-3)$, $N \geq 3$; the non-minimal $\bar{\chi} F \chi$ - contribution is $\frac{1}{12} (3N-14)$, $N \geq 5$; the contributions of scalars and pseudoscalars cancel each other. The analysis of diagrams also shows that there is no $L^2 F_{\mu\nu}^2$ infinities for all $N \geq 3$, i.e. $\rho_0 = 0$.

⁴⁾ Let us remind that the background non-covariant gauge $(\mathcal{L}_g = (\bar{\psi}_\mu \gamma_\mu) \partial (\gamma_\nu \psi_\nu))$ was used in [23]. The effective $(\Lambda = 0)$ gravitino contribution in $\bar{\psi}$ in our gauge can be obtained from that of [23] by adding the term $-\frac{5}{3} R_{\mu\nu} (R_{\mu\nu} - \frac{1}{2} g^2 T_{\mu\nu})$ vanishing on shell.

in (16). Thus there is no on shell quadratic divergences
in all ^{the} theories with $N \geq 3$.

The values of β_0 in Table II coincide with those of [7,8] while the values of β_1 (the trace anomaly coefficient) are calculated under the assumption that the field content of the theory is obtained by reduction of the $N = 1$, $d = 11$ supergravity [11, 1]. The last column of Table II corresponds to the theory obtained by reduction of the $N=1$, $d=10$ supergravity [11]. This theory (as well as the $N=8$ theory) is distinguished by having zero β_2 - coefficient, i.e. by being off shell finite in the gravitational $\mathcal{L}^2$ -sector. It seems that this result cannot be explained (in contrast with zeroes of β_0 and β_1) by the use of sum rules, cf. [8, 13]. It is also interesting to note that the $N = 8$ theory has a maximal degree of the off shell finiteness: $\beta_0 = \beta_1 = \beta_2 = 0$. One may even conjecture that it is completely off shell finite when treated in a suitable supergauge (where $\alpha_1 = \alpha_2 = 0$) with a preservation of the background supersymmetry (it may turn out that in this gauge also $\beta_3 = \beta_4 = 0$ if the $N = 8$ extension of R^2 does not exist). As a final remark let us note that the numbers of Table II may be of some help in attempts to understand the off shell structure of the extended supergravities. ⁵⁾

5) observe also the relation $\beta_2 = 2(\beta_3 - 1)$ which holds for $3 \leq N \leq 8$.

Table 1
Gaug and gravitational infinities for different
spins (see eq. (12), (13))

S	2	3/2	1	1/2	0
φ	2	-2	2	-2	1
B_0	-	-13/3	11/3	-2/3	-1/6
B_1	53/45	-233/720	-13/180	7/720	1/180
B_2	7/10	-77/60	7/5	1/20	1/60
B_3	3/4	-7/3	-	-	$1/24(1-6\gamma)^2$
B_4	-26/3	-4/3	-	-	$-\frac{1}{6}m_0^2\Lambda^{-1}(1-6\gamma)$
B_5	20	44/9	-	-	$(1/2)m_0^4\Lambda^{-2}$
B_6	-19/15	1/60	-2/10	+1/60	$+1/30(1-5\gamma)$
ρ_1	-23/3	1/6	-2/3	1/6	$1/6(1-6\gamma)$
ρ_2	20	16/3	-	-	$-m_0^2\Lambda^{-1}$

Table 2. One-loop cfr shell infinities in gauged $O(N)$ supergravities
(see eq. (16), (17))

N	0	1	2	3	4	5	6	7	$\frac{N-1}{2}$ $\frac{N-1}{2} \rightarrow 1$ $\frac{N-1}{2}$
H_1	-	-	1	3	6	10	15	20	12
$H_{3/2}$	-	-	-	1	4	11	25	56	25
H_0	-	-	-	-	2	10	30	70	30
C_0	-	-	-26/3	-5	-2	0	0	0	-
B_1	53/45	41/18	11/24	0	0	0	0	0	0
B_2	7/10	-7/12	-5/3	-5/2	-3	-3	-2	1	0
B_3	3/5	5/12	1/12	-1/4	-1/2	-1/2	0	1	13/12
B_5	-19/15	15/4	-4/3	-3/2	-21/15	-1/3	11/3	25/3	22/5
B_6	-	-	1/3	1/2	2/3	11/12	4/3	13/6	-
$\tilde{H}_1$	-	40	50	33	114	146	190	173	-
$\tilde{H}_2$	-	-	-2	-5/3	-17/6	-7/2	-5	-5	-3
$\tilde{H}_3$	-23/3	-15/2	-8	-9	-10	-10	-3	-4	-31/3
$\tilde{H}_5$	-32/3	-16/3	-2/3	0	0	0	0	0	0
$\tilde{H}_6$	-	-	0	0	0	0	0	0	-

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