Ref.TH.3301-CERN

UNRAVELLING HIGHER TWISTS

R.K. Ellis

CERN -- Geneva

W. Furmanski ^{*)}

CERN -- Geneva

and

Département de Physique
Université de Genève

and

R. Petronzio

CERN -- Geneva

A B S T R A C T

We present a novel method of deriving the first power corrections ($1/Q^2$) to the lepton production structure functions. The method is based directly on the manipulation of Feynman diagrams. Correspondence with the operator product expansion approach is established. We find that the transverse components of momenta and the gluon field control both the kinematic and dynamical power corrections. The full QCD result is similar in structure to the naïve parton model with intrinsic transverse momentum. We explain how the concept of parton transverse momentum is generalized in the presence of interaction. In the operator language our method defines a new basis in which all but the transverse components of covariant derivatives are eliminated by the equations of motion. We derive constraints on signs of the $1/Q^2$ terms.

^{*)} On leave of absence from the Institute of Physics,
Jagellonian University, Cracow, Poland.

1. - INTRODUCTION

A recurrent problem with attempts to make precision tests of QCD in inelastic processes is our ignorance of the corrections which fall like inverse powers of the large energy scale, Q . This is true for all hadronic processes which are formally calculable as a perturbation expansion in the QCD coupling constant, $\alpha_s(Q^2)$. Any serious comparison of theoretical predictions with presently available data, requires the introduction of phenomenological procedures (e.g., an intrinsic transverse momentum or a hadronization Monte Carlo program) to model the power corrections to the perturbative result. The corrections introduced in this way are not always small, so one often ends up testing the phenomenological model rather than the underlying QCD theory.

It is therefore a matter of some urgency to examine to what extent these procedures can be justified by QCD. Ultimately this will require knowledge of the way quarks and gluons bind to make hadrons, but even in the realm of perturbative QCD the theory makes definite statements about the correct implementation of the power corrections. As emphasized by Politzer¹⁾, stringent tests of QCD require that we include all the information which the theory gives us.

To investigate power corrections in QCD it is natural to turn first of all to lepton-hadron deep-inelastic scattering. The kinematics of this process is shown in Fig. 1. In this case the operator product expansion (OPE)²⁾ guarantees (at least to all orders in perturbation theory) that we may write $(Q^2 = -q^2, x_B = Q^2/2P \cdot q)$

$$W^{\mu\nu}(q, p) = \sum_{T=2}^{\infty} \left(\frac{\Lambda}{Q}\right)^{T-2} \sum_i \int d\{x\} C_{i,T}^{\mu\nu}(x_B, \alpha_s(Q^2), \{x\}) f_{i,T}(\{x\}, \frac{Q^2}{\Lambda^2}). \quad (1.1)$$

Equation (1.1), in which we have suppressed inessential details, shows that the structure function W factorizes into two parts. The coefficient functions C_i describe the short-distance part of the lepton-parton interaction and are calculable as a perturbation series in $\alpha_s(Q^2)$. The functions f_i , which we call parton correlation functions, contain non-perturbative information about the distribution of partons inside the struck hadron.

The equivalence of Eq. (1.1) to the operator product expansion is established by identifying the moments of the parton correlation functions with the reduced matrix elements of the local operators. The power at which the

operators contribute to deep inelastic scattering is controlled by their twist τ ³⁾. Operators of twist-two scale (up to logarithms)^{*)}. It is the terms of twist higher than two which are the subject of this paper.

In particular we shall deal with the $1/Q^2$ corrections corresponding to $\tau = 4$. We consider the case of massless quarks and limit our discussion to the non-singlet structure functions. For these terms we shall arrive directly at the form of Eq. (1.1) in the space of light-cone fractions, without passing through the intermediate step of the operator product expansion. We will, however, demonstrate the equivalence of our approach with the operator method. We unravel the description of power corrections provided by higher twist operators and construct a simple parton-like picture.

Previous analyses⁶⁾⁻⁸⁾ of higher twist operators have tended to concentrate on logarithmic scaling violations rather than the calculation of coefficient functions^{**)}. Because of the complexity of the problem many authors have chosen to restrict their attention to a limited number of operators. An important exception to the above statement is the work of Jaffe and Soldate¹⁰⁾ where a first complete analysis of the coefficient functions and operator basis for electroproduction is presented. Also noteworthy is the work of Shuryak and Vainshtein¹¹⁾ where detailed results for first and second moments of the structure functions are presented.

On the phenomenological side various functional forms have been proposed for the x_B dependence of the higher twist terms¹²⁾. These forms have been used to investigate the magnitude of the higher twist contributions and its correlation with the size of the logarithmic scaling violation parameter¹³⁾⁻¹⁵⁾. It is also clear that at least part of the answer is included in the CIM model and the QCD results which have evolved from it¹⁶⁾.

This paper contains the details of our approach to power corrections. We discuss only deep inelastic lepton-hadron scattering, but since our method is based directly on the manipulation of Feynman diagrams it offers the possibility

*) The Q^2 variation of the parton correlation functions is calculable in perturbative QCD⁴⁾. For example, at leading log level the parton correlation functions obey generalizations of the Altarelli-Parisi equation⁵⁾. These logarithmic variations are not considered in this paper.

**) Calculations of some coefficient functions are given in Ref. 9).

of extension to other processes. The basic idea of our treatment, introduced in Ref. 17), is that the transverse degrees of freedom - both momenta and components of the gluon field - provide a convenient and concise way of describing the power corrections to hard scattering processes. We shall find that there are striking similarities between the QCD analysis and the naïve parton model with transverse momentum. The parton model cannot represent the full result, since it relies on the impulse approximation which is not generally valid in field theory. Nevertheless, we find that it is both theoretically and phenomenologically desirable to augment and improve the parton model by refining the concept of an intrinsic transverse momentum.

At the leading twist level, the parton language can be established in a direct way : for each type of parton there is exactly one unknown function which has therefore a natural interpretation as a parton density. Beyond leading twist the number of unknown functions (parton correlation functions) increases with twist so that the parton interpretation of the higher twist contributions is less immediate.

A common feature of the diagrammatical analysis and the operator product expansion treatment is that one starts with an overcomplete set of correlation functions (a redundant basis of local operators) which is then reduced to a simpler basis by using the classical equations of motion ^{1),18)}, Lorentz invariance and gauge invariance.

However, even after the reduction has been completed, we are still left with several correlation functions. In consequence, the parametrization of the final result is not unique and the parton interpretation may be either simple or complicated depending on the particular choice of basis for the correlation functions.

A possible choice of the operator basis has been advocated by Politzer ¹⁾ and implemented by Jaffe and Soldate ¹⁰⁾. These authors eliminate all derivatives on fermion fields by use of the classical equations of motion. In terms of correlation functions (i.e., in the parton language) this procedure replaces the off-shellness and transverse momentum of the parton fields by higher Fock components of the hadronic wave functions. Transverse momentum and off-shellness are generated dynamically by the interaction of the partons. We call the resulting set of correlation functions the collinear basis.

We carry out this program in our language and find that the formulae are complicated in the collinear basis. Contrary to expectation, contact with the intuition provided by the parton model is lost and we feel that the over-all physical picture is unclear.

Our proposal is rather to emphasize the rôle of the transverse degrees of freedom. We therefore eliminate the off-shellness using the equations of motion but retain the transverse momentum. In this way we arrive at a set of parton correlation functions which we call the transverse basis. The transverse basis has simple coefficient functions and an attractive parton interpretation.

The simple picture of on-shell partons with intrinsic transverse momentum plays the rôle of a reference model, which will then be improved and refined by the complete QCD analysis. We therefore take this model as the starting point for our discussion.

The scattering of a highly virtual photon off a hadron target acquires a parton description in the infinite momentum frame in which the characteristic interaction time of partons increases due to the Lorentz dilatation. On the short-time scale of the deep inelastic scatter the partons appear in this frame to be almost free. We give this frame dependence a covariant guise by introducing an auxiliary light-like vector n in terms of which we may write the Sudakov¹⁹⁾ decomposition of the parton momentum k :

$$k_\mu = x p_\mu + \frac{k^2 + k_T^2}{2x} n_\mu + k_{T\mu}, \quad (1.2)$$

where

$$p^2 = M^2 = p k_T = n k_T = 0, \quad p n = 1. \quad (1.3)$$

The four momentum P of the parent hadron of mass M is given by

$$P_\mu = p_\mu + \frac{M^2}{2} n_\mu. \quad (1.4)$$

The parton interpretation of the variables x and k_T in Eq. (1.2),

$$x = k/M, \quad k_T^\mu = d^{\mu\nu} k_\nu, \quad d^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu \quad (1.5)$$

as the longitudinal momentum fraction and the transverse momentum in the infinite momentum frame becomes clear after rewriting the Sudakov decomposition of Eq. (1.2) in a class of Lorentz frames where

$$p \equiv (p_0, \vec{p}_T, p_3) = (\mathbb{P}, \vec{0}, \mathbb{P}), \quad m = \left(\frac{1}{2\mathbb{P}}, \vec{0}, -\frac{1}{2\mathbb{P}} \right) \quad (1.6)$$

and $\mathbb{P}$ is the boost parameter, specifying a particular frame. We have then

$$\mathbb{P} = \left(\mathbb{P} + \frac{M^2}{4\mathbb{P}}, \vec{0}, \mathbb{P} - \frac{M^2}{4\mathbb{P}} \right), \quad (1.7)$$

$$k = \left(x\mathbb{P} + \frac{k^2 + k_T^2}{4x\mathbb{P}}, \vec{k}_T, x\mathbb{P} - \frac{k^2 + k_T^2}{4x\mathbb{P}} \right) \quad (1.8)$$

so that in the limit $\mathbb{P} \rightarrow \infty$ the covariant decomposition in Eq. (1.2) looks very similar to the standard parametrization of the parton momenta in the infinite momentum frame.

Using the decomposition Eq. (1.2), we can specify the recipe for the introduction of an intrinsic transverse momentum in the parton model. The parton cross-section σ is evaluated with parton momenta on-shell, but not collinear with their parent hadron. In this model the structure functions are given by

$$W(q, \mathbb{P}) = \int \frac{dx}{x} d^2k_T \delta(\dot{k}) f(x, k_T^2) \quad (1.9)$$

where $\dot{k}$ is the on-shell part of the momentum k ,

$$\dot{k}_\mu = k_\mu - \frac{k^2}{2x} m_\mu, \quad \dot{k}^2 = 0. \quad (1.10)$$

The structure functions are readily calculated from Eq. (1.9) (cf. Fig. 2a) by parametrizing the virtual photon momentum q as follows

$$q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} m^\mu, \quad Q^2 = -q^2, \quad \xi = \frac{Q^2}{2p_2}, \quad x_B = \frac{Q^2}{2p_2}. \quad (1.11)$$

The result can be decomposed into longitudinal and transverse structure functions,

$$W^{\alpha\beta}(q, \mathbb{P}) = F_L(x_B, \frac{Q^2}{\Lambda^2}) e_L^{\alpha\beta} + F_T(x_B, \frac{Q^2}{\Lambda^2}) e_T^{\alpha\beta} \quad (1.12)$$

where

$$e_{\perp}^{\alpha\beta} = \frac{1}{2} \left(g^{\alpha\beta} - \frac{q^{\alpha} q^{\beta}}{q^2} \right), \quad (1.13)$$

$$e_T^{\alpha\beta} = \frac{\xi}{p_2} p^{\alpha} p^{\beta} + \frac{1}{2p_2} (p^{\alpha} q^{\beta} + p^{\beta} q^{\alpha}) - \frac{1}{2} g^{\alpha\beta}. \quad (1.14)$$

The result for one species of unit charge quark is

$$F_L(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4}{Q^2} \int d^2 k_T k_T^2 f(\xi, k_T^2) + O\left(\frac{1}{Q^4}\right) = \frac{4 \langle k_T^2 \rangle}{Q^2} + O\left(\frac{1}{Q^4}\right), \quad (1.15)$$

$$F_T(x_B, \frac{\Lambda^2}{Q^2}) = \int d^2 k_T f(\xi, k_T^2) + \frac{1}{Q^2} \int d^2 k_T k_T^2 \left[4f(\xi, k_T^2) + \xi \frac{\partial}{\partial \xi} f(\xi, k_T^2) \right] + O\left(\frac{1}{Q^4}\right). \quad (1.16)$$

We see that the presence of the intrinsic transverse momentum generates a longitudinal cross-section. Equations (1.15), (1.16) are the point of departure for our QCD analysis of power corrections.

Equations (1.15), (1.16) have several deficiencies which the full QCD treatment will rectify. They are : (a) the continued reliance on the impulse approximation beyond its range of applicability ; (b) the absence off-shellness and/or interactions for the partons ; (c) the mutilation of the Lorentz invariance because the arguments of the function $f(x, k_T^2)$ are defined in terms of the auxiliary vector n^μ .

A striking illustration of the inadequacy of Eq. (1.16) is the breakdown of the Adler sum rule (which should be valid to all orders in $1/Q^2$)

$$\int_0^1 \frac{dx_B}{x_B} \left[F_2^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) - F_2^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] = A, \quad (1.17)$$

where A is a pure number dependent only on the isospin of the target. The standard structure functions F_1, F_2 ²⁰⁾ are related to the transverse and longitudinal structure functions F_T, F_L as follows

$$\frac{1}{x_B} F_2(x_B, \frac{\Lambda^2}{Q^2}) = \left(\frac{1 - \mu \xi^2}{1 + \mu \xi^2} \right)^2 F_T(x_B, \frac{\Lambda^2}{Q^2}), \quad (1.18)$$

$$F_1(x_B, \frac{\Lambda^2}{Q^2}) = \frac{1}{2} [F_T(x_B, \frac{\Lambda^2}{Q^2}) - F_L(x_B, \frac{\Lambda^2}{Q^2})], \quad (1.19)$$

where $\mu = M^2/Q^2$, and the relation between the Bjorken scaling variable x_B and the Nachtmann variable ξ ²¹⁾ is

$$x_B = \frac{\xi}{1 - \mu\xi^2}, \quad \xi = \frac{2x_B}{1 + \sqrt{1 + 4\mu x_B^2}}. \quad (1.20)$$

Substituting these relations we see that the Adler sum rule implies that our formulae for F_T should satisfy

$$\int_0^1 \frac{d\xi}{1 + \mu\xi^2} F_T(x_B, \frac{\Lambda^2}{Q^2}) = 1. \quad (1.21)$$

From Eq. (1.16) we see that this is only true for special choices of the function $f(x, k_T^2)$.

This failure is due to the form of the parton model ansatz in Eq. (1.9). In general the function $f(x, k_T^2)$ depends on the vector n and the Lorentz invariance is mutilated. Restoring the full Lorentz invariance we find that the only acceptable form for $f(x, k_T^2)$ is

$$f(x, k_T^2) = \frac{1}{\pi M^2} \Phi\left(\frac{2Pk}{M^2}\right) \Theta((P-k)^2) = \frac{1}{\pi M^2} \Phi\left(x + \frac{k_T^2}{xM^2}\right) \Theta(x(1-x)M^2 - k_T^2). \quad (1.22)$$

The transverse momentum is bounded to be of order M^2 . Inserting this form in Eqs. (1.15), (1.16) and working out the kinematics exactly we obtain the ξ scaling formula ²²⁾ in terms of the quark distribution function defined by

$$A_0(\xi) = \xi \int_{\xi}^1 d\eta \Phi(\eta); \quad (1.23)$$

$$\begin{aligned} F_T(x_B, \frac{\Lambda^2}{Q^2}) = & \frac{A_0(\xi)}{1 + \mu\xi^2} + \frac{6\mu\xi^2}{(1 + \mu\xi^2)^2} \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta) + \\ & + \frac{12\mu^2\xi^3}{(1 + \mu\xi^2)^3} \int_{\xi}^1 d\eta \int_{\eta}^1 \frac{d\tau}{\tau} A_0(\tau), \end{aligned} \quad (1.24)$$

$$\bar{F}_\perp(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4\mu\xi^2}{(1+\mu\xi^2)^2} \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta) + \frac{8\mu^2\xi^3}{(1+\mu\xi^2)^3} \int_{\xi}^1 d\eta \int_{\eta}^1 \frac{d\tau}{\tau} A_0(\tau), \quad (1.25)$$

$$F_A(x_B, \frac{\Lambda^2}{Q^2}) = \frac{A_0(\xi)}{1+\mu\xi^2} - \frac{2\mu\xi^2}{(1+\mu\xi^2)^2} \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta). \quad (1.26)$$

The VA interference structure function quoted here (F_A) is for the weak current $J^\alpha = 1/\sqrt{2} \bar{q} \gamma^\alpha (1-\gamma_5) q$ and it is defined as the coefficient of the tensor ϵ_A [Eq. (5.11)]. Its relationship to the more usually defined structure function is $F_3 = (1-\mu\xi^2)/(1+\mu\xi^2) F_A$. Equation (1.21) is now satisfied because of the quark counting relation

$$\int_0^1 dx d^2k_T f(x, k_T^2) = \int_0^1 d\xi A_0(\xi) = 1. \quad (1.27)$$

ξ scaling is a consequence of the Lorentz invariant inclusion of intrinsic transverse momentum but without off-shellness²³⁾. We conclude that the only correct way to include intrinsic transverse momentum without including off-shellness is given by Eq. (1.22). Commonly used forms such as

$$f(x, k_T^2) = q(x) e^{-\beta k_T^2} \quad (1.28)$$

are not acceptable. The question of whether the inclusion of transverse momentum without off-shellness is a good approximation is a dynamical question which has been discussed extensively in the literature²³⁾⁻²⁵⁾. Here we rather want to stress the intimate connection between transverse momentum and ξ scaling. At order $1/Q^2$ this correspondence is given by the substitution

$$\langle k_T^2 \rangle \rightarrow M^2 \xi^2 \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta). \quad (1.29)$$

The rôle of ξ scaling and its inclusion in the final result is discussed in Section 7.

The purpose of this paper is to go beyond the parton model by including both off-shellness and interactions. We start with the full set of Feynman diagrams and work in the light-like axial gauge $n \cdot A = 0$. The similarity between

transverse momentum and the transverse (physical) components of the gluon field is maintained by choosing the same vector n_μ to define the transverse direction and to fix the gauge.

The plan of this paper is as follows. In Section 2 we present a graphical method of separating the short- and long-distance physics of the hard scattering. The final result is a short-distance cut amplitude linked to the parton correlation functions (describing the hadron structure) by a series of integrals over light cone fractions. The method is an extension of the techniques used to derive the QCD parton model at leading twist²⁶⁾. The reduction of the four-dimensional momentum integrals which link the upper and lower parts of the graph to one-dimensional integrals over light-cone fractions is achieved by Taylor expansion about the collinear direction. Denoting the upper part by S and the lower part by T this procedure may be written schematically as

$$\int \frac{d^4k}{(2\pi)^4} S(k) T(k) = \int dx S(xp) T(x) + \int dx \left. \frac{\partial S}{\partial k^\mu} \right|_{k=xp} T^\mu(x) + \dots \quad (1.30)$$

where

$$T(x) = \int \frac{d^4k}{(2\pi)^4} \delta(x - km) T(k), \quad (1.31)$$

$$T^\mu(x) = \int \frac{d^4k}{(2\pi)^4} \delta(x - km) (k^\mu - xp^\mu) T(k). \quad (1.32)$$

We see that the off-shellness k^2 and transverse momentum k_T^μ are not set to zero, but rather integrated over. By power counting²⁷⁾ only a limited number of terms in the Taylor expansion contribute at a given power of $1/Q^2$.

The reduction of this factorized form to the transverse basis is presented in Section 6. The final formula for the non-singlet structure functions which demonstrates explicitly the rôle of the transverse degrees of freedom is given by *)

$$F_L(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4\Lambda^2}{Q^2} T_1(\xi) + O\left(\frac{1}{Q^4}\right), \quad (1.33)$$

*) This formula does not contain four-fermion diagrams (Fig. 3). The contribution of these diagrams is given in Section 5.

$$\begin{aligned} F_T(x_B, \frac{\Lambda^2}{Q^2}) &= A_0(\xi) + \frac{\Lambda^2}{Q^2} \left[4T_1(\xi) - \xi \int dx_2 dx_1 \frac{\delta(\xi-x_2) - \delta(\xi-x_1)}{x_2-x_1} T_2(x_2, x_1) \right. \\ &\quad \left. + O\left(\frac{1}{Q^4}\right) \right], \\ F_A(x_B, \frac{\Lambda^2}{Q^2}) &= F_T(x_B, \frac{\Lambda^2}{Q^2}) - F_L(x_B, \frac{\Lambda^2}{Q^2}) + O\left(\frac{1}{Q^4}\right). \end{aligned} \quad (1.34)$$

In this formula A_0 is the leading twist quark distribution function and T_1 and T_2 are the next-to-leading twist parton correlation functions. They can be expressed by products of fields on the light-cone in the following way,

$$A_0(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\psi}(0) \not{x} \psi(\lambda) \} | P \rangle, \quad (1.35)$$

$$\Lambda^2 T_1(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\psi}(0) \delta_\mu \not{x} D_T^\mu(0) D_T^\nu(\lambda) \psi(\lambda) \} | P \rangle, \quad (1.36)$$

$$\Lambda^2 T_2(x_2, x_1) = \frac{1}{4} \int \frac{d\lambda d\eta}{(2\pi)^2} e^{i\eta(x_2-x_1)} e^{i\lambda x_1} \langle P | T \{ \bar{\psi}(0) \delta_\mu \not{x} D_T^\nu(\eta) D_T^\mu(\lambda) \psi(\lambda) \} | P \rangle. \quad (1.37)$$

In writing these equations we have introduced a short-hand notation for fields on the light cone,

$$\psi(\lambda) = \psi(\lambda n), \quad D^\mu(\eta) = i\partial^\mu - g A^\mu(\lambda n), \quad D_T^\mu = d^{\mu\nu} D_\nu. \quad (1.38)$$

The reader is invited to compare the form of the naïve parton model result [Eqs. (1.15), (1.16)] with Eqs. (1.33), (1.34). The transverse momenta of Eqs. (1.15), (1.16) have been replaced by transverse covariant derivatives involving the colour field A^μ . The ξ scaling part of the full answer is made manifest by defining the quantities T_1, T_2 which correspond to T_1 and T_2 in the zero target mass limit,

$$\Lambda^2 T_1(x) = M^2 x^2 \int_x^1 \frac{d\eta}{\eta} A_0(\eta) + \Lambda^2 T_1(x), \quad (1.39)$$

$$\Lambda^2 T_2(x_2, x_1) = \delta(x_2 - x_1) M^2 x_1^2 \int_{x_1}^1 \frac{d\eta}{\eta} A_0(\eta) + \Lambda^2 T_2(x_2, x_1). \quad (1.40)$$

The zero target mass correlation functions T_1 and T_2 are the dynamic complements of the kinematic power corrections given by ξ scaling. Both are manifestations of the transverse degrees of freedom. Substituting Eqs. (1.39), (1.40) into Eqs. (1.33), (1.40) we obtain

$$F_L(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4H^2\xi^2}{Q^2} \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta) + \frac{4\Lambda^2}{Q^2} T_1(\xi) + O\left(\frac{1}{Q^4}\right), \quad (1.41)$$

$$\begin{aligned} F_T(x_B, \frac{\Lambda^2}{Q^2}) = & A_0(\xi) + \frac{H^2}{Q^2} \left(4 + \xi \frac{\partial}{\partial \xi}\right) \xi^2 \int_{\xi}^1 \frac{d\eta}{\eta} A_0(\eta) + \\ & + \frac{\Lambda^2}{Q^2} \left[4T_1(\xi) - \xi \int dx_2 dx_1 \frac{\delta(\xi-x_2) - \delta(\xi-x_1)}{x_2-x_1} T_2(x_2, x_1)\right] + O\left(\frac{1}{Q^4}\right). \end{aligned} \quad (1.42)$$

This formula is the principal result of our paper. The power corrections to the structure functions are controlled by parton correlation functions which contain the non-perturbative information about the target structure. The relationship of these quantities to the matrix elements of local operators is given in Section 3.

In Sections 4 and 5 we describe the reduction to the collinear basis and the results in that basis. In this approach all reference to the transverse momentum is eliminated. In Section 4 we describe in detail the passage from transverse momentum to off-shellness and then from off-shellness to more partons. The basic ingredients are Lorentz invariance and the classical equations of motion. After these steps we recover the collinear basis of Ref. 10) in which all derivatives on fermion fields have been eliminated. The results in this basis for deep inelastic scattering are given in Section 5. Whereas the results in the transverse basis are simple, the derivation and final results in the collinear basis are algebraically complicated. We also feel that the final physical picture in the collinear basis is less persuasive. The less dedicated reader may therefore wish to skip Sections 4 and 5 on a first reading.

The transverse basis is discussed in Section 6. Section 7 reinstates the effects due to finite target mass which we suppressed in the intervening sections for clarity. Section 8 discusses positivity constraints on the parton correlation functions in the transverse basis.

2. - FACTORIZATION

The technical steps needed to separate the long and short distance physics are described in this section. The factorization is presented as a series of manipulations on Feynman diagrams. The first step is to write each diagram as the product of an upper part S and a lower part Γ , the precise definition of which will depend on the method of factorization. Independent of particular factorization scheme which we choose, the desired end-result is that the upper part S should contain the information about the hard interaction which is calculable perturbatively. Conversely the lower part Γ parametrizes the non-perturbative information about the structure of the target.

As discussed in Section 1, the final formula is a power series expansion in $1/Q^2$. The coefficients in this series vary logarithmically with Q^2 . Since the aim of this paper is to formulate a diagrammatic scheme to derive the power expansion, we will postpone the question of logarithmic radiative corrections. These effects are calculable and give slow logarithmic variations on top of the power law behaviour discussed here. In our gauge the calculation of logarithmic variations - (anomalous dimensions) - requires detailed analysis of generalized ladder diagrams, such as those shown in Fig. 4. For the treatment of the power corrections we will absorb the ladder part of all such diagrams into the lower part Γ and turn a blind eye to the logarithmic ultra-violet divergences which are consequently generated by the Taylor expansion. In the complete factorization scheme these divergences will be regularized and converted into logarithms of Q^2 .

The upper parts $\hat{S}$ are therefore the one-particle-irreducible tree graphs with n incoming partons. The lower parts $\hat{\Gamma}$ are the full n parton-hadron Green's functions.

Consider the forward amplitude for a virtual photon of momentum q scattering off a spin averaged target of momentum p

$$T^{\beta\alpha}(2,p) = \frac{1}{8\pi} \int d^4z e^{iqz} \langle p | T \{ J^\beta(0) J^\alpha(z) \} | p \rangle. \quad (2.1)$$

The structure functions of deep inelastic scattering are given by the absorptive part of this amplitude. In the approximation in which we work Eq. (2.1) is calculable from the diagrams of Figs. 2 and 3

$$\begin{aligned} T = & [\hat{S}(k)\hat{T}(k)] + [\hat{S}_\mu(k_2, k_1)\hat{T}^\mu(k_2, k_1)] + [\hat{S}_{\mu\nu}(k_2, k, k_1)\hat{T}^{\mu\nu}(k_2, k, k_1)] + \\ & + (4 \text{ fermion diagrams}) + O\left(\frac{1}{Q^4}\right). \end{aligned} \quad (2.2)$$

The restriction to diagrams involving at most four parton lines is valid at order $1/Q^2$ and will be justified later in this section. In Eq. (2.2) we have suppressed several details to make the notation compact. We have dropped the virtual photon indices α and β . The repeated momenta k_i imply the four momentum integrals which link the upper and lower parts. The spinor and colour indices are shown by a hat. Here and elsewhere the square brackets indicate the trace over these indices

$$[\hat{A}\hat{B}] \equiv \text{Tr}\{\hat{A}\hat{B}\}. \quad (2.3)$$

We adopt these conventions throughout the paper. The four fermion contributions coming from the diagrams of Fig. 3 will not be considered in detail in this section. The factorization of these four fermion diagrams (Fig. 3) is more straightforward than the factorization of the diagrams of Fig. 2. We therefore drop them for clarity. The result for the diagrams of Fig. 3 is given in Section 5.

The aim of the rest of this section is to reduce Eq. (2.2) to products of scalar functions coupled only by one dimensional integrals over light cone fractions as in Eq. (1.1). This requires factorization in spinor indices, colour and momentum space.

(a) Spinor Algebra

We expand each $\hat{T}$ in the standard basis of gamma matrices $\{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$. For the case of massless quarks which we consider in this paper the expansion is,

$$\hat{T} = \gamma_5 \hat{T}^S + \gamma_5 \gamma_5 \hat{T}^P. \quad (2.4)$$

This expansion allows a complete decoupling of the spinor trace over the upper and lower parts. As an example we quote the second term in Eq. (2.2) which becomes

$$[\hat{S}_\mu(k_2, k_1) \hat{T}^\mu(k_2, k_1)] = S_{\mathcal{Q}\mu}(k_2, k_1) T^{\mathcal{Q}\mu}(k_2, k_1) + \tilde{S}_{\mathcal{Q}\mu}(k_2, k_1) \tilde{T}^{\mathcal{Q}\mu}(k_2, k_1), \quad (2.5)$$

where

$$S_{\mathcal{Q}\mu}(k_2, k_1) = [\gamma_{\mathcal{Q}} \hat{S}_\mu(k_2, k_1)], \quad \tilde{S}_{\mathcal{Q}\mu}(k_2, k_1) = [\gamma_5 \gamma_{\mathcal{Q}} \hat{S}_\mu(k_2, k_1)], \quad (2.6)$$

and

$$T^{\mathcal{Q}\mu}(k_2, k_1) = \frac{1}{4} [\gamma_{\mathcal{Q}} \hat{T}^\mu(k_2, k_1)], \quad \tilde{T}^{\mathcal{Q}\mu}(k_2, k_1) = \frac{1}{4} [\gamma_{\mathcal{Q}} \gamma_5 \hat{T}^\mu(k_2, k_1)]. \quad (2.7)$$

In the following we will reserve the index ρ for the gamma matrix introduced in this spinor factorization to distinguish it from the vector indices associated with momenta and gluon fields. Quantities containing γ_5 are identified by a tilde.

(b) Colour

The general procedure is similar to the factorization of spinor indices. For each of the lower parts we should perform a general expansion in $SU(3)$ tensors. Making the colour indices explicit the expression for the forward amplitude is

$$T = \hat{S}_{ik} \hat{T}_{ik} + \hat{S}_{\mu, ik}^A \hat{T}_{ik}^{\mu, A} + \hat{S}_{\mu\nu, ik}^{AB} \hat{T}_{ik}^{\mu\nu, AB} \quad (2.8)$$

where i, k are indices in the fundamental representation and A, B are indices in the adjoint representation. We find that

$$\hat{T}_{ik} = \delta_{ik} \hat{T}, \quad (2.9)$$

$$\hat{T}_{ik}^{\mu,A} = t_{ik}^A \hat{T}^\mu, \quad (2.10)$$

$$\hat{T}_{ik}^{\mu\nu,AB} = \delta^{AB} \delta_{ik} \hat{T}_1^{\mu\nu} + i f^{ABC} t_{ik}^C \hat{T}_2^{\mu\nu} + d^{ABC} t_{ik}^C \hat{T}_3^{\mu\nu} \quad (2.11)$$

(The normalization of the colour matrices is fixed by the condition $\text{Tr } t^A t^B = \frac{1}{2} \delta^{AB}$.) For the purposes of our lowest order calculation this general decomposition is too cumbersome. It is more efficient to write explicitly the colour structure of the upper parts, which is simple at tree level,

$$\hat{S}_{ik} = \delta_{ik} \hat{S}, \quad \hat{S}_{\mu,ik}^A = -g t_{ik}^A \hat{S}_\mu, \quad \hat{S}_{\mu\nu,ik}^{AB} = g^2 (t^A t^B)_{ik} \hat{S}_{\mu\nu} \quad (2.12)$$

and to include the extracted colour factors in the lower parts,

$$\hat{T} = \delta_{ik} \hat{T}_{ik}, \quad \hat{T}_\mu^A = -g t_{ik}^A \hat{T}_{\mu,ik}^A, \quad \hat{T}_{\mu\nu}^{AB} = g^2 (t^A t^B)_{ik} \hat{T}_{\mu\nu,ik}^{AB}. \quad (2.13)$$

For book-keeping convenience we have also included in the lower parts the explicit powers of the coupling constant g . In the following the upper parts $\hat{S}$ are those defined on the right-hand side of Eq. (2.12) which are all of zero order in g and identical with the expressions which would be derived in QED with unit charge. Since the colour has been extracted explicitly the hat ($\hat{}$) placed over symbols now denotes only the spinor indices and the square brackets $[]$ indicate from now on the traces over these indices.

(c) Factorization of momentum integrals

The upper and lower parts in Eq. (2.2) are coupled by four dimensional integrations over the repeated loop momenta. We reduce these four dimensional integrations to one dimensional integrals over light cone fractions by Taylor expansion of the upper parts about the collinear direction. The upper parts then become,

$$\hat{S}(k) = \hat{S}(xp) + \left. \frac{\partial \hat{S}}{\partial k^\mu} \right|_{k=xp} (k-xp)^\mu + \frac{1}{2} \left. \frac{\partial^2 \hat{S}}{\partial k^\mu \partial k^\nu} \right|_{k=xp} (k-xp)^\mu (k-xp)^\nu + \dots \quad (2.14)$$

$$\hat{S}_\mu(k_2, k_1) = \hat{S}_\mu(x_2 p, x_1 p) + \left. \frac{\partial \hat{S}_\mu}{\partial k_1^\nu} \right|_{\substack{k_1=x_1 p, \\ k_2=x_2 p}} (k_1-x_1 p)^\nu + \left. \frac{\partial \hat{S}_\mu}{\partial k_2^\nu} \right|_{\substack{k_1=x_1 p, \\ k_2=x_2 p}} (k_2-x_2 p)^\nu + \dots \quad (2.15)$$

$$\hat{S}_{\mu\nu}(k_2, k, k_1) = \hat{S}_{\mu\nu}(x_2 p, y p, x_1 p) + \dots \quad (2.16)$$

The retention of only a limited number of terms in these Taylor expansions will be justified a posteriori. Since the upper parts now depend only on the light cone fractions x_i , we arrive at the desired form after performing the integrals over the other momentum components. Explicitly we rewrite,

$$\int d^4 k_i = \int d^4 k_i dx_i \delta(x_i - k_{iM}) \quad (2.17)$$

and perform the momentum integral over the lower part. For example, the second term on the right-hand side in Eq. (2.15) contributes to T (after spin, colour and momentum factorization) as,

$$T = \dots + \int dx_2 dx_1 \hat{G}_{\mu\nu}(x_2, x_1) T^{\mu\nu}(x_2, x_1) + \int dx_2 dx_1 \tilde{G}_{\mu\nu}(x_2, x_1) \hat{T}^{\mu\nu}(x_2, x_1) + \dots \quad (2.18)$$

where

$$\hat{G}_{\mu\nu}(x_2, x_1) = \left[\frac{\partial \hat{S}_\mu(k_2, k_1)}{\partial k_1^\nu} \gamma_5 \right] \bigg|_{k_i=x_i p}, \quad \tilde{G}_{\mu\nu}(x_2, x_1) = \left[\frac{\partial \hat{S}_\mu(k_2, k_1)}{\partial k_1^\nu} \gamma_5 \gamma_5 \right] \bigg|_{k_i=x_i p} \quad (2.19)$$

and

$$T_{\sigma\mu\nu}(x_2, x_1) = \frac{1}{4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x_1 - k_1 n) \delta(x_2 - k_2 n) (k_1 - x_1 p)_\nu [\delta_\sigma \hat{T}_\mu(k_2, k_1)], \quad (2.20)$$

$$\hat{T}_{\sigma\mu\nu}(x_2, x_1) = \frac{1}{4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x_1 - k_1 n) \delta(x_2 - k_2 n) (k_1 - x_1 p)_\nu [\delta_\sigma \delta_5 \hat{T}_\mu(k_2, k_1)]. \quad (2.21)$$

As seen from this example, the upper parts σ , $\tilde{\sigma}$ are calculated for $k_i = x_i p$ so that the only dimensional quantity they depend on at fixed x_i and x_B is Q^2 . Since both derivatives and gluon insertions decrease the canonical dimension of σ 's, the upper parts will contribute more and more powers of $1/Q^2$. The lower parts Γ contain integrals over transverse momenta and off-shellness of the form

$$(k_i - x_i p)_\mu = \frac{k_{i\perp}^2 + k_{T\perp}^2}{2x_i} n_\mu + k_{T\perp\mu} \quad (2.22)$$

so that the canonical dimension of Γ 's increases with the number of derivatives and/or gluon fields. Since, after Taylor expansion, there is no Q^2 dependence in the lower parts, the only dimensional factor contained in the Γ 's (in the massless limit) is Λ , the strong interaction scale. In this way we construct the power series in Λ^2/Q^2 .

For the purposes of the power counting it is convenient to introduce a projector which removes the collinear components of any vector,

$$(k - x p)_\mu^\perp = \omega_\mu^\perp k^\mu, \quad \omega_\mu^\perp = g_\mu^\perp - p^\perp n_\mu. \quad (2.23)$$

The projector ω can be introduced also in the sums over gluon indices which link the upper and lower parts, because in our gauge $n^\mu A_\mu = 0$. For example, Eq. (2.18) can be rewritten as

$$\begin{aligned} T &= \dots + \delta_{\sigma\mu\nu} T^{\sigma\mu\nu} + \tilde{\delta}_{\sigma\mu\nu} \tilde{T}^{\sigma\mu\nu} + \dots = \\ &= \dots + \delta_{\sigma\mu'\nu'} \omega_{\mu'}^{\mu'} \omega_{\nu'}^{\nu'} T^{\sigma\mu\nu} + \tilde{\delta}_{\sigma\mu'\nu'} \omega_{\mu'}^{\mu'} \omega_{\nu'}^{\nu'} \tilde{T}^{\sigma\mu\nu} + \dots \end{aligned} \quad (2.24)$$

Using an obvious extension of the notation given above we find that the generic term with $2F$ fermion lines, n_G gluon lines and n_D derivatives coming from Taylor expansion gives a contribution to T

$$T = \dots + \int d\{x\} \sigma_{\rho_1} \dots \sigma_{\rho_F} \mu_1 \dots \mu_B (\{x\}, Q^2, x_B) \omega_{\mu_1}^{\mu_1'} \dots \omega_{\mu_B}^{\mu_B'} T^{\rho_1 \dots \rho_F \mu_1 \dots \mu_B} (\{x\}, p, n, \Lambda) \quad (2.25)$$

The upper and lower parts are coupled by the integrals over the set of light cone fractions $\{x\}$ and by the summed four vector indices. The indices ρ belong to the gamma matrices introduced to factor the spinor indices of the $2F$ fermion lines present in the t channel. The μ indices represent both the n_D powers of momenta produced by Taylor expansion and the n_G gluon lines

$$B = n_G + n_D. \quad (2.26)$$

As in the explicit example, Eq. (2.24), all B indices enter in an equivalent way.

The reduction to scalar functions coupled only by integrals over the light cone fractions $\{x\}$ is completed by performing an expansion of the lower part in invariant amplitudes

$$T^{\rho_1 \dots \rho_F \mu_1 \dots \mu_B} (\{x\}, p, n, \Lambda) = \sum_i \Lambda_i^{\tau_i-2} e_i^{\rho_1 \dots \rho_F \mu_1 \dots \mu_B} (p, n) T_i(\{x\}). \quad (2.27)$$

The T_i in this equation are defined to be dimensionless. The dimensions of the left-hand side of this equation are fixed by dimensional analysis

$$d(T) = 3F + B - 2. \quad (2.28)$$

Hence the minimum value of twist τ at which a given invariant amplitude contributes is given by,

$$\tau_i \geq 3F + B - \max_{\{i\}} [d(e_i)]. \quad (2.29)$$

To find the maximal dimension of the polarizers e_i we have to consider all possible tensors with $F+B$ indices, constructed from p_μ , n_μ , $g_{\mu\nu}$ and $\epsilon_{\mu\nu\alpha\beta}$.

The relevant point is that the canonical mass dimension of the vector n_μ is $d(n_\mu) = -1$, whereas $d(p_\mu) = 1$. Formally, this follows from the constraint $pn = 1$, which is allowed since all formulae in the light-like gauge are invariant under rescaling $n_\mu \rightarrow n_\mu$. Equivalently, we could work without the constraint $pn = 1$, ascribing a standard dimension for the vector n , $d(n_\mu) = 1$. Then, however, n_μ will always appear in the form n_μ/pn so that again $d(n_\mu/pn) = -1$.

Since the polarizers e_i are always projected by $\omega_\mu^{u'}$ in all vector indices $\mu_1, \dots, \mu_B$, we can limit ourselves to tensors which are orthogonal to n_μ :

$$e^{\mu_1 \dots \mu_F} p^{\mu_1 \dots \mu_B} m_{\mu_k} = 0, \quad k=1, \dots, B. \quad (2.30)$$

By a simple argument we find²⁷⁾

$$\max[d(e_i)] = F - \frac{1}{2}[1 - (-1)^B]. \quad (2.31)$$

Thus the minimal value of τ_i is given by

$$\tau_i^{\text{MIN}} = 2F + B + \frac{1}{2}[1 - (-1)^B]. \quad (2.32)$$

Equation (2.32) implies that only a limited number of fermion lines, gluon lines and derivatives contributes to a given power of $1/Q^2$.

We emphasize that the above statement is gauge dependent. The gauge condition $nA = 0$ allows us to eliminate many polarizers which are present in the covariant gauges as, e.g.,

$$e^{\mu_1 \dots \mu_F} p^{\mu_1 \dots \mu_B} = p^{\mu_1 \dots \mu_F} p^{\mu_1 \dots \mu_B} \quad (2.33)$$

for $n_G = B$, $n_D = 0$. In the latter case we have $\dim(e) = B+F$, $\tau_i^{\text{MIN}} = 2F-2$ so that an arbitrarily large number of gluon lines contributes to each power of $1/Q^2$ in covariant gauges. These longitudinal gluons in the end make no contribution and can be eliminated by repeated use of the Ward identities. In the light-like gauge the organization of the contributions from various graphs is such that the longitudinal gluons are automatically absent.

Note that the power counting argument would also break down if the upper part continued to depend on a small invariant. This would be the case if the Taylor expansion were performed about an off-shell point with small virtual mass ($k^2 \neq 0$).

The polarizers e_i present at $\tau = 2$ and $\tau = 4$ are given in Table 1. At order $1/Q^2$ for non-singlet deep inelastic scattering we are linked to the graphs of Figs. 2a,b,c. Moreover in Figs. 2a,b,c we need take only 2, 1 and 0 momentum derivatives respectively.

The power counting argument which we have given is conventionally called naïve since it does not take account of possible divergences, generated by Taylor expansion. The ultra-violet divergences have been already discussed in Section 1 - we think that the standard oversubtraction procedure as in Refs. 18) and 26) or the equivalent removal of mass singularities as in Refs. 25) and 28) is all that is needed to convert the ultra-violet singularities into logarithms of Q^2 .

The gauge divergences coming from the $1/kn$ singularities of the gluon propagators in the light-like gauge are regularized by the principal value prescription²⁹⁾

$$\frac{1}{kn} \rightarrow \frac{kn}{(kn)^2 + \delta^2 (pn)^2} \quad (2.34)$$

We expect that for fixed $\delta \neq 0$ the naïve power counting is legitimate since all intermediate quantities are finite and the δ regulator does not interfere with the dimensional analysis ($d(\delta) = 0$)^{*)}.

In the final result for the physical quantities all δ singularities will cancel. Indeed, we demonstrate in the next subsection that the parton correlation functions we are working with are manifestly gauge invariant.

d) Ward identities for derivatives

The final step in the factorization is the recovery of manifest gauge invariance by use of the Ward identities. The forward amplitudes $\hat{S}$ satisfy the following low energy theorems

*) The consistency of this procedure through to second order in perturbation theory has been shown in Ref. 28).

$$\begin{aligned} \left. \frac{\partial \hat{S}(k)}{\partial k^\mu} \right|_{k=x_p} &= \hat{S}_\mu(x_p, x_p), \quad \left. \frac{1}{2} \frac{\partial^2 \hat{S}(k)}{\partial k^\mu \partial k^\nu} \right|_{k=x_p} = \hat{S}_{\mu\nu}(x_p, x_p, x_p), \\ \left. \frac{\partial \hat{S}_\mu(k_2, k_1)}{\partial k_1^\nu} \right|_{k_1=x_{1p}} &= \hat{S}_{\mu\nu}(x_2p, x_{1p}, x_{1p}), \quad \left. \frac{\partial \hat{S}_\nu(k_2, k_1)}{\partial k_2^\mu} \right|_{k_1=x_{1p}} = \hat{S}_{\mu\nu}(x_2p, x_2p, x_{1p}). \end{aligned} \quad (2.35)$$

Because the colour degrees of freedom have been included in the lower blobs, Eq. (2.35) are the same as the corresponding QED expressions. We remind the reader that $\hat{S}$, $\hat{S}^\mu$ and $\hat{S}^{\mu\nu}$ are all of zero order in the strong coupling g . Equations (2.35) allow one to combine the powers of momenta produced by Taylor expansion with terms containing more gluon fields.

As an example we take the second term of Eq. (2.14) and the first term of Eq. (2.15). These two terms give a contribution to T which we write as

$$\begin{aligned} T &= \dots + \int \frac{d^4 k}{(2\pi)^4} \left[\left. \frac{\partial \hat{S}(k)}{\partial k^\mu} \right|_{k=x_p} (k-x_p)^\mu \hat{T}(k) \right] + \int \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \left[\hat{S}_\mu(x_2p, x_{1p}) \hat{T}^\mu(k_2, k_1) \right] + \dots \\ &= \dots + \int dx_2 dx_1 \left\{ S_{\mathcal{G}\mu}(x_2p, x_{1p}) \omega_\mu^\dagger \mathcal{B}^{\mathcal{G}\mu'}(x_2, x_1) + \tilde{S}_{\mathcal{G}\mu}(x_2p, x_{1p}) \omega_\mu^\dagger \tilde{\mathcal{B}}^{\mathcal{G}\mu'}(x_2, x_1) \right\} + \dots \end{aligned} \quad (2.36)$$

where

$$S_{\mathcal{G}\mu} = \left[\hat{S}_\mu(x_2p, x_{1p}) \gamma_5 \right], \quad \tilde{S}_{\mathcal{G}\mu} = \left[\hat{S}_\mu(x_2p, x_{1p}) \gamma_5 \gamma_5 \right] \quad (2.37)$$

and

$$\begin{aligned} \mathcal{B}_{\mathcal{G}\mu}(x_2, x_1) &= \delta(x_2 - x_1) \int \frac{d^4 k_1}{(2\pi)^4} \delta(x_1 - k_{1n}) k_{1\mu} \frac{1}{4} \left[\gamma_5 \hat{T}(k_1) \right] + \\ &\quad + \int \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \delta(x_2 - k_{2n}) \delta(x_1 - k_{1n}) \frac{1}{4} \left[\gamma_5 \hat{T}_\mu(k_2, k_1) \right], \\ \tilde{\mathcal{B}}_{\mathcal{G}\mu}(x_2, x_1) &= \delta(x_2 - x_1) \int \frac{d^4 k_1}{(2\pi)^4} \delta(x_1 - k_{1n}) k_{1\mu} \frac{1}{4} \left[\gamma_5 \gamma_5 \hat{T}(k_1) \right] + \\ &\quad + \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta(x_2 - k_{2n}) \delta(x_1 - k_{1n}) \frac{1}{4} \left[\gamma_5 \gamma_5 \hat{T}_\mu(k_2, k_1) \right]. \end{aligned} \quad (2.38)$$

All other terms can be similarly assembled to give the combination of momentum and gluon field prescribed by the covariant derivative. In the following sections it will be convenient to have expressions for the B's in position space. These expressions can be derived from the position space expressions for the parton hadron amplitudes

$$T_g(k) = \frac{1}{4} \int d^4x e^{ikx} \langle P | T \{ \bar{\Psi}(0) \gamma_5 \Psi(x) \} | P \rangle, \quad (2.39)$$

$$T_{g\mu}(k_2, k_1) = \frac{1}{4} \int d^4x d^4x' e^{i(k_2 - k_1)x'} e^{ik_1x} \langle P | T \{ \bar{\Psi}(0) \gamma_5 D_\mu(x') \Psi(x) \} | P \rangle. \quad (2.40)$$

The covariant derivative in Eq. (2.40) is

$$D_\mu(x) = i\partial_\mu - gA_\mu(x). \quad (2.41)$$

Colour degrees of freedom are understood and the gluon field A^μ is a three by three matrix in colour space. The four momentum integrations are performed by writing

$$\int \frac{d^4k}{(2\pi)^4} e^{ikx} \delta(x - km) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \delta^4(x - \lambda m). \quad (2.42)$$

The expression for $B^{\rho\mu}$ is

$$B^{\rho\mu}(x_2, x_1) = \frac{1}{4} \int \frac{d\eta}{2\pi} \frac{d\lambda}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \langle P | T \{ \bar{\Psi}(0) \gamma^\rho D^\mu(\eta) \Psi(\lambda) \} | P \rangle. \quad (2.43)$$

As before we use the shorthand notation for fields on the light cone,

$$\Psi(\lambda) \equiv \Psi(\lambda m), \quad D_\mu(\lambda) \equiv i\partial_\mu - gA_\mu(\lambda m). \quad (2.44)$$

We therefore find that all quantities required for our factorized formula are given by Fourier transform of products of fields and covariant derivatives on the light cone. The complete list of light cone products which we require is,

$$A^\rho(\lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \gamma^\rho \Psi(\lambda) \} | P \rangle, \quad (2.45)$$

$$\tilde{A}^{\mu}(\lambda) = \frac{1}{4} \langle \mathbb{I} | T \{ \bar{\Psi}(0) \gamma^{\mu} \gamma_5 \Psi(\lambda) \} | \mathbb{I} \rangle, \quad (2.46)$$

$$\tilde{B}^{\mu}(\eta, \lambda) = \frac{1}{4} \langle \mathbb{I} | T \{ \bar{\Psi}(0) \gamma^{\mu} D^{\mu}(\eta) \Psi(\lambda) \} | \mathbb{I} \rangle, \quad (2.47)$$

$$\tilde{\tilde{B}}^{\mu}(\eta, \lambda) = \frac{1}{4} \langle \mathbb{I} | T \{ \bar{\Psi}(0) \gamma^{\mu} \gamma_5 D^{\mu}(\eta) \Psi(\lambda) \} | \mathbb{I} \rangle, \quad (2.48)$$

$$C^{\mu\nu}(\omega, \eta, \lambda) = \frac{1}{4} \langle \mathbb{I} | T \{ \bar{\Psi}(0) \gamma^{\mu} D^{\mu}(\omega) D^{\nu}(\eta) \Psi(\lambda) \} | \mathbb{I} \rangle, \quad (2.49)$$

$$\tilde{C}^{\mu\nu}(\omega, \eta, \lambda) = \frac{1}{4} \langle \mathbb{I} | T \{ \bar{\Psi}(0) \gamma^{\mu} \gamma_5 D^{\mu}(\omega) D^{\nu}(\eta) \Psi(\lambda) \} | \mathbb{I} \rangle. \quad (2.50)$$

The quantities which appear naturally in our calculation are the Fourier transforms of Eqs. (2.45)-(2.50) defined as,

$$A(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} A(\lambda), \quad (2.51)$$

$$B(x_2, x_1) = \int \frac{d\eta}{2\pi} \frac{d\lambda}{2\pi} e^{i\eta(x_2 - x_1)} e^{i\lambda x_1} B(\eta, \lambda), \quad (2.52)$$

$$C(x_2, y, x_1) = \int \frac{d\omega}{2\pi} \frac{d\eta}{2\pi} \frac{d\lambda}{2\pi} e^{i\omega(x_2 - y)} e^{i\eta(y - x_1)} e^{i\lambda x_1} C(\omega, \eta, \lambda). \quad (2.53)$$

e) Factorized form

We can perform a tensor decomposition of the light cone correlation functions of Eqs. (2.45)-(2.50) dropping terms which have scalar coefficients of mass dimension higher than two. This is the expansion into polarizers discussed previously.

$$A^S(\lambda) = A_0(\lambda) p^S + A_1(\lambda) m^S, \quad (2.54)$$

$$\tilde{A}^S(\lambda) = 0, \quad (2.55)$$

$$B^{\mu}(\eta, \lambda) = B_0(\eta, \lambda) p^S p^\mu + B_1(\eta, \lambda) d^{\mu} + B_2(\eta, \lambda) p^S \eta^\mu + B_3(\eta, \lambda) m^S p^\mu, \quad (2.56)$$

$$\tilde{B}^{\mu}(\eta, \lambda) = i \epsilon^{\mu} \tilde{B}_1(\eta, \lambda), \quad (2.57)$$

$$\begin{aligned} C^{\mu\nu}(\omega, \eta, \lambda) = & C_0(\omega, \eta, \lambda) p^S p^\mu p^\nu + \\ & + C_1(\omega, \eta, \lambda) p^S d^{\mu\nu} + C_2(\omega, \eta, \lambda) p^\mu d^{\nu} + C_3(\omega, \eta, \lambda) p^\nu d^{\mu} \\ & + C_4(\omega, \eta, \lambda) m^S p^\mu p^\nu + C_5(\omega, \eta, \lambda) \eta^\mu p^S p^\nu + C_6(\omega, \eta, \lambda) \eta^\nu p^S p^\mu, \end{aligned} \quad (2.58)$$

$$\tilde{C}^{\mu\nu}(\omega, \eta, \lambda) = i \epsilon^{\mu\nu} p^S \tilde{C}_1(\omega, \eta, \lambda) + i \epsilon^{\mu} p^\nu \tilde{C}_2(\omega, \eta, \lambda) + i \epsilon^{\nu} p^\mu \tilde{C}_3(\omega, \eta, \lambda), \quad (2.59)$$

where we have defined the transverse tensors as *)

$$d^{\mu\nu} = g^{\mu\nu} - p^\mu m^\nu - p^\nu m^\mu, \quad \epsilon^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} m_\alpha p_\beta. \quad (2.60)$$

It is clear that not all the scalar functions defined above will appear in the formula for the forward scattering amplitude because of the projection with the tensor ω which removes components along p^μ or p^ν . The parton correlation functions which survive are $\{A_0, A_1, B_1, B_2, \tilde{B}_1, C_1, \tilde{C}_1\}$.

To obtain the measured structure functions we now calculate the S channel discontinuity of the forward scattering amplitude. The lower parts are linked to the forward amplitude only by the light cone integrals x and are

*) Our ϵ tensor is defined such that $\epsilon^{0123} = +1$. All other conventions follow Bjorken and Drell.

hence independent of $s = Q^2(1-x_B)/x_B$. The discontinuity acts only on the upper parts. We therefore define the following s channel discontinuities (or equivalently discontinuities in x_B)

$$\begin{aligned} q_0(x) &= \frac{1}{2\pi} \text{disc} [\hat{S}(xp) \not{x}], \quad q_1(x) = \frac{1}{2\pi} \text{disc} [\hat{S}(xp) \not{x}], \\ b_1(x_2, x_1) &= \frac{1}{2\pi} \text{disc} \left\{ d^{\otimes \mu} [\hat{S}_\mu(x_2p, x_1p) \not{x}] \right\}, \quad \tilde{b}_1(x_2, x_1) = \frac{1}{2\pi} \text{disc} \left[i \epsilon^{\otimes \mu} [\hat{S}_\mu(x_2p, x_1p) \not{x} \not{s}] \right], \\ b_2(x_2, x_1) &= \frac{1}{2\pi} \text{disc} \left\{ m^{\otimes \mu} [\hat{S}_\mu(x_2p, x_1p) \not{x}] \right\}, \\ c_1(x_2, y, x_1) &= \frac{1}{2\pi} \text{disc} \left\{ d^{\otimes \mu\nu} [\hat{S}_{\mu\nu}(x_2p, yp, x_1p) \not{x}] \right\}, \\ \tilde{c}_1(x_2, y, x_1) &= \frac{1}{2\pi} \text{disc} \left\{ i \epsilon^{\otimes \mu\nu} [\hat{S}_{\mu\nu}(x_2p, yp, x_1p) \not{x} \not{s}] \right\}. \end{aligned} \quad (2.61)$$

Strictly speaking, we are allowed to take discontinuity in x_B under the light-cone fraction integrals only if the integration limits are finite. We discuss this point in the next section where our parton-like approach is compared with the operator product expansion. We discuss there constraints on the integration limits implied by the analytic properties of the hadron-parton amplitudes.

The final result of the factorization procedure, described in this section - the general form of the structure function of deep inelastic scattering - may be written as

$$\begin{aligned} W(Q, p) &= \frac{1}{2\pi} \text{disc} T(Q, p) = \int dx \left\{ q_0(x) A_0(x) + q_1(x) A_1(x) \right\} + \\ &+ \int dx_2 dx_1 \left\{ b_1(x_2, x_1) B_1(x_2, x_1) + \tilde{b}_1(x_2, x_1) \hat{B}_1(x_2, x_1) + b_2(x_2, x_1) B_2(x_2, x_1) \right\} + \\ &+ \int dx_2 dy dx_1 \left\{ c_1(x_2, y, x_1) C_1(x_2, y, x_1) + \tilde{c}_1(x_2, y, x_1) \tilde{C}_1(x_2, y, x_1) \right\}. \end{aligned} \quad (2.62)$$

$A_0(x)$ is the leading twist quark distribution function. All other parton correlation functions (denoted by capital letters) are of order Λ^2 . The corresponding perturbative amplitudes (small letters) are of order $1/Q^2$.

3. - OPERATOR PRODUCT EXPANSION AND THE PARTON MODEL

In the preceding section we constructed the factorized expression for deep inelastic scattering, Eq. (2.62). The upper parts (lepton-parton amplitudes) are connected to the lower parts (parton correlation functions) only by integrals over light-cone fractions $\{x\}$.

This formula is the starting point for the analysis of the power corrections in deep inelastic scattering. The main technical problem we have to resolve is the redundancy of the basis $\{A_1, B_1, B_2, \tilde{E}_1, C_1, \tilde{C}_1\}$. By using the equations of motion, Lorentz and gauge invariance we will reduce it to the collinear basis in Section 4 and to the transverse basis in Section 6.

Before starting the discussion of the reduction procedure, we make in this section a few general comments concerning our formalism. These remarks are relevant for any basis.

The further analysis of the factorized formula in Eq. (2.62) can be done either in the "operator mode" or in the "parton mode". Our approach is flexible enough to switch to either of these languages at any stage of the discussion. For this reason throughout this paper we use the light-cone position (λ, η, ω) and the light-cone fraction (x, x_1, x_2, y) representations in parallel.

They are related by Fourier transformation and display complementary aspects of factorization. The light-cone position representation is useful to make contact with the OPE approach. The light-cone fraction representation is appropriate for establishing the parton language.

We discuss first the "operator mode". To relate moments of the structure functions to hadronic matrix elements of Wilson operators one compares the OPE formula for the forward Compton amplitude $T(x_B, Q^2)$, with the dispersion relation for the same quantity. The comparison is performed in the deep Euclidean region $Q^2 \rightarrow \infty, x_B \rightarrow \infty$.

To write down the dispersion relations it is convenient to consider particular linear combinations of leptonic currents and/or hadronic targets for which the corresponding amplitudes T_i are even or odd under $s \leftrightarrow u$ crossing,

$$T_i(-x_B, Q^2) = (-1)^\omega T_i(x_B, Q^2), \quad \omega = \pm 1, \quad i = L, T, 3. \quad (3.1)$$

Thus the amplitudes

$$T_i = T_{L,T}^{eP} - T_{L,T}^{eN}, \quad T_i = T_3^{\nu P} - T_3^{\bar{\nu} P}$$

are even and

$$T_i = T_{L,T}^{\nu P} - T_{L,T}^{\bar{\nu} P}, \quad T_i = T_3^{\nu P} + T_3^{\bar{\nu} P}$$

are odd. The dispersion relation then has the form

$$T_i(x_B, Q^2) = \frac{1}{\pi x_B} \int_0^1 dx \left(\frac{x}{x_B} \right)^\eta \frac{F_i(x, Q^2)}{1 - \left(\frac{x}{x_B} \right)^2 - i\epsilon}, \quad \eta = \frac{1+\omega}{2} \quad (3.2)$$

where F_i is the corresponding structure function. More precisely, if $T^{\mu\nu}$ and $W^{\mu\nu}$ have the expansions

$$T^{\mu\nu} = \sum_i c_i^{\mu\nu} T_i, \quad W^{\mu\nu} = \sum_i c_i^{\mu\nu} F_i, \quad W^{\mu\nu} = \frac{1}{2\pi} \delta_{\mu\nu} T^{\mu\nu} \quad (3.3)$$

then F_i and T_i are related as in Eq. (3.2).

In the limit $x_B \rightarrow \infty$ the Compton amplitude has the power expansion

$$T_i(x_B, Q^2) = \frac{1}{\pi} \sum_{N=0}^{\infty} \left(\frac{1}{x_B} \right)^{2N+\eta+1} \int_0^1 dx x^{2N+\eta} F_i(x). \quad (3.4)$$

On the other hand, the OPE provides the expression

$$T_i(x_B, Q^2) = \frac{1}{\pi} \sum_{N=0}^{\infty} \left(\frac{1}{x_B} \right)^{2N+\eta+1} c_N^i O_N \quad (3.5)$$

which allows the identification of the moments of $F_i(x)$ with the matrix elements O_N of the Wilson operators ²⁾.

In our notation the information provided by the operator product expansion is equivalent to that contained in Eq. (2.62) for the structure function. To recover the full result coming from OPE we must include the u channel terms (crossed photon lines) and rewrite the formula analogous to Eq. (2.62) before the discontinuity has been taken (i.e., for the forward scattering amplitude T).

Let us consider first the leading twist term $a_0 A_0$ (for $i = T, 3$). The (uncut) lepton-parton amplitude with an appropriate crossing symmetry is of the form

$$\begin{aligned}
 a_{0,i}^{\mu\nu}(x) &= e_n^{\mu\nu} \frac{1}{2\pi x_B} \left[\frac{1}{1 - \frac{x}{x_B} - i\epsilon} - \omega \frac{1}{1 + \frac{x}{x_B} - i\epsilon} \right] \\
 &= e_n^{\mu\nu} \frac{1}{\pi x_B} \left(\frac{x}{x_B} \right)^\eta \frac{1}{1 - \left(\frac{x}{x_B} \right)^2 - i\epsilon}
 \end{aligned} \tag{3.6}$$

so that we get from Eq. (2.62)

$$T_i(x_B, Q^2) = \frac{1}{\pi x_B} \int_{-\infty}^{\infty} dx \left(\frac{x}{x_B} \right)^\eta \frac{1}{1 - \left(\frac{x}{x_B} \right)^2 - i\epsilon} A_0(x) + O\left(\frac{1}{Q^2}\right). \tag{3.7}$$

Comparing Eqs. (3.7), (3.2) one is tempted to identify $A_0(x)$ with $F_1(x)$. In general, however, this is not legitimate since the integration limits in Eqs. (3.2), (3.7) do not coincide. The dispersive integral in Eq. (3.2) extends over a finite range, corresponding to the s and u channel cuts, whereas the light-cone fraction integration in Eq. (3.7) is a priori unbounded. Therefore we are not allowed to take the discontinuity on both sides of Eq. (3.7) before doing the x integral.

Nevertheless, even if the local identification $F_1(x) = A_0(x)$ is not necessarily true, we can get relations between moments.

Since the left-hand side in Eq. (3.7) admits a power expansion in $1/x_B$, we can also formally expand the right-hand side. By comparing the coefficients of $(1/x_B)^N$ we get

$$\int_0^1 dx x^{N-1} F(x) = \int_{-\infty}^{\infty} dx x^{N-1} A_0(x) = \int_{-\infty}^{\infty} dx x^{N-1} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \not{x} \Psi(\lambda) \} | P \rangle \tag{3.8}$$

(to be precise, one should sum over flavours).

The tower of local Wilson operators can be reconstructed using the identity

$$\begin{aligned}
 \int_{-\infty}^{\infty} dx x^N \int \frac{d\lambda}{2\pi} e^{i\lambda x} \Psi(\lambda) &= \sum_{m=0}^{\infty} \frac{1}{m!} \frac{\partial^m}{\partial \lambda^m} \Psi(\lambda) \Big|_{\lambda=0} \int_{-\infty}^{\infty} dx x^N (-i)^m \frac{\partial^m}{\partial x^m} \delta(x) = \\
 &= i^N \frac{\partial^N}{\partial \lambda^N} \Psi(\lambda) \Big|_{\lambda=0}.
 \end{aligned} \tag{3.9}$$

In the light-like gauge, $nA = 0$, we have

$$\begin{aligned}\Psi(\lambda m) &= \Psi(0) + \lambda m^\mu \partial_\mu \Psi(0) + \frac{1}{2} \lambda^2 m^\mu m^\nu \partial_\mu \partial_\nu \Psi(0) + \dots \\ &= \Psi(0) + (-i\lambda) m^\mu D_\mu \Psi + \frac{1}{2} (-i\lambda)^2 m^\mu m^\nu D_\mu D_\nu \Psi + \dots\end{aligned}\quad (3.10)$$

so that finally

$$\int_0^1 dx x^{N-1} F(x) = m^{\mu_1} \dots m^{\mu_N} \langle P | \bar{\Psi}(0) \delta_{\mu_1} D_{\mu_2} \dots D_{\mu_N} \Psi(0) | P \rangle. \quad (3.11)$$

In the same way one can treat multiparton correlation functions. Let us consider, for example, one term which contributes to the forward Compton scattering amplitude at order $1/Q^2$

$$T(x_2, Q^2) = \dots + \int dx_2 dx_1 t_2(x_2, x_1) T_2(x_2, x_1) + \dots \quad (3.12)$$

where

$$t_2(x_2, x_1) = \frac{1}{2\pi x_B} \left[\frac{1}{1 - \frac{x_1}{x_B} - i\epsilon} \frac{1}{1 - \frac{x_2}{x_B} - i\epsilon} - \omega \frac{1}{1 + \frac{x_1}{x_B} - i\epsilon} \frac{1}{1 + \frac{x_2}{x_B} - i\epsilon} \right], \quad (3.13)$$

$$T_2(x_2, x_1) = \int \frac{d\lambda}{2\pi} \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \langle P | T \{ \bar{\Psi}(0) G(\eta) \Psi(\lambda) \} | P \rangle \quad (3.14)$$

and $G(\eta)$ is a local operator constructed from gluon fields. [In particular, for $G(\eta) = \gamma_\mu \not{A}_\nu D_T^V(\eta) D_T^H(\eta)$ the correlation function $T_2(x_2, x_1)$ represents one of the two elements of the transverse basis, constructed in Section 6 and t_2 in Eq. (3.13) is the corresponding lepton-parton amplitude.]

Taking, e.g., $\omega = 1$, N odd, expanding t_2 in powers of $1/x_B$ and constructing a formal Taylor expansion of the matrix element in Eq. (3.14) around $\eta = \lambda = 0$ we can express this particular contribution as a linear combination of twist-four operators

$$\int_0^1 dx x^{N-1} F(x) = \dots + \sum_{N=0}^{N-1} O_N^{(\tau)} \quad (3.15)$$

where

$$O_N^{(\eta)} = m^{\mu_1} \dots m^{\mu_{N-1}} m^{\nu_1} \dots m^{\nu_{N-1}} \langle P | \bar{\Psi}(0) D_{\mu_1} \dots D_{\mu_{N-1}} G(0) D_{\nu_1} \dots D_{\nu_{N-1}} \Psi(0) | P \rangle \quad (3.16)$$

To establish the connection between these formal operations and the parton language we note that the expansion in inverse powers of x_B is formally equivalent to the sequence of operations defined by first taking the discontinuity in x_B and then taking moments in x_B . For example, we have that ($\omega = 1$)

$$\frac{1}{2i} \text{Disc } t_2(x_2, x_1) = x_B \frac{\delta(x_B - x_2) - \delta(x_B - x_1)}{x_2 - x_1} \quad (3.17)$$

so that

$$\frac{1}{2i} \text{Disc } T(x_B, Q^2) = \dots + x_B \int_{-\infty}^{\infty} dx_2 dx_1 \frac{\delta(x_B - x_2) - \delta(x_B - x_1)}{x_2 - x_1} T_2(x_2, x_1) + \dots \quad (3.18)$$

Taking moments we find that

$$\begin{aligned} \int_0^1 dx_B x_B^{N-1} F(x_B, Q^2) &= \int_{-\infty}^{\infty} dx_B x_B^{N-1} \frac{1}{2i} \text{Disc } T(x_B, Q^2) = \\ &= \sum_{j=0}^{N-1} \int_{-\infty}^{\infty} dx_2 dx_1 x_2^j x_1^{N-1-j} T_2(x_2, x_1) = \sum_{j=0}^{N-1} O_N^{(j)}. \end{aligned} \quad (3.19)$$

If the intermediate step Eq. (3.17) makes sense by itself then the parton interpretation is established. We must therefore show that the discontinuity in x_B can be taken before the integrals over the light cone fractions are performed.

If the parton correlation functions vanish outside a finite range in light cone fraction space, the order of the two operations can be exchanged. The finite range in light cone fraction can be established to any order in perturbation theory because of the general analytic properties of Feynman diagrams.

A possible ansatz is therefore to postulate a similar analytic structure for the hadronic Green's functions ³⁰⁾. This is precisely the assumption which gives meaning to the parton model and it has been used extensively in the leading twist analysis.

The perturbative analyticity can be formulated as follows. For any connected N point Green's function $G(k)$ with the incoming momenta $k_1, \dots, k_N$, $\sum k_i = 0$ one considers all possible cuts which divide the Green's function into two parts with external momenta $\{k_1, \dots, k_{i_r}\}$, $\{k_{i_r+1}, \dots, k_N\}$ and one constructs the corresponding invariants $s_r = (\sum_{\ell=1}^{i_r} k_{i_\ell})^2$. The Green's function $G(k)$ then has the integral representation

$$G(k) = \int_0^\infty d\gamma \int_0^1 \prod d\alpha_i \delta(1 - \sum \alpha_i) \frac{S(\alpha, \gamma)}{\sum_i \alpha_i s_i - \gamma + i\epsilon} \quad (3.20)$$

In other words, $G(k)$ is an analytic function of the s_r , considered as independent variables and the singularities are located in the appropriate lower-half plane for all choices of the invariants s_r .

Using the representation in Eq. (3.20) one can derive limits for the light-cone fraction integrals. For this reason we consider the Green's function $G(k, p)$ with incoming momenta $\{k_1, \dots, k_{N-1}, -(k_1 + \dots + k_{N-1}), p, -p\}$. The parton correlation function $F(x)$ is defined as

$$F(x) = \int d^4k \delta(x - kn) \mathcal{G}(k, p) \quad (3.21)$$

where

$$x = \{x_1, \dots, x_{N-1}\}, \quad x_i = k_i \cdot n, \quad d^4k = \prod_i d^4k_i, \quad \delta(x - kn) = \prod_i \delta(x_i - k_i \cdot n).$$

Inserting the representation Eq. (3.20) into Eq. (3.21) we can perform d^4k_i integrals in the standard way (Wick rotation). The denominator is a quadratic form in the momenta k_i

$$D(k) = \sum_i \alpha_i s_i - \gamma + i\epsilon = \sum_{\alpha\beta} k_\alpha A_{\alpha\beta} k_\beta - 2 \sum_\alpha B_\alpha (p k_\alpha) + C + i\epsilon \quad (3.22)$$

where the matrix A is positive-definite. The linear terms in k can be eliminated by the shift $k_i = \hat{k}_i + r_i p$, where the vector $r_i(\alpha)$ is given by the solution of the linear equation

$$\sum_k A_{ik} r_k = B_i. \quad (3.23)$$

The term $\delta(x-kn)$ in Eq. (3.21) now has the form

$$\delta(x-kn) = \delta(x-nm - \hat{k}n) = \delta(x-nm) - \delta'(x-nm) \hat{k}n + \frac{1}{2} \delta''(x-nm) (\hat{k}n)^2 + \dots$$

so that only the first term survives after performing d^4k_i integrals ; the terms odd in $\hat{k}$ vanish after integrating over angles and for the terms even in $\hat{k}$ we get products of $g_{\mu\nu}$ which vanish when contracted with n_μ .

Finally we get

$$F(x) = \int_0^1 \prod_i d\alpha_i \delta(1 - \sum_i \alpha_i) \delta(x_i - r_i(\alpha)) \Phi(\alpha). \quad (3.24)$$

After solving the system in Eq. (3.23) we get an explicit expression for $r_i(\alpha)$ in terms of the parameters α_i . Since the latter are bounded we can put limits on the x_i . In the simplest case $n = 2$ (two-parton correlation function) we have

$$D(k) = \alpha_1 k^2 + \alpha_2 (p-k)^2 + \alpha_3 (p+k)^2 - \tilde{\sigma} + i\epsilon = k^2 - 2(\alpha_2 - \alpha_3)pk - \tilde{\sigma} + i\epsilon \quad (3.25)$$

so that $r(\alpha) = \alpha_2 - \alpha_3$ and consequently the limits on $x = kn$ are

$$-1 < x < 1. \quad (3.26)$$

For $N > 2$ the calculation is more complicated since $r_i(\alpha)$ is now given by a rational function of the parameters α_i . A more detailed discussion of the limits in the light cone fractions of multiparton correlation functions is given in Appendix A.

Let us summarize the above remarks. If we postulate that the analytic properties of the parton-hadron Green's functions are as suggested by perturbation theory, the light-cone integrals in Eq. (2.62) run over finite ranges. We can then take discontinuity in Eq. (2.62) under the light-cone integrals and work directly in the x space. Without the analyticity assumption only the analysis of moments is meaningful.

In this paper we are more oriented towards the parton approach. Nevertheless, all our formulae can be immediately reinterpreted in the more rigorous operator language. Since the limits of the light-cone integrals are never written explicitly, one can always consider the expressions in the x space as an intermediate step before calculating moments, as in Eq. (3.18). By taking the integral $\int_{-\infty}^{\infty} dx_B x_B^N$ and expanding the light-cone position correlation functions around the origin, one gets the corresponding OPE expressions.

4. - REDUCTION TO THE COLLINEAR BASIS

This section exploits the relations between the parton correlation functions in Eq. (2.62) to reduce the lower parts to the collinear basis. The reduction is a two-step process. The first step is the imposition of the constraints following from Lorentz invariance and the limited n dependence of the parton correlation functions. The second step is the application of the classical equations of motion. Schematically we can describe these processes as the reduction of transverse momentum to off-shellness followed by the replacement of off-shellness by more partons as a consequence of the interaction. This description is only approximate since the parton correlation functions in fact depend only on the light-cone fractions $\{x_i\}$ of the quarks and gluons. The transverse momentum and the off-shellness of the partons have been integrated over. In fact what we will show is that the components of the parton correlation functions in the transverse direction (e.g., those terms which before integration contained powers of transverse momenta) can be expressed in terms of parton correlation functions with greater numbers of partons.

Although the underlying ideas are quite simple the reduction to the collinear basis presents some degree of algebraic complexity. We therefore illustrate the technique using a simple scalar example. This model contains all the features of the QCD reduction except for the gauge invariance which we will discuss separately. The key equations and results for the full QCD case are presented later in this section.

Consider the simple hadronic blob in Fig. 5 giving the distribution of scalar partons inside a hadron. We may define the integrated "off-shellness" E as

$$E(x) = \int \frac{d^4k}{(2\pi)^4} \delta(x - km) k^2 F(k^2, \frac{2pk}{k^2}) \theta((p-k)^2) \quad (4.1)$$

[To simplify the discussion we take the $s = (p-k)^2$ channel cut as indicated by the θ function in Eq. (4.1).] In a similar way we can define the integrated transverse momentum

$$B_1(x) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \delta(x - km) k_T^2 F(k^2, \frac{2pk}{k^2}) \theta((p-k)^2). \quad (4.2)$$

Defining the variable ρ

$$\varrho = \frac{2pk}{k^2} = \frac{1}{x} \left(1 + \frac{k_T^2}{k^2} \right) \quad (4.3)$$

one finds that

$$E(x) = - \frac{1}{2^5 \pi^3} \int dk^2 (k^2)^2 \int_0^{\frac{1}{x}} d\varrho F(k^2, \varrho), \quad (4.4)$$

$$B_1(x) = - \frac{1}{2^5 \pi^3} \int dk^2 (k^2)^2 \int_0^{\frac{1}{x}} d\varrho \frac{1}{2} (1 - \varrho x) F(k^2, \varrho). \quad (4.5)$$

Thus one can easily show that

$$B_1(x) = - \frac{1}{2} \int_x^1 d\omega \left(1 - \frac{x}{\omega} \right) \left\{ \frac{d}{d\omega} E(\omega) \right\}. \quad (4.6)$$

After integration by parts we arrive at the final form

$$B_1(x) = \frac{1}{2} \int_0^1 dy \int_0^1 d\tau \tau \delta(x - \tau y) E(y). \quad (4.7)$$

Equation (4.7) expresses the "transverse momentum" B_1 in terms of the "off-shellness" E . However, the above method becomes cumbersome for the real QCD case, with several parton lines. The QCD reduction is most easily derived using the representation of the parton correlation functions as products of fields on the light-cone [Eqs. (2.45)-(2.50)].

For the scalar theory the light-cone reduction is as follows. Define the matrix elements

$$\langle P | T \{ \varphi(0) i\partial^\mu \varphi(\lambda m) \} | P \rangle = A_0(\lambda) p^\mu + A_1(\lambda) \eta^\mu \quad (4.8)$$

$$\begin{aligned} \langle P | T \{ \varphi(0) (i\partial^\mu) (i\partial^\nu) \varphi(\lambda m) \} | P \rangle = \\ = B_0(\lambda) p^\mu p^\nu + B_1(\lambda) d^{\mu\nu} + B_2(\lambda) (p^\mu m^\nu + p^\nu m^\mu). \end{aligned} \quad (4.9)$$

As before the object of this example is to show how A_1 , B_1 and B_2 are related to the "off-shellness" E

$$E(\lambda) = i^2 \langle P | T \{ \varphi(0) \partial^\mu \partial_\mu \varphi(\lambda n) \} | P \rangle \quad (4.10)$$

which in its turn can be reduced (using the equations of motion for the scalar field, $\partial^\mu \partial_\mu \phi = g\phi^3$), to the expression

$$E(\lambda) = -g \langle P | T \{ \varphi(0) \varphi^3(\lambda n) \} | P \rangle \quad (4.11)$$

In the language of light-cone products the relations between A_1 , B_1 , B_2 and E are derived as follows. Contracting Eq. (4.9) with n_ν we obtain the following light-cone relation

$$B_2(\lambda) = i \frac{\partial}{\partial \lambda} A_1(\lambda). \quad (4.12)$$

The second type of relation follows from the observation that all the n dependence of the left-hand side of Eq. (4.8) is contained in the field $\phi(\lambda n)$. Performing a variation of n^μ (compatible with the constraints $n^2 = 0$, $p \cdot n = 1$)

$$n^\mu \rightarrow n^\mu + m_T^\mu + \frac{m_T^2}{2} p^\mu, \quad m m_T = p m_T = 0 \quad (4.13)$$

and retaining terms of first order in m_T^μ we obtain

$$-i m_T^\nu \lambda \langle P | T \{ \varphi(0) (\partial^\mu \partial_\mu) (\partial^\nu) \varphi(\lambda n) \} | P \rangle = A_1(\lambda) m_T^\mu \quad (4.14)$$

which on substitution of the tensor decomposition Eq. (4.9) gives

$$-i \lambda B_1(\lambda) = A_1(\lambda). \quad (4.15)$$

Using the identity $(\frac{1}{2}E(\lambda) \equiv B_1(\lambda) + B_2(\lambda))$ we obtain a differential equation for A_1

$$\left(\lambda \frac{\partial}{\partial \lambda} + 1 \right) A_1(\lambda) = \frac{\lambda}{2i} E(\lambda). \quad (4.16)$$

Since all dependence of Eq. (4.8) on n^μ must vanish in the limit $\lambda \rightarrow 0$, we have the boundary condition $A_1(0) = 0$. The solution is

$$A_1(\lambda) = \frac{\lambda}{2i} \int_0^1 d\tau \tau E(\lambda\tau), \quad (4.17)$$

$$B_1(\lambda) = \frac{1}{2} \int_0^1 d\tau \tau E(\lambda\tau). \quad (4.18)$$

Eq. (4.18) is the Fourier transform of Eq. (4.7).

As a last illustration we derive the same results using the method of moments. We define the matrix elements of local operators,

$$i^N \langle P | \varphi(0) \partial^{\mu_1} \dots \partial^{\mu_N} \varphi(0) | P \rangle = S(N) p^{\mu_1} \dots p^{\mu_N} + \frac{E(N)}{4N} \sum_{i \neq j} g^{\mu_i \mu_j} p^{\mu_1} \dots p^{\mu_N} / p^{\mu_i} p^{\mu_j}. \quad (4.19)$$

Note that this formula has no n^μ dependence. By contraction with n^μ we obtain ($x \equiv (in \cdot \partial)$)

$$\begin{aligned} i \langle P | \varphi(0) \partial^{\mu_1} x^{N-1} \varphi(0) | P \rangle &= S(N) p^{\mu_1} + E(N) \frac{N-1}{2N} n^{\mu_1}, \\ i^2 \langle P | \varphi(0) \partial^{\mu_1} \partial^{\mu_2} x^{N-2} \varphi(0) | P \rangle &= S(N) p^{\mu_1} p^{\mu_2} + \frac{E(N)}{2N} d^{\mu_1 \mu_2} + (n^{\mu_1} p^{\mu_2} + n^{\mu_2} p^{\mu_1}) \frac{N-1}{2N} E(N), \\ i^2 \langle P | \varphi(0) \partial^{\mu} \partial_{\mu} x^{N-2} \varphi(0) | P \rangle &= E(N). \end{aligned} \quad (4.20)$$

Thus defining the inverse Mellin transforms

$$E(N) = \int dx x^{N-2} E(x), \quad (4.21)$$

$$\frac{E(N)}{2N} = \int dx x^{N-2} B_1(x), \quad (4.22)$$

we obtain the relation

$$B_1(x) = \frac{1}{2} \int dy \int_0^1 d\tau \tau \delta(x - \tau y) E(y). \quad (4.23)$$

From this derivation, it is clear that the integral over τ comes from the $1/N$ terms in Eq. (4.20).

To summarize we have shown in this simple scalar case that the parton correlation functions containing transverse momenta $k_T^\mu, k_T^\mu k_T^\nu$ can always be expressed in terms of those containing k^2 . This relationship makes it quite clear that a model which has one without the other cannot be correct ^{*)}.

To accomplish a similar reduction in QCD we must examine the additional n dependence coming from the choice of gauge. The operators in Eqs. (2.45)-(2.50) have an implicit n dependence because of the identification of the vector n with the gauge vector. In particular, the bilocal operators of Eqs. (2.45)-(2.50) coincide with gauge invariant operators only for our particular choice of gauge. We derive the Lorentz invariance identities by performing small variations of n on the surface of the light-cone. For this reason we must make all the n dependence explicit. This can be done by inserting the string of gluons integrated along the straight line between the fields (P is the path-ordering operator)

$$\bar{\Psi}(0) \gamma^\rho \Psi(\lambda) = \bar{\Psi}(0) \gamma^\rho P \left\{ e^{ig \int_0^\lambda d\eta n^\mu A_\mu(\eta)} \right\} \Psi(\lambda) \quad (4.24)$$

The equivalence is true in our gauge ($n \cdot A = 0$). Performing the variation of n , Eq. (4.13) and retaining only first order terms in m_T we have

$$-im_T^\mu \langle P | \bar{\Psi}(0) \gamma^\rho P \left\{ e^{ig \int_0^\lambda d\eta n^\mu A_\mu(\eta)} \int_0^\lambda d\omega D_\mu(\omega) \right\} \Psi(\lambda) | P \rangle = A_1(\lambda) m_T^\rho. \quad (4.25)$$

The two terms of order m_T have combined to build the covariant derivative. Reimposing the gauge condition $n \cdot A = 0$ we find that

$$A_1(\lambda) = \frac{1}{i} \int_0^\lambda d\eta B_1(\eta, \lambda). \quad (4.26)$$

Two other Lorentz invariance / n independence relations are derived in a similar fashion,

$$B_2(\eta, \lambda) - B_1(\eta, \lambda) = \frac{1}{i} \left[\int_0^\eta d\omega C_1(\omega, \eta, \lambda) + \int_\eta^\lambda d\omega C_1(\eta, \omega, \lambda) \right], \quad (4.27)$$

^{*)} Apart from ξ scaling effects (i.e., the case of non-zero target mass) discussed in Section 7.

$$\tilde{\mathcal{B}}_1(\eta, \lambda) = \frac{1}{i} \left[\int_{\eta}^{\lambda} d\omega \tilde{\mathcal{C}}_1(\eta, \omega, \lambda) - \int_0^{\eta} d\omega \tilde{\mathcal{C}}_1(\omega, \eta, \lambda) \right]. \quad (4.28)$$

After imposing all the light-cone relations [cf. Eq. (4.12)] and the Lorentz invariance relations given above we obtain a series of differential equations which relate A , B and C to the quantities E reducible by the classical equations of motion

$$\not{D}(z)\Psi(z) = 0, \quad [D_{ij}^{\mu}, f_{\mu}]_{ij} = -\frac{1}{2}g^2 \bar{\Psi}_j \not{A} \Psi_i \quad (4.29)$$

where f^{μ} is the light-cone projection of the field strength [in the second equation (i, j) are colour indices $(i, j=1, 2, 3)$ and only the traceless part should be taken]

$$f^{\nu} = -ig \eta_{\mu} F^{\mu\nu} \quad (4.30)$$

For QCD we define three matrix elements which are reducible by the equations of motion

$$E_1(\eta, \lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{D}(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.31)$$

$$E_2(\omega, \eta, \lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{A} D^{\alpha}(\omega) D_{\alpha}(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.32)$$

$$E_3(\omega, \eta, \lambda) = \frac{i}{4} \epsilon^{\alpha\beta\gamma\sigma} m_{\sigma} \langle P | T \{ \bar{\Psi}(0) \gamma_{\sigma} \gamma_5 D_{\alpha}(\omega) D_{\beta}(\eta) \Psi(\lambda) \} | P \rangle. \quad (4.33)$$

Using all this information together with the light-cone relations, such as

$$\mathcal{B}_3(\eta, \lambda) = \mathcal{B}_3(\lambda) = i \frac{\partial}{\partial \lambda} A_1(\lambda) \quad (4.34)$$

we obtain the following equations

$$E_1(\eta, \lambda) = 2\mathcal{B}_1(\eta, \lambda) + \mathcal{B}_2(\eta, \lambda) + \mathcal{B}_3(\lambda), \quad (4.35)$$

$$E_2(\omega, \eta, \lambda) = 2C_1(\omega, \eta, \lambda) + i \left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \lambda} \right) B_2(\eta, \lambda) + i \frac{\partial}{\partial \lambda} B_2(\omega, \lambda), \quad (4.36)$$

$$E_3(\omega, \eta, \lambda) = 2\tilde{C}_1(\omega, \eta, \lambda) - 2i \left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \lambda} \right) \tilde{B}_1(\eta, \lambda) + 2i \frac{\partial}{\partial \lambda} \tilde{B}_1(\omega, \lambda). \quad (4.37)$$

We solve this system of coupled integro-differential equations with the boundary conditions which follow from the n independence in the limit of vanishing arguments

$$B_1(0, 0) = B_2(0, 0) = B_3(0, 0) = 0, \quad (4.38)$$

$$\tilde{B}_1(0, 0) = 0. \quad (4.39)$$

The solution is

$$\begin{aligned} A_1(\lambda) &= \frac{1}{\pi} \int_0^\lambda d\eta B_1(\eta, \lambda) = \frac{1}{\pi} \int_0^\lambda d\eta B_3(\eta), \\ B_1(\eta, \lambda) &= \frac{1}{2} [E_1(\eta, \lambda) - B_2(\eta, \lambda) - B_3(\lambda)], \\ B_2(\eta, \lambda) &= B_3(\lambda) + B(\eta, \lambda), \\ 2C_1(\eta_2, \eta_1, \lambda) &= E_2(\eta_2, \eta_1, \lambda) - i \frac{\partial}{\partial \lambda} B_2(\eta_2, \lambda) - i \left(\frac{\partial}{\partial \eta_1} + \frac{\partial}{\partial \lambda} \right) B_2(\eta_1, \lambda), \\ 2\tilde{C}_1(\eta_2, \eta_1, \lambda) &= E_3(\eta_2, \eta_1, \lambda) - 2i \frac{\partial}{\partial \lambda} \tilde{B}_1(\eta_2, \lambda) + 2i \left(\frac{\partial}{\partial \lambda} + \frac{\partial}{\partial \eta_1} \right) \tilde{B}_1(\eta_1, \lambda), \end{aligned} \quad (4.40)$$

where

$$B_3(\lambda) = \int_0^1 d\tau \tau H_1(\lambda\tau), \quad (4.41)$$

$$B(\eta, \lambda) = \int_0^1 d\tau \tau H(\eta\tau, \lambda\tau), \quad (4.42)$$

$$\tilde{B}_1(\eta, \lambda) = \int_0^1 d\tau \tau H_3(\eta\tau, \lambda\tau), \quad (4.43)$$

and the functions H_i can be expressed by E_i ($i = 1, 2, 3$) as follows

$$H_3(\eta, \lambda) = \frac{1}{2i} \left[\int_{\eta}^{\lambda} d\sigma E_3(\eta, \sigma, \lambda) - \int_0^{\eta} d\sigma E_3(\sigma, \eta, \lambda) \right], \quad (4.44)$$

$$H_2(\eta, \lambda) = \frac{1}{2i} \left[\int_{\eta}^{\lambda} d\sigma E_2(\eta, \sigma, \lambda) + \int_0^{\eta} d\sigma E_2(\sigma, \eta, \lambda) \right], \quad (4.45)$$

$$H_1(\lambda) = \frac{1}{2} \frac{\partial}{\partial \lambda} \int_0^{\lambda} d\sigma \left[E_1(\sigma, \lambda) - \int_0^{\lambda} d\tau \tau H(\sigma, \tau, \lambda) \right], \quad (4.46)$$

$$H(\eta, \lambda) = 2H_2(\eta, \lambda) + E_1(\eta, \lambda) - \frac{\partial}{\partial \lambda} \int_0^{\lambda} d\sigma E_1(\sigma, \lambda). \quad (4.47)$$

The above solution written in terms of the light-cone fractions is given in Appendix B.

The last step in the reduction procedure is the use of the equations of motion. The classical equations of motion impose relations between on-shell operator matrix elements even in the fully interacting theory ¹⁾. We have set up the problem in such a way that the elimination of (covariant) derivatives using the equations of motion is almost immediate.

Before doing so we want to discuss briefly whether there is a more direct route to a parton-like formulation. In the case of leading twist the coefficient functions of the operator product expansion had a simple graphical interpretation as cross-sections for parton processes. Is the same true for $1/Q^2$ corrections? Our belief is that there may be physical processes which correspond to the coefficient functions at order $1/Q^2$, but that they are unlikely to have a simple interpretation in terms of purely perturbative Feynman graphs.

We support this belief by examination of the deep inelastic scattering graphs shown in Fig. 6. The incorporation of the information contained in the equations of motion corresponds to the inclusion of gluon attachments on the external quark lines. These attachments are also necessary for electromagnetic gauge invariance. The diagrams of Fig. 6 seem to provide a candidate physical process from which we could calculate the coefficient functions. In particular, the twist-four contributions of diagrams (a), (b) and (c) come from the contact term in which the pole due to the coalescing quark and gluon is cancelled. The

terms in which the pole is uncanceled correspond to quark-gluon interactions in the distant past which are contained in the leading twist parton distribution functions.

The practical problem with the diagrams of Fig. 6 is that we need to specify a Lorentz and gauge invariant state of a quark and gluon dependent on the only physical vector of the problem p^μ . Taking the quark and gluon on-shell and collinear is ill-defined ; the pole due to the coalescing quark and gluon is unregulated. The more sophisticated choice for the wave function with on-shell quark with momentum k_1 and gluon with momentum l_1

$$\psi(k_1, l_1) \sim \delta(k_1^2) \delta(l_1^2) \Phi(p, k_1, l_1) \delta^4(p - k_1 - l_1) \quad (4.48)$$

also suffers from the same problem due to the diagram of Fig. 7. This diagram would be absent if ψ were a genuine Bethe-Salpeter wave function. For the function ψ in Eq. (4.48) the diagram is infinite.

A better method would be to switch off the interaction adiabatically in the distant past and distant future ³²⁾. This introduces a momentum non-conservation at the interaction vertices which regulates the poles. Another way to regulate the coalescing poles is to retain the quark masses. However, the presence of the quark masses would necessitate a complete reassessment of the power counting argument.

These methods are certainly no easier (and probably much more complicated) than our general treatment where the hadrons are represented by full blobs. Working with the full hadronic blobs we will ultimately arrive at a method of calculating the contributions of multiparton processes from lowest order Feynman diagrams with on-shell partons. However, we have been unable to derive these rules without passing through the intermediary steps with the general hadronic blob.

The equations of motion may be applied to E_1 , E_2 and E_3 after shifting the covariant derivatives along the light cone. This is performed by using the identity

$$D^\mu(\omega) = D^\mu(\eta) + \frac{1}{2} \int_{\eta}^{\omega} d\sigma f^\mu(\sigma). \quad (4.49)$$

As before f is the projection of the field strength Eq. (4.30) and the covariant derivative satisfies the relation

$$[D^\mu(x), D^\nu(x)] = -igF^{\mu\nu}(x). \quad (4.50)$$

The reduction of E_2 using the equations of motion will serve as an example of the method. Using Eq. (4.49) we obtain the identity

$$\begin{aligned} D^\mu(\omega)D_\mu(\eta) &= \frac{1}{2}[D^2(\omega) + D^2(\eta)] + \frac{1}{2i} \int_{\omega}^{\eta} d\alpha [D^\mu(\alpha), f_\mu(\alpha)] + \\ &+ \int_{\omega}^{\eta} d\alpha \int_{\alpha}^{\eta} d\beta f^\mu(\alpha) f_\mu(\beta). \end{aligned} \quad (4.51)$$

The first two terms in Eq. (4.51) can be cast in a form suitable for the application of the fermion equation of motion by writing

$$D^2(\omega) = \not{D}(\omega)\not{D}(\omega) + \frac{ig}{4}[\delta_\mu, \delta_\nu]F^{\mu\nu}(\omega). \quad (4.52)$$

The covariant derivatives $\not{D}(\omega)$ can be now eliminated as follows

$$\bar{\Psi}(0)\not{D}(\omega) = \bar{\Psi}(0)\left[\not{D}(0) + \frac{1}{i} \int_0^{\omega} d\phi \not{A}(\phi)\right] = \frac{1}{i} \int_0^{\omega} d\phi \bar{\Psi}(0)\not{A}(\phi). \quad (4.53)$$

There are five correlation functions in the collinear basis. We define them as follows

$$G_1(\eta, \lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{A}(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.54)$$

$$\tilde{G}_1(\eta, \lambda) = -\frac{g}{4} \epsilon^{\mu\alpha\beta} \Gamma_{m_\alpha} \langle P | T \{ \bar{\Psi}(0) \delta_\beta \delta_5 F_{\mu\nu}(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.55)$$

$$G_2(\omega, \eta, \lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{A}^\alpha(\omega) f_\alpha(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.56)$$

$$\tilde{G}_2(\omega, \eta, \lambda) = -\frac{i}{4} \epsilon^{\mu\nu\alpha\beta} \Gamma_{m_\alpha} \langle P | T \{ \bar{\Psi}(0) \delta_\beta \delta_5 f_\mu(\omega) f_\nu(\eta) \Psi(\lambda) \} | P \rangle, \quad (4.57)$$

$$J(\eta, \lambda) = \frac{1}{4} g^2 \langle P | T \{ \bar{\Psi}(0) \not{A} t^\Lambda \Psi(\lambda) \bar{\Psi}(\eta) \not{A} t^\Lambda \Psi(\eta) \} | P \rangle. \quad (4.58)$$

To simplify our formulae we define also

$$\Delta G_2(\omega, \eta, \lambda) = \tilde{G}_2(\omega, \eta, \lambda) - G_2(\omega, \eta, \lambda). \quad (4.59)$$

Using the equations of motion Eq. (4.29), we obtain the following relations for E_1 , E_2 and E_3

$$E_1(\eta, \lambda) = -i \int_0^\eta d\alpha G_1(\alpha, \lambda) = i \int_0^\lambda d\alpha G_1(\alpha, \lambda), \quad (4.60)$$

$$\begin{aligned} E_2(\omega, \eta, \lambda) = & \frac{1}{2} \left(\frac{\partial}{\partial \eta} + 2 \frac{\partial}{\partial \lambda} \right) \int_0^\eta d\alpha G_1(\alpha, \lambda) + \frac{1}{2} \left(\frac{\partial}{\partial \omega} + 2 \frac{\partial}{\partial \lambda} \right) \int_0^\omega d\alpha G_1(\alpha, \lambda) + \\ & + \frac{i}{2} \int_0^\eta d\alpha J(\alpha, \lambda) + \frac{1}{4} [\tilde{G}_1(\eta, \lambda) + \tilde{G}_1(\omega, \lambda)] + \\ & + \int_0^\eta d\alpha \int_0^\omega d\beta G_2(\beta, \alpha, \lambda) + \\ & + \frac{1}{2} \left[\int_0^\omega d\beta \int_0^\lambda d\alpha + \int_0^\eta d\beta \int_0^\lambda d\alpha \right] \Delta G_2(\beta, \alpha, \lambda), \end{aligned} \quad (4.61)$$

$$\begin{aligned} E_3(\omega, \eta, \lambda) = & E_2(\omega, \eta, \lambda) - \frac{\partial}{\partial \lambda} \int_0^\omega d\alpha G_1(\alpha, \lambda) - \left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \lambda} \right) \int_0^\eta d\alpha G_1(\alpha, \lambda) + \\ & + \int_0^\omega d\beta \int_0^\eta d\alpha \Delta G_2(\beta, \alpha, \lambda). \end{aligned} \quad (4.62)$$

As a consequence of Eq. (4.60) the correlation function G_1 satisfies the constraint

$$\int_0^\lambda d\alpha G_1(\alpha, \lambda) = 0. \quad (4.63)$$

We have found that working directly in co-ordinate space on the light cone leads to more compact expressions for the reduction equations. In many circumstances the Fourier transformed expressions which are functions of light-cone momentum fractions may be more suggestive. For example, for G_1 we have

$$G_1(x_2, x_1) = \int \frac{d\eta}{2\pi} \frac{d\lambda}{2\pi} e^{i\eta(x_2 - x_1)} e^{i\lambda x_1} G_1(\eta, \lambda). \quad (4.64)$$

The constraint Eq. (4.63) in momentum space gives

$$\int dx_2 dx_1 \frac{\sigma(x_2) - \sigma(x_1)}{x_2 - x_1} G_1(x_2, x_1) = 0, \quad (4.65)$$

where $\sigma(x)$ is an arbitrary test function.

The final step in the reduction to the collinear basis consists in combining Eqs. (4.39)-(4.47) and Eqs. (4.60)-(4.62). It is enough to reduce the correlation functions B_3 , B and $\tilde{B}_1$ since the other functions ($A_1, B_1, B_2, C_1, \tilde{C}_1$) can be expressed by the former in a simple way [see Eqs. (4.40)].

The explicit reduction formulae for B_3 , B and $\tilde{B}_1$ are

$$B_3(\lambda) = \frac{i}{2\lambda^2} \int_0^\lambda d\omega \left[1 + \lambda \omega \frac{\omega}{\lambda} \right] \left\{ \int_0^\omega d\alpha \left[i\alpha(\omega - \alpha) \mathcal{J}(\alpha, \omega) + \frac{1}{2} \omega \tilde{G}_1(\alpha, \omega) \right] + \right. \\ \left. + \int_0^\omega d\alpha \int_0^\alpha d\beta \left[2\beta(\omega - \alpha) G_2(\beta, \alpha, \omega) + \omega(\beta - \alpha) \Delta G_2(\beta, \alpha, \omega) \right] \right\}, \quad (4.66)$$

$$B(\eta, \lambda) = i \left\{ \int_0^1 d\tau \tau \int_0^{\eta\tau} d\alpha G_1(\alpha, \lambda\tau) + \left(\eta - \frac{\lambda}{2} \right) \int_0^1 d\tau \tau^2 G_1(\eta\tau, \lambda\tau) - \int_0^\eta d\alpha G_1(\alpha, \lambda) \right\} + \\ + \frac{1}{2} \int_0^1 d\tau \tau \left\{ \int_0^{\eta\tau} d\alpha \alpha - \int_0^{\lambda\tau} d\alpha (\alpha - \lambda\tau) \right\} \mathcal{J}(\alpha, \lambda\tau) + \\ - \frac{i}{4} \int_0^1 d\tau \tau \left\{ \lambda\tau \tilde{G}_1(\eta\tau, \lambda\tau) + \int_0^{\lambda\tau} d\alpha \tilde{G}_1(\alpha, \lambda\tau) \right\} + \\ + i \int_0^1 d\tau \tau \left\{ \int_0^{\eta\tau} d\alpha \int_0^{\eta\tau} d\beta (\alpha - \lambda\tau) - \int_0^{\eta\tau} d\alpha \int_0^\alpha d\beta \beta \right\} G_2(\beta, \alpha, \lambda\tau) + \\ + \frac{i}{2} \int_0^1 d\tau \tau \left\{ \lambda\tau \int_0^{\lambda\tau} d\alpha \int_0^{\eta\tau} d\beta + \int_0^{\lambda\tau} d\alpha \int_0^\alpha d\beta (\alpha - \beta) \right\} \Delta G_2(\beta, \alpha, \lambda\tau), \quad (4.67)$$

$$\tilde{B}_1(\eta, \lambda) = -\frac{i\lambda}{4} \int_0^1 d\tau \tau^2 G_1(\eta\tau, \lambda\tau) + \frac{1}{4} \int_0^1 d\tau \tau \left\{ \lambda\tau \int_0^{\eta\tau} d\alpha - \int_0^{\lambda\tau} d\alpha \right\} \mathcal{J}(\alpha, \lambda\tau) - \\ - \frac{i}{8} \int_0^1 d\tau \tau \left\{ (\lambda - 2\eta)\tau \tilde{G}_1(\eta\tau, \lambda\tau) + \left[\int_0^{\lambda\tau} d\alpha - 2 \int_0^{\eta\tau} d\alpha \right] \tilde{G}_1(\alpha, \lambda\tau) \right\} +$$

(4.68)

$$\begin{aligned}
 & + \frac{i}{2} \int_0^1 d\tau \tau \left\{ \int_{\lambda\tau}^{\eta\tau} d\alpha \int_{\alpha}^{\eta\tau} d\beta (\alpha - \lambda\tau) + \int_0^{\eta\tau} d\alpha \int_0^{\alpha} d\beta \beta \right\} G_2(\beta, \alpha, \lambda\tau) + \\
 & + \frac{i}{4} \int_0^1 d\tau \tau \left\{ \int_{\lambda\tau}^{\lambda\tau} d\alpha \int_0^{\eta\tau} d\beta (\lambda\tau - 2\alpha) + \right. \\
 & \quad \left. + \left[\int_0^{\lambda\tau} d\alpha - 2 \int_0^{\eta\tau} d\alpha \right] \int_0^{\alpha} d\beta (\alpha - \beta) \right\} \Delta G_2(\beta, \alpha, \lambda\tau).
 \end{aligned}$$

(4.68)
Contd.

The corresponding reduction formulae written in terms of the light-cone fractions are given in Appendix B.

5. - RESULTS IN THE COLLINEAR BASIS

We report here our results for deep inelastic processes produced by interaction with a space-like photon or W boson. We work in the frame in which ^{*})

$$Q^\mu = -x_B p^\mu + \frac{Q^2}{2x_B} n^\mu, \quad \nu = \frac{Q^2}{2x_B} \quad (5.1)$$

and we choose the current to which the gauge boson couples to be of the general form,

$$J^\alpha(x) = \bar{Q}(x) \gamma^\alpha (v + a \gamma_5) Q(x). \quad (5.2)$$

To order $1/Q^2$ there are two classes of diagrams which we have to calculate. The diagrams of Fig. 2 require all the techniques of the last section. We shall consider them first. The four fermion diagrams of Fig. 3 are obtained by calculating with on-shell partons so the results are automatically in the collinear basis without the need for further reduction. The total structure function is the sum of the contributions of Figs. 2 and 3

$$F_k(x_B, Q^2) = F_k^{(a)}(x_B, Q^2) + F_k^{(b)}(x_B, Q^2). \quad (5.3)$$

For the graphs of Fig. 2 ($F^{(a)}$) the only remaining technical step is the explicit evaluation of the cut amplitudes defined in Eq. (2.61). From the diagrams of Fig. 8a we obtain, ($S = (|v|^2 + |a|^2)$, $R = (v \cdot a + a \cdot v)$)

$$Q_0^{\alpha\beta}(x) = (S e_T^{\alpha\beta} + R e_A^{\alpha\beta}) \delta(x - x_B), \quad (5.4)$$

$$Q_1^{\alpha\beta}(x) = S m_f^{\alpha\beta} \delta(x - x_B). \quad (5.5)$$

The one gluon diagram Fig. 8b gives,

$$b_1^{\alpha\beta}(x_2, x_1) = -\frac{S}{\nu} [p_m^{\alpha\beta} \delta(x_1 - x_B) + p_m^{\beta\alpha} \delta(x_2 - x_B)], \quad (5.6)$$

^{*}) This is for zero target mass. Throughout Sections 5 and 6 we consider only the case $M = 0$. Target mass effects are included in our formalism in Section 7.

$$\begin{aligned}\tilde{b}_1^{\alpha\beta}(x_2, x_1) &= -\frac{S}{\nu} \left[p_m^\alpha \delta(x_1 - x_B) - p_m^\beta \delta(x_2 - x_B) \right], \\ \tilde{b}_2(x_2, x_1) &= 0.\end{aligned}\quad (5.7)$$

Finally from the two gluon diagram of Fig. 8c we obtain,

$$c_1^{\alpha\beta}(x_2, y, x_1) = \frac{1}{\nu} \left[(S\epsilon_T^{\alpha\beta} + R\epsilon_A^{\alpha\beta}) \Delta(x_2, x_1) - S \frac{p^\alpha p^\beta}{\nu} \delta(y - x_B) \right], \quad (5.8)$$

$$\tilde{c}_1^{\alpha\beta}(x_2, y, x_1) = \frac{1}{\nu} \left[(S\epsilon_T^{\alpha\beta} + R\epsilon_A^{\alpha\beta}) \Delta(x_2, x_1) + S \frac{p^\alpha p^\beta}{\nu} \delta(y - x_B) \right]. \quad (5.9)$$

The distribution Δ is defined as

$$\Delta(x_2, x_1) = \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1}. \quad (5.10)$$

We remind the reader that the conserved tensors in terms of which we wish to express the results are,

$$\begin{aligned}\epsilon_T^{\alpha\beta} &= \frac{1}{2} (p_m^\alpha p_m^\beta + p_m^\beta p_m^\alpha - g^{\alpha\beta}) = \frac{x_B}{\nu} p^\alpha p^\beta + \frac{1}{2\nu} (p_2^\alpha p_2^\beta + p_2^\beta p_2^\alpha) - \frac{1}{2} g^{\alpha\beta}, \\ \epsilon_L^{\alpha\beta} &= \frac{1}{2} \left(g^{\alpha\beta} - \frac{2p^\alpha p^\beta}{\nu^2} \right), \\ \epsilon_A^{\alpha\beta} &= -\frac{i}{2} \epsilon^{\alpha\beta\mu\nu} p_\mu p_\nu = -\frac{i}{2\nu} \epsilon^{\alpha\beta\mu\nu} p_\mu p_\nu.\end{aligned}\quad (5.11)$$

The contributions from individual diagrams cannot be expressed exclusively in terms of these conserved tensors. The gauge invariance is restored only after adding all diagrams and performing the reduction as described in the previous section. This is a long but straightforward procedure.

We write *)

$$W^{(a)\alpha\beta} = S(\epsilon_L^{\alpha\beta} F_L + \epsilon_T^{\alpha\beta} F_T^{(a)}) + R \epsilon_A^{\alpha\beta} F_A^{(a)} \quad (5.12)$$

where $K = (L, T, A)$ and

$$\begin{aligned} F_K^{(a)}(x_B, Q^2) = & \int dx \sigma_K^{A_0}(x) A_0(x) + \\ & + \int dx_2 dx_1 \left\{ \sigma_K^{G_1}(x_2, x_1) G_1(x_2, x_1) + \sigma_K^J(x_2, x_1) J(x_2, x_1) + \sigma_K^{\tilde{G}_1}(x_2, x_1) \tilde{G}_1(x_2, x_1) \right\} + \\ & + \int dx_2 dy dx_1 \left\{ \sigma_K^{G_2}(x_2, y, x_1) G_2(x_2, y, x_1) + \sigma_K^{\Delta G_2}(x_2, y, x_1) \Delta G_2(x_2, y, x_1) \right\} + O\left(\frac{1}{Q^4}\right). \end{aligned} \quad (5.13)$$

The parton correlation functions are defined in Eqs. (4.54)-(4.59). The results for the cut amplitudes (coefficient functions) are,

$$\begin{aligned} \sigma_L^{A_0}(x) &= 0, \\ \sigma_T^{A_0}(x) &= \delta(x - x_B), \\ \sigma_A^{A_0}(x) &= \delta(x - x_B), \\ \sigma_L^{G_1}(x_2, x_1) &= \sigma_T^{G_1}(x_2, x_1) = \sigma_A^{G_1}(x_2, x_1) = 0, \\ \sigma_J^J(x_2, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{8\ln\tau\} \delta_2(x_2, x_1; \tau) + \right. \\ & \quad \left. + \{4 + 4\ln\tau\} [\delta(x_2\tau - x_B) - \delta(x_1\tau - x_B)] \right], \\ \sigma_T^J(x_2, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{12\ln\tau - 2\} \delta_2(x_2, x_1; \tau) + \right. \\ & \quad \left. + \{\delta(1-\tau) + 5 + 6\ln\tau\} [\delta(x_2\tau - x_B) + \delta(x_1\tau - x_B)] \right], \\ \sigma_A^J(x_2, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{4\ln\tau - 2\} \delta_2(x_2, x_1; \tau) + \right. \\ & \quad \left. + \{\delta(1-\tau) + 1 + 2\ln\tau\} [\delta(x_2\tau - x_B) + \delta(x_1\tau - x_B)] \right], \end{aligned} \quad (5.14)$$

*) The longitudinal structure function F_L requires no superscript since four-fermion diagrams do not contribute (i.e., $F_L^{(b)} = 0$).

$$\begin{aligned}
 \tilde{\sigma}_L^{\hat{G}_1}(x_2, x_1) &= \frac{1}{2\nu} \int_0^1 d\tau \tau \{-2 - 2\ln\tau\} \delta_1(x_2, x_1; \tau), \\
 \tilde{\sigma}_T^{\hat{G}_1}(x_2, x_1) &= \frac{1}{2\nu} \int_0^1 d\tau \tau \left\{ \frac{1}{2} \delta(1-\tau) - \frac{5}{2} - 3\ln\tau \right\} \delta_1(x_2, x_1; \tau), \\
 \tilde{\sigma}_A^{\hat{G}_1}(x_2, x_1) &= \frac{1}{2\nu} \int_0^1 d\tau \tau \left\{ \frac{1}{2} \delta(1-\tau) - \frac{1}{2} - \ln\tau \right\} \delta_1(x_2, x_1; \tau), \\
 \sigma_L^{G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{-8 - 8\ln\tau\} \Delta_3(x_2, y, x_1; \tau) + \{8\ln\tau\} \Delta_2(x_2, y, x_1; \tau) \right], \\
 \sigma_T^{G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{-2\delta(1-\tau) - 10 - 12\ln\tau\} \Delta_3(x_2, y, x_1; \tau) + \right. \\
 &\quad \left. + \{12\ln\tau - 2\} \Delta_2(x_2, y, x_1; \tau) \right], \\
 \sigma_A^{G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \left[\{-2\delta(1-\tau) - 2 - 4\ln\tau\} \Delta_3(x_2, y, x_1; \tau) + \right. \\
 &\quad \left. + \{4\ln\tau - 2\} \Delta_2(x_2, y, x_1; \tau) \right], \\
 \sigma_L^{\Delta G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(y - x_1)(y - x_2)} \int_0^1 d\tau \tau \{4 + 4\ln\tau\} \Delta_1(x_2, y, x_1; \tau), \\
 \sigma_T^{\Delta G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(y - x_1)(y - x_2)} \int_0^1 d\tau \tau \{5 + 6\ln\tau\} \Delta_1(x_2, y, x_1; \tau), \\
 \sigma_A^{\Delta G_2}(x_2, y, x_1) &= \frac{1}{2\nu} \frac{1}{(y - x_1)(y - x_2)} \int_0^1 d\tau \tau \{1 + 2\ln\tau\} \Delta_1(x_2, y, x_1; \tau). \tag{5.14}
 \end{aligned}$$

cont.

The various distributions are defined as :

$$\delta_1(x_2, x_1; \tau) = \frac{\delta(x_2\tau - x_B) - \delta(x_1\tau - x_B)}{x_2 - x_1},$$

$$\delta_2(x_2, x_1; \tau) = \frac{x_2\delta(x_2\tau - x_B) - x_1\delta(x_1\tau - x_B)}{x_2 - x_1},$$

$$\Delta_1(x_2, y, x_1; \tau) = \left\{ \delta_1(y, x_1; \tau) + \delta_1(y, x_2; \tau) - \delta_1(x_2, x_1; \tau) - \tau\delta'(y\tau - x_B) \right\},$$

$$\begin{aligned}
 \Delta_2(x_2, y, x_1; \tau) &= \left\{ \frac{1}{y - x_2} - \frac{1}{y - x_1} \right\} [\delta_2(y, x_1; \tau) - \delta_2(y, x_2; \tau)] - \\
 &\quad - \left\{ \frac{1}{y - x_2} + \frac{1}{y - x_1} \right\} \delta_2(x_2, x_1; \tau),
 \end{aligned}$$

$$\Delta_3(x_2, y, x_1; \tau) = \left\{ \frac{\delta(x_2\tau - x_B)}{y - x_2} + \frac{\delta(x_1\tau - x_B)}{y - x_1} \right\}.$$

(5.15)

It is probably superfluous to note that the coefficient functions in this basis are complicated. Other remarks to be made are that the parton correlation function $G_1(x_2, x_1)$ is entirely absent, despite its presence in the intermediate stages of the calculation. The longitudinal cross-section is non-zero. This is because we have included the effect of parton transverse momentum. Note also that up to order $(1/Q^2)$ we have

$$\sigma_T^X - \sigma_L^X = \sigma_A^X \quad (5.16)$$

for all choices of the parton correlation functions X .

To establish contact with the work of other authors¹⁰⁾ we take moments of Eq. (5.14)

$$\begin{aligned} Q^2 \int_0^1 dx_B x_B^{N-1} W^{(\alpha)}(x_B, Q^2) &= Q^2 \int dx x^{N-1} (\epsilon_T^{\alpha\beta} + \epsilon_A^{\alpha\beta}) A_0(x) + \\ &+ \int dx_2 dx_1 \left\{ -\frac{8}{(N+2)^2} \epsilon_L^{\alpha\beta} - \frac{2(N+8)}{(N+2)^2} \epsilon_T^{\alpha\beta} - \frac{2(N+4)}{(N+2)^2} \epsilon_A^{\alpha\beta} \right\} S^J(x_2, x_1; N) J(x_2, x_1) + \\ &+ \int dx_2 dx_1 \left\{ -\frac{2(N+1)}{(N+2)^2} \epsilon_L^{\alpha\beta} + \frac{N(N-1)}{2(N+2)^2} \epsilon_T^{\alpha\beta} + \frac{N^2+3N+4}{2(N+2)^2} \epsilon_A^{\alpha\beta} \right\} S^{\tilde{G}_1}(x_2, x_1; N) \tilde{G}_1(x_2, x_1) + \\ &+ \int dx_2 dy dx_1 \left\{ -\frac{8}{(N+2)^2} \epsilon_L^{\alpha\beta} - \frac{2(N+8)}{(N+2)^2} \epsilon_T^{\alpha\beta} - \frac{2(N+4)}{(N+2)^2} \epsilon_A^{\alpha\beta} \right\} S^{G_2}(x_2, y, x_1; N) G_2(x_2, y, x_1) + \\ &+ \int dx_2 dy dx_1 \left\{ \frac{4(N+1)}{(N+2)^2} \epsilon_L^{\alpha\beta} + \frac{5N+4}{(N+2)^2} \epsilon_T^{\alpha\beta} + \frac{N}{(N+2)^2} \epsilon_A^{\alpha\beta} \right\} S^{\Delta G_2}(x_2, y, x_1; N) \Delta G_2(x_2, y, x_1) \end{aligned} \quad (5.17)$$

where the quantities S are,

$$\begin{aligned} S^J(x_2, x_1; N) &= \frac{1}{(x_2 - x_1)^2} \left\{ \frac{x_2^{N+1} - x_1^{N+1}}{x_2 - x_1} - \frac{1}{2}(N+1)(x_2^N + x_1^N) \right\}, \\ S^{\tilde{G}_1}(x_2, x_1; N) &= \frac{x_2^N - x_1^N}{x_2 - x_1}, \end{aligned} \quad (5.18)$$

$$S^{G_2}(x_2, y, x_1; N) = \frac{1}{(x_2 - x_1)^2} \left\{ \left(\frac{1}{y - x_2} - \frac{1}{y - x_1} \right) \left\{ \frac{y^{N+1} - x_1^{N+1}}{y - x_1} - \frac{y^{N+1} - x_2^{N+1}}{y - x_2} \right\} - \right. \\ \left. - \left(\frac{1}{y - x_2} + \frac{1}{y - x_1} \right) \frac{x_2^{N+1} - x_1^{N+1}}{x_2 - x_1} + (N+1) \left(\frac{x_2^N}{y - x_2} + \frac{x_1^N}{y - x_1} \right) \right\},$$

$$S^{\Delta G_2}(x_2, y, x_1; N) = \frac{1}{(y - x_2)(y - x_1)} \left\{ \frac{y^N - x_1^N}{y - x_1} + \frac{y^N - x_2^N}{y - x_2} - \frac{x_2^N - x_1^N}{x_2 - x_1} - N y^{N-1} \right\}. \quad (5.18)$$

In the above we have set the current coupling factors Eq. (5.2) equal to $v = -a = 1/\sqrt{2}$.

The functions $S(\{x\}, N)$ are not singular in the limit when two of their arguments coincide. In fact they can be written as finite sums

$$S^J(x_2, x_1; N) = -\frac{1}{2} \sum_{k=0}^{N-2} (k+1)(N-k-1) x_2^k x_1^{N-k-2},$$

$$S^{\tilde{G}_1}(x_2, x_1; N) = \sum_{k=0}^{N-1} x_2^k x_1^{N-k-1},$$

$$S^{G_2}(x_2, y, x_1; N) = -\sum_{k=0}^{N-3} \sum_{l=0}^{N-3-k} (k+1)(N-2-k-l) x_2^k y^l x_1^{N-3-k-l}, \quad (5.19)$$

$$S^{\Delta G_2}(x_2, y, x_1; N) = -\sum_{k=0}^{N-3} \sum_{l=0}^{N-3-k} (l+1) x_2^k y^l x_1^{N-3-k-l}.$$

Using these identities we can show the agreement of Eq. (5.17) with the operator product expansion results of Refs. 10 and 11). From Eq. (5.17) we can derive the contribution of Fig. 2 to lowest moments,

$$\int_0^1 dx_B W^{(a)}(x_B, Q^2)^{\alpha\beta} = \left(\epsilon_T^{\alpha\beta} + \epsilon_A^{\alpha\beta} \right) \int dx A_0(x) - \\ - \frac{4}{9} \frac{1}{Q^2} (\epsilon_\perp^{\alpha\beta} - \epsilon_A^{\alpha\beta}) \int dx_2 dx_1 \tilde{G}_1(x_2, x_1), \quad (5.20)$$

$$\begin{aligned}
 \int_0^1 dx_B x_B W^{(\alpha)\alpha\beta}(x_B, Q^2) &= (\epsilon_T^{\alpha\beta} + \epsilon_A^{\alpha\beta}) \int dx x A_0(x) + \\
 &+ \left(\frac{1}{4} \epsilon_{\perp}^{\alpha\beta} + \frac{5}{8} \epsilon_T^{\alpha\beta} + \frac{3}{8} \epsilon_A^{\alpha\beta} \right) \frac{1}{Q^2} \int dx_2 dx_1 J(x_2, x_1) \\
 &+ \left(-\frac{3}{8} \epsilon_{\perp}^{\alpha\beta} + \frac{1}{16} \epsilon_T^{\alpha\beta} + \frac{7}{16} \epsilon_A^{\alpha\beta} \right) \frac{1}{Q^2} \int dx_2 dx_1 (x_2 + x_1) \tilde{G}_1(x_2, x_1).
 \end{aligned}
 \tag{5.21}$$

We now consider the four fermion diagrams of Fig. 3. These diagrams have a very different flavour structure than the diagrams of Fig. 2 because the scattering occurs on different quarks. For simplicity we suppress the flavour structure in the following. The calculation is straightforward and the result can be written as

$$W^{(b)\alpha\beta} = \int dx_2 dy dx_1 \left\{ \epsilon_T^{\alpha\beta} F_T^{(b)}(x_B, Q^2) + \epsilon_A^{\alpha\beta} F_A^{(b)}(x_B, Q^2) \right\} \tag{5.22}$$

where the structure functions can be expressed in terms of the four quark correlation function as follows

$$Q(\omega, \eta, \lambda) = \frac{g^2}{16} \langle P | T \{ [\bar{\Psi}(0) \gamma^\mu t^\mu \Psi(\omega)] [\bar{\Psi}(\eta) \gamma^\mu t^\mu \Psi(\lambda)] \} | P \rangle \tag{5.23}$$

$$\tilde{Q}(\omega, \eta, \lambda) = \frac{g^2}{16} \langle P | T \{ [\bar{\Psi}(0) \gamma^\mu \gamma_5 t^\mu \Psi(\omega)] [\bar{\Psi}(\eta) \gamma^\mu \gamma_5 t^\mu \Psi(\lambda)] \} | P \rangle \tag{5.24}$$

and

$$\begin{aligned}
 F_K^{(b)}(x_B, Q^2) &= \int dx_2 dy dx_1 \sigma_K^Q(x_2, y, x_1) Q(x_2, y, x_1) + \\
 &+ \int dx_2 dy dx_1 \sigma_K^{\tilde{Q}}(x_2, y, x_1) \tilde{Q}(x_2, y, x_1).
 \end{aligned}
 \tag{5.25}$$

The results for the coefficient functions^{9),10)} are [the numbers v, a specify the vector and axial coupling Eq. (5.2)]

$$\begin{aligned} \sigma_T^Q(x_2, y, x_1) = \frac{1}{8v} \left\{ v^2 \left[\Delta_4(y_2, y_1, y) + \Delta_4(x_2, x_1, y) - \Delta_4(y_2, x_1, y) - \Delta_4(x_2, y_1, y) \right] \right. \\ \left. + a^2 \left[\Delta_4(y_2, y_1, y) + \Delta_4(x_2, x_1, y) + \Delta_4(y_2, x_1, y) + \Delta_4(x_2, y_1, y) \right] \right\}, \end{aligned} \quad (5.26)$$

$$\sigma_A^Q(x_2, y, x_1) = \frac{1}{8v} \left\{ 2av \left[\Delta_4(y_2, y_1, y) - \Delta_4(x_2, x_1, y) \right] \right\}, \quad (5.27)$$

where

$$\Delta_4(y_2, y, y_1) = \frac{1}{y_2 - y_1} \left\{ \frac{\delta(y_2 - x_B) - \delta(y - x_B)}{y_2 - y} - \frac{\delta(y_1 - x_B) - \delta(y - x_B)}{y_1 - y} \right\} \quad (5.28)$$

and

$$y_1 = y - x_1, \quad y_2 = y - x_2. \quad (5.29)$$

The coefficient functions of the parton correlation functions containing γ_5 are given by ($K = T, A$)

$$\tilde{\sigma}_K^Q(x_2, y, x_1; v, a) = \sigma_K^Q(x_2, y, x_1; a, v). \quad (5.30)$$

The moments of Eq. (5.25) can be written in a compact fashion by defining,

$$S(x_2, y, x_1; N) = \int dx_B x_B^{N-1} \Delta_4(x_2, y, x_1) = \sum_{j=1}^{N-1} y^{N-j-1} \sum_{k=0}^{j-1} x_2^{j-k-1} x_1^k \quad (5.31)$$

Because of the relations

$$S(x_2, y, x_1; 1) = 0, \quad S(x_2, y, x_1; 2) = 1 \quad (5.32)$$

these operators make no contribution to the $N = 1$ moment and contribute to the $N = 2$ moment as follows

$$\begin{aligned} \int_0^1 dx_B x_B W^{(6)\alpha\beta} &= \frac{1}{Q^2} \epsilon_T^{\alpha\beta} a^2 \int dx_2 dy dx_1 Q(x_2, y, x_1) \\ &+ \frac{1}{Q^2} \epsilon_T^{\alpha\beta} v^2 \int dx_2 dy dx_1 \tilde{Q}(x_2, y, x_1). \end{aligned} \quad (5.33)$$

6. - TRANSVERSE BASIS

In the preceding two sections we have studied the representation of the power corrections in the collinear basis. Following the chain of reasoning from transverse momentum to off-shellness and then from off-shellness to more partons we have arrived at an expression for the power corrections in the collinear basis in which all reference to transverse momentum and off-shellness has been eliminated.

In this section we derive a complementary description of the hadron structure in which the transverse degrees of freedom are retained. The advantages of this transverse basis are that the coefficient functions are simple and that contact can be established with the intuitive ideas of the parton model.

To convince the reader that the transverse components of momenta and gluon polarizations play a special rôle in the analysis of the higher twist contributions, we start our discussion of the transverse basis by considering the expansion of the matrix element of the Wilson operator $\bar{\psi}_{\mu_1} D_{\mu_2} \dots D_{\mu_N} \psi$ up to terms of order Λ^2 . We have

$$\begin{aligned} \langle P | \bar{\psi}_{\mu_1} D_{\mu_2} \dots D_{\mu_N} \psi | P \rangle = & A_0(N) p_{\mu_1} p_{\mu_2} \dots p_{\mu_N} + \\ & + \frac{1}{2} \sum_{i \neq j} B_{ij}(N) g^{\mu_i \mu_j} p_{\mu_1} \dots p_{\mu_N} / p_{\mu_i} p_{\mu_j} \\ & + \text{higher twists } (\tau > 4) \end{aligned} \quad (6.1)$$

By dimensional counting we see that the reduced matrix elements $A(N)$ control the leading twist structure functions, whereas the $B_{ij}(N)$ contain the information about the next-to-leading twist. The leading twist part can be isolated by contracting all indices in Eq. (6.1) with the light-like vector n_μ/p_n . After identifying n_μ with the vector which fixes the axial gauge, we get a simple expression

$$A(N) = \langle P | \bar{\psi} \not{x}^{N-1} \psi | P \rangle \quad (6.2)$$

where $x = i n_\mu \partial_\mu$ in configuration space ($x = kn$ in momentum space).

The above relation was the starting point for the parton model interpretation of the quantity $x = kn$ as the longitudinal fraction of the parton momentum. Therefore, a natural question one may ask to establish the parton

language for the twist-four sector is : what is the projector which isolates a given term $B_{ij}(N)$ in Eq. (6.1)? The answer is that the $B_{ij}(N)$ are projected by the tensor

$$d^{\mu_i \mu_j} m^{\mu_1} \dots m^{\mu_n} / m^{\mu_i} m^{\mu_j} \quad (6.3)$$

in which only transverse components of the covariant derivatives D_{μ_i}, D_{μ_j} appear

$$B_{ij}(N) = \frac{1}{2} d^{\mu_i \mu_j} \langle P | \bar{\Psi} \not{x}^{i-2} D_{T_{\mu}} x^{j-i-1} D_{T_{\nu}} x^{N-j} \Psi | P \rangle. \quad (6.4)$$

A similar statement is true for the operator containing $\gamma_{\mu} \gamma_5$.

This simple argument shows that the scale of the power corrections in the two-fermion, n -gluon sector is determined exclusively by the transverse degrees of freedom. Our next task is to make this feature manifest on the light cone.

The elimination of all but the transverse covariant derivatives using the equations of motion proceeds as follows. In the light-cone representation we may write the six higher twist parton correlation functions appearing in Eq. (2.62) as

$$A_1(\lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{x} \Psi(\lambda) \} | P \rangle, \quad (6.5)$$

$$B_1(\eta, \lambda) = \frac{1}{8} d^{\mu} \langle P | T \{ \bar{\Psi}(0) \not{x} \not{\eta} D_{\mu}(\eta) \Psi(\lambda) \} | P \rangle, \quad (6.6)$$

$$B_2(\eta, \lambda) = \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \not{x} \not{\eta} D_{\mu}(\eta) \Psi(\lambda) \} | P \rangle, \quad (6.7)$$

$$\tilde{B}_1(\eta, \lambda) = \frac{1}{8i} \epsilon^{\mu} \langle P | T \{ \bar{\Psi}(0) \not{x} \not{\eta} \not{\eta} D_{\mu}(\eta) \Psi(\lambda) \} | P \rangle, \quad (6.8)$$

$$C_1(\omega, \eta, \lambda) = \frac{1}{8} d^{\mu} \langle P | T \{ \bar{\Psi}(0) \not{x} D_{\mu}(\omega) D_{\nu}(\eta) \Psi(\lambda) \} | P \rangle, \quad (6.9)$$

$$\tilde{C}_1(\omega, \eta, \lambda) = \frac{1}{8i} \epsilon^{\mu} \langle P | T \{ \bar{\Psi}(0) \not{x} \not{\eta} D_{\mu}(\omega) D_{\nu}(\eta) \Psi(\lambda) \} | P \rangle. \quad (6.10)$$

By the transverse basis we mean the set of two correlation functions $C_1, \tilde{C}_1$. Our aim now is to demonstrate that all other correlation functions, $A_1, B_1, B_2, \tilde{B}_1$ can be expressed in terms of C_1 and $\tilde{C}_1$. For this reason it will be convenient to define the related quantities

$$B_L(\eta, \lambda) = -\frac{1}{8} \langle P | T \{ \bar{\Psi}(0) \not{D}_T(\eta) \not{X} \Psi(\lambda) \} | P \rangle = B_1(\eta, \lambda) - \tilde{B}_1(\eta, \lambda), \quad (6.11)$$

$$B_R(\eta, \lambda) = -\frac{1}{8} \langle P | T \{ \bar{\Psi}(0) \not{X} \not{D}_T(\eta) \Psi(\lambda) \} | P \rangle = B_1(\eta, \lambda) + \tilde{B}_1(\eta, \lambda), \quad (6.12)$$

$$C_L(\omega, \eta, \lambda) = -\frac{1}{8} \langle P | T \{ \bar{\Psi}(0) \delta_\mu \not{X} \delta_\nu D_T^\mu(\omega) D_T^\nu(\eta) \Psi(\lambda) \} | P \rangle = C_1(\omega, \eta, \lambda) - \tilde{C}_1(\omega, \eta, \lambda), \quad (6.13)$$

$$C_R(\omega, \eta, \lambda) = -\frac{1}{8} \langle P | T \{ \bar{\Psi}(0) \delta_\mu \not{X} \delta_\nu D_T^\mu(\omega) D_T^\nu(\eta) \Psi(\lambda) \} | P \rangle = C_1(\omega, \eta, \lambda) + \tilde{C}_1(\omega, \eta, \lambda). \quad (6.14)$$

To apply the equations of motion we use the identity

$$\frac{1}{2i} (\not{X} \not{D}(\eta) + \not{D}(\eta) \not{X}) \hat{O}(\lambda) = \frac{\partial}{\partial \lambda} \hat{O}(\lambda) \quad (6.15)$$

valid for any $\hat{O}(\lambda)$. In particular, by taking $\hat{O}(\lambda) = \psi(\lambda)$, $\eta = \lambda$ and using the equations of motion we get

$$\frac{1}{2i} \not{D}(\lambda) \not{X} \Psi(\lambda) = \frac{\partial}{\partial \lambda} \Psi(\lambda). \quad (6.16)$$

Inserting this relation in the representation for $A_1(\lambda)$ we obtain

$$\begin{aligned} \frac{\partial}{\partial \lambda} A_1(\lambda) &= \frac{1}{8i} \langle P | T \{ \bar{\Psi}(0) \not{D}_T(\lambda) \not{X} \Psi(\lambda) \} | P \rangle = i B_L(\lambda, \lambda) \\ &= \frac{1}{8i} \langle P | T \{ \bar{\Psi}(0) \not{X} \not{D}_T(0) \Psi(\lambda) \} | P \rangle = i B_R(0, \lambda). \end{aligned} \quad (6.17)$$

Performing this insertion in all possible places we obtain, in addition to Eq. (6.17),

$$\frac{\partial^2}{\partial \lambda^2} A_1(\lambda) = C_L(0, \lambda, \lambda), \quad (6.18)$$

$$\frac{\partial}{\partial \lambda} B_R(\eta, \lambda) = \frac{1}{i} C_L(\eta, \lambda, \lambda), \quad (6.19)$$

$$\left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \lambda}\right) \mathcal{B}_\perp(\eta, \lambda) = \frac{1}{i} C_\perp(0, \eta, \lambda). \quad (6.20)$$

Constraints on $\mathcal{B}_2$ can be obtained by using the relation [Eq. (4.35)]

$$\mathcal{B}_2(\eta, \lambda) = E_1(\eta, \lambda) - 2\mathcal{B}_1(\eta, \lambda) - i \frac{\partial}{\partial \lambda} A_1(\lambda) \quad (6.21)$$

and exploiting the equations of motion

$$E_1(0, \lambda) = E_1(\lambda, \lambda) = 0. \quad (6.22)$$

We therefore obtain the two conditions for $\mathcal{B}_2$ using Eqs. (6.17), (6.21), (6.22)

$$\mathcal{B}_2(0, \lambda) = -\mathcal{B}_\perp(0, \lambda), \quad \mathcal{B}_2(\lambda, \lambda) = -\mathcal{B}_R(\lambda, \lambda). \quad (6.23)$$

An equivalent derivation of the above relations can be obtained by observing that there are only two independent dynamical components of the spinor ψ in the light-like gauge³³⁾. The other two components of the fermion field can be eliminated by using equations of motion.

We define these to be the "plus" and "minus" components of the Dirac field

$$\psi^{(+)}(\lambda) = \hat{e}^{(+)} \psi(\lambda), \quad \psi^{(-)}(\lambda) = \hat{e}^{(-)} \psi(\lambda), \quad (6.24)$$

where the spinor projection matrices $\hat{e}^{(\pm)}$ are

$$\begin{aligned} \hat{e}^{(+)} &= \frac{1}{2} \not{n} \not{\lambda}, \quad \hat{e}^{(-)} = \frac{1}{2} \not{\lambda} \not{n}, \\ \hat{e}^{(+)} \hat{e}^{(-)} &= \hat{e}^{(-)} \hat{e}^{(+)} = 0, \quad \hat{e}^{(+)} \hat{e}^{(+)} = \hat{e}^{(-)} \hat{e}^{(-)} = \hat{e}^{(+)} + \hat{e}^{(-)} = 1. \end{aligned} \quad (6.25)$$

From Eq. (6.15) we find

$$\frac{\partial}{\partial \lambda} \psi^{(-)}(\lambda) = -\frac{1}{2i} \not{n} \not{\lambda}(\lambda) \psi^{(+)}(\lambda). \quad (6.26)$$

By using this equation we can systematically eliminate all $\psi^{(-)}$ components in favour of $\psi^{(+)}$ components and transverse covariant derivatives. In this way we can derive Eqs. (6.18)-(6.20).

Using $\psi^{(\pm)}$ formalism, we can also derive the constraint on B_2 , Eq. (6.23), by observing that $\psi^{(+)}$ satisfies the following equation of motion

$$p(D_\mu(\lambda)\psi^{(+)}(\lambda) = -\frac{1}{2}pD_T(\lambda)\psi^{(-)}(\lambda). \quad (6.27)$$

By taking the Fourier transforms of the relations in Eqs. (6.18)-(6.20) and Eq. (6.23), we get

$$\int dx x^2 \sigma(x) A_1(x) = - \int dx_2 dy dx_1 \sigma(y) C_\perp(x_2, y, x_1), \quad (6.28)$$

$$\int dx_2 dx_1 x_1 \sigma(x_2, x_1) B_R(x_2, x_1) = \int dx_2 dy dx_1 \sigma(x_2, y) C_\perp(x, y, x_1), \quad (6.29)$$

$$\int dx_2 dx_1 x_2 \sigma(x_2, x_1) B_\perp(x_2, x_1) = \int dx_2 dy dx_1 \sigma(y, x_1) C_\perp(x_2, y, x_1), \quad (6.30)$$

$$\int dx_2 dx_1 \sigma(x_1) B_2(x_2, x_1) = - \int dx_2 dx_1 \sigma(x_1) B_\perp(x_2, x_1), \quad (6.31)$$

$$\int dx_2 dx_1 \sigma(x_2) B_2(x_2, x_1) = - \int dx_2 dx_1 \sigma(x_2) B_R(x_2, x_1), \quad (6.32)$$

where σ is an arbitrary test function. By suitable choices of the function σ we can eliminate the parton correlation functions A_1 , B_1 and $\tilde{B}_1$ from Eq. (2.62). The result is

$$\begin{aligned} N^{(a)}(q, p) = & \int dx a_0(x) A_0(x) + \\ & + \int dx_2 dx_1 \left\{ b_2(x_2, x_1) - \frac{1}{2} b_2(x_2, x_2) - \frac{1}{2} b_2(x_1, x_1) \right\} B_2(x_2, x_1) \\ & - 2 \int dx_2 dy dx_1 \sigma_\perp(x_2, y, x_1) C_\perp(x_2, y, x_1) \\ & - 2 \int dx_2 dy dx_1 \sigma_R(x_2, y, x_1) C_R(x_2, y, x_1) \end{aligned} \quad (6.33)$$

where

$$\begin{aligned} \sigma_L(x_2, y, x_1) = \frac{1}{2} \left\{ \frac{a_1(y)}{y^2} - \frac{1}{2x} \left[b_1(x_2, y) + b_1(y, x_1) + \tilde{b}_1(x_2, y) - \tilde{b}_1(y, x_1) \right. \right. \\ \left. \left. - b_2(x_2, x_2) - b_2(x_1, x_1) \right] - \right. \\ \left. - \frac{1}{2} [c_1(x_2, y, x_1) - \tilde{c}_1(x_2, y, x_1)] \right\}, \end{aligned} \quad (6.34)$$

$$\sigma_R(x_2, y, x_1) = -\frac{1}{2} [c_1(x_2, y, x_1) + \tilde{c}_1(x_2, y, x_1)]. \quad (6.35)$$

Equation (6.33) is identical with our result obtained directly using a graphical method in Ref. 17). We have thus succeeded in eliminating many of the parton correlation functions and shown the equivalence of our more formal methods with the graphical technique in Ref. 17).

The reduction of the correlation function B_2 to the transverse basis is more complicated. Before discussing this point we present the lowest-order results for the deep inelastic structure functions. Here the reduction formula for B_2 is not relevant since the coefficient $b_2(x_2, x_1)$ in Eq. (6.33) vanishes to lowest order in α_s .

By explicit calculation we find the following results in lowest order,

$$a_0^{\alpha\beta}(x) = \epsilon_T^{\alpha\beta} \delta(x - x_B), \quad (6.36)$$

$$b_2^{\alpha\beta}(x_2, x_1) = 0, \quad (6.37)$$

$$\sigma_R^{\alpha\beta}(x_2, y, x_1) = -\frac{x_B}{Q^2} \epsilon_T^{\alpha\beta} \left\{ \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1} \right\}, \quad (6.38)$$

$$\sigma_L^{\alpha\beta}(x_2, y, x_1) = \frac{4}{Q^2} (\epsilon_T^{\alpha\beta} + \epsilon_L^{\alpha\beta}) \delta(y - x_B). \quad (6.39)$$

The results for the deep inelastic scattering cross-section are therefore (for unit charge quark) *)

$$W^{(a)\alpha\beta}(q, p) = e_T^{\alpha\beta} F_T^{(a)}(x_B, \frac{\Lambda^2}{Q^2}) + e_L^{\alpha\beta} F_L(x_B, \frac{\Lambda^2}{Q^2}), \quad (6.40)$$

*) We present the formulae for the electromagnetic current. The results for the weak current can be reconstructed using Eq. (5.16).

where

$$F_L(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4\Lambda^2}{Q^2} T_1(x_B) + O\left(\frac{1}{Q^4}\right), \quad (6.41)$$

$$F_T^{(a)}(x_B, \frac{\Lambda^2}{Q^2}) = A_0(x_B) + \frac{\Lambda^2}{Q^2} \left\{ 4T_1(x_B) - x_B \int dx_2 dx_1 \left\{ \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1} \right\} T_2(x_2, x_1) \right\} + O\left(\frac{1}{Q^4}\right). \quad (6.42)$$

The first power corrections to deep inelastic scattering are determined by the two functions $T_1(x)$ and $T_2(x_2, x_1)$ which are defined as

$$\Lambda^2 T_1(x) = -2 \int dx_2 dx_1 C_L(x_2, x, x_1), \quad (6.43)$$

$$\Lambda^2 T_2(x_2, x_1) = -2 \int dy C_R(x_2, y, x_1). \quad (6.44)$$

The dimensions of C_L and C_R are carried by the strong interaction scale parameter Λ so that the functions T_1 and T_2 are dimensionless.

The explicit expressions for T_1 and T_2 in terms of the parton fields on the light cone are

$$\Lambda^2 T_1(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \gamma_\mu \not{x} \gamma_\nu D_T^\mu(0) D_T^\nu(\lambda) \Psi(\lambda) \} | P \rangle, \quad (6.45)$$

$$\Lambda^2 T_2(x_2, x_1) = \frac{1}{4} \int \frac{d\lambda d\eta}{(2\pi)^2} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \langle P | T \{ \bar{\Psi}(0) \gamma_\mu \not{x} \gamma_\nu D_T^\mu(\eta) D_T^\nu(\lambda) \Psi(\lambda) \} | P \rangle. \quad (6.46)$$

We now return to the general case and complete the construction of the transverse basis by eliminating B_2 . This can be achieved by combined use of the equations of motion and the Lorentz invariance relations given in Section 4. In this section we will reproduce the particular combination of those relations which we require, using a quick method which relies on the n independence of the final answer.

To do so we return to the forward amplitudes of Section 2, and derive the following "collinear" Ward identities

$$p^\mu S_{g\mu}(x_2, x_1) = \frac{S_g(x_2) - S_g(x_1)}{x_2 - x_1}, \quad (6.47)$$

$$p^\mu S_{g\mu\nu}(x_2, y, x_1) = \frac{S_{g\nu}(y, x_1) - S_{g\nu}(x_2, x_1)}{y - x_2}, \quad (6.48)$$

$$p^\nu S_{g\mu\nu}(x_2, y, x_1) = \frac{S_{g\mu}(x_2, y) - S_{g\mu}(x_2, x_1)}{y - x_1}. \quad (6.49)$$

These relations state that, in the collinear limit, p^μ acting on a gluon line can be related to the differences of amplitudes with fewer gluon lines. These relations can be inserted into Eq. (2.62) after expanding out the tensors $d^{\mu\nu}$ and $\epsilon^{\mu\nu}$ to isolate the n dependent pieces. For the antisymmetric part there is a useful identity which follows from the existence of only two transverse directions,

$$d_{\mu\mu'} d_{\nu\nu'} d_{\rho\rho'} \epsilon^{\mu'\nu'\rho'\alpha} p_\alpha = 0. \quad (6.50)$$

Expanding this relation we obtain

$$\epsilon_{\mu\nu} p_\rho - \epsilon_{\mu\rho} p_\nu + \epsilon_{\nu\rho} p_\mu = \epsilon_{\mu\nu\rho\alpha} p^\alpha. \quad (6.51)$$

We now isolate all the explicitly n independent terms in the upper parts which we define as follows

$$g_2(x_2, x_1) = \frac{1}{2n} \text{disc} \left\{ [\hat{S}_\mu(x_2, x_1)] \sigma^\mu \right\}, \quad (6.52)$$

$$g_1(x_2, y, x_1) = \frac{1}{2n} \text{disc} \left\{ g^{\mu\nu} [\hat{S}_{\mu\nu}(x_2, y, x_1)] \right\}, \quad (6.53)$$

$$\tilde{g}_1(x_2, y, x_1) = \frac{1}{2n} \text{disc} \left\{ i \epsilon^{\mu\nu\rho\alpha} p_\alpha [\hat{S}_{\mu\nu}(x_2, y, x_1)] \delta s \delta \bar{s} \right\}. \quad (6.54)$$

Applying the collinear Ward identities we find that

$$b_1(x_2, x_1) = g_2(x_2, x_1) - \frac{a_1(x_2) - a_1(x_1)}{x_2 - x_1} - b_2(x_2, x_1), \quad (6.55)$$

$$c_1(x_2, y, x_1) = g_1(x_2, y, x_1) - \frac{b_2(y, x_1) - b_2(x_2, x_1)}{y - x_2} - \frac{b_2(x_2, y) - b_2(x_2, x_1)}{y - x_1}, \quad (6.56)$$

$$\tilde{c}_1(x_2, y, x_1) = \tilde{g}_1(x_2, y, x_1) + \frac{\tilde{b}_1(y, x_1) - \tilde{b}_1(x_2, x_1)}{y - x_2} - \frac{\tilde{b}_1(x_2, y) - \tilde{b}_1(x_2, x_1)}{y - x_1}. \quad (6.57)$$

The result for the structure functions may now be written as

$$\begin{aligned}
 W(q,p) = & \int dx a_o(x) A_o(x) + \frac{1}{2} \int dx_2 dy dx_1 \left[g_1(x_2, y, x_1) + \tilde{g}_1(x_2, y, x_1) \right] C_R(x_2, y, x_1) + \\
 & + \frac{1}{2} \int dx_2 dy dx_1 \left[g_1(x_2, y, x_1) - \tilde{g}_1(x_2, y, x_1) + \frac{1}{y} \left\{ g_2(x_2, y) + g_2(y, x_1) \right\} \right] C_L(x_2, y, x_1) - \\
 & - \int dx_2 dy dx_1 \left[\frac{a_1(y)}{y^2} + \frac{1}{2y} \left\{ \frac{a_1(x_2) - a_1(y)}{x_2 - y} + \frac{a_1(y) - a_1(x_1)}{y - x_1} \right\} \right] C_L(x_2, y, x_1) + \\
 & + \int dx_2 dx_1 \left[b_2(x_2, x_1) - \frac{1}{2} b_2(x_2, x_2) - \frac{1}{2} b_2(x_1, x_1) \right] B_2(x_2, x_1) - \\
 & - \int dx_2 dy dx_1 \frac{1}{2y} \left[b_2(x_2, y) + b_2(y, x_1) + b_2(x_2, x_2) + b_2(x_1, x_1) \right] C_L(x_2, y, x_1) - \\
 & - \int dx_2 dy dx_1 \left[\frac{b_2(y, x_1) - b_2(x_2, x_1)}{y - x_2} + \frac{b_2(x_2, y) - b_2(x_2, x_1)}{y - x_1} \right] C_1(x_2, y, x_1) + \\
 & + \int dx_2 dy dx_1 \frac{1}{2y} \left[\tilde{b}_1(x_2, y) - \tilde{b}_1(y, x_1) \right] C_L(x_2, y, x_1) + \\
 & + \int dx_2 dy dx_1 \left[\frac{\tilde{b}_1(y, x_1) - \tilde{b}_1(x_2, x_1)}{y - x_2} - \frac{\tilde{b}_1(x_2, y) - \tilde{b}_1(x_2, x_1)}{y - x_1} \right] \tilde{C}_1(x_2, y, x_1)
 \end{aligned}
 \tag{6.58}$$

The definitions of the cut amplitudes $a_1, \tilde{b}_1, b_2$ all involve the vector n_μ . Since the full result for $W(q,p)$ is n independent, the coefficients of the functions $a_1, b_2, \tilde{b}_1$, given by integrals over the correlation functions C_L, C_R, B_2 , must vanish ^{*)}. This leads to the following constraints (n independence relations)

^{*)} This feature is not manifest in our explicit lowest order calculation Eqs. (5.4)-(5.9), (6.26)-(6.39) because we adopted there a special choice for the vector n Eq. (5.1).

$$\int dx_2 dy dx_1 \left[\frac{\sigma(y)}{y^2} + \frac{1}{2y} \left\{ \frac{\sigma(x_2) - \sigma(y)}{x_2 - y} + \frac{\sigma(y) - \sigma(x_1)}{y - x_1} \right\} \right] C_{\perp}(x_2, y, x_1) = 0, \quad (6.59)$$

$$\begin{aligned} & \int dx_2 dx_1 \left\{ \sigma(x_2, x_1) - \frac{1}{2} \sigma(x_2, x_2) - \frac{1}{2} \sigma(x_1, x_1) \right\} B_2(x_2, x_1) = \\ & \int dx_2 dy dx_1 \left[\frac{1}{2y} \left\{ \sigma(x_2, y) + \sigma(y, x_1) + \sigma(x_2, x_2) + \sigma(x_1, x_1) \right\} \right] C_{\perp}(x_2, y, x_1) + \\ & + \int dx_2 dy dx_1 \left[\frac{\sigma(y, x_1) - \sigma(x_2, x_1)}{y - x_2} + \frac{\sigma(x_2, y) - \sigma(x_2, x_1)}{y - x_1} \right] C_1(x_2, y, x_1), \end{aligned} \quad (6.60)$$

$$\begin{aligned} & \int dx_2 dy dx_1 \frac{1}{2y} [\sigma(x_2, y) - \sigma(y, x_1)] C_{\perp}(x_2, y, x_1) = \\ & \int dx_2 dy dx_1 \left\{ \frac{\sigma(x_2, y) - \sigma(x_2, x_1)}{y - x_1} - \frac{\sigma(y, x_1) - \sigma(x_2, x_1)}{y - x_2} \right\} \tilde{C}_1(x_2, y, x_1). \end{aligned} \quad (6.61)$$

Here, as in Eqs. (6.28)-(6.32), σ denotes an arbitrary test function.

After imposition of the constraints it is only the first two lines of Eq. (6.58) which remain. This is yet another representation for the answer, although it is not a particularly useful one. The purpose of this exercise was rather to derive the constraint equations [Eqs. (6.59)-(6.61)].

Equation (6.60) allows us to eliminate B_2 altogether from the formula. We therefore see explicitly that the power corrections are controlled entirely by C_L and C_R . [For the lowest order example which we have calculated this substitution is only of academic importance since $b_2(x_2, x_1) \equiv 0$.] Having eliminated B_2 from the basis we may investigate the constraint on the remaining elements of the basis coming from Eq. (6.60) by setting the test function $\sigma(x_2, x_1) \equiv \sigma(x_2) + \sigma(x_1)$. In this way, Eq. (6.60) becomes

$$\begin{aligned} & \int dx_2 dy dx_1 \frac{1}{2y} \{ 3\sigma(x_2) + 3\sigma(x_1) + 2\sigma(y) \} C_{\perp}(x_2, y, x_1) = \\ & = - \int dx_2 dy dx_1 \left\{ \frac{\sigma(y) - \sigma(x_2)}{y - x_2} + \frac{\sigma(y) - \sigma(x_1)}{y - x_1} \right\} C_1(x_2, y, x_1). \end{aligned} \quad (6.62)$$

The constraints, Eqs. (6.59), (6.60), (6.61) can also be derived using more formal techniques developed in Section 4. There we derived an equivalent set of n independence relations, Eqs. (4.26), (4.27), (4.28), which can be written in the following way in terms of the light-cone fractions

$$\int dx \sigma(x) A_1(x) = \int dx_2 dx_1 \frac{\sigma(x_2) - \sigma(x_1)}{x_2 - x_1} B_1(x_2, x_1), \quad (6.63)$$

$$\begin{aligned} & \int dx_2 dx_1 \sigma(x_2, x_1) [B_2(x_2, x_1) - B_1(x_2, x_1)] = \\ & = \int dx_2 dy dx_1 \left[\frac{\sigma(y, x_1) - \sigma(x_2, x_1)}{y - x_2} + \frac{\sigma(x_2, y) - \sigma(x_2, x_1)}{y - x_1} \right] C_1(x_2, y, x_1), \end{aligned} \quad (6.64)$$

$$\begin{aligned} & \int dx_2 dx_1 \sigma(x_2, x_1) \tilde{B}_1(x_2, x_1) = \\ & = \int dx_2 dy dx_1 \left[\frac{\sigma(x_2, y) - \sigma(x_2, x_1)}{y - x_1} - \frac{\sigma(y, x_1) - \sigma(x_2, x_1)}{y - x_2} \right] \tilde{C}_1(x_2, y, x_1). \end{aligned} \quad (6.65)$$

By combining the above relations with those coming from the equations of motion, Eqs. (6.28)-(6.32), we can rederive the constraints on the elements of the transverse basis, Eqs. (6.59)-(6.62).

From Eqs. (6.43), (6.44), we see that C_L and C_R in their full generality do not appear in the final answer (at least at the tree graph level). The constraint equations tend to give relations between the quantities appearing in the answer and other integrals over C_L and C_R which are not directly of interest at the tree graph level. An important exception to this statement is the relation

$$4 \int dx T_1(x) = \int dx_2 dx_1 T_2(x_2, x_1), \quad (6.66)$$

which can be derived from Eqs. (6.59)-(6.62). Substituting this constraint into Eq. (6.42), we see that it ensures the validity of the Adler sum rule (for the case of a massless target).

7. - ξ SCALING

The first analysis of power corrections to scaling processes came from the operator product expansion^{21),22)}. In this context, classification of the operators in terms of their twist requires that the operators be traceless and symmetric in their tensor indices. This introduces a calculable dependence on the target mass M . For example, the matrix element of the operator of spin two has the tensor structure,

$$\langle P | O^{\{\mu\nu\}} | P \rangle = \langle O^{(2)} \rangle \left\{ P^\mu P^\nu - \frac{1}{4} M^2 g^{\mu\nu} \right\}. \quad (7.1)$$

After constructing similar expressions for any spin, the complete M dependence (to all orders in M^2/Q^2) can be included by taking combinations of moments in the variable ξ ,

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4\mu x_B^2}}, \quad \mu = \frac{M^2}{Q^2}, \quad x_B = \frac{Q^2}{2P_0}. \quad (7.2)$$

In our parton-like treatment ξ scaling is a kinematic consequence of the inclusion of transverse momentum. Since we have taken full account of the transverse momentum the information which leads to ξ scaling is already present in our formulae.

The only adjustment which we have to make so that our formulae are completely correct also for a massive target is to reinterpret the vector p . For a massless target p was identified directly with the target momentum. For a massive target p now becomes an auxiliary vector related to the target momentum P (and the other auxiliary vector n) by,

$$P^\mu = p^\mu + \frac{M^2}{2} n^\mu. \quad (7.3)$$

For practical purposes it is most convenient to continue to work in terms of p^μ and n^μ , rather than to eliminate p using Eq. (7.3). For the massive case the vector q is expressed as

$$Q^\mu = -\xi p^\mu + \frac{Q^2}{2\xi} n^\mu \quad (7.4)$$

[i.e., completely equivalent to Eq. (5.1) with the replacement $(x_B \rightarrow \xi)$]. From Eqs. (7.3) and (7.4) we have that

$$x_B = \frac{Q^2}{2P_2} - \frac{\xi}{1-\mu\xi^2}, \quad (7.5)$$

and consequently Eq. (7.2). To take account of finite target mass, x_B should be replaced by ξ everywhere in our previous formulae.

This distinction between the vectors p and P also influences the derivation of the n independence relations. We continue to define the tensor decomposition of the parton correlation functions in terms of the vectors p and n according to Eqs. (2.54)-(2.59). To distinguish the correlation functions corresponding to zero and non-zero target mass we denote the correlation functions in the latter case by bold face letters. The relation between these correlation functions are modified by terms of order M^2 , because before imposing n independence we must express all formulae in terms of the physical vector of the problem P . The n dependence implicit in p because of Eq. (7.3) should be made explicit.

For example, the n independence relations Eq. (4.26) relate the coefficient of n^0 in Eq. (2.54) to the coefficient of $d^{0\mu}$ in Eq. (2.56). In terms of the physical vector P , Eq. (2.54) should be rewritten as,

$$\begin{aligned} \frac{1}{4} \langle P | T \{ \bar{\Psi}(0) \gamma^S \Psi(\lambda) \} | P \rangle &= \mathbf{A}_0(\lambda) p^S + \mathbf{A}_1(\lambda) n^S = \\ &= \mathbf{A}_0(\lambda) P^S + \left\{ \mathbf{A}_1(\lambda) - \frac{M^2}{2} \mathbf{A}_0(\lambda) \right\} n^S. \end{aligned} \quad (7.6)$$

For the massive case the n independence relation [corresponding to Eq. (4.26)] becomes

$$\mathbf{A}_1(\lambda) - \frac{M^2}{2} \mathbf{A}_0(\lambda) = -i \int_0^\lambda d\eta \mathbf{B}_1(\eta, \lambda) \quad (7.7)$$

In a similar way Eq. (4.27) is modified by the replacement

$$\mathbf{B}_2(\lambda) \rightarrow \mathbf{B}_2(\lambda) - \frac{M^2}{2} i \frac{\partial}{\partial \lambda} \mathbf{A}_0(\lambda). \quad (7.8)$$

Eq. (4.28) remains unchanged.

Using the mnemonic method of deriving the n independence relations presented in the last section, the inclusion of the M^2 terms is even more immediate. To order M^2 the only change is the replacement,

$$\begin{aligned} a_0(x) &= \frac{1}{2\hat{n}} \text{Disc} [\hat{S}(xp)] = \\ &= \text{Disc} [\hat{S}(xI)] - \frac{M^2}{2} a_1(x) - \frac{xM^2}{2} b_2(x, x) + O(M^4). \end{aligned} \quad (7.9)$$

As a result of these changes the constraint equations [Eqs. (6.59) and (6.60)] become

$$\begin{aligned} \int dx_2 dy dx_1 \left\{ \frac{\sigma(y)}{y^2} + \frac{1}{2y} \left[\frac{\sigma(x_2) - \sigma(y)}{x_2 - y} + \frac{\sigma(y) - \sigma(x_1)}{y - x_1} \right] \right\} \mathbf{C}_L(x_2, y, x_1) = \\ = -\frac{1}{2} M^2 \int dy \sigma(y) \mathbf{A}_0(y), \end{aligned} \quad (7.10)$$

$$\begin{aligned} \int dx_2 dx_1 \left\{ \sigma(x_2, x_1) - \frac{1}{2} \sigma(x_2, x_2) - \frac{1}{2} \sigma(x_1, x_1) \right\} \mathbf{B}_2(x_2, x_1) = \\ \int dx_2 dy dx_1 \left[\frac{1}{2y} \left\{ \sigma(x_2, y) + \sigma(y, x_1) + \sigma(x_2, x_2) + \sigma(x_1, x_1) \right\} \right] \mathbf{C}_L(x_2, y, x_1) + \\ + \int dx_2 dy dx_1 \left[\frac{\sigma(y, x_1) - \sigma(x_2, x_1)}{y - x_2} + \frac{\sigma(x_2, y) - \sigma(x_2, x_1)}{y - x_1} \right] \mathbf{C}_1(x_2, y, x_1) + \\ + \frac{1}{2} M^2 \int dx x \sigma(x, x) \mathbf{A}_0(x). \end{aligned} \quad (7.11)$$

Eq. (6.61) is unchanged.

The two changes allow us to include the effects of a massive target. The results for the electroproduction structure functions are

$$\begin{aligned} F_L(x_B, \frac{\Lambda^2}{Q^2}) &= \frac{4\Lambda^2}{Q^2} \mathbf{T}_1(\xi) + O\left(\frac{1}{Q^4}\right), \\ F_T^{(a)}(x_B, \frac{\Lambda^2}{Q^2}) &= \mathbf{A}_0(\xi) + \frac{\Lambda^2}{Q^2} \left[4\mathbf{T}_1(\xi) - \xi \int \frac{\sigma(x_2, \xi) - \sigma(x_1, \xi)}{x_2 - x_1} \mathbf{T}_2(x_2, x_1) dx_2 dx_1 \right] + \\ &\quad + O\left(\frac{1}{Q^4}\right) \end{aligned} \quad (7.12)$$

and for the interference structure functions ($v = -a = 1/\sqrt{2}$)

$$F_A^{(a)}(x_3, \frac{\Lambda^2}{Q^2}) = \mathbf{A}_0(\frac{x}{2}) - \frac{\Lambda^2}{Q^2} \frac{x}{2} \int dx_2 dx_1 \frac{\delta(x_2 - \xi) - \delta(x_1 - \xi)}{x_2 - x_1} \mathbf{T}_2(x_2, x_1) + O(\frac{1}{Q^4}). \quad (7.13)$$

The formulae for the parton correlation functions $\mathbb{T}$ are still given by covariant derivatives on the light cone,

$$\Lambda^2 \mathbf{T}_1(x) = \frac{1}{4} \int \frac{d\lambda}{\lambda} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \gamma_\mu \gamma_\nu D_1^\mu(0) D_1^\nu(\lambda) \Psi(\lambda) \} | P \rangle, \quad (7.14)$$

$$\Lambda^2 \mathbf{T}_2(x_2, x_1) = \frac{1}{4} \int \frac{d\eta d\lambda}{(\lambda\eta)^2} e^{i\eta(x_2 - x_1)} e^{i\lambda x_1} \langle P | T \{ \bar{\Psi}(0) \gamma_\mu \gamma_\nu D_1^\mu(\eta) D_1^\nu(\lambda) \Psi(\lambda) \} | P \rangle, \quad (7.15)$$

but the fields are now sandwiched between massive hadron states. As before $\mathbb{T}_1$ and $\mathbb{T}_2$ are integrals over the correlation functions of the transverse basis

$$\begin{aligned} \Lambda^2 \mathbf{T}_1(x) &= -2 \int dx_2 dx_1 \mathbf{C}_L(x_2, x, x_1), \\ \Lambda^2 \mathbf{T}_2(x_2, x_1) &= -2 \int dx \mathbf{C}_R(x_2, x, x_1). \end{aligned} \quad (7.16)$$

$\mathbf{C}_L$ and $\mathbf{C}_R$ are defined in analogy with the corresponding quantities for the massless target Eqs. (6.13) and (6.14).

The implication of the constraint equations Eqs. (6.61), (7.10) and (7.11) for the parton correlation functions $\mathbb{T}_1$ and $\mathbb{T}_2$ is

$$4 \int dx \mathbf{T}_1(x) - \int dx_2 dx_1 \mathbf{T}_2(x_2, x_1) = \int_0^1 dx x^2 \mathbf{A}_0(x). \quad (7.17)$$

This equation is the generalization of Eq. (6.66) to the case of a massive target.

The kinematical and dynamical components of the correlation functions $\mathbf{C}_L$, $\mathbf{C}_R$ are isolated by taking the $g = 0$ limit in the covariant derivatives. In this limit,

$$\mathbf{C}_L(x_2, y, x_1) = \mathbf{C}_R(x_2, y, x_1) = -\frac{1}{2} \delta(y - x_1) \delta(y - x_2) \mathbf{C}(y). \quad (7.18)$$

The arbitrary function $\mathbf{C}(y)$ can be fixed by inserting Eq. (7.18) into the constraint Eq. (7.10)

$$\mathbf{C}(y) = \int_0^1 dx x^2 \int_y^1 \frac{dz}{z} \mathbf{A}_0(z). \quad (7.19)$$

As expected the kinematic power corrections are of order M^2 . In the limit $g \rightarrow 0$, $M \rightarrow 0$ there are no power corrections. For a massive target Eq. (7.19) reproduces ξ scaling (up to order M^2) and $\mathbb{C}(y)$ is the transverse momentum squared [cf. Eq. (1.29)].

A general decomposition of $\mathbb{C}_L, \mathbb{C}_R$ into kinematical and dynamical components can be written as follows,

$$\begin{aligned} \mathbb{C}_L(x_2, y, x_1) &= -\frac{1}{2} \delta(y-x_1) \delta(y-x_2) M^2 y^2 \int_y^1 \frac{dz}{z} \mathbf{A}_0(z) + C_L(x_2, y, x_1), \quad (7.20) \\ \mathbb{C}_R(x_2, y, x_1) &= -\frac{1}{2} \delta(y-x_1) \delta(y-x_2) M^2 y^2 \int_y^1 \frac{dz}{z} \mathbf{A}_0(z) + C_R(x_2, y, x_1). \end{aligned} \quad (7.21)$$

These relations imply that T_1 and T_2 are given by,

$$\Lambda^2 T_1(x) = M^2 x^2 \int_x^1 \frac{dz}{z} \mathbf{A}_0(z) + \Lambda^2 T_1(x), \quad (7.22)$$

$$\Lambda^2 T_2(x_2, x_1) = \delta(x_2 - x_1) M^2 x_1^2 \int_{x_1}^1 \frac{dz}{z} \mathbf{A}_0(z) + \Lambda^2 T_2(x_2, x_1). \quad (7.23)$$

The first terms on the right-hand side of these equations (proportional to M^2) correspond to the transverse momentum in the absence of interaction. The second terms contain the transverse momentum generated by the interaction together with its gauge covariant partners - the transverse components of the gluon field. Inserting the forms Eqs. (7.20) and (7.21) into the full constraint equations we find that C_L and C_R (and hence T_1 and T_2) satisfy the constraint equations for the massless case Eqs. (6.59)-(6.62) and (6.66).

Equations (7.22) and (7.23) separate the dynamic and kinematic power corrections, keeping the latter to order M^2 . However, the kinematic power corrections can be taken into account to all orders. This target mass dependence is removed by taking Nachtmann moments^{(21), (34), *)}

*) In these formulae the standard linear combinations of structure functions should be taken to project out the non-singlet parts.

$$\begin{aligned}
 & \int_0^1 \frac{dx_B}{x_B} \xi^{N-1} \left\{ 1 + 2\mu \xi^2 \frac{N-1}{N+2} + \frac{N(N-1)}{(N+2)(N+3)} \mu^2 \xi^4 \right\} F_2(x_B, \frac{\Lambda^2}{Q^2}) = \\
 & = \int d\xi \xi^{N-1} A_0(\xi) + \frac{\Lambda^2}{Q^2} \left[4 \int dx x^{N-1} T_1(x) - \int dx_2 dx_1 \frac{x_2^N - x_1^N}{x_2 - x_1} T_2(x_2, x_1) \right] + \\
 & \quad + O\left(\frac{1}{Q^4}\right), \\
 & \int_0^1 \frac{dx_B}{x_B^2} \xi^{N-1} \left\{ N + \mu \xi^2 \frac{N(N-1)}{N+1} \right\} F_1(x_B, \frac{\Lambda^2}{Q^2}) = \tag{7.24}
 \end{aligned}$$

$$\begin{aligned}
 & = 4\mu \int d\xi \xi^{N-1} A_0(\xi) + \frac{4\Lambda^2}{Q^2} \int dx x^{N-1} T_1(x) + O\left(\frac{1}{Q^4}\right), \\
 & \int_0^1 dx_B \xi^{N-1} \left\{ 1 - \frac{2}{N+2} \mu \xi^2 - \frac{N}{N+2} \mu^2 \xi^4 \right\} F_3(x_B, \frac{\Lambda^2}{Q^2}) = \tag{7.25}
 \end{aligned}$$

$$\begin{aligned}
 & = \int d\xi \xi^{N-1} A_0(\xi) - \frac{\Lambda^2}{Q^2} \int dx_2 dx_1 \frac{x_2^N - x_1^N}{x_2 - x_1} T_2(x_2, x_1) + O\left(\frac{1}{Q^4}\right).
 \end{aligned}$$

(7.26)

In these moment equations there is a well-known difficulty²³⁾ associated with the range of ξ . All values of ξ such that

$$\xi > \frac{2}{1 + \sqrt{1 + 4\mu}} \tag{7.27}$$

correspond to unphysical values of x_B ($x_B \geq 1$). The paradox arises because the natural range for ξ is $0 \leq \xi \leq 1$ and simple sum rules follow only from Eqs. (7.24)-(7.26) if ξ spans this range. This effect is caused by the mismatch between the spectrum of states present in perturbative QCD (quarks and gluons) and the observed spectrum of physical particles. The problem remains for all finite twists and is resolved only after inclusion of the effects which bind the quarks and gluons to give s channel resonances.

Ignoring this problem, taking the full range for ξ and using the constraint equation (6.66) we can derive the Adler sum rule,

$$\int \frac{dx_B}{x_B} \left[F_2^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) - F_2^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] = A. \quad (7.28)$$

A is a pure number. All Q^2 dependence drops out because (7.28) is an exact current algebra sum rule.

Including the first logarithmic corrections and the dynamic corrections we may write the Gross-Llewellyn Smith sum rule as,

$$\begin{aligned} \int dx_B \left(1 - \frac{2}{3}\mu\xi^2 - \frac{1}{3}\mu^2\xi^4 \right) \left[F_3^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) + F_3^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] = \\ = L_0 \left[1 - \frac{\alpha_s(Q^2)}{\pi} - \frac{\Lambda^2}{Q^2} L_1 + O\left(\frac{1}{Q^4}\right) \right]. \end{aligned} \quad (7.29)$$

L_0 counts the number of quarks in the target H and L_1 is given by

$$L_1 L_0 = 4 \sum_{\text{flavours}} \int dx T_1(x). \quad (7.30)$$

Since we have taken Nachtmann moments the term L_1 contains only dynamical power corrections. In Section 8 we discuss the positivity constraint: $T_1(x) \geq 0$. Using Eq. (7.22) we see that this implies that the dynamical term is bounded from below,

$$\Lambda^2 L_0 L_1 \geq - \frac{4M^2}{3} \int_0^1 dx x^2 A_0(x) = - \frac{4M^2}{3} \int_0^1 dx x_B x_B^2 \left[F_3^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) + F_3^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] + O\left(\frac{1}{Q^4}\right). \quad (7.31)$$

Certain caveats must be made about the use of Eq. (7.29). As before the derivation in the context of perturbative QCD requires that ξ run from zero to one. Unknown higher order corrections in $\alpha_s(Q^2)$ which are formally always more important than powers have been omitted.

8. - POSITIVITY

As we have seen the result in the transverse basis is expressed in terms of only two correlation functions T_1, T_2 (apart from four fermion contributions $Q, \tilde{Q}$) and has a simple form. This is to be contrasted with the corresponding result in the collinear basis where there are four correlation functions $\tilde{G}_1, J, G_2, \tilde{G}_2$ and the coefficient functions are complicated. In a certain sense this change of basis is trivial since we know that at tree graph level the end result for any structure function is

$$F(x_B, \frac{\Lambda^2}{Q^2}) = L(x_B) + \frac{\Lambda^2}{Q^2} N(x_B) + O\left(\frac{1}{Q^4}\right). \quad (8.1)$$

For every structure function we can write the function $N(x_B)$ as the matrix element of a single operator.

However, the physical significance of the operators of the transverse basis is that they have simple expressions in terms of fields. The interpretation in terms of dynamical transverse momentum and the connection with the parton model¹⁷⁾ follows from these simple forms. Reduction to the collinear basis, although mathematically equivalent, obscures this interpretation.

In this section we consider the positivity constraints on the correlation functions of the transverse basis. In particular we shall use the explicit form of T_1 to establish the positivity condition $T_1 > 0$. T_1 can be expressed as a product of fields on the light cone which from Eq. (6.18) can be reduced to

$$\Lambda^2 T_1(\lambda) = -2C_1(0, \lambda, \lambda) = -2 \frac{\partial^2}{\partial \lambda^2} A_1(\lambda) \quad (8.2)$$

or equivalently in momentum space,

$$\Lambda^2 T_1(x) = 2x^2 A_1(x). \quad (8.3)$$

To study the properties of T_1 it is therefore enough to analyze A_1 which has very simple expression in terms of the fields on the light-cone

$$A_1(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\psi}(0) \not{x} \psi(\lambda) \} | P \rangle. \quad (8.4)$$

The similarity with the leading twist quark distribution function is apparent:

$$A_0(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \not{x} \Psi(\lambda) \} | P \rangle \quad (8.5)$$

The quantities, A_0 and A_1 , correspond to the two different spin projections of the hadronic blob with only two fermion lines

$$\hat{T}_{\alpha\beta}^i(k) = \int d^4x e^{ikx} \langle P | T \{ \bar{\Psi}_\alpha(0) \Psi_\beta(x) \} | P \rangle \quad (8.6)$$

It is therefore convenient to introduce a common notation

$$T_i(k) = \frac{1}{4} [\not{x} \hat{T}^i(k)], \quad A_i(x) = \int \frac{d^4k}{(2\pi)^4} \delta(x - km) T_i(k), \quad i=0,1 \quad (8.7)$$

where

$$l_0^\mu = m^\mu, \quad l_1^\mu = p^\mu. \quad (8.8)$$

We will perform d^4k integral in Eq. (8.7) by exploiting the analyticity properties of the blobs $T_i(k)$.

As discussed in Section 3, perturbation theory suggests that $T_i(k)$ is an analytic function in the variables k^2 , $s = (p-k)^2$, $u = (p+k)^2$ with all singularities located in the corresponding lower half-planes. For the present purposes it will be more convenient to write down the dispersion relation rather than use the integral representation, Eq. (3.20). We have

$$T_i(k) = \frac{i}{2\pi} \int_0^\infty d\sigma^2 \frac{S_i(\sigma^2, k^2)}{(p-k)^2 - \sigma^2 + i\epsilon} + u \text{ channel cut.} \quad (8.9)$$

The integral $d^4k \delta(x - km)$ can be now performed using the light cone parametrization,

$$d^4k \delta(x - km) = \frac{1}{2|x|} dk^2 d^2k_T. \quad (8.10)$$

Since $(p-k)^2 = -(1-x)k^2/x - k_T^2/x$ and the singularities in k^2 , hidden in the spectral function $\rho_i(\sigma^2, k^2)$ are below the real axis, we conclude that the non-zero contribution to the dk^2 integral comes only from the region $(1-x)/x > 0$, i.e., $0 < x < 1$. Otherwise we get zero by closing the contour

in the upper half-plane. For $0 < x < 1$ we close the contour around the dispersive denominator and we get

$$T_i(k) = \Theta(x(1-x)) \int \frac{d^4k}{(2\pi)^4} \delta(x-km) \mathcal{G}_i((p-k)^2, k^2). \quad (8.11)$$

In a similar way we find that the u channel cut in Eq. (8.9) gives a non-zero contribution for $-1 < x < 0$. Since for both A_0 and A_1 , the photon-parton cut amplitude contains the factor $\delta(x-x_B)$, only the s channel term written explicitly in Eq. (8.9) contributes for $0 < x_B < 1$. By using Cutkosky rules³⁵⁾ we can write a more explicit expression for the spectral function ρ_i :

$$\begin{aligned} \mathcal{G}_i((p-k)^2, k^2) &= \text{Disc}_{(p-k)^2} T_i(k) = \\ &= \sum_{\sigma} \langle p | \bar{\Psi}_{\alpha}(0) | \sigma \rangle (2\pi)^4 \delta^4(p-k-\sigma) \langle \sigma | \Psi_{\beta}(0) | p \rangle \mathcal{L}_{i\alpha\beta}. \end{aligned} \quad (8.12)$$

In Eq. (8.12) the sum goes over all physical states $|\sigma\rangle$ with the total four-momentum σ (i.e., $\sigma^2 > 0$, $\sigma_0 > 0$) and the normalization is such that $\sum_{\sigma} |\sigma\rangle\langle\sigma| = 1$.

Performing d^4k integral in Eq. (8.11) we get,

$$A_i(x) = \Theta(x(1-x)) \sum_{\sigma} \delta(\sigma m - (1-x)) \Phi_{\alpha}^+(\sigma, p) H_i^{\alpha\beta} \Phi_{\beta}(\sigma, p), \quad (8.13)$$

where

$$\Phi_{\alpha}(\sigma, p) = \langle \sigma | \Psi_{\alpha}(0) | p \rangle, \quad (8.14)$$

and

$$H_i^{\alpha\beta} = [\gamma_0 \mathcal{L}_i]^{\alpha\beta}. \quad (8.15)$$

Equation (8.13) implies that $A_i(x)$ is positive-definite

$$A_i(x) \geq 0, \quad i=0, 1. \quad (8.16)$$

To demonstrate it, we choose a frame in which

$$p = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \quad m = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right) \quad (8.17)$$

[It corresponds to $P = 1/\sqrt{2}$ in Eq. (1.6)]. We may now write a representation for γ_0

$$\gamma_0 = \frac{1}{\sqrt{2}} (\not{p} + \not{m}) \quad (8.18)$$

which implies

$$\hat{H}_0 = \frac{1}{\sqrt{2}} \not{p} \not{m} = \sqrt{2} \hat{e}^{(+)}, \quad \hat{H}_1 = \frac{1}{\sqrt{2}} \not{m} \not{p} = \sqrt{2} \hat{e}^{(-)}. \quad (8.19)$$

The matrices $\hat{e}^{(\pm)}$ project onto the "plus" and "minus" components of the spinor field Eq. (6.25). In consequence, the matrices $\hat{H}_i$ are semi-positive definite and so are the quadratic forms in Eq. (8.13),

$$\Phi_\alpha^+ \hat{H}^{\alpha\beta} \Phi_\beta \geq 0. \quad (8.20)$$

which completes the proof.

Let us finally note that the expression (8.13) can be written again in a compact form in terms of the fields on the light cone. Writing the Fourier representation of the δ function in Eq. (8.13), absorbing the n, σ dependence by shifting the argument of the fermion field and using the completeness of the states $|\sigma\rangle$ we get,

$$A_i(x) = G(x) \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | \bar{\psi}(0) \not{x}_i \psi(\lambda) | p \rangle + u \text{ channel} \quad (8.21)$$

The positivity constraint for A_1 fixes the sign of T_1 , i.e., the sign of the $1/Q^2$ contribution to the longitudinal structure function. Also, due to the constraint $4 \int dx T_1(x) = \int dx_2 dx_1 T_2(x_2, x_1)$, we can now fix signs of the $1/Q^2$ corrections to the first moments of all structure functions (F_L, F_T, F_3).

So far, we have discussed the massless case ($M=0$). For a massive target the positivity condition is valid for the full $T_1(x)$, i.e., with the ξ scaling part included. For the dynamical partner of the kinematical M^2/Q^2 correction we get therefore the lower bound

$$T_1(x) > 0 \Rightarrow \Lambda^2 T_1(x) > - M^2 x^2 \int_x^1 \frac{dz}{z} A_0(z). \quad (8.22)$$

This version of the positivity constraint should be used when the data analysis is done in terms of Nachtmann moments. In particular, for the $1/Q^2$ correction to the GLS sum rule we get Eq. (7.31).

9. - CONCLUSIONS

We have presented a complete study of the power corrections to deep inelastic scattering at the tree graph level. The principal point which our analysis brings out is the importance of the transverse degrees of freedom. The kinematical power corrections (ξ scaling) are given by the transverse momentum of on-shell partons; the dynamical power corrections are controlled by the transverse components of momentum and the gluon field. The similarity of this picture with the naive parton model encourages the belief that the transverse basis will provide a phenomenologically viable method of parametrizing the power corrections to deep inelastic scattering.

Our graphical method suggests several topics which deserve further study. The first is the treatment of loops and associated logarithms. It is necessary to prove that all mass singularity logarithms can be factored also in order $1/Q^2$ and beyond. A method of proof has been proposed¹⁾, largely based on the proof for the terms which scale (twist 2)²⁶⁾. It is only now that we have a valid graphical method of dealing with multiparton processes that these suggestions can be carried through.

On the more practical side there is the question of the lowest order coefficients of these logarithms. Although partial calculations of anomalous dimensions exist in the literature, a complete analysis is lacking. The problem is daunting because of its technicality. We believe that working directly in the space of light cone fractions in the light-like axial gauge will provide a considerable simplification.

One may argue with some justification that logarithmic scale violations to power corrections are of minor phenomenological interest. We believe that such calculations would give two interesting pieces of information. The first would be to show to what extent the simplicity of the transverse basis description is preserved under renormalization. Secondly, we would discover the correct scale for dynamical higher twists. The dynamical higher twists begin at order g but we do not know whether the coupling constant should be evaluated at a large scale $g(Q^2)$ or a non-perturbative scale $g(\Lambda^2)$. These two choices differ in perturbation theory by a logarithm and the calculation of the anomalous dimensions will settle the question.

In order to apply our methods to other processes we will have to understand in detail the cut structure of the multiparton correlation functions. This was not necessary for deep inelastic scattering because the structure functions are given by the total discontinuity of the forward scattering amplitude. The formalism which we have built up lends itself to extension to other processes. To convince ourselves that it is really applicable we must reproduce the results for deep inelastic scattering treating the structure function as a sum of cuts rather than as the total discontinuity of the forward amplitude.

We have therefore not considered the power corrections to other processes in this paper. In particular we have left the Pandora's box containing multiparton contributions to lepton pair production³⁶⁾ unopened.

ACKNOWLEDGEMENTS

We are happy to acknowledge helpful discussions with R.L. Jaffe, P.V. Landshoff, A.H. Mueller, W. Scott, M. Soldate and R. Stora.

τ	F , B	
2	1 0	p^0
4	1 0	n^0
	1 1	$d^{0\mu}, i \epsilon^{0\mu}, p^0 n^\mu$
	1 2	$p^0 d^{\mu_1\mu_2}, i p^0 \epsilon^{\mu_1\mu_2}$
	2 0	$p^{01} p^{02}$

TABLE - Polarizers present for $\tau = 2, 4$ in the non-singlet sector. $d^{\mu\nu} = g^{\mu\nu} - n^\mu p^\nu - n^\nu p^\mu$,
 $\epsilon^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} n_\alpha p_\beta$.

APPENDIX A

We discuss here the limits on the light cone fractions x_1, x_2 for the three-parton correlation functions implied by the perturbative analyticity.

We use the standard notation for the momenta k_1, k_2 as in Fig. 2b. The relevant invariants are

$$\sigma_1^2, \sigma_2^2, \sigma_3^2, (p+\sigma_i)^2, (p-\sigma_i)^2, \quad i=1,2,3$$

where $\sigma_1 = k_1$, $\sigma_2 = k_2$, $\sigma_3 = k_2 - k_1$.

It is convenient to write down the denominator present in the representation Eq. (3.20) in the following way,

$$\begin{aligned} D = & y_1 [\alpha_1 \sigma_1^2 + \alpha_2 (p-\sigma_1)^2 + \alpha_3 (p+\sigma_1)^2] + y_2 [\beta_1 \sigma_2^2 + \beta_2 (p-\sigma_2)^2 + \beta_3 (p+\sigma_2)^2] + \\ & + y_3 [\gamma_1 \sigma_3^2 + \gamma_2 (p-\sigma_3)^2 + \gamma_3 (p+\sigma_3)^2], \end{aligned} \quad (A.1)$$

where

$$\sum_i y_i = \sum_i \alpha_i = \sum_i \beta_i = \sum_i \gamma_i = 1 \quad (A.2)$$

and each of the parameters $y_i, \alpha_i, \beta_i, \gamma_i$ is in the range (0,1). The sets $\{y_i\}, \{\alpha_i\}, \{\beta_i\}, \{\gamma_i\}$ are independent. We express now σ_i 's in terms of k_i 's and solve the system of linear equations in Eq. (3.23). The result is

$$\begin{aligned} x_1 &= \frac{\alpha(y_1 y_2 + y_1 y_3) + (\beta - \gamma) y_2 y_3}{y_1 y_2 + y_1 y_3 + y_2 y_3}, \\ x_2 &= \frac{\beta(y_1 y_2 + y_2 y_3) + (\alpha + \gamma) y_1 y_3}{y_1 y_2 + y_1 y_3 + y_2 y_3}, \\ x_2 - x_1 &= \frac{\gamma(y_1 y_3 + y_2 y_3) + (\beta - \alpha) y_1 y_2}{y_1 y_2 + y_1 y_3 + y_2 y_3}, \end{aligned} \quad (A.3)$$

where $\alpha = \alpha_2 - \alpha_3$, $\beta = \beta_2 - \beta_3$, $\gamma = \gamma_2 - \gamma_3$. Since y_i are positive, we can write the result in the following form

$$\begin{aligned} x_1 &= \alpha(1-\omega_1) + (\beta-\gamma)\omega_1, \\ x_2 &= \beta(1-\omega_2) + (\alpha+\gamma)\omega_2, \\ x_2-x_1 &= \gamma(1-\omega_3) + (\beta-\alpha)\omega_3 \end{aligned} \quad (A.4)$$

where

$$\omega_1 = y_2 y_3 / \Delta, \quad \omega_2 = y_1 y_3 / \Delta, \quad \omega_3 = y_1 y_2 / \Delta, \quad \Delta = y_1 y_2 + y_1 y_3 + y_2 y_3.$$

Since ω_i are in the interval $(0,1)$ we get the bounds

$$\begin{aligned} \min\{\alpha, \beta-\gamma\} &\leq x_1 \leq \max\{\alpha, \beta-\gamma\}, \\ \min\{\beta, \alpha+\gamma\} &\leq x_2 \leq \max\{\beta, \alpha+\gamma\}, \\ \min\{\gamma, \beta-\alpha\} &\leq x_2-x_1 \leq \max\{\gamma, \beta-\alpha\}. \end{aligned} \quad (A.5)$$

The parameters α, β, γ are in the range $(-1,1)$. We therefore find that the absolute limits on x_i which we can derive from the Nakanishi representation are

$$-2 \leq x_1, x_2, x_2-x_1 \leq 2 \quad (A.6)$$

We have therefore shown that the light-cone fractions x_i span a finite range. However, the limit on the x_i is less stringent than one would expect from parton model ideas ($|x_i| \leq 1$). In particular if the limit Eq. (A.6) were saturated in deep inelastic scattering it would imply that the discontinuity in x_B would exceed the permissible range $-1 \leq x_B \leq 1$.

We shall therefore consider some simple diagrams to investigate the ranges of the light cone fractions x_i . The simplest topology is the one in Fig. 9a. In this case we have only $k_1^2, k_2^2, (p-k_1)^2, (p-k_2)^2, (k_2-k_1)^2$ singularities which correspond to $\alpha_3 = \beta_3 = \gamma_2 = \gamma_3 = \gamma = 0$. Now $0 < \alpha, \beta < 1$ and we get

$$0 < x_i < 1, \quad -1 < x_2-x_1 < 1. \quad (A.7)$$

The naïve parton model limit $0 < x_i < 1$ is exceeded when we include $(p \pm k_2 - k_1)^2$ singularities. The simplest topology of this kind is presented in Figs. 9b,c. In Fig. 9b we have $\alpha = 0$, $0 < \beta < 1$, $0 < \gamma < 1$ which implies the limits

$$-1 < x_1 < 1, \quad 0 < x_2 < 1, \quad 0 < x_2 - x_1 < 1. \quad (\text{A.8})$$

The corresponding limits for the diagram in Fig. 9c are

$$0 < x_1 < 1, \quad -1 < x_2 < 1, \quad -1 < x_2 - x_1 < 0. \quad (\text{A.9})$$

We see that the bound $|x_i| \leq 1$ is never violated at tree graph level.

We have also examined all one-loop diagrams contributing to the three-parton correlation function. For these graphs we find that there are correlations between the $\{\alpha_i, \beta_i, \gamma_i, y_i\}$ of Eq. (A.1) which are apparent after Feynman parametrization of the graph. For simplicity, we may consider the diagram with truncated parton legs since the diagram with the parton propagators included respects the same bounds. The correlations between the parameters $\{\alpha_i, \beta_i, \gamma_i, y_i\}$ restrict the bounds of Eq. (A.5) such that

$$|x_i| \leq 1 \quad (\text{A.10})$$

for all one-loop graphs.

We expect that Eq. (A.10) is true in general. The Nakanishi representation is not strong enough to prove Eq. (A.10) in general because it ignores all properties of the parameters $\{\alpha_i, \beta_i, \gamma_i, y_i\}$ apart from their positivity. Nevertheless we believe that it should be possible to prove Eq. (A.10) using the full perturbative analyticity of the lower blob alone.

In the case of deep inelastic scattering the problem of the ranges for the light cone fractions $\{x_i\}$ can be analyzed in the operator mode by examination of the forward Compton amplitude ³⁷⁾ (cf. Section 3). We illustrate the method by reference to Eq. (3.12). The procedure is to separate the various time orderings in Eq. (3.14), insert complete sets of states between the local fields and perform the co-ordinate space integrals. This leads to a spectral representation for $T_2(x_2, x_1)$ which allows the integrals over the light cone fractions x_i to be performed. In the operator mode the integrals span the range $(-\infty < x_i < \infty)$ and if the circle at infinity gives no contribution they can be evaluated by closing the contour. The consequent limit on the x_i (for $x_B > 0$)

is a co-operative effect of the spectral properties of the lower blob ($x_1 < 1$) and the analytic properties of the uncut photon parton amplitude ($x_1 > 0$).

Neither of the approaches so far presented is entirely satisfactory. In the operator mode one is restricted to deep inelastic scattering. The method does not determine the support of the correlation function but only fixes the range which is probed in DIS. The treatment using the Nakanishi representation fixes a finite range for the light-cone fractions and therefore justifies the parton approach. But the explicit limits are not as stringent as the examination of zero- and one-loop diagrams would suggest. The problem clearly requires further study.

APPENDIX B

In this Appendix we present the set of formulae which are relevant for reduction to the collinear basis in the light-cone fraction space. The corresponding results in the text were in co-ordinate space on the light cone.

The results presented here are therefore Fourier transforms of the relations given in the text. As such they contain no new information, but since the performance of the Fourier transform is straightforward but tedious, the content of this Appendix may be of use to the reader who wants to work with our results.

The general relations between the light-cone fraction space and the light-cone position space are

$$\begin{aligned} A(x) &= \frac{1}{2\pi} \int d\lambda e^{i\lambda x} A(\lambda), \\ B(x_2, x_1) &= \frac{1}{(2\pi)^2} \int d\eta d\lambda e^{i\eta(x_2-x_1)} e^{i\lambda x_1} B(\eta, \lambda), \\ C(x_2, y, x_1) &= \frac{1}{(2\pi)^3} \int d\omega d\eta d\lambda e^{i\omega(x_2-y)} e^{i\eta(y-x_1)} e^{i\lambda x_1} C(\omega, \eta, \lambda). \end{aligned} \tag{B.1}$$

The starting point for the reduction is the factorized formula, Eq. (2.62), written in the redundant basis $\{A_1, B_1, \tilde{B}_1, B_2, C_1, \tilde{C}_1\}$. The first step is to express all these correlation functions in terms of E_i , $i = 1, 2, 3$, Eqs. (4.31)-(4.33), for which one can then impose in a natural way the equations of motion.

We define first

$$B_3(x) = x A_1(x), \quad B(x_2, x_1) = B_2(x_2, x_1) - \delta(x_2 - x_1) B_3(x). \tag{B.2}$$

All other functions can be related to B_3 , B_1 , $\tilde{B}_1$ and E_i in the simple way

$$\begin{aligned} B_1(x_2, x_1) &= \frac{1}{2} [E_1(x_2, x_1) - B(x_2, x_1) - 2\delta(x_2 - x_1) B_3(x_1)], \\ B_2(x_2, x_1) &= B(x_2, x_1) + \delta(x_2 - x_1) B_3(x), \end{aligned} \tag{B.3}$$

$$A_1(x) = \frac{1}{x} B_3(x),$$

$$C_1(x_2, y, x_1) = \frac{1}{2} [E_2(x_2, y, x_1) - x_2 \delta(y-x_2) B_2(x_2, x_1) - x_1 \delta(y-x_1) B_2(x_2, x_1)],$$

$$\tilde{C}_1(x_2, y, x_1) = \frac{1}{2} [E_3(x_2, y, x_1) - 2x_2 \delta(y-x_2) \tilde{B}_1(x_2, x_1) - 2x_1 \delta(y-x_1) \tilde{B}_1(x_2, x_1)].$$

(B.3)
Contd.

The relations $B_3, B, \tilde{B}_1 \rightarrow E_1, E_2, E_3$ are as follows

$$\begin{aligned} \int dx \delta(x) B_3(x) &= \int dx_2 dx_1 E_1(x_2, x_1) \left[\int_0^1 d\tau \tau (1 + \lambda \tau) \frac{x_2 \delta(x_2 \tau) - x_1 \delta(x_1 \tau)}{x_2 - x_1} \right] + \\ &+ \int dx_2 dy dx_1 \frac{E_2(x_2, y, x_1)}{x_2 - x_1} \int_0^1 d\tau \tau \lambda \tau \left\{ \frac{y \delta(y \tau) - x_2 \delta(x_2 \tau)}{y - x_2} - \frac{y \delta(y \tau) - x_1 \delta(x_1 \tau)}{y - x_1} \right\}, \end{aligned}$$

(B.4)

$$\begin{aligned} \int dx_2 dx_1 \delta(x_2, x_1) B(x_2, x_1) &= \\ \int dx_2 dx_1 E_1(x_2, x_1) \int_0^1 d\tau \tau \left\{ \delta(x_2 \tau, x_1 \tau) - \frac{x_2 \delta(x_2 \tau, x_2 \tau) - x_1 \delta(x_1 \tau, x_1 \tau)}{x_2 - x_1} \right\} &+ \\ + \int dx_2 dy dx_1 E_2(x_2, y, x_1) \int_0^1 d\tau \tau \left\{ \frac{\delta(y \tau, x_1 \tau) - \delta(x_2 \tau, x_1 \tau)}{y - x_2} + \frac{\delta(x_2 \tau, y \tau) - \delta(x_2 \tau, x_1 \tau)}{y - x_1} \right\}, \end{aligned}$$

(B.5)

$$\begin{aligned} \int dx_2 dx_1 \delta(x_2, x_1) \tilde{B}_1(x_2, x_1) &= \\ \frac{1}{2} \int dx_2 dy dx_1 E_3(x_2, y, x_1) \int_0^1 d\tau \tau \left\{ \frac{\delta(x_2 \tau, y \tau) - \delta(x_2 \tau, x_1 \tau)}{y - x_1} - \frac{\delta(y \tau, x_1 \tau) - \delta(x_2 \tau, x_1 \tau)}{y - x_2} \right\}. \end{aligned}$$

(B.6)

Here and in the following σ is an arbitrary test function. For example, by taking $\sigma(x) = x^N$, $\sigma(x_2, x_1) = x_2^i x_1^{N-i}$, etc., we get the relations for moments and by taking $\sigma(x) = \delta(x-\hat{x})$, $\sigma(x_2, x_1) = \delta(x_2-\hat{x}_2)\delta(x_1-\hat{x}_1)$ we get the relations in the light-cone fraction space.

In deriving Eqs. (B.4)-(B.6) we used the n independence relations, Eqs. (4.26)-(4.28). The x space version of the latter was given in the text [Eqs. (6.36)-(6.65)].

The next step is to express the functions E_i in terms of the correlation functions in the collinear basis, $G_1, \tilde{G}_1, J, \Delta G_2, G_2$. The relations are

$$\begin{aligned} & \int dx_2 dx_1 \sigma(x_2, x_1) E_1(x_2, x_1) = \\ & \int dx_2 dx_1 \frac{G_1(x_2, x_1)}{x_2 - x_1} \left\{ \sigma(x_2, x_1) - \alpha \sigma(x_1, x_1) - (1-\alpha) \sigma(x_2, x_2) \right\} \end{aligned} \quad (B.7)$$

[where α is arbitrary, cf. Eq. (4.65)]

$$\begin{aligned} & \int dx_2 dy dx_1 \sigma(x_2, y, x_1) E_2(x_2, y, x_1) = \\ & \int dx_2 dx_1 \frac{G_1(x_2, x_1)}{x_2 - x_1} \left\{ \frac{1}{2}(x_2 + x_1) [\sigma(x_2, x_2, x_1) + \sigma(x_2, x_1, x_1)] - x_1 \sigma(x_1, x_1, x_1) - \right. \\ & \quad \left. - x_2 \sigma(x_2, x_2, x_2) \right\} - \\ & - \frac{1}{2} \int dx_2 dx_1 \frac{J(x_2, x_1)}{x_2 - x_1} \left\{ \sigma(x_2, x_2, x_1) - \sigma(x_2, x_1, x_1) \right\} + \\ & + \frac{1}{4} \int dx_2 dx_1 \tilde{G}_1(x_2, x_1) \left\{ \sigma(x_2, x_2, x_1) + \sigma(x_2, x_1, x_1) \right\} - \\ & - \int dx_2 dy dx_1 \frac{G_2(x_2, y, x_1)}{x_2 - x_1} \left\{ \frac{\sigma(x_2, y, x_1) - \sigma(x_2, x_2, x_1)}{y - x_2} - \frac{\sigma(x_2, y, x_1) - \sigma(x_2, x_1, x_1)}{y - x_1} \right\} + \end{aligned} \quad (B.8)$$

$$+ \int dx_2 dy dx_1 \frac{\Delta G_2(x_2, y, x_1)}{2(y-x_2)(y-x_1)} \left\{ 2\sigma(y, y, y) + \sigma(x_2, x_2, x_1) + \sigma(x_2, x_1, x_1) - \right. \\ \left. - \sigma(x_2, y, y) - \sigma(y, y, x_1) - \sigma(x_2, x_2, y) - \sigma(y, x_1, x_1) \right\},$$

(B.8)
cont.

$$\int dx_2 dy dx_1 \sigma(x_2, y, x_1) E_3(x_2, y, x_1) = \int dx_2 dy dx_1 \sigma(x_2, y, x_1) E_2(x_2, y, x_1) + \\ + \int dx_2 dx_1 \frac{G_1(x_2, x_1)}{x_2 - x_1} \left\{ x_2 [\sigma(x_2, x_2, x_2) - \sigma(x_2, x_2, x_1)] - x_1 [\sigma(x_2, x_1, x_1) - \sigma(x_1, x_1, x_1)] \right\} - \\ - \int dx_2 dy dx_1 \frac{\Delta G_2(x_2, y, x_1)}{(y-x_2)(y-x_1)} \left\{ \sigma(y, y, y) - \sigma(x_2, y, y) - \sigma(y, y, x_1) + \sigma(x_2, y, x_1) \right\}.$$

(B.9)

As a final step we have to combine Eqs. (B.4)-(B.6), (B.7)-(B.9) to obtain the reduction for $\{B_3, B, \tilde{B}_1\} \rightarrow \{G_1, \tilde{G}_1, J, \Delta G_2, G_2\}$. To write the expressions for this step we introduce the notations

$$\sigma'(x_2, x_1) = \left(\frac{d}{dx_2} + \frac{d}{dx_1} \right) \sigma(x_2, x_1),$$

(B.10)

$$\sigma^{(1)}(x_2, x_1) = \frac{d}{dx_1} \sigma(x_2, x_1),$$

(B.11)

$$\sigma^{(2)}(x_2, x_1) = \frac{d}{dx_2} \sigma(x_2, x_1).$$

(B.12)

$$\begin{aligned}
 \int dx \sigma(x) \mathcal{B}_3(x) = & \\
 & - \frac{1}{4} \int dx_2 dx_1 \tilde{G}_1(x_2, x_1) \int_0^1 d\tau \tau (1 + \ell u \tau) \frac{\sigma(x_2 \tau) - \sigma(x_1 \tau)}{x_2 - x_1} + \\
 & + \int dx_2 dx_1 \frac{J(x_2, x_1)}{(x_2 - x_1)^2} \left[\int_0^1 d\tau \tau \ell u \tau \frac{x_2 \sigma(x_2 \tau) - x_1 \sigma(x_1 \tau)}{x_2 - x_1} + \right. \\
 & \quad \left. + \frac{1}{2} \int_0^1 d\tau \tau (1 + \ell u \tau) \{ \sigma(x_2 \tau) + \sigma(x_1 \tau) \} \right] + \\
 & + \frac{1}{2} \int dx_2 dy dx_1 \frac{\Delta G_2(x_2, y, x_1)}{(y - x_2)(y - x_1)} \int_0^1 d\tau \tau (1 + \ell u \tau) \left\{ \frac{\sigma(y \tau) - \sigma(x_2 \tau)}{y - x_2} + \frac{\sigma(y \tau) - \sigma(x_1 \tau)}{y - x_1} - \right. \\
 & \quad \left. - \frac{\sigma(x_2 \tau) - \sigma(x_1 \tau)}{x_2 - x_1} - \tau \sigma'(y \tau) \right\} + \\
 & + \int dx_2 dy dx_1 \frac{G_2(x_2, y, x_1)}{(x_2 - x_1)^2} \int_0^1 d\tau \tau \ell u \tau \left\{ \frac{\Delta(x_2, x_2) - \Delta(x_2, x_1) - \Delta(y, x_2) + \Delta(y, x_1)}{y - x_2} + \right. \\
 & \quad \left. + \frac{\Delta(x_1, x_1) - \Delta(x_2, x_1) + \Delta(y, x_2) - \Delta(y, x_1)}{y - x_1} \right\},
 \end{aligned}$$

(B.13)

$$\Delta(x_2, x_1) = \frac{x_2 \sigma(x_2 \tau) - x_1 \sigma(x_1 \tau)}{x_2 - x_1},$$

$$\Delta(x, x) = \sigma(x \tau) + x \tau \sigma'(x \tau),$$

$$\begin{aligned}
 & \int dx_2 dx_1 \sigma(x_2, x_1) B(x_2, x_1) = \\
 & \int dx_2 dx_1 \frac{G_1(x_2, x_1)}{x_2 - x_1} \left[\frac{1}{2}(x_2 + x_1) \int_0^1 d\tau \tau^2 \sigma'(x_2\tau, x_1\tau) - \sigma(x_1, x_1) + \int_0^1 d\tau \tau \{ \sigma(x_2\tau, x_1\tau) + \sigma(x_1\tau, x_1\tau) \} \right] + \\
 & + \frac{1}{4} \int dx_2 dx_1 \tilde{G}_1(x_2, x_1) \int_0^1 d\tau \tau \left[\frac{\sigma(x_2\tau, x_2\tau) - \sigma(x_1\tau, x_1\tau)}{x_2 - x_1} + \tau \sigma'(x_2\tau, x_1\tau) \right] + \\
 & + \frac{1}{2} \int dx_2 dx_1 \frac{J(x_2, x_1)}{x_2 - x_1} \int_0^1 d\tau \tau \left[\frac{2\sigma(x_2\tau, x_1\tau) - \sigma(x_2\tau, x_2\tau) - \sigma(x_1\tau, x_1\tau)}{x_2 - x_1} + \right. \\
 & \quad \left. + \tau \{ \sigma^{(1)}(x_2\tau, x_1\tau) - \sigma^{(2)}(x_2\tau, x_1\tau) \} \right] + \\
 & + \frac{1}{2} \int dx_2 dy dx_1 \frac{\Delta G_2(x_2, y, x_1)}{(y - x_2)(y - x_1)} \left[\int_0^1 d\tau \tau^2 \{ 2\sigma'(y\tau, y\tau) + \sigma'(x_2\tau, x_1\tau) - \sigma'(x_2\tau, y\tau) - \sigma'(y\tau, x_1\tau) \} + \right. \\
 & \quad \left. + \int_0^1 d\tau \tau \left\{ \frac{\sigma(x_2\tau, x_2\tau) - \sigma(x_1\tau, x_1\tau)}{x_2 - x_1} - \frac{\sigma(y\tau, y\tau) - \sigma(x_1\tau, x_1\tau)}{y - x_1} - \frac{\sigma(y\tau, y\tau) - \sigma(x_2\tau, x_2\tau)}{y - x_2} \right\} \right] + \\
 & + \int dx_2 dy dx_1 \frac{G_2(x_2, y, x_1)}{x_2 - x_1} \left[\frac{1}{y - x_2} \left(\int_0^1 d\tau \tau \left\{ \tau \sigma^{(2)}(x_2\tau, x_1\tau) + \frac{\sigma(x_2\tau, x_2\tau) - \sigma(x_2\tau, x_1\tau)}{x_2 - x_1} - \right. \right. \right. \\
 & \quad \left. \left. - \frac{\sigma(x_2\tau, y\tau) - \sigma(x_2\tau, x_1\tau)}{y - x_1} - \frac{\sigma(y\tau, x_1\tau) - \sigma(x_2\tau, x_1\tau)}{y - x_2} \right\} \right) - \frac{1}{y - x_1} \left(\int_0^1 d\tau \tau \left\{ \tau \sigma^{(1)}(x_2\tau, x_1\tau) + \right. \right. \\
 & \quad \left. \left. + \frac{\sigma(x_2\tau, x_1\tau) - \sigma(x_1\tau, x_1\tau)}{x_2 - x_1} - \frac{\sigma(x_2\tau, y\tau) - \sigma(x_2\tau, x_1\tau)}{y - x_1} - \frac{\sigma(y\tau, x_1\tau) - \sigma(x_2\tau, x_1\tau)}{y - x_2} \right\} \right) \right],
 \end{aligned}$$

(B.14)

$$\begin{aligned}
 & \int dx_2 dx_1 \sigma(x_2, x_1) \tilde{B}_1(x_2, x_1) = \\
 & \frac{1}{4} \int dx_2 dx_1 G_1(x_2, x_1) \int_0^1 dt \tau^2 \sigma'(x_2 \tau, x_1 \tau) + \\
 & + \frac{1}{8} \int dx_2 dx_1 \tilde{G}_1(x_2, x_1) \int_0^1 dt \tau \left\{ \tau [\sigma^{(1)}(x_2 \tau, x_1 \tau) - \sigma^{(2)}(x_2 \tau, x_1 \tau)] + \right. \\
 & \quad \left. + \frac{\sigma(x_1 \tau, x_1 \tau) + \sigma(x_2 \tau, x_2 \tau) - 2\sigma(x_2 \tau, x_1 \tau)}{x_2 - x_1} \right\} + \\
 & + \frac{1}{4} \int dx_2 dx_1 \frac{J(x_2, x_1)}{x_2 - x_1} \int_0^1 dt \tau \left\{ \tau \sigma'(x_2 \tau, x_1 \tau) - \frac{\sigma(x_2 \tau, x_2 \tau) - \sigma(x_1 \tau, x_1 \tau)}{x_2 - x_1} \right\} + \\
 & + \frac{1}{4} \int dx_2 dy dx_1 \frac{\Delta G_2(x_2, y, x_1)}{(y - x_2)(y - x_1)} \left[\int_0^1 dt \tau^2 \left\{ \sigma'(x_2 \tau, y \tau) - \sigma'(y \tau, x_1 \tau) + \right. \right. \\
 & \quad \left. \left. + \sigma^{(1)}(x_2 \tau, x_1 \tau) - \sigma^{(2)}(x_2 \tau, x_1 \tau) \right\} + \right. \\
 & + \int_0^1 dt \tau \left\{ \frac{\sigma(y \tau, y \tau) - \sigma(x_1 \tau, x_1 \tau)}{y - x_1} + \frac{\sigma(x_2 \tau, x_2 \tau) - \sigma(y \tau, y \tau)}{y - x_2} + \right. \\
 & \quad \left. + \frac{\sigma(x_1 \tau, x_1 \tau) + \sigma(x_2 \tau, x_2 \tau) - 2\sigma(x_2 \tau, x_1 \tau)}{x_2 - x_1} + \right. \\
 & \quad \left. + 2 \frac{\sigma(x_2 \tau, x_1 \tau) - \sigma(x_2 \tau, y \tau)}{y - x_1} + 2 \frac{\sigma(y \tau, x_1 \tau) - \sigma(x_2 \tau, x_1 \tau)}{y - x_2} \right\} \Big] + \\
 & + \frac{1}{2} \int dx_2 dy dx_1 \frac{G_2(x_2, y, x_1)}{x_2 - x_1} \int_0^1 dt \tau \left[\frac{1}{y - x_1} \left\{ \frac{\sigma(x_2 \tau, x_1 \tau) - \sigma(x_1 \tau, x_1 \tau)}{x_2 - x_1} + \right. \right. \\
 & + \frac{\sigma(x_2 \tau, y \tau) - \sigma(x_2 \tau, x_1 \tau)}{y - x_1} - \frac{\sigma(y \tau, x_1 \tau) - \sigma(x_2 \tau, x_1 \tau)}{y - x_2} - \tau \sigma^{(1)}(x_2 \tau, x_1 \tau) \Big\} + \\
 & + \frac{1}{y - x_2} \left\{ \frac{\sigma(x_2 \tau, x_2 \tau) - \sigma(x_2 \tau, x_1 \tau)}{x_2 - x_1} - \frac{\sigma(x_2 \tau, y \tau) - \sigma(x_2 \tau, x_1 \tau)}{y - x_1} \right\} + \\
 & \quad \left. \frac{\sigma(y \tau, x_1 \tau) - \sigma(x_2 \tau, x_1 \tau)}{y - x_2} - \tau \sigma^{(2)}(x_2 \tau, x_1 \tau) \right\} \Big].
 \end{aligned}$$

(B.15)

REFERENCES

- 1) H.D. Politzer, Nucl. Phys. B172 (1980) 349.
- 2) K. Wilson, Phys. Rev. 179 (1969) 1499;
R. Brandt and G. Preparata, Nucl. Phys. B27 (1971) 541;
N. Christ, B. Hasslacher and A.H. Mueller, Phys. Rev. D6 (1972) 3543.
- 3) D. Gross and S. Treiman, Phys. Rev. D4 (1971) 1059.
- 4) H. Georgi and H.D. Politzer, Phys. Rev. D9 (1974) 416;
D. Gross and F. Wilczek, Phys. Rev. D9 (1974) 980.
- 5) G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
- 6) S. Gottlieb, Nucl. Phys. B139 (1978) 125.
- 7) S.P. Luttrell and S. Wada, Bologna preprint IFUB 81/17 (1981);
S. Wada, Bologna preprint IFUB 81/18 (1981).
- 8) M. Okawa, Nucl. Phys. B172 (1980) 481; B187 (1981) 71; Brookhaven preprint
BNL 30858 (1981).
- 9) S.P. Luttrell, S. Wada and B.R. Webber, Nucl. Phys. B188 (1981) 219.
- 10) R.L. Jaffe and M. Soldate, in "Perturbative Quantum Chromodynamics",
(Tallahassee, 1981), (ed. D.W. Duke and J.F. Owens), Phys. Lett. 105B
(1981) 467; MIT preprint 954 (1981).
- 11) E.V. Shuryak and A.I. Vainshtein, Phys. Lett. 105B (1981) 65; Novosibirsk
preprints 81-77, 106 (1981).
- 12) A. De Rújula, H. Georgi and H.D. Politzer, Ann. Phys. (N.Y.) 103 (1977) 315.
- 13) L.F. Abbott and R.M. Barnett, Annals of Physics 125 (1980) 276;
L.F. Abbott, W.B. Atwood and R.M. Barnett, Phys. Rev. D22 (1980) 582;
R.M. Barnett, SLAC preprint SLAC-PUB 2883 (1982).
- 14) A. Donnachie and P.V. Landshoff, Phys. Lett. 95B (1980) 437;
D.W. Duke and R.G. Roberts, Nucl. Phys. B166 (1980) 243.
- 15) F. Eisele et al., Dortmund preprint DO-TH-81-11 (1981).
- 16) R. Blankenbecler, S.J. Brodsky and J. F. Gunion, Phys. Rev. D18 (1978) 900;
I.A. Schmidt and R. Blankenbecler, Phys. Rev. D16 (1977) 1318;
E.L. Berger and S.J. Brodsky, Phys. Rev. Lett. 42 (1979) 940;
E.L. Berger, Phys. Lett. 89B (1980) 241, Z. Phys. C4 (1980) 289 and references
therein.
- 17) R.K. Ellis, W. Furmanski and R. Petronzio, CERN preprint TH.3254 (1982).
- 18) W. Zimmerman, Lectures on Elementary Particles and Quantum Field Theory,
Brandeis University Summer Institute (MIT Press, Cambridge, 1970).
- 19) V. Sudakov, JETP (Sov. Phys.) 3 (1956) 65.
- 20) J.D. Bjorken, Phys. Rev. 179 (1969) 1547.

- 21) O. Nachtmann, Nucl. Phys. B63 (1973) 237.
- 22) H. Georgi and H.D. Politzer, Phys. Rev. D14 (1976) 1829.
- 23) R.K. Ellis, R. Petronzio and G. Parisi, Phys. Lett. 64B (1976) 97;
R. Barbieri et al., Phys. Lett. 64B (1976) 17; Nucl. Phys. B117 (1976) 50.
- 24) D.J. Gross, S.B. Treiman and F. Wilczek, Phys. Rev. D15 (1977) 2486;
A. De Rújula, H. Georgi and H.D. Politzer, Phys. Rev. D15 (1977) 2495.
- 25) W.E. Caswell, R.R. Horgan and S.J. Brodsky, Phys. Rev. D18 (1978) 2415.
- 26) A.H. Mueller, Phys. Rev. D18 (1978) 3705.
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Phys. Lett. 78B (1978) 281; Nucl. Phys. B152 (1979) 285;
D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54; B146 (1978) 29.
- 27) W. Furmanski, Jagellonian University Habilitation Thesis, Cracow preprint TPJU 12/81; Trieste preprint ICTP 81-152 (1981).
- 28) G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27.
- 29) W. Konetschny and W. Kummer, Nucl. Phys. B100 (1975) 106;
W. Kummer, Acta Phys. Austriaca 41 (1975) 315;
D.J. Pritchard and W.J. Stirling, Nucl. Phys. B165 (1980) 237.
- 30) See for example P.V. Landshoff and J.C. Polkinghorne, Phys. Rep. 5 (1972) 1.
- 31) N. Nakanishi, "Graph Theory and Feynman Integrals", Gordon and Breach, Science Publishers Inc., N.Y. (1971).
- 32) F.J. Dyson, Phys. Rev. 83 (1951) 608.
- 33) J.B. Kogut and D.E. Soper, Phys. Rev. D1 (1970) 2901.
- 34) S. Wandzura, Nucl. Phys. B122 (1977) 412.
- 35) R.E. Cutkosky, J. Math. Phys. 1 (1960) 429.
- 36) A.V. Efremov, A.V. Radyushkin, Dubna preprint E2-80-521 (1980);
G.T. Bodwin, S.J. Brodsky and G.P. Lepage, Phys. Rev. Lett. 47 (1981) 1799;
A.H. Mueller, Phys. Lett. 108B (1982) 355;
J.C. Collins, D.E. Soper and G. Starman, Stony Brook preprint ITP SB 81-59 (1981);
P.V. Landshoff and W.J. Stirling, Cambridge preprint DAMTP 82/3 (1982);
W.W. Lindsay, D.A. Ross and C.T. Sachrajda, Southampton preprint SHEP 81/82-4 (1982).
- 37) R.L. Jaffe - MIT Preprint CTP-970 (1982).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

a)

b)

c)

Fig. 8

a)

b)

c)

Fig. 9