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CONSISTENCY OF MODELS WITH CP VIOLATION PREDOMINANTLY
IN THE $\Delta S = 1$ HAMILTONIAN

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A B S T R A C T

It is emphasized that models with small or vanishing CP violation in the $\Delta S = 2$ "superweak" sector can be compatible with experiment provided there is "direct" $\Delta S = 1$ CP violation and a "soft" contribution to ΔM_K . Such models must possess the following special features: 1) ϵ'/ϵ negative and close to the experimental lower bound of -0.018 ; 2) a large negative "soft" contribution to ΔM_K such that $\text{Re} M_{12}^{\text{soft}} \leq -1.7 \text{Re} M_{12}^{\text{exp}}$, consistent with some current algebra estimates. The Higgs-boson-exchange models of Lee and Weinberg are examples of this class of model. The standard KM model can be another.

There are in general two possible sources which contribute to the observed non-invariance of CP in the neutral kaon system ¹⁾ : 1) "direct" CP violation in the $\Delta S=1$ Hamiltonian responsible for $K \rightarrow 2\pi$ decay, and 2) a "superweak" contribution which is purely $\Delta S=2$, contributing to CP violation in the $K^0 - \bar{K}^0$ transition amplitude. Direct CP violation in $K \rightarrow 2\pi$ can give rise to different phases for the weak amplitudes for $K \rightarrow 2\pi$ ($I=0$) and $K \rightarrow 2\pi$ ($I=2$) decays, and is parametrized by the quantity ϵ' where

$$\epsilon' = \sqrt{\frac{1}{2}} e^{i(\pi/2 + \delta_2 - \delta_0)} \frac{\text{Im } A_2}{A_0} \quad (1)$$

in the standard phase convention where A_0 is chosen to be real and positive ²⁾. Superweak CP violation cannot produce different phases for A_0 and A_2 , but contributes to the CP impurity parameter ϵ which measures the departure of the mass eigenstates K_L, K_S from CP eigenstates :

$$K_{L,S} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon) K^0 \pm (1-\epsilon) \bar{K}^0 \right] \quad (2)$$

where

$$\epsilon = \frac{\frac{1}{2} \text{Im } \Gamma_{12} + i \text{Im } M_{12}}{\Delta M - \frac{i}{2} \Delta \Gamma} \quad (3)$$

and M_{12} and Γ_{12} are the off-diagonal elements of a phenomenological Hamiltonian in the flavour eigenbasis :

$$\begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} . \quad (4)$$

In the standard convention where A_0 is real, $\text{Im } \Gamma_{12}$ is negligible [resulting from the fact that the 2π ($I=0$) contribution dominates the width difference] ³⁾ which together with the approximate relation $\Delta M = \frac{1}{2} \Delta \Gamma$ allows one to write

$$\epsilon = \frac{e^{i\pi/4}}{2\sqrt{2}} \left(\frac{\text{Im } M_{12}}{\text{Re } M_{12}} \right)^{A_0^R} \equiv \frac{e^{i\pi/4}}{2\sqrt{2}} \epsilon_m^{A_0^R} \quad (5)$$

where the superscript A_0^R is intended to emphasize that the quantities are to be evaluated in the convention where A_0 is real and positive.

The experimental values for δ_0 and δ_2 ⁴⁾ [Eq. (1)] place ϵ' and ϵ nearly parallel (or antiparallel) in the complex plane, and their relative magnitude can be taken from Eqs. (1) and (5) :

$$\frac{\epsilon'}{\epsilon} = \frac{2}{\epsilon_m^{A_0^R}} \frac{\text{Im} A_2}{A_0} . \quad (6)$$

ϵ' and ϵ are determined from the experimental quantities η_{+-} and η_{00} , where

$$\eta_{+-} \equiv \frac{\langle \pi^+ \pi^- | H_w | K_L \rangle}{\langle \pi^+ \pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0 \pi^0 | H_w | K_L \rangle}{\langle \pi^0 \pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon' \quad (7)$$

The experimental ratio $\eta_{00}/\eta_{+-} = 1.008 \pm 0.041$ puts an upper bound on the magnitude of ϵ'/ϵ ²⁾:

$$\frac{\epsilon'}{\epsilon} = -0.003 \pm 0.015 . \quad (8)$$

In general, the conventional choice of quark phases in which the weak couplings to light quarks are real is not consistent with the convention in which A_0 is real, unless the $\Delta S=1$ Hamiltonian responsible for $K \rightarrow 2\pi$ ($I=0$) decay receives no CP violating contribution from loop diagrams. We know, however, that in the Kobayashi-Maskawa (KM)⁵⁾ model as well as in Higgs exchange models of CP violation such as those of Lee⁶⁾ and Weinberg⁷⁾, "penguin" diagrams generate an effective $\Delta S=1$ interaction which is CP violating^{8),9)} (Fig.1). To the extent that such "direct" CP violation is confined to penguin-type operators, it is purely $\Delta I = \frac{1}{2}$ and contributes only to A_0 :

$$A_o^Q = e^{i\xi} |A_o^Q| \quad (9)$$

where here the superscript Q is used to emphasize a quark convention in which A_o is complex. In general there could be CP violating contributions to $K \rightarrow 2\pi$ ($I=2$) decay as well, in which case

$$A_2^Q = e^{i\eta} |A_2^Q|. \quad (10)$$

The standard phase convention in which A_o is real is restored by letting $|K^0\rangle \rightarrow e^{-i\xi} |K^0\rangle$, $|\bar{K}^0\rangle \rightarrow e^{i\xi} |\bar{K}^0\rangle$ under which ⁸⁾

$$\begin{aligned} A_o^Q &\rightarrow A_o^{A_o^R} = e^{-i\xi} A_o^Q \\ A_2^Q &\rightarrow A_2^{A_o^R} = e^{-i\xi} A_2^Q \end{aligned} \quad (11)$$

and

$$\epsilon_m^Q \rightarrow \epsilon_m^{A_o^R} = e^{2i\xi} \epsilon_m^Q. \quad (12)$$

Using the experimental value for $|A_2/A_o| = 1/20$ and working to first order in CP violating quantities, Eq. (6) for ϵ'/ϵ becomes

$$\frac{\epsilon'}{\epsilon} = \frac{1}{20} \frac{2(\eta - \xi)}{\epsilon_m^{A_o^R}} = \frac{1}{20} \frac{2(\eta - \xi)}{\epsilon_m^Q + 2\xi}. \quad (13)$$

For simplicity, we consider the interesting and relevant case where the CP violating contribution to the $\Delta S=1$ Hamiltonian is predominantly $\Delta I = \frac{1}{2}$, so that η is negligible. Then Eqs. (8) and (13) imply

$$\frac{\xi}{\epsilon_m^{A_o^R}} = 0.03 \pm 0.15 \quad (14.a)$$

or equivalently

$$\frac{\xi}{\epsilon_m^Q} = 0.03 \begin{matrix} +0.25 \\ -0.13 \end{matrix}. \quad (14.b)$$

To explore the consequences of this familiar constraint, we divide the contributions to M_{12} into two pieces. The first comprises the contribution to M_{12} from an (effective) short-distance $\Delta S = 2$ interaction generated, for example, in the standard model by the $2W^\pm$ exchange "box" diagrams involving heavy quark loops ¹⁰⁾, and in the Lee-Weinberg models by similar graphs plus diagrams involving one or more charged Higgs scalars (Fig.2) ⁹⁾ :

$$M_{12}^{sd} \equiv \frac{1}{2m_K} \langle K^0 | H^{\Delta S=2} | \bar{K}^0 \rangle. \quad (15)$$

The second piece is the contribution from the dispersive part of

$$\int d^4x \langle K^0 | T \{ H^{\Delta S=1}(x) H^{\Delta S=1}(0) \} | \bar{K}^0 \rangle$$

which is dominated by low-lying intermediate states ($\pi^0, \eta, 2\pi, \dots$), the so-called "large-distance" or "soft" contribution :

$$M_{12}^{soft} = \frac{1}{2m_K} \int_{\substack{\text{dispersive} \\ \pi^0, \eta, 2\pi, \dots}} d^4x \langle K^0 | T \{ H^{\Delta S=1}(x) H^{\Delta S=1}(0) \} | \bar{K}^0 \rangle. \quad (16)$$

It has long been known that there are contributions to M_{12} from low-lying intermediate states which are of the order of ΔM_K itself ¹¹⁾, and the sensitivity of these contributions to the precise values of m_π , etc., demonstrates that they are not accounted for with any accuracy whatsoever by M_{12}^{sd} . Therefore the separation of M_{12} into a short-distance piece (due to an effectively local $\Delta S = 2$ interaction) and a "soft" contribution seems natural ^{12), 13)}.

Using this notation one can write

$$\epsilon_m^{A_0^R} \equiv \left(\frac{\text{Im } M_{12}}{\text{Re } M_{12}} \right)^{A_0^R} = \frac{(\text{Im } M_{12}^{sd} + \text{Im } M_{12}^{soft})^{A_0^R}}{\text{Re } M_{12}^{sd} + \text{Re } M_{12}^{soft} = \frac{1}{2} \Delta M_K^{exp}} \quad (17)$$

where $\text{Re } M_{12}$ is independent of phase convention (to lowest order in CP violating quantities) and has been set equal to its experimental value of $\frac{1}{2} \Delta M_K$. One can argue, however, that M_{12}^{soft} ought to be nearly real in the convention where A_0 is real [to the extent it is dominated by 2π ($I=0$) and other intermediate states whose matrix elements $\langle i | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ can be derived from $\langle 2\pi(I=0) | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ by current algebra]. This is not meant to be an exact statement, but rather an assertion that the convention in which A_0 is real

provides a best guess at the convention which will minimize $\text{Im}M_{12}^{\text{soft}}$, and is certainly much better in this regard than the standard quark convention in which A_0 is complex¹³⁾. Neglecting $\text{Im}M_{12}^{\text{soft}A_0^R}$ and using Eq. (12) to write $\text{Im}M_{12}^{\text{sd}A_0^R} = \text{Im}M_{12}^{\text{sd}Q} + 2\xi \text{Re}M_{12}^{\text{sd}}$, we have

$$\epsilon_m^{A_0^R} = \frac{\text{Im}M_{12}^{\text{sd}Q}}{\frac{1}{2}\Delta M_K} + 2\xi \frac{\text{Re}M_{12}^{\text{sd}}}{\frac{1}{2}\Delta M_K} \quad (18.a)$$

or equivalently

$$\begin{aligned} \epsilon_m^Q &= \epsilon_m^{A_0^R} - 2\xi \\ &= \frac{\text{Im}M_{12}^{\text{sd}Q}}{\frac{1}{2}\Delta M_K} - 2\xi \frac{\text{Re}M_{12}^{\text{soft}}}{\frac{1}{2}\Delta M_K}. \end{aligned} \quad (18.b)$$

Because $\epsilon'/\epsilon \ll 1$, it is widely held that CP violation in neutral K decays requires a dominant contribution from an (effective) $\Delta S=2$ "superweak" interaction. In the light of Eq. (18) it becomes evident that this is not the case. Considering for example Eq. (18.b), one observes that even if the $\Delta S=2$ contribution $\text{Im}M_{12}^{\text{sd}Q}$ were small or vanishing, the experimental bound on ξ/ϵ_m^Q is sufficiently weak [Eq. (14.b)] that the second term proportional to ξ can account by itself for ϵ_m^Q provided $\text{Re}M_{12}^{\text{soft}} \leq -1.7 \text{Re}M_{12}^{\text{exp}}$. In the limit of a vanishing "superweak" contribution, Eq. (18) implies

$$\frac{\xi}{\epsilon_m^{A_0^R}} = \frac{1}{2} \frac{\text{Re}M_{12}^{\text{exp}}}{\text{Re}M_{12}^{\text{sd}}} \quad (19)$$

$$\frac{\xi}{\epsilon_m^Q} = \frac{-1}{2} \frac{\text{Re}M_{12}^{\text{exp}}}{\text{Re}M_{12}^{\text{soft}}}$$

which is not in conflict with Eq. (14) provided $\text{Re}M_{12}^{\text{soft}} \leq -1.7 \text{Re}M_{12}^{\text{exp}}$ ($\text{Re}M_{12}^{\text{sd}} \geq 2.7 \text{Re}M_{12}^{\text{exp}}$) [or $\text{Re}M_{12}^{\text{soft}} \geq 5.2 \text{Re}M_{12}^{\text{exp}}$ ($\text{Re}M_{12}^{\text{sd}} \leq -4.2 \text{Re}M_{12}^{\text{exp}}$) which is ruled out in the KM and the Lee-Weinberg models by the calculated sign of $\text{Re}M_{12}^{\text{sd}}$].

A "soft" contribution to the K_L-K_S mass difference $\text{Re}M_{12}^{\text{soft}} \leq -1.7$ $\text{Re}M_{12}^{\text{exp}}$ requires a larger positive contribution from $\text{Re}M_{12}^{\text{sd}}$, but is otherwise consistent with all known constraints. In fact, the current algebra estimate of Itzykson et al. ¹¹⁾ finds that the η plus π pole terms alone provide a contribution to ΔM_K equal to -1.4 times the experimental value. The associated requirement that $\text{Re}M_{12}^{\text{sd}} \geq 2.7 \text{Re}M_{12}^{\text{exp}}$ also poses no threat, but does require a somewhat larger contribution from the top quark in the $2W^\pm$ exchange "box" diagrams for models without significant additional contributions.

Thus we find that the experimental uncertainty in ϵ'/ϵ , together with the possibility of a non-negligible (and negative) large-distance contribution to ΔM_K , allows models of CP violation with little or no "superweak" contribution at all - models with CP violation predominantly in the $\Delta S=1$ Hamiltonian.

Examples of models with CP violation predominantly in the $\Delta S=1$ Hamiltonian can include both the Higgs-exchange models of Lee and Weinberg and the KM model. Deshpande and Sanda ⁹⁾ have independently argued that CP violation in the Higgs models is more effective in the $\Delta S=1$ Hamiltonian than in the $\Delta S=2$ sector, and conclude that ξ/ϵ_m^Q is too large in such models. Recently Dupont and Pham ¹⁴⁾ have estimated the additional "soft" contribution to ϵ_m^Q , and found that it can be large enough to reduce ξ/ϵ_m^Q to within experimental errors. Whereas this result may have appeared to be in obvious conflict with the "nearly superweak" pattern of CP violation in K^0 decays, we have shown that it is actually permissible in the light of Eq. (18.b), provided $|\text{Re}M_{12}^{\text{soft}}|$ is sufficiently large.

Dupont and Pham estimate $\epsilon'/\epsilon \sim -0.015$, which is consistent with the experimental lower bound but requires $\text{Re}M_{12}^{\text{soft}} = -2.2 \text{Re}M_{12}^{\text{exp}}$ by Eq. (19). The precise value for ϵ'/ϵ they quote is rough and depends upon pole dominance and the fractional importance of penguins in $\Delta I = \frac{1}{2}$ decays, so even an improvement on the experimental lower bound for ϵ'/ϵ would not appear to rule out the model. What does rule out the model as the experimental bound on ϵ'/ϵ is improved, together with all other models with CP violation predominantly in the $\Delta S=1$ Hamiltonian, is the general relation

$$\frac{\epsilon'}{\epsilon} = -\frac{1}{20} \frac{\text{Re} M_{12}^{\text{exp}}}{\text{Re} M_{12}^{\text{sd}} + \text{Im} M_{12}^{\text{sd}Q}/2} \quad (20)$$

which, when the superweak contribution $\text{Im}M_{12}^{\text{sdQ}}$ is negligible, requires an increasingly large contribution from $\text{Re}M_{12}^{\text{sd}}$ and an unlikely cancellation between $\text{Re}M_{12}^{\text{sd}}$ and $\text{Re}M_{12}^{\text{soft}}$ in order to reproduce the observed ΔM_K .

Finally, we comment that even the standard KM model can be a predominantly $\Delta S=1$ CP violating theory. CP violation in the low-energy effective $\Delta S=1$ Hamiltonian is proportional to the KM angles $s_1 s_2 c_2 s_3 \sin\delta$ ¹⁵⁾, whereas the superweak contribution $\text{Im}M_{12}^{\text{sd}}$ is proportional to the same combination times an additional coefficient which can vanish for $\cos\delta < 0$ and s_3 non-negligible ¹³⁾. In these regions where the superweak contribution is small or vanishing, the soft contribution to $\text{Im}M_{12}$ can itself account for the CP impurity parameter ϵ_m [Eq. (18)] and still be compatible with the ϵ'/ϵ constraint provided $\text{Re}M_{12}^{\text{soft}} \leq -1.7 \text{ Re}M_{12}^{\text{exp}}$.

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FIGURE CAPTIONS

- Figure 1 : "Penguin" diagrams responsible for direct CP violation in $K \rightarrow 2\pi$ decays in the KM model and Higgs-boson-exchange models, respectively.
- Figure 2 : Diagrams which contribute to an effective short-distance $\Delta S = 2$ interaction in the KM and Higgs-boson-exchange models of CP violation.

Fig . 1

Fig. 2