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IMPOSSIBILITY OF PERTURBATIVE RESTORATION OF
F TYPE SPONTANEOUSLY BROKEN SUPERSYMMETRY

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ABSTRACT

We prove that higher order corrections cannot restore supersymmetry if it is spontaneously broken at the tree level by an F type mechanism even if not all terms consistent with the symmetries have been included.

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The gauge hierarchy problem is perhaps the most serious weakness of currently known grand unified models. There are two aspects to this problem: firstly, why is the ratio M_W/M_X so small ($\sim 10^{-15}$)? Secondly, why does it remain small when radiative corrections are taken into account? A possible solution to the second problem is provided by $N = 1$ rigid supersymmetry¹⁾, which protects light scalars from getting large mass corrections. However, since fermions and bosons are not degenerate in mass at low energies, it follows that supersymmetry has to be broken. Explicit soft breaking²⁾, though efficient from a technical point of view, is rather arbitrary and not entirely satisfactory, and it would be preferable if one could implement the breaking spontaneously.

Here, there are two alternatives: F and D type breaking^{3),4)}. D type breaking requires an additional $U(1)$ gauge group and attempts to build low energy models of this type exist in the literature⁵⁾. So far no satisfactory anomaly-free model of this type has been constructed. Recently there have been attempts to build models based on the F type mechanism⁶⁾ and in these schemes the mass splittings between the light fermions and their superpartners, as well as the $SU(3) \times U(1)$ breaking, arise as radiative effects. In constructing models of this type, it has become common practice to write down superpotentials which are not the most general ones which are consistent with the symmetries of the models. For example, one cannot invoke R invariance in order to explain the absence of terms which could in principle be present. Indeed, the absence of R invariance is essential since in this way gluinos can get a Majorana mass; spontaneously broken R symmetry would imply the existence of Goldstone bosons which have not been observed.

A possible problem with these models is that terms which have been neglected could restore supersymmetry. For example, consider a theory with three chiral fields A , B and C and a superpotential

$$W = g AB + \lambda_1 C (A^2 - \mu^2) + \lambda_2 A^3 \quad (1)$$

which has no R symmetry because of the A^3 term. The potential breaks supersymmetry and μ characterizes the breaking. Notice that if a C^3 term were present then supersymmetry would no longer be broken. However, at the quantum level there is mixing of the A and C fields via the graph of the figure. This produces divergent terms proportional to $\int d^4\theta (A\bar{C} + \bar{A}C)$ which in turn necessitate non-diagonal wave function renormalizations. This means that beyond the tree approximation the fields have to be redefined in order to represent the physical degrees of freedom correctly. In particular, this mixing may induce

terms which are not present at the tree level, such as C^3 terms in (1). It is easy to see that the classical potential is still bounded below by a strictly positive constant after such a redefinition because of its invariance under orthogonal transformations. However, we know that the classical potential will be modified by higher order contributions in perturbation theory and it is therefore not a priori clear that supersymmetry breaking will persist, especially if terms that cannot be ruled out by any symmetry have been omitted at the classical level.

It is the purpose of this note to show that higher order corrections cannot restore supersymmetry if it is spontaneously broken at the tree level by an F type mechanism even if not all terms consistent with the symmetries have been included. The proof of this assertion follows by a simple extension of the argument that was given in Ref. 7) to show that supersymmetry breaking cannot be induced by radiative corrections. From Ref. 8), we take over the result that the effective action of any $N = 1$ supersymmetric theory containing chiral superfields $\phi_a(x, \theta)$ and vector superfields $V_A(x, \theta)$ is always of the form

$$\begin{aligned} \Gamma_{\text{eff}}(V, \phi) = & \Gamma_{\text{class}}(V, \phi) + \\ & + \sum_n \int d^4\theta \, dx_1 \dots dx_n \, \Gamma(x_1, \dots, x_n) \cdot \\ & \cdot f_n(\phi(x_i, \theta), D_\alpha \phi(x_i, \theta), \dots, V_A(x, \theta), \dots) \end{aligned} \quad (2)$$

The remarkable feature of (1) is that the quantum corrections are local in θ space and can always be written as an integral over all of superspace $d^4\theta$. To obtain the effective potential from (2), we put all external momenta equal to zero and ignore external fermion lines. The special form (2) then implies that the effective potential can be expressed as follows

$$\begin{aligned} V_{\text{eff}}(A, F, D) = & -F_a \bar{F}_a + (\bar{F}_a \mathcal{F}_a(A) + \text{h.c.}) + \\ & + F_a \bar{F}_b \mathcal{G}_{ab}(A, F) - \frac{1}{2} D_A D_A + D_A \mathcal{Q}_A(A, F, D) \end{aligned} \quad (3)$$

where (for a renormalizable theory)

$$\mathcal{F}_a(A) = \lambda_a + m_{ab} A_b + g_{abc} A_b A_c \quad (4)$$

gives the classical potential while the quantum corrections are contained in the functions $\mathcal{G}$ and $\mathcal{D}$. The structure imposed by the gauge group further restricts the form of the effective potential, but this circumstance is irrelevant for our subsequent considerations.

Supersymmetry is spontaneously broken at the tree level if

$$\mathcal{F}(A) \neq 0 \text{ for all } A \quad (5)$$

which implies that $\langle F_a \rangle \neq 0$ for at least one a . At the minimum, the potential (3) is subject to the following conditions

$$\begin{aligned} \frac{\partial V_{\text{eff}}}{\partial F_a} = & -\bar{F}_a + \mathcal{F}_a(A) + \bar{F}_b \mathcal{G}_{ab}(A, F) + \\ & + F_b \bar{F}_c \frac{\partial \mathcal{G}_{bc}(A, F)}{\partial F_a} + D_A \frac{\partial \mathcal{D}_A(A, F, D)}{\partial F_a} = 0 \end{aligned} \quad (6)$$

$$\frac{\partial V_{\text{eff}}}{\partial A_a} = \frac{\partial V_{\text{eff}}}{\partial D_A} = 0 \quad (7)$$

If supersymmetry were restored by quantum corrections to the full potential (3), there would have to be a solution to (6) and (7) satisfying

$$\langle F_a \rangle = \langle D_A \rangle = 0 \quad (8)$$

Clearly, condition (5) implies that (6) cannot be satisfied in this case. Hence supersymmetry cannot be restored by quantum corrections. We also note that the argument does not carry over straightforwardly to D type breaking since higher order corrections need not be quadratic in D .

To recapitulate, F type model builders should not lose any sleep if they forbid terms at the classical level allowable by the symmetries of the theory because technically these terms are harmless to all orders in perturbation theory. No danger of SUSY restoration is in sight.

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FIGURE