Near Forward p-p and pbar-p Elastic Scattering:
Slope Analysis of Existing Data

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ABSTRACT

An analysis of nuclear slope parameters b_{p-p} and b_{pbar-p} evaluated at low $|t|$ as a function of s , is given. We use $t=0$ amplitudes found in a previous communication [1], and analyse elastic scattering data in the low $|t|$ region for both p-p and pbar-p reactions, in terms of even and odd hadronic slope parameters, parametrized as $b_+ = C_+ + D_+ \ln(s) + E_+ \ln^2(s)$ and $b_- = C_- + D_- \ln(s)$. Extrapolations are made to collider energies.

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The near-forward hadronic amplitude of nucleon-nucleon (anti-nucleon) elastic scattering is reflected in 3 experimentally determined quantities, the total cross section σ_{tot} , the ratio ρ of the real to the imaginary part of the forward amplitude, and the nuclear slope parameter

$$b(s,t) = d[\ln(d\sigma/dt)]/dt, \quad (1)$$

where t is the squared 4-momentum transfer.

In the preceding communication^[1], we analysed the high energy data for $\sigma_{\text{tot}}(p-p)$, $\sigma_{\text{tot}}(p\bar{p}-p)$, $\rho(p-p)$ and $\rho(p\bar{p}-p)$ simultaneously, in order to extract the forward hadronic scattering amplitudes. It was useful to express the amplitudes in terms of an even amplitude M_+ and an odd amplitude M_- , such that

$$M_{\pm} = (M_{p\bar{p}-p} \pm M_{p-p})/2, \quad (2)$$

with the normalization

$$\sigma_{\text{tot}} = -\text{Im } M/s, \quad (3)$$

and a very satisfactory fit was obtained using

$$M_+ = -is[A+B(\ln(s/s_0)-i\pi/2)^2], \quad (4)$$

and

$$M_- = Ds^{\alpha} e^{i\pi(1-\alpha)/2}, \quad (5)$$

where $A = 41.8$ mb, $B = 0.68$ mb, $s_0 = 343$ (GeV)², $D = -39$ mbGeV^{2(1-\alpha)} and $\alpha = 0.48$, with s in (GeV)² and σ_{tot} in mb.

In this communication, we use these results to obtain the s dependence of the b parameters in the near-forward direction (defined as the small $|t|$ region, $|t| \sim 0.02$ (GeV/c)²). We write the elastic differential scattering cross section as:

$$d\sigma/dt = |M_+f_+(t,s) \pm M_-f_-(t,s)|^2/16\pi s^2, \quad (6)$$

and assume real, exponential^[2] form factors $f_+(t,s)$ and $f_-(t,s)$ for the even and odd amplitudes, respectively. Using $f_{\pm}(t,s) = \exp(b_{\pm}(s)t/2)$, we write:

$$d\sigma/dt = ([\text{Re } M_+ e^{b_+ t/2} \pm \text{Re } M_- e^{b_- t/2}]^2 + [\text{Im } M_+ e^{b_+ t/2} \pm \text{Im } M_- e^{b_- t/2}]^2)/16\pi s^2, \quad (7)$$

with the + sign for $p\bar{p}-p$ and the - sign for $p-p$ ^[3]. For very small $|t|$, the t behavior of (7), for both $p-p$ and $p\bar{p}-p$, is ap-

proximately exponential, although we actually define the slope parameters b by (1), using the appropriate $d\sigma/dt$, where we have evaluated the derivatives at $|t|=0.02$ (Gev/c)², a relatively low value of $|t|$. We have included for analysis only the higher energy data, i.e., $\sqrt{s} > 5$ Gev, which is the energy region used in the cross section and ρ fit made earlier [1].

In order to determine the form of the s dependence of $b_+(s)$, we note that as $s \rightarrow \infty$, $b_{pp} = b_{p\bar{p} \rightarrow p} = b_+$, since $M_-/s \rightarrow 0$. Since the high energy elastic scattering crosssection is known to be diffractive, with an approximately exponential slope b , we can write $\sigma_{el} = (d\sigma/dt)_{t=0}/b$, and further, $\sigma_{tot} > \sigma_{el}$. Thus, it is trivial to show that $b > \text{const.} \cdot \sigma_{tot}$. We therefore parametrize the even slope [4] as

$$b_+ = C_+ + D_+ \ln(s) + E_+ \ln^2(s), \quad (8)$$

while we choose for the odd slope the normal Regge behavior

$$b_- = C_- + D_- \ln(s). \quad (9)$$

We emphasize the importance at high energy of including the term $\ln^2(s)$ in b_+ . In order for the slope parameter to follow the trend of the total crosssection, this term is essential. It has not been included in a recent slope analysis by Burg et al [5], and its absence can seriously distort the predictions for high s . In particular, if one plots b vs. $\ln(s)$, one expects positive curvature similar to that measured for σ_{tot} as a function of $\ln(s)$, for large s .

Unlike the situation for ρ and σ_{tot} , where the measurements are of high quality and are experimentally very smooth in s , with various groups mainly getting consistent results, the experimental situation for the slopes (even after being corrected for curvature to $|t|=0.02$ (Gev/c)²) is quite confused, with many of the groups quoting very inconsistent values. It is a Solomonian task to decide which results are correct, and which are false and thus should not be included in our analysis. We have been guided in our judgment by the principle that roughly, the s dependence of the b values resembles the shape

of the total cross section curve, with its positive curvature. The data seem to fall in three distinct groups, listed respectively in ref.[6], [7] and [8]. The data of [6] will be used in all of our analyses. The p-p data of Bartenov et al^[7] are clearly inconsistent with the p-p data^[8] of Fajardo et al and Burq et al. In order to decide which set to use, we fit the observed (corrected to $|t|=0.02(\text{Gev}/c)^2$) slopes b_{p-p} and $b_{p\bar{p}-p}$, using (8) and (9) in (7), with the logarithmic derivative given in (1), using a χ^2 minimization procedure. The results are shown in Fig.1 and Fig.2, where the data set of ref.[7] has been used in Fig.1, whereas the set of ref.[8] has been used in Fig.2. In both cases, the data of ref.[6] have been used. The data have been fitted using as input b_{p-p} , $b_{p\bar{p}-p}$ and $\Delta b = b_{p\bar{p}-p} - b_{p-p}$, together with their associated errors. The Δb values of Fajardo et al^[8] have been used in both fits.

The $\chi^2/\text{d.f.} = 1.52$ for 52 degrees of freedom (d.f.) for Fig. 1, whereas the $\chi^2/\text{d.f.} = 4.17$ for 39 degrees of freedom for Fig. 2. Further, Fig. 2 has negative curvature, with b decreasing asymptotically, and thus does not follow the cross section trend. We thus are forced to conclude that we must choose the results of Fig. 1, which are, in $(\text{Gev}/c)^{-2}$:

$$\begin{aligned} C_+ &= 10.94 \pm 5.3, \quad D_+ = -.09 \pm 1.8, \quad E_+ = .044 \pm 0.016, \\ C_- &= 23.2 \pm 1.6, \quad D_- = .94 \pm 1.6 \quad (\text{see Table I for errors}). \end{aligned}$$

The value predicted at $\sqrt{s}=540 \text{ Gev}$ (the SPS collider) is $b_{p\bar{p}-p} = 16.7 \pm 7 (\text{Gev}/c)^{-2}$, and it is expected to be $19.6 \pm 1.4 (\text{Gev}/c)^{-2}$ at the Fermilab Tevatron ($\sqrt{s}=2000 \text{ Gev}$).

The data are rather poor, and even the best fit is not terribly convincing statistically (a $\chi^2/\text{d.f.} \sim 1.5$ is barely satisfactory). Clearly, the situation is only clarifiable experimentally by direct measurements at high s , and only an accurate SPS Collider measurement will settle this question.

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TABLE I

Squared Error Matrix of Fitted Coefficients, in $(\text{Gev}/c)^{-4}$

	C_+	D_+	E_+	C_-	D_-
C_+	.281	-.096	.0080	-.29	.0076
D_+		.034	-.0028	.127	-.027
E_+			.00025	-.011	.0024
C_-				2.44	-.19
D_-					.027

REFERENCES

- [1] M. M. Block and R. N. Cahn, preceding letter.
- [2] Since we are at very small $|t|$, the form factors can be replaced by $1+bt/2$. From conventional Regge theory, one expects $|\text{Im}(b_+-b_-)| \sim \pi\alpha' \sim 2 \text{ (Gev/c)}^{-2}$, whereas the real parts b_+ and b_- are $\sim 10-30 \text{ (Gev/c)}^{-2}$.
- [3] A simplified form of (1), valid in the limit of $p=0$ and large s , is $b(s) = b_+(s) \pm \Delta b(s)\sigma_-/\sigma_+$, where the upper sign is for $p\bar{p}$ and the lower sign is for p - p , and $\Delta b(s) = b_-(s) - b_+(s)$. The result assumes that $\sigma_-/\sigma_+ \ll 1$.
- [4] Since the "standard" Regge theory is not used in the parametrization of b_+ , which is the dominant term for $b(s)$ (see ref.[3]), it is not clear that slope "universality", a concept stressed in the analysis of Burq et al [5], has any obvious meaning.
- [5] p - p data and slope analysis of Burq et al, Phys. Lett. 39B (1982) 124.
- [6] The first data sets are p - p data: Foley et al, Phys. Rev. Lett. 11 (1963) 425; Belletini et al, Phys. Lett. 14 (1965) 164; Amaldi et al, Phys. Lett. 36B (1971) 504; Chervnev et al, Phys. Lett. 36B (1971) 266; Holder et al, Phys. Lett. 36B (1971) 400; Barbiellini et al, Phys. Lett. 39B (1972) 663; Ayres et al, Phys. Rev. D 15 (1977) 3105; Baksay et al, Nucl. Phys. B 141 (1978) 1; Louvain-Northwestern Collaboration, Talk of S. Zucchelli, XVIIth Rencontre de Moriond, Les Arcs, France, March, 1982 and private communication--the following are $p\bar{p}$ data--Antipov et al, Nucl. Phys. B 57 (1973) 333; Fajardo et al, Phys. Rev. D 24 (1981) 46 (used as $\Delta b = b_{p\bar{p}} - b_{p-p}$ only); Louvain-Northwestern Collaboration, Talk of S. Zucchelli, XVIIth Rencontre de Moriond, Les Arcs, France, March, 1982 and private communication.
- [7] p - p data of Bartenev et al, Phys. Rev. Lett. 29 (1972) 1755; Bartenev et al, Phys. Rev. Lett. 31 (1972) 1088.
- [8] p - p data of ref.[5] and of Fajardo et al, Phys. Rev. D 24 (1981) 46.

Figure Captions

Fig. 1. Nuclear slope parameters $b_{\text{pbar-p}}$ and $b_{\text{p-p}}$, evaluated at $|t|=.02 \text{ (Gev/c)}^{-2}$, using data sets of ref.[6] and [7]. The dashed curve is for pbar-p and the solid one for p-p. The appropriate fitted parameters C_+, D_+, E_+, C_- , and D_- are given in the text, and their squared error matrix is given in Table I. The 4 Δb values of ref.[6] were used in the fit, but are not shown in the Figure.

Fig. 2. Nuclear slope parameters $b_{\text{pbar-p}}$ and $b_{\text{p-p}}$, evaluated at $|t|=.02 \text{ (Gev/c)}^{-2}$, using data sets of ref.[6] and [8]. The dashed curve is for pbar-p and the solid one for p-p. The 4 Δb values of ref.[6] were used in the fit, but are not shown in the Figure.

Data File #20CUR6-57-0

o=p x=pbar

Fig. 1

$\sqrt{s}$ (GeV)

Data File #20CUR6-44-0

$\sigma = p$ $x = pbar$

$\sqrt{s}$ (GeV)

Fig. 2