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AXIONS AND SUPERSYMMETRY

J.-M. Frère
Université Libre, Bruxelles *)
and
CERN -- Geneva

and

J.-M. Gérard
CERN -- Geneva

ABSTRACT

Most of the proposed supersymmetric low-energy models contain at least one accidental axial symmetry which at first sight could solve the strong CP problem. Unfortunately, experimental, astrophysical and cosmological constraints on one side and asymptotic freedom on the other are never simultaneously respected. We propose to consider the supersymmetric realization of the Kim, Shifman, Vainshtein and Zakharov mechanism as a new guide for model building.

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The strong CP problem is currently well known and has raised considerable speculation¹⁾. The following list of possible solutions is not exhaustive:

- i) The problem is evacuated by considering the strong CP θ angle as just another parameter to be tuned (like the electron mass).
- ii) A symmetry is imposed such that the bare angle is naturally zero in the technical sense, and consequently the radiative corrections are calculable and finite²⁾.
- iii) The basic Lagrangian is invariant under the Peccei-Quinn (PQ) symmetry whose associated (pseudo) Goldstone boson, the axion, has undergone an extensive but negative search^{3),4)}.
- iv) The "invisible axion" solution of Dine, Fischler and Srednicki⁵⁾ (DFS) keeps a PQ symmetry acting on all the known coloured fermions. It relies on the introduction of an extra scalar with large vacuum expectation value to make the associated axion superlight and virtually uncoupled to ordinary matter. This solution can be made unbeatable from an experimental point of view, including astrophysical considerations⁶⁾. A possible counter-argument based on cosmology has, however, been suggested by P. Sikivie⁷⁾ *).
- v) The Kim, Shifman, Vainshtein and Zakharov⁹⁾ (KSVZ) solution also requires the introduction of new particles, especially a coloured, flavour singlet, which is the only fermion sensitive to the PQ symmetry. As a consequence, the cosmological argument raised by Sikivie proves here inoperative.

In this paper we will be concerned with possible realizations of these various mechanisms in the framework of supersymmetry, basing our study on minimal extensions of the Weinberg-Salam model and also allowing the possible inclusion of an extra gauged axial $\tilde{U}(1)$ group¹⁰⁾. It is quite interesting indeed to realize that most of these mechanisms find a natural framework in supersymmetric models. For instance, the minimal supersymmetrical extension of the Weinberg-Salam model implies the doubling of the Higgs structure, a fact suggestive of a PQ symmetry. In fact the model builder has to go somewhat out his way to prevent this now unwanted symmetry from arising: this automatically puts him on the track of the DFS solution. On the other hand, the Fayet extension, proposed essentially to get a realistic mass spectrum (after symmetry breaking) needs extra coloured, flavour singlet particles. These can serve the double purpose of establishing a KSVZ solution without having to abandon the requirement of asymptotic freedom.

*) See however Ref. 8).

Recently, the possibility of a "technically natural" solution in the supersymmetric framework was put forward by Ellis, Ferrara and Nanopoulos¹¹⁾ (EFN). Basing their argument on the non-renormalization theorem, they show that if the θ_{CP} parameter is set equal to zero, it will remain so to any order in perturbation. Finite corrections due to the breakdown of supersymmetry may then be calculable and, depending on the precise model, be shown to be small. We remark that this solution presupposes that all "PQ-like" symmetries (which appear naturally in a supersymmetric context) have effectively been eliminated. This, in general, implies the introduction of more particles (see below). We will not consider this kind of solution further but rather look for the PQ alternative where the θ_{CP} angle remains zero, due to a global symmetry.

Let us first consider the most naïve and minimal supersymmetric extension of the Weinberg-Salam model. This necessarily involves left-handed matter superfields with the following usual quantum numbers:

	$SU(3)_c$	$SU(2)$	$U(1)$
Q	3	2	1/3
U^c	$\bar{3}$	1	-4/3
D^c	$\bar{3}$	1	2/3
L	1	2	-1
E^c	1	1	2
H	1	2	1
$\bar{H}$	1	2	-1

The supersymmetric Lagrangian in general contains the so-called F terms:

$$(\alpha Q U^c H + b Q D^c \bar{H} + c L E^c \bar{H} + d H \bar{H})_{100} \quad (1)$$

where contraction of internal symmetry indices is understood. The doublet H is needed to provide the "up" quarks with masses; although the $\bar{H}$ superfield has the same quantum numbers as L , its presence is necessary to cancel anomalies. Even in the presence of the $H\bar{H}$ term, the Lagrangian is invariant under a global axial transformation¹²⁾ (commonly named R-symmetry) on:

- the matter superfields :

$$\phi_a(x, \theta, \bar{\theta}) \rightarrow \exp[i n(\phi_a) \alpha] \phi_a(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}) \quad (2)$$

- the gauge superfields :

$$V(x, \theta, \bar{\theta}) \rightarrow V(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha})$$

such that the invariance of

$$\phi_a \phi_b |_{\theta\theta}$$

$$n(\phi_a) + n(\phi_b) = 2$$

requires

$$\phi_a \phi_b \phi_c |_{\theta\theta}$$

$$n(\phi_a) + n(\phi_b) + n(\phi_c) = 2$$

This global R-symmetry is in general plagued by the strong anomaly coming from the triangular diagram involving the $U(1)_R$ current and two gluons. This feature is essential if we want to use it to rotate away the strong CP angle. The anomaly associated with the current defined by (2) is in fact proportional to

$$\begin{aligned} A &= \left\{ 2 [n(Q) - 1] + [n(U') - 1] + [n(D') - 1] \right\} N_g + 6 \\ &= - [n(H) + n(\bar{H})] N_g + 6 \end{aligned}$$

for N_g "generations" of quarks. The last contribution is that of gluinos, the fermionic superpartners of the coloured gauge bosons. In our naïve model, (1) imposes $n(H) + n(\bar{H}) = 2$ such that

$$A = -2 N_g + 6$$

It is amusing to note that the strong anomaly associated with this current precisely vanishes for three generations. As a consequence, in this minimal case, the global symmetry mentioned above cannot be used to solve the strong CP problem. It must, however, be broken to generate quark masses^{*)} and thus still leads to a (nearly) massless particle, phenomenologically unacceptable. So we have all the drawbacks of the standard PQ axion without its advantages.

The above conclusion about the existence of a strong anomaly-free axial symmetry may seem to be contingent to the presence of the $H\bar{H}$ term. It is, however, obvious that in the absence of such a term, the global symmetry and the associated phenomenological problems persist a fortiori. Moreover, in this case, a second PQ-like symmetry appears, acting globally on the full chiral superfields; we will refer to it as a "super-PQ" transformation:

^{*)} This R-symmetry must also be broken to provide the gauginos with a mass.

$$\begin{aligned}
 Q &\rightarrow e^{i\beta} Q \\
 U^c &\rightarrow e^{i\beta} U^c & D^c &\rightarrow e^{i\beta} D^c \\
 H &\rightarrow e^{-2i\beta} H & \bar{H} &\rightarrow e^{-2i\beta} \bar{H}
 \end{aligned} \tag{3}$$

Phenomenologically this tells us that the model described in (1) is untenable. Even if we wanted to turn back to the EFN solution, we would still have to forbid both the "super-PQ" and the R-symmetry. This can be achieved at the cost of introducing at least one new chiral superfield behaving as a singlet under $SU(3)_C \times SU(2) \times U(1)^{*}$. The replacement of $H\bar{H}$ in (1) by

$$\alpha H\bar{H}S + \beta S + \gamma SS + \delta SSS$$

forbids any PQ-symmetry. If we set β and γ to zero we find the DFS resolution of the axion problem because the potential contains the term $h\bar{h}s^2 + \text{h.c.}$ where we use lower case letters for the scalar component in each matter superfield. Let us note that this $U(1)_{\text{DFS}}$ may never be a "super-PQ"-like symmetry because of the way the scalar potential is derived: it can only be realized as an R-symmetry. The simultaneous presence of $H\bar{H}S$ and S^3 makes this R-symmetry anomalous even for three generations of quarks because now, $n(H) + n(\bar{H}) = 2n(S) = 4/3$ and

$$A = -\frac{4}{3} N_g + 6$$

This of course extends to grand unified models where the vacuum expectation value of the scalar s is of the order of the highest mass scale ($\sim 10^{15}$ GeV) and the axion becomes invisible. A simple application is given in the framework of $SU(5)$ by Nilles and Raby¹³⁾. Nevertheless, in this kind of scenario, the global symmetry still has to be put in by hand and is not a consequence of the supersymmetric invariance of the Lagrangian.

Let us now consider the possibility of introducing new coloured particles in connection with Fayet's extension of the standard model based on the $SU(2) \times U(1) \times \tilde{U}(1)$ gauge groups. This was motivated by the wish to avoid too light coloured scalars commonly met in the spontaneous D-term breaking of the supersymmetric $SU(2) \times U(1)$ model. Since an explicit $H\bar{H}$ term is now forbidden by gauge invariance, we must again include a chiral supermultiplet S with quantum numbers such that a $H\bar{H}S$ F-term now provides a scalar potential susceptible to generate vacuum expectation values for H and $\bar{H}$. To ensure the

*) In addition to the solution considered here, we should keep in mind that a new stronger interaction (supercolour?) may also provide a new anomalous current.

absence of anomalies for the extra axial $\tilde{U}(1)$ gauge current, we need to introduce at least one new real representation under $SU(3)_C$ with a non-zero hypercharge $\tilde{Y}$. Indeed, if we choose $\tilde{Y}(Q) = +1$ then

$$\tilde{Y}(U^c) = \tilde{Y}(D^c) = +1 \quad (\text{axial interaction})$$

$$\tilde{Y}(H) = \tilde{Y}(\bar{H}) = -2 \quad (\text{gauge invariance})$$

The new anomaly associated with the triangular diagram involving $U(1)$ and two gluons is proportional to $4 N_g$ and can be cancelled in two simple ways. One may introduce, for each generation, one triplet ϕ and one antitriplet $\bar{\phi}$ of coloured superfields with $\tilde{Y}(\phi) = \tilde{Y}(\bar{\phi}) = -2$ ¹⁴⁾. As $\tilde{Y}(S) = +4$, this automatically allows a $\phi\bar{\phi}S$ F-term. A second way valid only for three generations of quarks, consists of one new octet χ of $SU(3)_C$ with $\tilde{Y}(\chi) = -2$ ¹⁵⁾. Again, a χ^2S F-term will in general be present in the Lagrangian. Now, coming back to the strong CP problem, one easily sees that the above-mentioned (3) "super-PQ" symmetry acting on light quarks is simply enlarged to become the $\tilde{U}(1)$ gauge symmetry. This takes care of the corresponding Goldstone boson which will be absorbed by a gauge particle. But what about the R-symmetry realization of PQ? In the first case with ϕ and $\bar{\phi}$, if the R-symmetry is not explicitly broken by additional F-terms, we have:

$$\begin{aligned} A &= - [m(H) + m(\bar{H})] N_g + [m(\phi) + m(\bar{\phi}) - 2] N_g + 6 \\ &= - 2 N_g + 6 \end{aligned}$$

such that for three generations of quarks the current is again anomaly free. Likewise, with the octet, we obtain

$$\begin{aligned} A &= - [m(H) + m(\bar{H})] \times 3 + 6 [m(\chi) - 1] + 6 \\ &= 0 \end{aligned}$$

Of course, this problem might be cured by adding again new coloured fields but then we would certainly lose asymptotic freedom. Indeed, the presence of the gluinos and coloured scalars reduces the modulus of the β -function:

$$\beta_g^{(1)} = - \frac{g^3}{16\pi^2} \left[9 - \sum_{\text{chiral supermultiplet}} C_2 \right]$$

with $\text{Tr} T_a T_b = \delta_{ab} C_2$.

We note that in both cases considered above, one actually loses asymptotic freedom, since the one-loop contribution to the β -function vanishes, while the second loop contribution turns out to be positive. Although this circumstance is unfortunate, it is not necessarily excluded by experiment (the theory stays apparently asymptotically free as long as the heavy flavours are not excited) or by the constraints of grand unification which happen to be somewhat weaker¹⁶⁾. If we still want to insist on strict asymptotic freedom, we can use only one or two new flavours; the latter case brings us naturally to consider the Kim solution. This offers a direct solution to the cosmological problem which plagues both the standard PQ and the DFS solution. We briefly summarize the argument due to P. Sikivie. A PQ symmetry acting on the N light quark flavours leaves when broken, a discrete Z_N symmetry corresponding to axial phase rotation of $2\pi/N$ for each of the quarks. While of no relevance for the local dynamics, this circumstance proves disastrous in the cosmological context since it leads to the existence of domains characterized by different orientations of the chiral phases associated with the $N-1$ degenerate vacua. The real problem then comes from the energy stored in the walls separating those domains. Two obvious solutions to the problem exist in the present framework, namely:

- completely forbidding any PQ symmetry (see EFN)
- trying to implement the Kim mechanism

Indeed, as we pointed out, this model is exempt from trouble since only one coloured Dirac fermion undergoes the PQ rotation therefore restricting the surviving discrete group to the trivial one.

The KSVZ mechanism seems of course completely artificial in usual models. However, we precisely find in the Fayet $SU(2) \times U(1) \times \tilde{U}(1)$ extension a natural framework for its application, since extra coloured particles are anyway needed (see above). We will now investigate the possibility of actually implementing it (keeping in mind the risk always present in supersymmetric models of violating asymptotic freedom). Let us consider only three generations of quarks and restrict ourselves to one single new chiral triplet φ and one new antitriplet $\bar{\varphi}$. We first point out that the solution we are looking for can only be realized as a "super PQ" symmetry since it is no longer allowed to act on light quarks. In order to give the fermionic component of φ and $\bar{\varphi}$ a mass, we also need a scalar, electroweak singlet, superfield K (describing essentially the "Kim axion" k and its spin 1/2 partner, the Kim-ino) which also decouples from the quark sector. Remarkably, this decoupling is imposed by the hypercharge assignment for more than one generation. Moreover, φ and $\bar{\varphi}$ must be electrically neutral, otherwise the "super-PQ" would be gauged.

A rather simple example satisfying all the anomaly conditions (renormalizability) and trace constraints (cancellation of the quadratic divergences¹⁷⁾) may be found. The Yukawa sector of the Lagrangian is

$$(a Q U^c H + b Q D^c \bar{H} + c L N^c H + d L E^c \bar{H} + e H \bar{H} S + f \psi \bar{\psi} K + g E' \bar{E}' S)_{100}$$

if we impose the conservation of the number of E' minus the number of $\bar{E}'$. The quantum numbers for all superfields are given in the following table:

	$SU(3)_C$	$SU(2)$	$U(1)$	$\tilde{U}(1)$
Q_i	3	2	1/3	1/4
U_i^c	$\bar{3}$	1	-4/3	1/4
D_i^c	$\bar{3}$	1	2/3	1/4
L_i	1	2	-1	1/4
N_i^c	1	1	0	1/4
E_i^c	1	1	2	1/4
H_i	1	2	1	-1/2
$\bar{H}_i$	1	2	-1	-1/2
S	1	1	0	1
φ	3	1	0	-1
$\bar{\varphi}$	$\bar{3}$	1	0	-2
K	1	1	0	3
E'	1	1	-2	-1/2
$\bar{E}'$	1	1	2	-1/2

The global $U(1)$ symmetry which appears is given by:

$$\psi \rightarrow e^{-3i\beta} \psi$$

$$\bar{\psi} \rightarrow e^{2i\beta} \bar{\psi}$$

$$K \rightarrow e^{i\beta} K$$

all other superfields remaining unchanged.

In this present form, the Lagrangian still has an R-symmetry but in a now standard way this can be easily cured by adding one extra gauge singlet superfield. The choice of different $\tilde{Y}$ hypercharges for φ and $\bar{\varphi}$ is imposed by the condition that all anomalies be cancelled. It proves essential in the present context since otherwise the "super-PQ" current would be anomaly free. Supersymmetry must of course be broken. A detailed investigation of the possible mechanism goes beyond the scope of this paper.

Exploiting the new coloured particles introduced by Fayet in the $SU(2) \times U(1) \times \tilde{U}(1)$ model, we suggest a supersymmetric realization of the KSVZ mechanism. This gives a natural solution to the strong CP problem in agreement with experimental, astrophysical and cosmological constraints, keeping asymptotic freedom for the colour gauge group.

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