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TEMPERATURE DEPENDENCE OF COUPLING CONSTANTS AND THE PHASE
TRANSITION IN THE COLEMAN-WEINBERG THEORY

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TEMPERATURE DEPENDENCE OF COUPLING CONSTANTS AND THE
PHASE TRANSITION IN THE COLEMAN-WEINBERG THEORY

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It is shown that the temperature dependence of the effective Higgs boson coupling constants leads to a considerable modification of the theory of the SU(5) Coleman-Weinberg phase transition.

There is now an ever growing interest in cosmological consequences of symmetry breaking phase transitions in grand unified theories. In some cases these phase transitions proceed from the strongly supercooled symmetric state $\varphi = 0$ [1 - 6]. According to the inflationary universe scenario [7], supercooled universe expands exponentially, which may help us to solve the horizon, flatness and primordial monopole problems. Unfortunately after the phase transitions in the theories studied in [7] the universe becomes greatly inhomogeneous and anisotropic [8]. In our paper [9] (see also [10-15]) a new inflationary universe scenario was suggested, based on the investigation of the phase transition in the Coleman-Weinberg theory [16]. This scenario may provide us with a natural solution of the homogeneity and isotropy problems [9], and also may explain the origin of inhomogeneities, which are necessary for the galaxy formation [17]. In the context of this scenario it is possible to obtain the observed baryon asymmetry $\frac{n_B}{n_\gamma} \sim 10^{-8}$ independently of the initial baryon asymmetry of the universe [14] (this condition is not satisfied in the standard scenario [18] in the absence of sufficiently strong B-L violating interactions).

For the realization of this scenario the metastable vacuum state $\varphi = 0$ should be supercooled down to some temperature $T \lesssim H$ [9, 13]. Here H is the Hubble constant at $\varphi = 0$, $H = \sqrt{\frac{8\pi}{3M_P^2} V(0)}$; $M_P \sim 1.2 \cdot 10^{19}$ GeV is the Planck mass, and $V(\varphi)$ is the effective potential of the field φ . In the minimal $SU(5)$ Coleman-Weinberg theory $V(0) = \frac{9M_X^4}{32\pi^2}$. At the X-boson mass $M_X \sim 5 \cdot 10^{14}$ GeV the Hubble constant is given by

$$H = \frac{M_x^2}{2M_P} \sqrt{\frac{3}{\pi}} \approx 10^{10} \text{ GeV.} \quad (1)$$

Investigation of the phase transition in this theory performed in [3-6] shows that the phase transition may occur only at $T \ll H$. However in [3-6] the temperature dependence of the effective coupling constants of the Higgs bosons [19] has not been taken into account. As it will be shown below, this leads to a considerable modification of the results of refs. [3-6].

In order to analyse this question let us first recall that the interaction Lagrangian of the Higgs field Φ in the SU(5) theory is given by

$$L_{\text{int}} = - \frac{a}{4} (T_2 \Phi^2)^2 - \frac{\beta}{2} T_2 \Phi^4. \quad (2)$$

After symmetry breaking $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ the field Φ acquires a diagonal classical part $\Phi_i = \text{diag } \Phi$, where

$$\Phi_i = \sqrt{\frac{2}{15}} \varphi \left(1, 1, 1, -\frac{3}{2}, -\frac{3}{2} \right). \quad (3)$$

Note, that in our normalization $T_2 \Phi^2 = \varphi^2$, and $M_x = \frac{g\varphi_0}{2} \sqrt{\frac{5}{3}}$, where φ_0 is the value of φ in the minimum of $V(\varphi)$.

At $T \neq 0$ the φ boson mass in the SU(5) Coleman-Weinberg theory is given by [20]

$$\mu^2(T) = \left. \frac{d^2 V(\varphi, T)}{d\varphi^2} \right|_{\varphi=0} = \left[\frac{5}{4} g^2 + \frac{13}{6} a + \frac{47}{30} \beta \right] T^2. \quad (4)$$

However since in the Coleman-Weinberg theory the constants a and β usually assumed to be of the order of g^4 , eq.(4) in [3-6] was written simply as

$$\mu^2(T) = \frac{5}{4} g^2 T^2. \quad (5)$$

From (5) it follows that $V(\varphi)$ has a local minimum with $\frac{d^2V}{d\varphi^2}|_{\varphi=0} = \mu^2(T) > 0$ at all T , at which the effective gauge coupling constant $g^2(T)$ is positive. Therefore it was concluded in [3-6], that the phase transition from $\varphi=0$ to $\varphi=\varphi_0$ may occur only due to the bubble formation [3,4] or due to other nonperturbative effects [5], which are greatly suppressed at small $g^2(T)$, and may occur only at $g^2(T) \gtrsim 1$. According to the renormalization group (r.g.) equation in the minimal SU(5) theory with three fermion generations [4]

$$g^2(T) = \frac{3\pi^2}{5} \ln^{-1} \frac{T}{\Lambda}, \quad (6)$$

where $\Lambda \approx 4 \cdot 10^6 \text{ GeV}^x$. From eq. (6) it follows that

^{x)} In fact, eq.(6) is not quite precise. It can be argued that a more accurate equation for $g^2(T)$ (6) should contain $\ln \frac{g^2(T)T}{\Lambda}$ instead of $\ln \frac{T}{\Lambda}$. Indeed, usual r.g. equations contain $\ln \frac{p^2}{\Lambda^2}$, where p^2 is a typical momentum squared of interacting particles. At $T \neq 0$ a typical momentum of the Yang-Mills particles is $O(T)$, but a typical momentum squared (the "mass"^{squared} of the Yang-Mills particle) is $O(g^4(T)T^2)$ [21]). In the present paper we assume for simplicity that eq.(6) is valid, bearing in mind that the critical temperature T_c obtained that way may differ from the true one by the factor $O(g^2(T))$. We shall slightly improve this approach by assuming that $g^2(T=0, \varphi=\varphi_0) = g^2(T=\varphi_0, \varphi=0) \approx 0.3$, contrary to the assumption that $g^2(T=0, \varphi=\varphi_0) = g^2(T=M_x, \varphi=0)$ [4]. This leads to some small increase of Λ (6) as compared with the value used in [4].

$g^2(T)$ becomes large at $T \sim \Lambda$. It was concluded therefore [4-6] that only at $T_c \lesssim \Lambda \sim 10^6$ GeV (or, according to [11], at $T_c \lesssim 10^8$ GeV) nonperturbative effects may lead to the symmetry breaking phase transition in the SU(5) Coleman-Weinberg theory. Let us take into account, however, the r.g. equations for the effective coupling constants $a(T)$ and $b(T)$ at $\varphi = 0$, which in our case look as follows [22]:

$$4\pi^2 \frac{da}{dt} = 16a^2 + \frac{94}{5}ab + \frac{168}{25}b^2 - 15g^2a + \frac{9}{4}g^4, \quad (7)$$

$$4\pi^2 \frac{db}{dt} = \frac{32}{5}b^2 + 6ab - 15g^2b + \frac{15}{8}g^4, \quad (8)$$

where $t = \ln \frac{T}{\Lambda}$. Since analogous equations could be written for the case $T=0$, $\varphi \neq 0$, $t = \ln \frac{\varphi}{\Lambda}$, one may argue that $a(T=\varphi_0, \varphi=0) \approx a(T=0, \varphi=\varphi_0)$ and $b(T=\varphi_0, \varphi=0) \approx b(T=0, \varphi=\varphi_0)$. Let us consider for simplicity the case $a(T=0, \varphi=\varphi_0) = b(T=0, \varphi=\varphi_0)$, and recall that at $T=0, \varphi=0$ the effective coupling constant $\lambda = \frac{1}{6} \frac{d^4 V}{d\varphi^4} = a + \frac{7}{15}b$ in the SU(5) Coleman-Weinberg model is equal to $\frac{275}{192\pi^2}g^4$. This means that $a(T=\varphi_0, \varphi=0) \approx b(T=\varphi_0, \varphi=0) \approx a(T=0, \varphi=\varphi_0) = b(T=0, \varphi=\varphi_0) = \frac{125}{128\pi^2}g^4 \approx 0.01$. Now we solve eqs. (6)-(8) numerically with these boundary conditions and with $\varphi_0 = \frac{2M_x\sqrt{3}}{g} \approx 1.4 \cdot 10^{15}$ GeV. The resulting temperature dependence of $g^2(T)$, $a(T)$ and $b(T)$ as well as the function $M^2(T)/T^2 = \frac{5}{4}g^2 + \frac{13}{6}a + \frac{47}{30}b$ (4) is shown in fig.1. It is seen that at $T \sim \varphi_0$ $|a|, |b| \ll g^2$, and eq.(5) is approximately valid. However at smaller values of T parameters a and b become negative and rapidly grow in magnitude, so that eq.(5) becomes inapplicable and

$\mu^2(T)$ (4) becomes negative at $T_c \sim 10^{-5} \varphi_0 \sim 10^{10} \text{ GeV}$. This means that the phase transition may start not at $T \sim 10^6 \text{ GeV}$ [4-6], and not at $T \sim 10^8 \text{ GeV}$ as it was claimed in [11], but much earlier, at $T_c \sim H \sim 10^{10} \text{ GeV}$, and may occur without any tunneling in the region where perturbative results are valid. It is necessary, however, to make some comments to these results.

1) The behaviour of $a(T)$, $\beta(T)$ and $\mu^2(T)$ depends crucially on the type of the theory and its field content. Therefore the value of T_c may appear either much greater or much smaller than H . Our result $T_c \sim H$ is therefore somewhat accidental.

2) At $T_c \gg H$ the phase transition occurs too early for the new inflationary universe scenario [9] to be realized. However for $T_c \sim 10^{10} - 10^{11} \text{ GeV}$ at least the primordial monopole problem can be solved (or softened) in this scenario, and the new scenario of the baryon asymmetry generation [14] remains true.

3) At $T_c \ll H$ all the abovementioned effects, as well as the effects considered in [4-6, 11] are unimportant for the theory of the phase transition.

Indeed, the time interval, which is necessary for the phase transition to start due to high-temperature effects exceeds $T_c^{-1} \gg H^{-1}$, and during this interval temperature falls down to $T_c e^{-\frac{H}{T_c}}$, i.e. practically vanishes. Kinetics of the phase transition in this case is discussed in [13].

4) In the case $T_c \sim H$ the phase transition starts due to the growth of quantum fluctuations of the field φ

(i.e. of the $\langle \varphi^2 \rangle$) at $\mu^2(T) < 0$. At $|\mu^2(T)| \ll H^2$ the value of $\langle \varphi^2 \rangle$ grows approximately as $\frac{3H^4}{8\pi^2|\mu^2|} (\exp(\frac{2|\mu^2|}{3H}) - 1)$ [15]. However, during the time $\Delta t \sim H^{-1}$ the value of $\mu^2(T)$ exponentially decreases, and a further growth of the fluctuations of the field φ proceeds just as at $T=0$. This may appear to be even a more efficient way of realization of the new inflationary universe scenario than the bubble formation at $T = 0$ [15].

We see therefore that the temperature dependence of the Higgs boson coupling constants may lead to a considerable modification of the theory of the phase transition in GUTs with the Coleman-Weinberg mechanism of symmetry breaking. Depending on the type of the theory and its field content, either the new inflationary universe scenario [9] can be realized, or this scenario is realized incompletely, but the primordial monopole problem can be solved (or softened) even in such a "truncated" scenario.

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Fig.I. Temperature dependence of the effective coupling constants g^2 , a and β , and of the Higgs boson mass μ^2 .