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SPIN-ISOSPIN RESPONSE FUNCTIONS IN NUCLEI

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ABSTRACT

Nuclear spin-isospin excitations are surveyed in terms of a unifying response function approach. At low momentum, the strong repulsive ρ force in addition to creating a collective mode, produces a quenching of the Gamow-Teller strength due to a Lorentz-Lorenz effect similar to that of electromagnetism. The missing strength is not displaced in the region of the Δ resonance. The implication of the quenching for the Gamow-Teller nuclear quark sum rules is discussed, as well as the nuclear Adler-Weinberger relation. The marked difference between the longitudinal and transverse responses at moderate momenta is stressed, the former being closely linked to pion degrees of freedom.

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INTRODUCTION

In this talk I will give a coherent survey of the question of the spin-isospin excitations in nuclei. It is a proper time for such a review since a unity to this field has emerged, which was not originally apparent.

My presentation will be oversimplified. While other speakers will give the detailed and quantitative treatments, it nevertheless contains the main physical features.

Let us consider as a starting point the response function of the nucleus to a spin-isospin perturbation. A concrete example is, for instance, the axial current of the weak interactions

$$\vec{A}^+(\vec{x}) = g_A \sum_i \vec{\sigma}_i \cdot \vec{\tau}_i \delta(\vec{x} - \vec{x}_i)$$

or the isovector magnetic transition operator

$$\vec{J}_s(\vec{x}) = g_M \sum_i (\vec{\sigma}_i \times \vec{\nabla}) \cdot \vec{\tau}_i \delta(\vec{x} - \vec{x}_i) ,$$

in general any perturbation which couples to the spin isospin density fluctuations.

When such a perturbation is applied to the nuclear medium, this medium responds in such a way as to adjust to the presence of the perturbing field. If all interactions between the nucleons are switched off, they respond individually to the external field. The response will then occur in two ways. Firstly, the nucleons can flip

their spin and isospin. This process represents an adjustment of the nucleonic orbits, in a way which is of course restricted by the Pauli principle. Secondly, the field can flip the spin and isospin of an internal constituent of the nucleons, of a quark, transforming the nucleon into a Δ resonance for instance.

Let us consider the case of infinite nuclear matter. The total response $\pi^0(q, \omega)$, which is a function of the momentum and frequency of the exciting field, is the sum of these two components

$$\pi^0(q, \omega) = \pi^N(q, \omega) + \pi^\Delta(q, \omega) . \quad (1)$$

What is then the behaviour of π^0 ? Its imaginary part is fixed by the excitations of the system. Restricting to single particle excitations these are of two kinds. Firstly, we have the one nucleon excitations at low energies. Secondly, we have the states with a real Δ excited which are located at high energies, around $\omega_\Delta \approx 300$ MeV. Between these two types of excitations there is a no man's land where $\text{Im } \pi$ vanishes (see Fig. 1). The real part of π^0 , on the contrary, extends everywhere and has the usual dispersive behaviour, so that $\text{Re } \pi^N$ changes sign at low energy and $\text{Re } \pi^\Delta$ at high energy (see Fig. 2).

When the interaction between the nucleons is switched on, the force acting on one nucleon is transmitted to its neighbours. The response then becomes collective. The random phase approximation (RPA) takes into account the transmission of the force by summing the chain of the ring diagrams shown in Fig. 3, where the heavy line represents the particle-hole (p.h.) interaction. As you see, the

Fig. 1

Fig. 2

Fig. 3

field can act directly on one nucleon or on a neighbouring one which transmits the force and so on. By summing this geometrical series one obtains the RPA expression

$$\pi(q, \omega) = \frac{\pi^0(q, \omega)}{1 - V(q, \omega)\pi^0(q, \omega)}, \quad (2)$$

where $V(q, \omega)$ is the p.h. interaction. This expression is oversimplified in many respects. Firstly, it is a nuclear-matter expression, while in practice one has to deal with finite systems. Secondly, it is based on the RPA which does not represent all the physics. The effect of two particle-two-hole states (2p.2h.) for instance, is ignored. Finally, even in RPA the ring expression is not exact because it omits the antisymmetrization. In the following we omit in addition the Hartree-Fock effects and use the free nucleon mass. In spite of these restrictions, this expression is useful since it is so simple and compact. Keeping in mind its quantitative limitations we will use it to predict or interpret many physical phenomena.

A very obvious remark: if the RPA denominator is close to unity, the response keeps its single-particle character, while if it deviates appreciably from unity, the response becomes collective. In particular, it may happen that the denominator vanishes. If this is the case, there will be a finite response, even for an infinitesimal perturbation with all nucleons participating. This is the condition for a collective state which is written:

$$1 - V(q, \omega)\pi^0(q, \omega) = 0. \quad (3)$$

PARTICLE-HOLE INTERACTION

The quantity V depends on the p.h. quantum numbers. It is not sufficient to fix the total spin and isospin of the p.h. pair, which in our case gives $S = T = 1$. In addition, one must also specify the magnetic quantum number M_S . The longitudinal channel corresponds to $M_S = 0$. It is explored with a $\vec{\sigma} \cdot \vec{q}$ coupling, which probes the component of the magnetization along the momentum $\vec{q}$. The transverse channel explores the transverse magnetization ($\vec{\sigma} \times \vec{q}$ coupling) and corresponds to $M_S = \pm 1$. In nuclear matter these two channels are completely decoupled and do not communicate.

In the longitudinal case the pion-exchange force is effective, providing the following force which is attractive at small energies:

$$-\frac{f^2}{m_\pi^2} \frac{q^2}{q^2 + m_\pi^2 - \omega^2}$$

where f is the πN coupling constant, $f^2/4\pi = 0.08$.

Since the pion mass is so small the attraction which vanishes at $q = 0$ builds up very quickly with increasing q . Pion exchange is not the only possible one. There are also heavy exchanges (equivalently short-range effects). These are simulated in the Landau-Migdal approach by a repulsive contact force characterized by a parameter g' . In this phenomenological approach one does not enquire about the origin of the force. It can be due to multimeson exchange as well as to the overlap of two quark bags: g' is taken as phenomenological and should be determined from experiments. The overall force is then

$$V(q, \omega) = \frac{f^2}{m_\pi^2} \left[g' - \frac{q^2}{q^2 + m_\pi^2 - \omega^2} \right]. \quad (4)$$

In the transverse channel there is no light meson exchange to provide an attraction so readily (the lightest exchange is the ρ meson) and the force remains repulsive over a larger range of momenta.

The determination of the g' has been the subject of very careful and thorough studies^{1,2}. From the position of the nuclear states, in particular those of unnatural parity, which are sensitive to these forces, it was concluded that the repulsion is large with $g' \approx 0.6$ to 0.7 (in the same units, the maximum attraction from the pion is -1). Such large values have been accounted for theoretically^{3,4}. The forces in the two channels are featured in Fig. 4, which displays a remarkable contrast between the two at moderate momenta, $1 < q < 2 \text{ fm}^{-1}$. The longitudinal force is mildly attractive, while the transverse one stays repulsive.

Fig. 4. The p.h. interaction at $\omega = 0$ as a function of q in the transverse (continuous line) and longitudinal (dot-dashed line) channel. The value of g' is fixed at 0.7 .

We can exploit this contrast. Suppose we suspect that an effect in the spin-isospin response originates from the p.h. force. We propose as a clean experimental signature for this the contrast between the responses in the two channels.

Let us now explore how the responses are affected by the forces, following the evolution with increasing momentum.

LOW MOMENTUM REGION ($q \leq 0.3 \text{ fm}^{-1}$)

The forces being identical in the two channels there is no need to distinguish between the two responses. The strong repulsion creates a collective state, the zero sound, predicted by Ikeda et al.⁵. Since V is real at low energies, the condition (3) for a collective state requires $\text{Im } \pi^0 = 0$. It is fulfilled outside the p.h. continuum, i.e. in the no man's land region of Fig. 1, where $\text{Im } \pi^0 = 0$. The collective state is pushed above the p.h. continuum by the repulsive interaction. The presence of the collective state produces a complete reshaping of the response as shown in Fig. 5. The single-particle states are completely depopulated in favour of the collective one.

The existence of this collective state has been beautifully displayed in the $(p,n)^{6,7}$ and $(pp')^8$ reactions. The (p,n) experiments have in addition revealed a very interesting feature, which concerns the total Gamow-Teller (GT) strength. Since all final states are energetically accessible in the (p,n) experiments, contrary to β decay, it is possible to extract the total GT strength.

Fig. 5. The imaginary part of the response function of infinite nuclear matter at $q = 0.3 \text{ fm}^{-1}$ as a function of ω for a free Fermi gas (dot-dashed line) and in the ring approximation (continuous line). From Ref. 29.

The result could be compared with an RPA calculation, but an even simpler comparison has been performed⁹. Take the difference of the total GT strengths for the β^+ branches, associated with the spin isospin operators $\sigma^3 \tau^\pm$.

$$S = S^- - S^+ \\ = g_A^2 \sum_I \left\{ \left| \langle I | \sum_i \sigma_i^3 \tau_i^+ | 0 \rangle \right|^2 - \left| \langle I | \sum_i \sigma_i^3 \tau_i^- | 0 \rangle \right|^2 \right\} . \quad (5)$$

Performing closure over the intermediate states I , one is left with the commutator of $(\tau^+, \tau^-) = \tau^3$ and

$$S = g_A^2 (N - Z) . \quad (6)$$

The sum S thus depends only on the neutron excess of the nucleus. It is totally independent of the detailed structure of the initial state and its correlations.

The β^+ branch which would be obtained from an (n,p) reaction is not accessible at present. In nuclei with a neutron excess, S is small due to the Pauli blocking, and $S^- \approx S = g_A^2 (N - Z)$. Even if S^+ cannot be neglected, the sum rule provides, in the case of a neutron excess, a lower limit for S^- (if $Z > N$, it gives a limit for S^+ instead).

This comparison has been performed. The remarkable result is that the experimental values which should be above the $(N - Z)$ line, lie in fact a factor ≈ 2 below⁹. What are the reasons for this discrepancy? One could try to blame the (p,n) reaction itself. After all it is not straightforward to extract the GT strength from a (p,n) reaction which involves a strongly interacting probe. However, similar quenching had already

Fig. 6. Quenching factor γ of the g_s factor as a function of the mass number A from the M_2 and M_1 strengths. From Ref. 10.

been observed in the M1 and M2 strengths¹⁰, as shown in Fig. 6 (the quenching factor shown applies to the amplitude and should be squared for the strength). Here there is no similar sum rule and the quenching refers to a RPA evaluation. This comparison shows in addition that the quenching is A-dependent, disappearing in light nuclei. It is therefore natural to seek for a universal mechanism which applies to all spin transitions.

A POSSIBLE INTERPRETATION: THE LORENTZ-LORENZ (LL) EFFECT^{11,12}

What could have gone wrong in our sum rule? We have assumed that the axial current couples to the nucleons with their free coupling constant, while the nucleons are not free in the nucleus. They are imbedded in the nuclear medium, which is not inert and becomes polarized under the influence of the weak field. Can this affect the sum rule? Suppose for a moment that there is no Δ excitation. The polarization represents the virtual excitations of the system, i.e. the mixing of nuclear configurations. But we have seen that the sum rule is independent of this mixing. Therefore as long as it respects the integrity of the nucleon the nuclear polarization does not affect the sum rule. This is no longer the case when the nucleonic excitations are introduced. Their effect can be pictured in the following way: under the influence of the weak field there appear polarization charges (axial charges) in the medium. In particular, there are surface charges at the surface of the correlation hole which surrounds a nucleon. This has the consequence that the field which couples to the nucleonic spins is not the average field in the medium but it is an effective one, as occurs for the e.m. field in a dielectric: $\vec{E}_{\text{eff}} = \vec{E}/(1 + \alpha/3)$ where α is the polarizability of the medium. This entails a renormalization of the axial coupling constant in the nucleus and therefore a modification of the sum rule*. Although the polarizability π in the expression of E_{eff} in principle involves all types of excitations, in the sum rule there survives only the effect of the Δ , the purely nuclear excitations cancelling their effect on the average. The renormalization factor of the sum is $1/(1 + f^2\pi\Delta/3m_\pi^2)$. This renormalization appears clearly in the RPA expression, which allows the generalization to a phenomenological p.h. force^{17,21}. The classical LL effect corresponds

*One may enquire how this geometrical description translates into a delta-hole (ΔN^{-1}) model. It is well known that the usual LL effect of pion scattering has an exact equivalence in that formalism¹³. The same occurs for the axial current. Indeed the geometrical picture originally proposed (in the first-order approximation) by Ericson et al.¹¹ was translated into the ΔN^{-1} formalism independently by Rho¹⁴, and by Ohta and Wakamatsu¹⁵. Restricting to infinite nuclear matter they generalized our results to all orders. In the geometrical picture the A-dependence immediately follows from the existence of axial charges at the nuclear surface which suppress the LL effect in a small nucleus, as was shown in Ref. 11. In the ΔN^{-1} language this suppression requires the full RPA formalism.

to the value $g' = 1/3$ of the Migdal parameter. We want to separate the roles of π^N and π^Δ in the RPA expression (2). It is rather natural to perform this separation since they behave quite differently: at low momentum and small energy π^Δ is purely real and practically constant with energy $\pi^\Delta(\omega) \approx \pi^\Delta$, while π^N , on the contrary, has both a real and an imaginary part which vary rapidly with energy. We consider for simplicity the very small momentum region where V reduced to $V \approx G' = (f^2/m_\pi^2)g'$.

The imaginary part of the response function, proportional to the inelastic cross-section is written:

$$\begin{aligned} \text{Im } \pi(q, \omega) &= \frac{\text{Im } \pi^0(q, \omega)}{[1 - G' \text{Re } \pi^0(q, \omega)]^2 + [G' \text{Im } \pi^0(q, \omega)]^2} \\ &= \frac{1}{(1 - G' \pi^\Delta)^2} \frac{\text{Im } \pi^N(q, \omega)}{\left[1 - \frac{G'}{1 - G' \pi^\Delta} \text{Re } \pi^N(q, \omega)\right]^2 + \left[\frac{G'}{1 - G' \pi^\Delta} \text{Im } \pi^N(q, \omega)\right]^2} \end{aligned} \quad (7)$$

In this form the RPA denominator has been split in two pieces. The second part depends on the energy through $\pi^N(\omega)$. It mixes the different single-particle configurations with a force $G'/(1 - G' \pi^\Delta)$ renormalized by the presence of the Δ . But we have seen that the GT sum rule is totally independent of this mixing and of the force. If it were only for this term the sum rule would be conserved. But there is the first constant factor $1/(1 - G' \pi^\Delta)^2$ which produces a renormalization of the GT strength (LL effect). Since π^Δ is negative it represents a quenching of the strength.

The renormalization factor depends strongly on the force, which mixes NN^{-1} and ΔN^{-1} states (which for simplicity has been taken here to be the same as the NN^{-1} force). For $g' \approx 0.6$ and $\pi^\Delta = -0.8$ the quenching of the strength is strong: a factor ≈ 2 . The LL effect offers a likely interpretation for at least part of the missing strength. If correct, it is also another indication for a strong collectivity, involving this time Δ resonances. One cannot flip the spin of one nucleon without flipping those of the virtual Δ cloud which surrounds it. However, the present experimental situation cannot conclude decisively in favour of the LL interpretation. Other mechanisms can also be partly responsible for the quenching^{2,2}.

Summarizing the previous results, there exists a nice sum rule which is independent of the mixing force as long as the space is restricted to nucleonic degrees of freedom. When the Δ degree of freedom is introduced, the sum rule is violated and the result becomes dependent on the force.

It is then natural to ask the question: Is it possible to find another sum rule which is independent of this mixing as well. For this one has to enlarge the traditional perspective of nuclear physics so as to incorporate the Δ degree of freedom in the basic description. The natural framework to perform this is the quark model, where the N and Δ are different configurations of the same three quarks.

GAMOW TELLER NUCLEAR QUARK SUM RULES²³

We take the simplest quark model, the constituent quark one where the axial current couples non-relativistically to the spin and isospin of the constituent quarks with a unit coupling constant

$$(A^3)^\pm(\vec{x}) = \sum_{\lambda} \sigma_{\lambda}^3 \tau_{\lambda}^{\pm} \delta(\vec{x} - \vec{x}_{\lambda}) . \quad (8)$$

This operator acts on the initial nucleus, which consists of $3A$ quarks. We do not assume that these quarks are free, but that they are instead strongly correlated due to the organization of the nucleus in nucleons and Δ . This idea is to find a sum rule which is independent of this structure.

We take as before the difference Σ of the total GT strengths for the β^- and β^+ branches, but this time for the operator (8). Performing closure we obtain

$$\Sigma = \Sigma^- - \Sigma^+ = N - Z . \quad (9)$$

As before the sum depends only on the neutron excess of the nucleus, but it is reduced as compared to the classical sum rule by the factor g_A^2 , which should be taken in the quark model ($g_A = 5/3$), $g_A^2 = 25/9$. What is the origin of this difference? It is clear; we are not talking of the same sum as before, $\Sigma \neq S$. When we apply closure the intermediate states automatically incorporate all possible excitations. Since the quark operator can transform a N into a Δ , the intermediate states include the excitations of a real Δ , the high-energy part of the GT strength which was not incorporated in our previous quantity S .

The different contributions to the sum are illustrated by the example of a neutron as initial state. With the $\sigma\tau^+$ operator one can reach either a proton or a Δ^+ . With $\sigma\tau^-$ one can reach a Δ^{--} . The contribution to Σ^- from the proton intermediate state is clearly $g_A^2 = (25/9)$, since the matrix corresponding element is that of the axial current between neutron and proton. The various contributions are given below:

$$\Sigma^- = \frac{25}{9} (p) + \frac{8}{9} (\Delta^+) = \frac{33}{9}$$

$$\Sigma^+ = \frac{8}{3} (\Delta^{--})$$

$$\Sigma = \Sigma^- - \Sigma^+ = 1 .$$

The two branches are clearly necessary to saturate the sum. In a nucleus, if there is no interaction between nucleons and Δ , and with total Pauli blocking, the respective contributions are the previous ones multiplied by $N - Z$. When the interaction between N and Δ is switched on the respective contributions are modified. The nucleon sector is quenched by the LL effect, but this is compensated by a corresponding modification of the high-energy sector in such a way as to respect the sum rule.

With our quark sum rule we have thus linked the low-energy and high-energy part of the GT strength. This result applies only to the difference of the β^- and β^+ branches. A measurement of the GT strength at high energy would allow a check of this sum rule. This is not feasible at present, but we can make the following observation: the new sum rule is reduced by a factor of $25/9$ as compared to the classical one. This happens to be close to the quenching factor of the classical sum observed in the (p,n) reactions. It appears then that the low-energy nucleon sector by itself nearly saturates the quark sum rule. This suggests a blocking of the Δ sector. The mixing forces seem to be such that the high-energy parts of the $\beta^\pm$ branches acquire more or less equal strength so as to approximately cancel in the quark sum rule. At present we have no possibility to check this point, but in some way we can come close to it, in the following sense.

The constituent quark model is rather crude, since it predicts a wrong value for g_A , $5/3$ instead of 1.25 . So the first question is: Can we believe in a sum rule based on this model? One could try to take a better quark model, but the result would still be model-dependent. What one would like to find instead is a model-independent sum rule. Such a sum rule exists in fact. It is the nuclear Adler-Weisberger (AW) relation which is based on the algebra of charges. The commutation relation of two axial charges follows from the quark model, but it has a validity which extends well beyond this model. While I do not want to enter into details, the AW relation also provides a link between the low- and high-energy axial strengths. This link is not along the line $\vec{q} = 0$ as our previous quark sum rule, but it is along the soft pion line where the energy and momentum are linked by the relation $|\vec{q}|^2 = \omega^2$. In the nuclear case the AW relation is written²⁴:

$$N - Z = (N - Z) (g_A)_{\text{eff}}^2 + \frac{f_\pi^2}{\pi} \int \frac{d\omega}{\omega} [\sigma_0^+(\omega) - \sigma_0^-(\omega)] . \quad (10)$$

Here f_π is the pion decay constant, $\sigma_0^\pm$ are the total cross-sections for soft pions ($\pi^\pm$) on the nucleus; $(g_A)_{\text{eff}}^2 (N - Z)$ is the total GT strength at low energy [the difference of the two branches, the β^- branch being the quantity measured experimentally in the (p,n) reaction].

The pion cross-sections in the AW sum rule are not physical. They refer to soft pions, but this is not a long way off shell and we believe that the extrapolation from the physical cross-sections can be performed reliably. The dispersive integral is dominated by the Δ_{33} resonance. If only the Δ is retained and its width neglected, then the dispersive term is proportional to the square of $f_{\pi N\Delta}$, the $\pi N\Delta$ coupling constant. Through the Goldberger-Treiman relation the dispersive term is then related to the axial strength leading to the Δ . The superiority of the nuclear AW relation is that it takes into account the width of the Δ and the contribution of the other resonances in a model-independent way. The physics however, is basically the same as that of our quark sum rule.

In addition, the AW sum rule involves (nearly) measurable cross-sections and has therefore a predictive power about $(g_A)_{\text{eff}}$, which our quark sum rule did not have.

Can we understand on these new grounds the quenching of the low-energy sector and the blocking of the high-energy one? It is remarkable that this last effect comes very naturally in this description. The Δ region is dominant in the dispersive integral. In that region the cross-sections which are large are also strongly shadowed. In particular, the difference $\sigma^- - \sigma^+$ does not increase linearly with the neutron excess. There is an A-dependent quenching factor, which reaches zero in heavy nuclei²⁵⁻²⁷. We believe that the same quenching factor should hold for soft pions at least in the Δ resonance region. This quenching has implications for the low-energy sector, due to the existence of the AW sum rule.

In a heavy nucleus, if we suppress completely the dispersive integral on the argument of total shadowing, then we obtain the result that the low-energy strength by itself saturates the sum rule, while it represents 156% of the sum in the nucleon case (the dispersive contribution is negative). Therefore, in the nucleus $g_{A\text{eff}}^2$ is reduced, reaching the value 1: $g_{A\text{eff}}^2/g_A^2 = 0.64$. If one assumes total shadowing in the Δ region only and no shadow above, one obtains $g_{A\text{eff}}^2/g_A^2 = 0.48$. In all cases the AW relation can account for the observed quenching which comes naturally in this description, and was the subject of an early prediction²⁴. Subsequently a detailed investigation of the experimental data then available, by Wilkinson²⁸, indicated the existence of a quenching.

In the AW relation the quenching is obviously A-dependent since there is no shadow in very light nuclei. What is not yet clear is

the link between the shadow and the microscopic description of the LL effect. Maybe if we understand this link, we can obtain an independent determination of the force which mixes NN^{-1} and ΔN^{-1} states.

MISSING STRENGTH: WHERE HAS IT GONE?

Let me now come to another question, which comes up naturally in relation to the missing strength. The question is: Where has the strength gone? A statement is commonly made that the strength has gone in the high-energy (Δ -resonance) region. This statement is misleading, since it evokes, at least to me, the following picture. When the interaction between NN^{-1} and ΔN^{-1} is switched on, the low-energy sector is quenched, but the high-energy one should be enhanced accordingly to conserve the total strength. This is indeed what we had found with our quark sum rule, but it applied only to the difference of the β^- and β^+ branches. In each branch separately there is no conservation of strength. Let me illustrate this by the example of symmetric nuclear matter. Here you can see immediately that the mixing with NN^{-1} states cannot influence the high energy strength for the following reason. The quantity $\pi^N(\omega)$ which determines this influence in the RPA expression vanishes in the high-energy limit since its crossing properties are such that

$$\pi^N(\omega) \propto \frac{1}{\omega + \omega_{\text{exc}}} - \frac{1}{\omega - \omega_{\text{exc}}},$$

where ω_{exc} is a typical nuclear excitation energy. In the Δ region, which is well above the nuclear energies $\omega_{\Delta} \gg \omega_{\text{exc}}$, π^N becomes very small. We have here an example where the strength missing at low energy is not at all compensated at high energy and is totally lost. With a neutron excess there may be some compensation but there is absolutely no reason for a strict conservation, since it is well known to be destroyed by the RPA correlations.

This feature is illustrated in Fig. 7 which displays the strength $S_T(q)$ in the nuclear sector of the response (including the collective state) as a function of the momentum in the free case and in the ring approximation, with and without Δ . Even when the Δ excitation is absent, i.e. when all the axial strength is concentrated in the low-energy nuclear sector, there is a quenching of the strength due to the RPA correlations. The quenching effect is enhanced by the introduction of the Δ (LL effect).

We will now follow the evolution of the response when the momentum increases.

Fig. 7. Strength $S_T(q)$ in the transverse channel for a free Fermi gas (dot-dashed line) and in the ring approximation, without (dotted line) and with (continuous line) Δ . The arrows indicate the momentum at which the collective state disappears. From Ref. 29.

RESPONSE AT MODERATE MOMENTA ($0.3 \text{ fm}^{-1} \lesssim q \lesssim 2 \text{ fm}^{-1}$)

We have now to distinguish between the two channels, since the corresponding forces are quite different.

Transverse channel^{29,30}

The force remains appreciably repulsive. The collective features of the response displayed in Fig. 8 are still pronounced. There is no longer a collective state in the strict sense of condition (3). It has dissolved in the p.h. continuum. But it survives in the form of a hardening of the response²⁹⁻³¹. The high-energy side of the response is enhanced, while the low-energy states are depopulated. In addition, the response is quenched by the repulsive interaction, as was shown in Fig. 7.

Fig. 8. The imaginary part of the response function of infinite nuclear matter for the transverse channel at $q = 1.3 \text{ fm}^{-1}$ as a function of ω for a free Fermi gas (dot-dashed line) and in the ring approximation (continuous line). From Ref. 29.

We have investigated whether this collective character of the response appears in the experimental data or not. We have explored in particular the inelastic electron scattering process which provides the isovector transverse spin response. Indeed the separation between the longitudinal (charge) and the transverse (magnetic) responses has been performed for ^{12}C ³² and ^{56}Fe ³³. For the magnetic part the coupling is $\vec{\sigma} \times \vec{q}$, a transverse one. It is essentially isovector due to the nearly perfect cancellation between proton and neutron magnetic moments.

Figures 9a and 9b show the experimental cross-section on ^{56}Fe as a function of the energy transfer for two values of the momentum $q = 1.45 \text{ fm}^{-1}$ and 1.65 fm^{-1} . The curves are the theoretical predictions. The upper one corresponds to the case of a free Fermi gas with $k_F = 1.15 \text{ fm}^{-1}$ and the lower one is our ring approximation. The agreement between the last curve and the experiments is satisfactory, apart from the region of high excitation where our description totally fails. The experimental data show a filling of the no man's land between the low-energy nuclear sector and the Δ sector. This filling is believed^{34,35} to be due to the 2p.2h. contribution not incorporated in our description. At low energy, where the omission of the 2p.2h. contribution is less crucial, our description reproduces well the slope at the origin (as well as the position of the maximum).

Although encouraging, this agreement should not be taken as conclusive evidence for the collective character of the response. Our theory applies to infinite nuclear matter and in addition it neglects the antisymmetrization and Hartree-Fock effects. It is quite possible that a single-particle description which incorporates these properly accounts for the experimental points, without collectivity. In this case we would have to understand why the collective features, which are so pronounced at low momentum, have already faded away at $q \approx 1 \text{ fm}^{-1}$.

In order to settle this question we suggest using the proposed experimental signature: the contrast between the transverse and longitudinal responses which we will discuss shortly.

Another place where we believe¹² that collectivity may show up is the μ -capture process that we are investigating again at present. The total rate is dominated by the axial contribution. The momentum transfer is small, $q \approx 0.5 \text{ fm}^{-1}$, and corresponds to the strongly repulsive region of the p.h. force. The quenching of the axial strength should influence the total rate.

Response in the longitudinal channel³⁶

Below 1 fm^{-1} the longitudinal force is still repulsive, but less so than the transverse one. It produces some quenching and hardening of the response. Beyond $q \gtrsim 1 \text{ fm}^{-1}$ the force turns attractive. It

Fig. 9. The separated transverse magnetic response in ^{56}Fe at $q = 1.45 \text{ fm}^{-1}$ (Fig. 9a) and 1.65 fm^{-1} (Fig. 9b) as a function of ω . The dot-dashed line refers to the free Fermi gas response with $k_F = 1.15 \text{ fm}^{-1}$. The continuous line is the ring approximation. From Ref. 29.

is only mildly attractive owing to the strong repulsion of the Migdal force. We will discuss the collective features associated with this attraction.

It had been suggested³⁷ that the attraction can produce a collective state, called a pion condensate. If it exists, the attractive force pulls it below the p.h. continuum, which starts itself at $\omega = 0$. The collective state thus has to occur at $\omega = 0$ ($\text{Im } \pi(q, 0) = 0$). This makes it of a different nature from the zero sound. It corresponds to a phase transition, a reordering of the system.

The condition for its realization would be

$$1 - V(q, 0)\pi^0(q, 0) = 0. \quad (11)$$

However, this condition cannot be met at the ordinary density, for the force is not sufficiently attractive. One has to increase $|\pi^0|$, i.e. the density (the estimates for the critical density are $\rho_c \geq 3\rho_0$).

Suppose that we are at ρ_c . The condition (11) for pion condensation is fulfilled for a value q_c of the momentum. Then at $q = q_c$ the single-particle states are depopulated in favour of the collective one at $\omega = 0$. If the density is decreased below ρ_c the collective state disappears but it survives in the form of a softening of the response: the high-energy states are depopulated in favour of the low-energy ones as shown in Fig. 10. Here the value of g' has been set at $g' = 0.7$, for which the softening is mild. A lower value, such as $g' = 0.6$, considerably enhances the collective character of the response and the softening effect.

In addition the strength $S_L(q)$ is enhanced in the vicinity of q_c by the attractive force as shown in Fig. 11.

These two effects, enhancement and softening, are the precursors of pion condensation. At the critical density the softening would be total (the response becoming singular at $\omega = 0$). In addition the enhancement becomes infinite (critical opalescence^{38,39}). This feature characterizes the occurrence of a second-order phase transition. A normal collective state instead, the zero sound, does not affect the total strength. When it disappears nothing special happens as was shown in Fig. 7.

Fig. 10. The imaginary part of the response function of infinite nuclear matter for the longitudinal channel at $q = 2.15 \text{ fm}^{-1}$ (critical value of the momentum) as a function of ω for a free Fermi gas (dot-dashed line) and in the ring approximation (continuous line). The value of g' is taken to be 0.7. From Ref. 36.

Fig. 11. Strength $S_L(q)$ in the longitudinal channel for a free Fermi gas (dot-dashed line) and in the ring approximation, without (dotted line) and with (continuous line) Δ . The arrows indicate the momentum at which the collective state disappears. From Ref. 29.

Between 1 and 2 fm^{-1} the contrast between the longitudinal and transverse responses is striking, owing to the different character of the forces. The spin longitudinal response functions are not yet available experimentally. The difficulty is to find probes which explore the longitudinal spin density. The precursors have been searched for only in discrete low-lying states. Softening in this case corresponds to the fact that states of unnatural parity 0^- , 1^+ , etc., are pulled down. This has been suggested and investigated from the very beginning of the history of pion condensation⁴⁰. However, no state has been observed to be pulled down, at best they are at their unperturbed position. As for the critical opalescence it has been searched for in the form factor of 1^+ transitions, with (p, p') and (p, n) scattering⁴¹⁻⁴⁴. The effect, if it is there, is mild (a factor of two enhancement).

These negative findings should not discourage the exploration of the longitudinal spin channel. The fact that no clear precursors are seen in discrete levels does not exclude the possibility of a detectable effect in the response function (see Fig. 10). Even in the case where the softening and enhancement of the longitudinal re-

sponse would be too weak to be detectable by themselves the longitudinal response will serve as a contrast to the transverse one, as is shown in Fig. 12. An experimental display of this sharp contrast would provide indisputable evidence for the influence of the p.h. force and hence of the collective character of the response. This opportunity should be exploited. There are still exciting developments ahead of us in this field.

Fig. 12. Contrast between the imaginary parts of the response functions in the ring approximation at $q = 1.3 \text{ fm}^{-1}$ for the longitudinal channel (dotted line) and the transverse one (continuous line). From Ref. 29.

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