A Study of Missing Transverse Energy in Minimum Bias Events with In-time Pile-up at The Large Hadron Collider using The ATLAS Detector and $\sqrt{s}=7$ TeV Proton-Proton Collision Data

by

Kuhan Wang
B.Sc. Honours, Queen’s University, 2009

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

in the Department of Physics and Astronomy

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ABSTRACT

A sample of $\int L\, dt = 3.67$ pb$^{-1}$ of minimum bias events observed using the ATLAS detector at the Large Hadron Collider at $\sqrt{s}=7$ TeV is analyzed for Missing Transverse Energy (MET) response in the presence of in-time pile-up. We find that the MET resolution ($\sigma_{X,Y}$) is consistent with a simple model of the detector response for minimum bias events, scaling with respect to the sum of the scalar energy ($\sum E_T$) as $\sigma_{X,Y} = A\sqrt{\sum E_T}$. This behavior is observed in the presence of in-time pile-up and does not vary with global calibration schemes. We find a bias in the mean ($\mu_{X,Y}$) of the MET that is linear with respect to $\sum E_T$, leading to an asymmetry in the $\phi_{X,Y}$ distribution of the MET. We propose an explanation for this problem in terms of a misalignment of the nominal center of the ATLAS detector with respect to its real...
center. We contrast the data with a Monte Carlo sample produced using PYTHIA. We find that the resolution, bias and asymmetry are all approximately reproduced in simulation.
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ACKNOWLEDGEMENTS

I would like to thank:

**Richard Keeler** my supervisor, for his guidance, patience and support - academically and vocationally.

**Michel Lefebvre** for his comments and guidance on the research.

*One evening in October,*  
*When I was one-third sober,*  
*An’ taking home a load’ with manly pride;*  
*My poor feet began to stutter,*  
*So I lay down in the gutter,*  
*And a pig came up an’ lay down by my side;*

*Then we sang It’s all fair weather*  
*When good fellows get together,’*  
*Till a lady passing by was heard to say:*  
*You can tell a man who ”boozes”*  
*By the company he chooses’*  
*And the pig got up and slowly walked away.*  
*...*

- Benjamin Hapgood Burt, *The Pig.*
DEDICATION

To my parents and grandparents.

To Matthias Le Dall, for eating 52 chicken wings at Monkey Tree. I can’t believe I made that bet. Now I am paying for it.
Chapter 1

Introduction

The Large Hadron Collider (LHC) is the most powerful particle accelerator built to date. With the ability to collide protons together at a center of mass energy ($\sqrt{s}$) of 14 TeV, the LHC promises to peer beyond the current frontiers of fundamental physics.

The LHC hosts four experiments with large detectors. A Toroidal LHC ApparatuS (ATLAS) is one of two general purpose experiments designed to search for the Higgs Boson and new physics such as Supersymmetry. ATLAS holds the promise of detecting a variety of predicted particles. Due to the conservation of momentum, new particle candidates will often leave signatures of their existence via the detection of missing momentum in the detector.

As the transverse component of the momenta of the initial state particles are well understood, a key methodology for searching for new physics is to look for statistically significant deviations from the conservation of momentum in the direction transverse to the proton beams in collision events.

In addition to reaching a peak center of mass energy of 14 TeV, the LHC will also be colliding proton beams at unprecedented luminosities. It is expected that, at the
design luminosity of $L = 10^{34}$ cm$^{-2}$s$^{-1}$, the majority of interaction events will consist of “minimum bias” events of little physics interest. Minimum bias events consist of a category of low momentum transfer “soft hadronic” interactions. These minimum bias events will dominate and overlay events of physical importance in a phenomenon called pile-up.

Minimum bias pile-up forms a major source of background at the LHC. The magnitude of the momentum of a highly relativistic particle is nearly equal to its energy. In this thesis we adopt the convention that $\hbar = c = 1$. The part of the detector system that is most inclusive with respect to detecting the energy of the final state particles is known as the calorimeter. Therefore, the task of searching for “missing” momentum is replaced with searching for “missing” energy. The missing transverse energy (MET) resolution of minimum bias events directly impacts the statistical significance of MET measurements for almost any event. Therefore, the understanding of the effects of pile-up in minimum bias events is of great importance in establishing the validity and statistical significance of true MET in an event.

Data taking with $\sqrt{s} = 7$ TeV collisions, at the Large Hadron Collider, began in the spring of 2010. In this thesis, $\int L \, dt = 3.67 \text{ pb}^{-1}$ of data taken at the LHC as a part of the ATLAS collaboration are analyzed for minimum bias events with regards to their MET response.

We apply data selection on the minimum bias events to remove unwanted background and noise. We examine the MET response after event selection with regards to the effects of pile-up on the MET. The MET resolution, mean and asymmetry are examined and compared with Monte Carlo simulation.
1.1 Overview

This thesis is divided into several parts.

Chapter 1 contains the introduction and an overview of the dissertation.

Chapter 2 reviews the Standard Model of particle physics, minimum bias physics and the prospects for new physics at the LHC.

Chapter 3 outlines the physical machinery of the LHC, we discuss some technical details of the collider, we provide a description of the phenomenon of pile-up.

Chapter 4 discusses the basic principles of calorimeter physics and its detail and relations to the ATLAS detector and the trigger system.

Chapter 5 contains the concept of MET and its construction in ATLAS.

Chapter 6 reviews the data samples used for the analysis both recorded at the LHC and simulated using Monte Carlo methods. We discuss the event selection and data cleaning techniques, presenting results with respect to both and we briefly overview the data preparation and production methods.

Chapter 7 forms the body of the analysis. We discuss the effects of in-time pile-up on minimum bias events with respect to resolution, bias, and asymmetry. In each case we compare the results of data taking with Monte Carlo simulation.

Chapter 8 closes the argument by summarizing the conclusions of the study.
Chapter 2

Physics at the Large Hadron Collider

In this section, we survey the current theory of particle physics, the Standard Model. In addition, we discuss the concept of hadronic interactions via Quantum Chromodynamics (QCD) in terms of minimum bias events and the creation of high transverse momentum ($p_t$) jets. We close the chapter with a brief overview of the prospects for new physics discovery at the LHC in the context of Supersymmetry.
2.1 The Standard Model

The Standard Model (SM), Figure 2.1, is the widely accepted current description of the fundamental nature of particle physics[1],[2]. The SM provides for three generations of fermions and three fundamental forces mediated by bosons. With the exception of gravity, the SM provides an accurate description of particle physics within the limits of our experimental capacity to verify it.

The bosons carry the fundamental forces and mediate particle interactions by being exchanged between particles. The SM describes the weak, strong and electromagnetic forces which are mediated by the $W^\pm$ and $Z$ bosons, the gluons and the photon respectively. The photon is charge neutral while the gluons are color charged. Therefore, gluons will interact amongst themselves. The $W^\pm$ carry electric charge while the $Z$ is electrically neutral. In addition, the $W^\pm$ and $Z$ all carry the weak charge. As such, they mediate the weak force and the $W^\pm$ are responsible for mixing down and across generations.

The fermions obey the Pauli Exclusion principle[2] and fermions constitute matter. Fermions are divided into two types called quarks and leptons. Quarks possess fractional electric charge and color charge and will interact through all three described forces. The leptons consist of the electron, muon and tau and their nearly massless neutrino partners. The electron, muon and tau will interact via the electromagnetic and weak forces. The neutrinos only interact via the weak force. In addition to the particles shown in Figure 2.1, there are the antiparticles, which are identical with respect to their particle counterparts but with opposite charge$^1$. Also not shown is the Higgs Boson. In the SM, particle mass is generated by interaction with the Higgs field[3].

With the exception of gravity, the SM has proven to be a good description of

---

$^1$In some cases the particle is its own antiparticle - for example the photon.
Figure 2.1: The Standard Model of Particle Physics.
the fundamental nature of the universe\textsuperscript{2}. At the time of writing, all of the predicted particles in the SM have been experimentally verified with the exception of the Higgs Boson, the discovery of which would fulfill one of the primary goals of the LHC project.

\textsuperscript{2}With the exception of dark matter, which may or may not be composed of fundamental particles.
2.2 Hadronic Physics

Hadrons constitute the class of composite particles composed of quarks and held together by the strong force. The quark carries both a partial electric charge ($\frac{2}{3}e$ or $-\frac{1}{3}e$) and a color charge (aptly named red, green or blue). The condition that quark combinations always exists as color singlets\cite{2} leads to the effect that quarks cannot exist independently of one another and therefore individual quarks can never be studied. This effect is known as color confinement. Hadrons are divided into baryons (fermions) and mesons (bosons), defined by the number of quarks that form them (3 or 2 respectively). The baryons are composed of three quarks, while the mesons are formed of quark anti-quark pairs. In addition, the baryons and mesons have antiparticle partners.

The condition of color confinement leads to the result that in the event a hadronic structure is being pulled apart (as in a proton-proton collision in a particle accelerator), it will at some point become energetically more favorable to create new particles rather than to allow the quarks to separate further. This is due to the fact that within the interacting radius between two strongly coupled particles the strong force is proportional to the distance of their separation. Therefore, as one attempts to separate two strongly coupled particles the strength of their bond grows. This is remarkably different from electromagnetism and gravity for which the bond energy are inversely proportional to the separation distance between two interacting particles.

Hadronic interactions are broadly placed into two categories - soft and hard processes\cite{4}. In soft hadronic processes, the square of the four momentum transfer of the interaction, $t$, is roughly related to the size of the hadron\cite{3}, $R$, by $t \sim 1/R^2$. The square of the momentum transferred between the interacting particles, $t$, is small

\footnote{This is to be understood in terms of the de Broglie wavelength $\lambda = \frac{h}{p}$, for interaction size $\lambda$ and momentum $p$. For very relativistic particles $p \simeq E$.}
(typically of the order of a hundred MeV$^2$). Perturbative QCD fails to describe soft hadronic processes because the hadronic coupling constant is too large to expand around in this regime. The standard approach to describing soft hadronic processes is known as Regge Theory[4]. In addition, the rate of change of the cross section for soft processes with respect to momentum scales as an exponential,

$$\frac{d\sigma}{dt} \sim e^{-R^2|t|},$$

(2.1)

Therefore, the cross section for high momentum transfer events is highly suppressed.

In the event of a high momentum transfer ($\geq 1$ GeV$^2$) collision between two hadrons, the process is said to be hard and the relationship between the cross section and momentum transfer is approximately a power law. One example of a hard hadronic process is high transverse momentum jet production. The attempt to separate two quarks within the hadrons that take part in the collision will lead to the creation of new particles. This process of creating a collimated "shower" of particles in the attempt to separate two quarks is called hadronization and leads to the production of jets.

A jet is a narrow "cone" of hadronic and non-hadronic secondary particles created by a high momentum collision between, for instance, two quarks[4]. In a particle detector, jets are defined operationally based on a reconstruction algorithm that takes into account the energy deposited into the detector, the angle of impact and other factors.

Within soft hadronic processes are a category of events characterized by diffraction. A diffractive interaction is defined as an interaction where no quantum numbers are exchanged between the interacting particles. Thus, the process $1 + 2 \rightarrow 1' + 2'$ is a diffractive interaction, as is $1 + 2 \rightarrow 1' + X$ where $X$ is a collection of particles that in sum preserve the quantum numbers of 2. The exchange particle of a diffractive event
is called a pomeron[4]. The pomeron is defined as a mediator that has the quantum numbers of the vacuum but is not in any way a fundamental particle. Diffractive events are characterized by a constant distribution in the pseudorapidity gap, $\Delta \eta$, between final state particles, $\frac{dN}{d\Delta \eta} \sim A$, where $N$ is the number of events and $\Delta \eta$ the separation of the final state particles in pseudorapidity.

The term "minimum bias" refers to observing all proton-proton (p-p) collisions with minimal selections (kinematic, topological or otherwise) imposed. Minimum bias events are typically low transverse momentum collisions that cannot be described by perturbative methods in QCD. Minimum bias events consists of soft hadronic diffractive events in addition to non-diffractive inelastic events. These latter events are characterized by constant distributions in pseudorapidity and can be broadly described as $1 + 2 \rightarrow X$, where $X$ is a collection of final state particles.

Since the cross section for soft hadronic processes is proportional to the exponential of the negative of the squared momentum transferred between the interacting particles and the cross section for hard processes as a power law, at low momentum scales the cross section is dominated by soft hadronic processes. As $t$ increases, soft processes fall off exponentially. At the same time, the cross section for jet production is falling as a power law. Therefore, at some value in $t$, jet production (a hard hadronic process) will dominate over soft hadronic processes.

In a hadron collider one will always see many minimum bias events for every jet event. Another way to put this is, the cross section for minimum bias events is much bigger than the cross section for jet events.

Figure 2.2 shows the four processes that are usually associated with minimum bias events, while Figure 2.3 shows the $\eta$-$\phi$ parameter space of the final state particles of some minimum bias events, where $\phi$ is the azimuthal angle. Figure 2.2 shows from

---

[4] Pseudorapidity is defined as $\eta = -\ln(\tan \frac{\theta}{2})$, where $\theta$ is the polar angle.
Figure 2.2: These four processes define minimum bias events in ATLAS. Left to right: non-diffractive inelastic (NSD), single diffractive (SD), double diffractive (DD) and central diffractive (CD) processes. In each of these processes, two protons ($p_1$ and $p_2$) form the initial state and in the latter three diagrams the interaction is drawn in terms of pomeron exchange (IP).

Figure 2.3: These figures show the parameter space ($\eta - \phi$) of elastic (top left), single diffractive (top right), double diffractive (bottom left) and non-diffractive (bottom right) interactions[5].

Figure 2.3 shows the final state distributions of three of the processes of Figure 2.2.
(excluding central diffractive events, whose contribution to the overall cross section for minimum bias events is negligible) and contrasts them with elastic events. The difference is apparent, where elastic events have well defined final state positions in $\eta$ and $\phi$, minimum bias events produce a scattering of particles distributed throughout the space of $\eta$ and $\phi$.

Figure 2.4: These plots show the $\eta$ distribution for events with 2 or more charged particles with transverse momentum greater than 100 MeV at 0.9 (left) and 7 (right) TeV[6].

Figure 2.4 illustrates the pseudorapidity distribution of charged particles with greater than two tracks per event and transverse momentum greater than 100 MeV, taken at the LHC as a part of the ATLAS[6] collaboration. These plots help to illustrate the $\eta$ distribution of non-diffractive inelastic events, the particle distribution in $\eta$ is roughly a constant.
Figure 2.5: These plots show the average transverse momentum as a function of the number of charged particles, for events with at least 2 charged particles and transverse momentum greater than 100 MeV, compared with Monte Carlo at 0.9 (left) and 7 (right) TeV[6].

Figure 2.5 shows the relationship between the average transverse momentum, $\langle p_t \rangle$ and the number of charged particles, $n_{ch}$, in minimum bias events. In minimum bias events, the average momentum, $\langle p_t \rangle$, is only a weak function of $n_{ch}$.

The cross section, a measure of the likelihood for a given type of interaction, for minimum bias events is defined as the difference between the total cross section for proton-proton (p-p) collisions and the cross section for elastic events. Breaking this down further in equation (2.2), from left to right the total cross section is equal to the sum of the elastic, single diffractive, double diffractive, non-diffractive inelastic and central diffractive cross sections,

$$\sigma_{\text{Total}} = \sigma_{E} + \sigma_{SD} + \sigma_{DD} + \sigma_{ND} + \sigma_{CD}. \quad (2.2)$$
Therefore, we define the minimum bias cross section as the contribution from diffractive events and non-diffractive inelastic events. In ATLAS the minimum bias cross section is \[7\],

\[
\sigma_{MB} = \sigma_{SD} + \sigma_{DD} + \sigma_{ND} + \sigma_{CD}. \tag{2.3}
\]
2.3 New Physics at the LHC

While being able to make precision tests of the SM, (which includes the search for the Higgs Boson) the Large Hadron Collider project also was undertaken to search for new physics beyond the SM. The LHC, for instance, holds the promise of producing evidence of Supersymmetry (SUSY), one of the foremost theories in contemporary physics.

We know from the radial velocity profiles of spiral galaxies that a significant portion of the mass present in the universe does not interact electromagnetically[8]. This ”dark matter” forms nearly a quarter of the total mass-energy in the universe. One of most important problems in physics is to search for candidate particles that would be the constituents of dark matter.

A problem of significant concern in particle physics is known as the Hierarchy problem. This problem has to do with the mass of the Higgs Boson. The first loop diagram of Figure 2.6 shows a correction to the mass of the Higgs boson by a top quark loop diagram. A process of this kind is motivated by a term in the interaction Lagrangian of the form of $-\lambda_t H \bar{t}_L t_R$, where $\lambda_t$ is the coupling constant for the top quark, $H$ is the Higgs field, $t_L$ and $t_R$ represent the left-handed and right-handed components of the top quark. The process shown in Figure 2.6 (the $t$ quark diagram) will make a contribution of the form,

$$\delta m_H^2 = -\frac{3|\lambda_t|^2}{8\pi^2}(\Lambda^2 - 3m_t^2 \ln(\frac{\Lambda^2 + m_t^2}{m_t^2})) + \ldots,$$

(2.4)

to the mass of the Higgs, where the ellipses represent additional terms that are finite as $\Lambda \to \infty$ and $m_t$ is the mass of the top quark. $\Lambda$ represents a momentum cutoff used to regulate the calculations performed to obtain equation (2.4)[9]. It is clear that

$^5\bar{t}_L = t_L^\dagger \gamma^0$ where $t_L^\dagger$ is the hermitian conjugate of $t_L$ and $\gamma^0$ is the first gamma matrix.
Figure 2.6: In Supersymmetry, the quadratic divergences to the Higgs Boson mass can be canceled in loop diagrams between the loop particles and their superpartners. In these Feynman diagrams, the top quark, $t$, contribution to the Higgs mass is shown along with the contributions of its superpartner, $\tilde{t}$.

The magnitude of the correction, for an interaction term of the form\(^6\) of $-\lambda_t H f \bar{f}$, to the Higgs mass is determined by $\Lambda$. $\Lambda$, as a momentum cutoff, can be interpreted as the minimum energy scale at which new physics will arise so as to make the theory untenable\([10]\). Therefore, the Higgs mass is "tuned" in the sense that the value of $\Lambda$ is adjusted such that the mass of the Higgs boson ($\sim 130$ GeV) and the

---

\(^6\)One may imagine a similar correction for each of the six quarks, $f$. In fact, there will be loop corrections to the Higgs mass for every particle that couples to the Higgs field.
vacuum expectation value of the Higgs field ($<0|H|0> \sim 246$ GeV) are consistent with experimental results of the measurements of the masses of the $W^\pm$ and $Z$ bosons[3]. There is, however, no intrinsic reason for $\Lambda$ to be such a value. As we move to higher energies, due to the form of equation (2.4), $m_H$ will grow without some sort of adjustments such that it will excessively deviate from the actual mass needed in the SM. This is known as the Hierarchy problem[3], [9], [10], [11]. In order to deal with this problem, we must either accept extremely precise fine-tuning of $\Lambda$ or expand to supersymmetric theories.

Supersymmetry (SUSY) postulates the existence of fermion-boson pairs that differ by a spin 1/2, such that,

$$Q|\text{boson} > = |\text{fermion} >,$$  

(2.5)

$$Q|\text{fermion} > = |\text{boson} >,$$  

(2.6)

where $Q$ is the operator that relates fermions to bosons with commutation relations,

$$\{Q, \bar{Q}\} = -2\gamma_\mu P^\mu,$$  

(2.7)

$$\{Q, P^\mu\} = \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0,$$  

(2.8)

here $P^\mu$ is the momentum operator and $\gamma_\mu$ are the gamma matrices. Figure 2.7 shows the particle family of Supersymmetry. In SUSY, the partners of the quarks and leptons are known as the squarks and sleptons (shown in light yellow and light red respectively). They differ from their spin 1/2 SM counterparts by having spin 0. The gauginos (in light green) are the counterparts to the photon, gluon, $W^\pm$ and $Z$
bosons. They have spin 1/2. Finally the counterpart to the Higgs boson (spin 0) is the Higgsino (light blue). It has spin 1/2.

Figure 2.7: The Supersymmetry particle family contrasted with the SM particle family. Anti-particles are not shown. ©DESY.

Supersymmetry solves the Hierarchy problem by maintaining that there are equal number of bosons and fermions in nature, which give opposite signs in the quantum corrections to the Higgs mass loop diagrams and thereby cancel all divergences. An example of this is shown in Figure 2.6. The term in equation (2.4) is canceled by loop corrections arising from the scalar $\tilde{t}[9]$ in the second and third diagrams$^7$ of Figure 2.6.

In order to conserve baryon and lepton numbers at low energy (LHC scales), we impose $R$-Parity conservation in SUSY theories,

$$R = (-1)^{3(B-L)+2S},$$  \hspace{1cm} (2.9)

$^7$The second and third diagrams represent quartic and trilinear coupling to the Higgs field. Under certain circumstances, it is possible to cancel the quadratic divergence in $\Lambda$ (quartically) and the logarithmic divergence in $\Lambda$ (trilinearly) with respect to the $t$ quark contribution to the Higgs mass of equation (2.4).
where $B$, $L$ and $S$ are the baryon number, lepton number and spin respectively of a particle. This means that $R=1$ for Standard Model particles and $R=-1$ for Supersymmetric particles and implies that SUSY particles are always produced in pairs and that the lightest SUSY particle (LSP) is completely stable.

Figure 2.8 shows a possible LSP ($\tilde{\chi}^0_1$) production process arising from a quark-quark collision. The two output particles at the vertex of the $W^{\pm}$, $\tilde{\chi}^{\pm}_1$ and $\tilde{\chi}^0_2$ are mixed states of the gauginos and higgsinos. They are called the chargino, $\tilde{\chi}^{\pm}_1$ and the neutralino, $\tilde{\chi}^0_2$. There are four neutralinos, the lightest is denoted by the symbol $\tilde{\chi}^0_1$ and is assumed to be stable and the LSP. Here, $R$ parity ensures that the LSP is produced in a pair. The interaction signature would be three leptons, $\ell$, plus missing transverse energy from the LSP’s and the neutrino, $\nu_\ell$.

A LSP with no electric charge is a prime candidate for dark matter as it would gravitate so as the solve the radial velocity profile problem in galaxies but remain undetectable through electromagnetic means. The LSP could be produced at the
LHC and would be detected through analysis of its decay chain and missing energy in the detector.

SUSY solves two important problems in physics and astronomy in addition to other additional unexplained phenomena[11]. It is therefore one of the leading candidates for extensions to the SM and its discovery is a sought after prize for the LHC project.
Chapter 3

The Large Hadron Collider

This section begins with a brief background and technical overview of the LHC and some basic principles of collider physics that are of relevance. We continue with a discussion of the beam structure and event rate generation at the LHC, naturally leading to a description of the phenomenon of pile-up.


3.1 Background

The Large Hadron Collider is a hadronic synchrotron collider, of 26.7 km in circumference, built by the European Organization for Nuclear Research (CERN)[12]. The LHC is the most powerful particle accelerator ever built. At design specifications, the LHC will collide proton beams at a center of mass energy of 14 TeV with a luminosity of $10^{34}$ cm$^{-2}$s$^{-1}$. In addition to proton-proton interactions, the LHC will also collide lead ions at energies of 2.8 TeV/nucleon at a luminosity of $10^{27}$ cm$^{-2}$s$^{-1}$.

The LHC is built in the tunnel originally used to house the Large Electron-Positron Collider (LEP)[12]. Final approval for the LHC project was given in December 1994 and the first proton beams were injected in September 2008. A faulty electrical connection between two of the superconducting magnets within the collider created an accident that caused the LHC to experience a maintenance period lasting from late 2008 to November 2009. On November 30th, 2009 the LHC officially became the most powerful particle accelerator in the world by circulating proton beams of 1.18 TeV, beating the Tevatron record of 0.98 TeV per beam. On March 30th of the following year, the research program at the LHC officially began with the collision of two proton beams at 7 TeV center of mass energy, $\sqrt{s}$.

The research program at the LHC consists of two high luminosity, $L = 10^{34}$ cm$^{-2}$s$^{-1}$, experiments; A Toroidal LHC ApparatuS (ATLAS) and Compact Muon Solenoid (CMS) which are designed for new physics searches[12]. In addition, there are two experiments optimised for low luminosity, LHC beauty (LHCb), running at $L = 10^{32}$ cm$^{-2}$s$^{-1}$, and TOTal Elastic and diffractive cross section Measurement (TOTEM), running at $2 \times 10^{29}$ cm$^{-2}$s$^{-1}$, which are dedicated to Bottom quark and small angle elastic scattering physics respectively. The LHC is also capable of colliding Pb (lead) ions together for the dedicated observation of heavy ion physics and the study of dense forms of matter in A Large Ion Collider Experiment (ALICE).
3.2 Machine Details

The luminosity\cite{12}, equation (3.1), represents an important measure of the performance of any collider. For the LHC, a two beam collider, with a gaussian beam shape and assuming bunch sizes are the same in both beams, the luminosity is defined by,

\[
L = \frac{N_b^2 n_b f_{\text{rev}} \gamma_r}{4\pi \epsilon_n \beta^*} F \text{[cm}^{-2}\text{s}^{-1}],
\]

where the luminosity, \(L\), is defined by the number of particles per bunch, \(N_b\), the number of bunches per beam, \(n_b\), the revolution frequency, \(f_{\text{rev}}\), the Lorentz factor of the beams, \(\gamma_r\), the normalized transverse beam emittance, \(\epsilon_n\), the beta function at the collision point, \(\beta^*\) and \(F\), a correction factor for the luminosity due to the crossing angles of the beams at the interaction point\cite{12}.

The integrated luminosity taken over a time period \(\Delta t\) is defined by,

\[
L_{\text{int}} = \int_{\Delta t} L \, dt \text{[cm}^{-2}],
\]

The design beam structure at the LHC consists of proton beams organized into bunch trains of 2808 bunches per beam with a nominal spacing of 25 ns per bunch, where a bunch is a discrete packet of protons of a nominal length of 5.87 cm\cite{1}. This is contrary to the expectation of a beam as a continuous series of protons. These bunches are further collected into trains. A train is a series of bunches equally spaced apart. Therefore, as evident from equation (3.1), the luminosity will scale according to both the number of protons in each bunch (broadly the beam intensity) and the number of bunch trains filled.

The center of mass energy, given by \(\sqrt{s}\), refers to the collision energy in the center of mass frame of the two beams. At peak performance, each beam will have an energy of 7 TeV, creating 14 TeV collisions in the center of mass frame.
The average bunch crossing rate, $R_c$, in the detector is given by the product of the number of proton bunches and the revolution frequency,

$$R_c = 2808 \times f_{\text{rev}} \, [s^{-1}].$$  \hfill (3.3)

The luminosity of any collider is directly related to its capacity for event generation by

$$N_{\text{event}} = L\sigma_{\text{event}} \left[ \frac{\text{events}}{\text{s}} \right],$$  \hfill (3.4)

where the number of events recorded per second is proportional to the product of the luminosity and the cross section $\sigma_{\text{event}}$ for the event. Therefore, the number of events per bunch crossing, $N_c$, for a given interaction type is determined from equations (3.3) and (3.4) to be

$$N_c = \frac{N_{\text{event}}}{R_c} = 3.17 \times 10^{26} \sigma_{\text{event}} \left[ \frac{\text{events}}{\text{bunch crossing}} \right],$$  \hfill (3.5)

where we have inserted the nominal operating luminosity $10^{34} \, \text{cm}^{-2}\text{s}^{-1}$, bunch number 2808 and revolution frequency 11236 s$^{-1}$ and assumed equal spacing of the bunches throughout the circumference of the detector\(^1\). If we assume an inelastic proton-proton (p-p) cross section of 57.2 ± 6.3 mb[13] then we find that $N_c = 18 ± 2$ for inelastic events.

For the data taking period of 2010, the luminosity was $L \simeq 10^{30} \, \text{cm}^{-2}\text{s}^{-1}$ and therefore, $N_c \simeq \frac{5.1}{N_B}$ where $N_B$ is the number of filled bunches for a particular run of data taking. We have assumed here that the cross section for inelastic p-p collisions is the same at 7 TeV as it is at 14 TeV. The phenomenon of multiple independent

\(^1\)This is not actually true because gaps in the bunches must exist to allow for their insertion and extraction. This is why naively entering the circumference for the collider and the speed of light will not yield the nominal 40 MHz collision rate in equation (3.3). We emphasize that (3.3) is the \textit{average} bunch crossing rate.
events occurring in one bunch crossing is known as pile-up.
3.3 Pile-up

The term pile-up can refer to two different phenomena, in-time pile-up and out-of-time pile-up. In-time pile-up is the situation where more than one interaction occurs in a bunch crossing and is recorded by the detector. In other words, when more than one pair of protons collide in a bunch crossing as seen in Figure 3.1. Out-of-time pile-up, in contrast, occurs when bunch groups are spaced closely together and the rate at which bunches collide exceed that of the ability of the detector to process them. In other words, before the calorimetry (for instance Liquid Argon detector) finishes processing the final state particles of the current bunch crossing the p-p collisions of the next bunch crossing occur. In this situation, the energy signal of the next collision will be distorted by an addition of the residue of the signal of the current collision.

At peak performance of the detector, both in-time and out-of-time pile-up will occur. In this dissertation, it is in-time pile-up that is of concern because in the data taking periods analyzed the proton bunches are not so closely spaced as to produce any out-of-time pile-up.

The distribution of the number of events per bunch crossing is described by Poisson statistics,

\[ P(n) = \frac{e^{-\lambda} \lambda^n}{n!}, \quad (3.6) \]

where the probability, \( P(n) \), to obtain \( n \) events in a bunch crossing is related to \( \lambda \) the mean value of pile-up events. The mean, \( \lambda \), represents the mean number of events per bunch crossing and is given by\[14]",

\[ \lambda = LT_c \sigma_{pp}, \quad (3.7) \]

where \( L \) is the instantaneous luminosity, \( T_c \), the bunch separation and \( \sigma_{pp} \) the cross
Figure 3.1: This figure shows pile-up as seen using ATLAS Event Display. Two vertices are clearly distinguishable in the longitudinal plane (along the beam-pipe). The figure represents the event in the X-Y plane (top left), $\phi$-$\eta$ plane (top right) and $\rho$-$z$ plane (bottom). The MET is seen as the green arrow head in the X-Y plane.

The number spectrum for pile-up events is therefore a linear function of the luminosity.
Figure 3.2: Equation (3.8), showing the relationship between $\xi$ and $\lambda$.

We form the ratio,

$$\xi = \frac{P(n > 1)}{P(n > 0)} = \frac{1 - P(0) - P(1)}{1 - P(0)} = 1 - \frac{\lambda e^{-\lambda}}{1 - e^{-\lambda}},$$

(3.8)

defining the quotient of pile-up events to total recorded events. This is because the $n = 0$ contribution of equation (3.6) is not detected by ATLAS. As (3.8) is transcendental, we plot the equation in Figure 3.2 to show the relation between $\xi$ and $\lambda$.

Figure 3.3 shows the relationship between luminosity and $\xi$. To extract the number spectrum we use the numerical results of Figure 3.2 to search for $\lambda$ in terms of $\xi$. Therefore,

$$\xi = 1 - \frac{LT_c \sigma e^{-LT_c \sigma}}{1 - e^{-LT_c \sigma}},$$

(3.9)
Figure 3.3: This plot shows $\xi$ as a function of instantaneous luminosity (scaled to $10^{30}$ cm$^{-2}$s$^{-1}$) using data from period F run 162347.

where $\xi$ is what the detector can see and the luminosity is what we can control. Figure 3.3 is to be understood by noting that at small values of $\lambda$, (i.e. luminosity), $\xi$ is approximately linear with respect to $\lambda$ and luminosity while at large values of $\lambda$, $\xi \rightarrow 1$. For the data seen in Figure 3.3, $\lambda$ can be read off from Figure 3.2 to be between 1 and 2. This is to be contrasted with respect to the average expectation of $\sim 18$ events per bunch crossing when the LHC operates at design.
Chapter 4

Calorimetry and The ATLAS Detector

In this chapter, we review the physics of particle interactions with matter and in particular the theory of calorimetry. We discuss the mechanics of electromagnetic and hadronic interactions with matter. These developments are then placed in perspective with respect to the ATLAS detector and its hardware components.
4.1 Calorimetry

A calorimeter is built to surround the interaction point in a collider and is in broad terms a block of matter that absorbs the final state particles of a p-p collision and is of sufficient thickness to force the incident particles to deposit all of their energy within the calorimeter[15]. It is the job of a calorimeter to give a measurement of the incident particle’s energy using techniques that measure the energy loss within its absorbing material. The calorimeter must turn the energy lost by the incident particle into a measurement. This is usually done via one of several methods such as, by the collection of visible light via scintillators, by the detection of Cherenkov radiation or by the collection of ionization electrons of the traversed medium. A calorimeter takes the collected signal and converts it to a measurement of energy. The calorimeter response is the ratio between the average signal and the energy of the particle that caused it. The fact that the calorimeter response is linear as a function of incident particle energy for electromagnetic particles is an important property of all calorimeters. A calorimeter is an energy measurement device whose quality is in part characterized by its energy resolution, $\sigma$. We describe the calorimeter separately in terms of electromagnetic and hadronic interactions.

4.1.1 Electromagnetic Interactions

The physics of electromagnetic interactions is completely described by the theory of Quantum Electrodynamics (QED). An electron or photon incident on a bulk material loses energy via interactions with the constituent particles of the material through either radiation or collision. These two processes roughly dominate the high and low energy scales of the energy spectrum respectively. As the particle propagates through the material, it will create new particles (electrons and photons) through
pair production and bremsstrahlung. These particle "showers" cascade through the medium until all of the incident particle energy is lost as heat in the bulk material.

At the high energy scale, we define the radiation length, $X_0$, in terms of the energy loss, $\Delta E_r$, for a particle of initial energy $E$ via radiation over a length scale $\Delta x$ such that

$$\frac{-\Delta E_r}{E} = \frac{\Delta x}{X_0}. \quad (4.1)$$

In equation (4.1) the radiation length is well approximated by $X_0 \simeq 180A/Z^2$ where $Z$ is the atomic number and $A$ the mass number for a single element material\textsuperscript{1}.

At the low end of the energy spectrum, defined by the critical energy $\epsilon_c$, collision processes dominate. The critical energy is defined in terms of the energy, $E_c$, lost via collisions by electrons or positrons with energy $\epsilon_c$ in one radiation length in a bulk material\textsuperscript{15} such that,

$$dE_c = -\epsilon_c \frac{dx}{X_0}. \quad (4.2)$$

An approximate empirical expression for $\epsilon_c$ is given by $\epsilon_c \simeq 500/Z$. Figure 4.1 compares the efficiency of various energy loss mechanisms in a bulk material as a function of the electron/positron energy\textsuperscript{1}.

A simple equation\textsuperscript{16} exists to describe the rate of energy, $E$, loss of an initial particle of energy $E_0$ as a function of longitudinal propagation length, $x$, within the detector,

$$-\frac{dE}{dt} = E_0 b \frac{(bt)^{\alpha-1}e^{-bt}}{\Gamma(\alpha)}, \quad (4.3)$$

where $t = x/X_0$ and $\Gamma(\alpha)$ is the gamma function defined by $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t} \, dt$.

\textsuperscript{1}A more accurate empirical expression for the radiation length in terms of the material type is $X_0 = \frac{716.4A}{Z(Z+1)\ln(\frac{88}{Z})} \, g/cm^2$. 

Figure 4.1: Fractional energy loss per radiation length as a function of electron/positron particle energy in Lead\cite{1}.

The values of $\alpha$ and $b$ are obtained from empirical fits to data. Given knowledge of the initial conditions and the composition of the bulk material, equation (4.3) determines the longitudinal energy spectrum of electromagnetic shower cascades. Figure 4.2 shows this for simulation data using a 30 GeV electron incident on an iron absorber\cite{1}. The left axis shows the fractional energy loss while the right axis shows the number of photons/electrons with energy $E \geq 1.5$ GeV passing a given number of crossing planes. A crossing plane is defined as half a radiation length. The photons are normalized so as to be comparable to the electron distribution shape. The horizontal axis represents the longitudinal depth into the material.
4.1.2 Hadronic Interactions

The concepts behind hadronic showers are broadly analogous to electromagnetic ones, but owing to their complexities there is no comprehensive analytical formalism that describes them. Hadronic interactions are characterized by approximately half of the initial energy going into multiple particle production and the other half into a few highly energetic forward moving particles. The resulting shower cascade is primarily composed of nucleons and pions of which about 1/3, on average per collision, will be $\pi^0$’s, which will rapidly decay into photons and further interact electromagnetically. This places limits on the inherent resolution of the energy measurement. A considerable portion of the energy of the incident particle is converted into work for the excitation or break-up of atomic nuclei within the bulk material and is lost. Therefore, a hadronic shower has two components - an electromagnetic component due to
$\pi^0$ production and characterized by $X_0$ and a longer range component due to hadronic activity characterized by the nuclear interaction length $\lambda_{\text{int}}$.

The nuclear interaction length is defined as the mean length inside a medium that a high-energy hadron will travel before a nuclear interaction occurs. The probability that the particle will travel a distance $x$ inside a bulk material without causing a nuclear interaction is given by,

$$P(x) = e^{-\frac{x}{\lambda_{\text{int}}}}. \quad (4.4)$$

In general, $\lambda_{\text{int}}$ is larger than $X_0$.

The parametrization of hadronic showers is non-trivial and the differential equation that relates energy loss to longitudinal length within the medium is given as[16],

$$-\frac{1}{E} \frac{dE}{dx} = \alpha \frac{b^{a+1}}{\Gamma(a+1)} x^a e^{-bx} + (1 - \alpha)ce^{-cx}, \quad (4.5)$$

from equation (4.5) the two component nature of the hadronic shower is evident. Equation (4.5) can be broadly seen in terms of an electromagnetic term similar to equation (4.3) and an exponential term that forms the hadronic component. In equation (4.5) $x$ represents longitudinal depth, $E$ represents energy and the constants $\alpha$, $a$, $b$ and $c$ are found empirically. Figure 4.3 shows the longitudinal profile of hadronic showers for incident pions and protons at 50 and 180 GeV[17].
Figure 4.3: Longitudinal profile of hadronic showers for incident pion (left) and proton (right). $\lambda$ represents the nuclear interaction[17].
4.1.3 Energy Resolution

The energy resolution of a calorimeter determines the degree of precision of an energy measurement. The resolution of a good calorimeter improves as the energy of the incident particles increases[18]. This is in contrast to other energy measurement devices such as magnetic spectrometers. The energy resolution, as the determinant of uncertainty in energy measurements, is the most important parameter of a calorimeter. Calorimeters consist of two types - homogeneous and sampling. Homogeneous calorimeters are uniformly composed of one material that the incident particle cascades in while at the same time providing the particle energy measurements. In sampling calorimeters, the tasks of absorption and detection are separated into alternating layers. Sampling calorimeters have the advantage of reduced material cost and the ability to separately optimize the tasks of absorption and detection based on differing materials. As ATLAS is a sampling calorimeter, we will discuss the resolution in terms of them.

The energy resolution of a calorimeter is characterized by a series of terms that relate the statistical nature of energy cascades, noise and instrumentation response of the device.

In electromagnetic interactions the incident electron or photon loses energy via either radiation or collision. Above 1 GeV the main energy loss mechanism is bremsstrahlung and pair production respectively for electrons and photons. Therefore, the number of cascade particles grows as a function of depth in the calorimeter \(^2\). The maximum number of particles is reached when the cascade particle’s energy fall to the critical energy \(\epsilon_c\), at which point ionization will dominate over radiation and the shower begins to terminate. We define the concept of track length in terms of the radiation

\(^2\)This is because bremsstrahlung photons will pair produce. Thus, until we reach the critical energy, we always achieve a net profit in the number of electrons produced, but with progressively less energy per particle.
length, incident particle energy and critical energy as,

\[ T = X_0 \frac{E_0}{\epsilon_c}. \]  

(4.6)

In real life there always exists some cut off energy, \( E_{\text{cut}} \), below which the calorimeter is insensitive to the cascade particles. Therefore, the measurable track length is given by,

\[ T_d = F(\zeta)T, \]  

(4.7)

where \( F(\zeta) \) is found by Rossi’s “Approximation B”\([16]\) such that \( T_d \leq T \). As such, it is clear that \( T_d \) is proportional to the incident particle energy. Therefore, the relative error on the energy measurement is proportional to the relative error on the measurement of the number of track lengths \( (N_T = \frac{T_d}{X_0}) \). Since the number of track lengths, \( N_T \), is discrete it is Poisson distributed and the intrinsic energy resolution is given by,

\[ \frac{\sigma_E}{E} \sim \frac{\sigma_{N_T}}{<N_T>} = \frac{\sqrt{N_T}}{N_T} \sim \frac{1}{\sqrt{E}}. \]  

(4.8)

It is evident that the intrinsic relative energy resolution of a calorimeter improves with energy.

An additional "sampling" fluctuation exists in sampling calorimeters. Its contribution to the resolution is of the same form as the intrinsic resolution term. This sampling fluctuation is due to the fact that in a sampling calorimeter energy measurement only occurs in the active planes (as opposed to the entire device of a homogeneous calorimeter), therefore, only a portion of the total energy of the particle cascade is sampled. The energy measured is determined from the signal given by the sum of all the signals induced in all the active planes. The limiting effect on the
energy measurement then becomes the fluctuation in the number of particles that transverse each active plane. As this quantity is Poisson distributed the intrinsic sampling fluctuation contributes,

\[ \frac{\sigma_{\text{sampling}}}{E} \propto \frac{1}{\sqrt{E}}. \quad (4.9) \]

At the same time, the effects of intrinsic noise in the detection and instrumental effects will put a lower bound on the detectable signal. For example, the signal produced by the ATLAS liquid argon calorimeters is obtained by collecting electric charges in the picocoulomb range within a certain time frame (known as the gate time) in the sampling layers of the calorimeter. Since the detector has an unavoidable capacitance, electronic noise will exist. Therefore, because both the signal and the noise are both measured in charges, the noise will contribute to the spread in the calorimeter energy measurement. As the variance of the electronic noise is a fixed term, the contribution to the resolution given by this term scales as,

\[ \frac{\sigma_{\text{noise}}}{E} \propto \frac{1}{E}. \quad (4.10) \]

Finally, part of the ATLAS detector uses a Lead-Liquid Argon "accordion" detector design which will have inherent sampling fraction fluctuations. This is due to the fact that at the micro length scales of the sampling layers the fraction of the particle shower that is sampled becomes position dependent. This effect is highly dependent on geometry and the particular design of the sampling layer. This effect is energy scale invariant and in ATLAS amounts to about \( \frac{\sigma_{\text{cal}}}{E} \sim 0.4[18] \) and therefore,

\[ \frac{\sigma_{\text{constant}}}{E} \sim \text{constant}. \quad (4.11) \]
Combining the terms above, we can express the resolution of the calorimeter as,

\[ \sigma = c \oplus a\sqrt{E} \oplus bE, \]  

(4.12)

where we have absorbed equations (4.8) and (4.9) together and \( \oplus \) indicates addition in quadrature. In equation (4.12) \( c, a \) and \( b \) represent the noise, stochastic, and constant terms respectively.

While we have discussed the above primarily in terms of electromagnetic interactions, they apply similarly to hadronic interactions.

Lastly, the different response of the calorimeter to electromagnetic and hadronic interactions introduces a problem. Remember, a portion of the energy in hadronic propagation through the calorimeter is consumed in interactions with the nuclei of the absorbing material. Since a calorimeter is calibrated in terms of the signal per energy of the particle, that is a calorimeter converts charges (in coulombs) seen by electronics into an energy (in MeV) measurement, one would not expect that the same calibration for electromagnetic interactions would hold for hadronic ones. Since the signal of hadronic interactions is partially suppressed by effects such as the production of particles that escape the detector (neutrinos, muons) and losses due to the binding energy of the nuclei of the bulk material, a portion of the energy of the initial hadron is always invisible to the calorimeter. A calorimeter that has a different response to electromagnetic and hadronic interactions is called a non-compensating calorimeter. The ATLAS calorimeter is one such device. This different response, on the one hand, can be exploited for particle identification purposes but, on the other hand, it means that hadronic interactions have to be scaled with calibration weights with respect to electromagnetic interactions in order to reflect the true energy of the initial particles.

A number of calibration schemes exist in ATLAS to improve the energy measurement. These are made at the software level. We discuss this issue in Chapter 5.
4.2 ATLAS Detector

A Toroidal LHC ApparatuS (ATLAS)[19] is a general purpose detector of proton-proton collisions designed primarily to search for new physics. A variety of proposed physical processes and phenomena are expected to be observable with the ATLAS detector and as such, stringent technical requirements are needed (as seen in Table 4.1). The detector measures the energy, momentum, mass and direction of physics objects such as electrons, muons, jets, $\tau$ decaying leptons, and missing energy that result from the p-p collisions.

Figure 4.4: The relative position of the ATLAS Detector with respect to other primary detector experiments at the LHC[19].

The ATLAS detector is mounted in the first octant of the LHC tunnel as seen in Figure 4.4. Figure 4.5 labels the primary components of the ATLAS detector. It
measures 25 m by 44 m in length and weighs approximately 7000 tonnes. The ATLAS coordinate system is defined such that the beam pipe direction is the z axis. The x-y plane is normal to the z axis with the interaction point as its nominal origin. The positive x direction points towards the center of the ring while positive y points, up, towards the surface as seen in Figure 4.6. The end-caps of the detector are designated A and C such that the former labels positive z and the latter negative z. The spherical coordinates $\theta$ and $\phi$ are measured from the beam axis and transverse to it respectively. The pseudorapidity $\eta$ and rapidity $y$ are defined as,

\[ \eta = -\ln \tan \left( \frac{\theta}{2} \right) \]  
\[ y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right), \]

where $E$ is the energy and $p_z$ is the component of the momentum along the beam.

Figure 4.5: The ATLAS detector and its primary components[19].
axis. In the relativistic limit $\eta \simeq y$. The rapidity has the property of differing by an additive constant under Lorentz boosts along the $z$ axis between any two frames of reference. The variables $\eta, y$ are preferred over the polar angle $\theta$ because the shape of cross sections, $\frac{d\sigma}{dy}$, are invariant with respect to boosts and in addition, the charged particle distribution, in minimum bias events, is roughly a constant as a function of pseudorapidity (recall Chapter 2).

The term "cone radius" is defined by the distance in the parameter space of $\eta$ and $\phi$ by $R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$. The term transverse momentum, $p_t$, is defined as the momentum projected onto to the $x$-$y$ plane. The term “transverse energy” refers to the energy deposited into the calorimeter projected onto the transverse plane.

![Figure 4.6: The ATLAS coordinate system.](image)

The stringent requirements of the physics program at ATLAS results in a detector built with ultra-fast radiation resistant electronics, high granularity, wide acceptance in pseudorapidity and good calorimetry resolution. Table 4.1 [19] lists the principle performance requirements of the ALTAS detector with respects to its primary detector components.
<table>
<thead>
<tr>
<th>Detector Component</th>
<th>Required Resolution</th>
<th>$\eta$ coverage*</th>
<th>$\eta$ coverage*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking</td>
<td>$\sigma_{p_t}/p_t = 0.05% p_t \oplus 1%$</td>
<td>$\pm 2.5$</td>
<td>-</td>
</tr>
<tr>
<td>EM Calorimetry</td>
<td>$\sigma_{E/E} = 10% /\sqrt{E} \oplus 0.7%$</td>
<td>$\pm 3.2$</td>
<td>$\pm 2.5$</td>
</tr>
<tr>
<td>Hadronic Calorimeter barrel, endcap</td>
<td>$\sigma_{E/E} = 50% /\sqrt{E} \oplus 3%$</td>
<td>$\pm 3.2$</td>
<td>$\pm 3.2$</td>
</tr>
<tr>
<td>Hadronic Calorimeter forward</td>
<td>$\sigma_{E/E} = 100% /\sqrt{E} \oplus 10%$</td>
<td>$3.1 &lt;</td>
<td>\eta</td>
</tr>
<tr>
<td>Muon Spectrometer</td>
<td>$\sigma_{p_t}/p_t = 10%$ at $p_t = 1$ $\text{TeV}$</td>
<td>$\pm 2.4$</td>
<td>$\pm 2.7$</td>
</tr>
</tbody>
</table>

Table 4.1: This table lists the principle performance specifications of ATLAS, the right two columns represent parameters for measurement (left) and trigger (right).

### 4.2.1 Inner Detector

The ATLAS detector is in principle forward-back symmetrical about the nominal interaction point. The overall design of the ATLAS detector is determined by its magnetic configuration. A superconducting solenoid surrounds the inner detector cavity and three large superconducting toroids, two at the end caps and one surrounding the barrel complete the magnetic system of ATLAS.

![Figure 4.7: A cut out view of the ATLAS inner detector](image)

Within the 2 Tesla superconducting solenoid is the inner detector [19] (Figure
4.7), at 2.1 m x 6.2 m, which fulfills the tasks of particle track finding, momentum and vertex measurements and electron identification. At design luminosity, approximately 1000 final state particles will emerge from the interaction point every 25 ns. The inner detector is responsible for tracking these particles, and identifying their origins (vertices) and momentum with high resolution. This has great importance for determining pile-up because the number of primary vertices identified, as determined by particle tracks consistent with a single point of origin, is an estimate on the number of interactions that have occurred. These tasks are handled by the silicon Pixel detector, the SemiConductor Tracker (SCT) and the Transition Radiation Tracker (TRT). These devices measure charged particle position within the inner detector[19]. In addition, particle momentum is measured by analyzing the curvature of the tracks within the magnetic field.

The pixel detector and SCT cover the range $|\eta| < 2.5$ and are mounted around the barrel region in concentric cylinders and at the end-caps perpendicular to the beam pipe. The TRT aids position measurement in the region $|\eta| < 2.0$ for charged particle tracks with transverse momentum above 0.5 GeV. The specification for the momentum resolution in terms of tracking is summarized in Table 4.1.

The design of the inner detector is as seen in Figure 4.7, the inner most detector is the pixel detector followed by the SCT. These devices operate on the principle of the creation of electron-hole pairs by incident particles interacting with the semiconductor material. Beyond the pixel detector and SCT is the TRT, which consists of thousands of gas filled tubes that provide additional tracking and electron identification via transition radiation.
4.2.2 Calorimetry

Figure 4.8: A cut out view of the ATLAS calorimeter[19].

The calorimeter system[19] encloses the inner detector as seen in Figure 4.8. It consists of Pb/LAr sampling calorimeters mounted along the barrel of the beam pipe and at the end-caps that provide coverage up to $|\eta| = 4.9$ and a hadron layer that follows the electromagnetic layer providing barrel (Tile Barrel) and endcap (LAr hadronic end-cap - HEC, LAr Forward - FCal) region coverage for hadronic interactions. The energy resolution requirements of the calorimeter components are as summarized in Table 4.1.

A cryostat is a device used to maintain cryogenic temperatures. The ATLAS calorimeter system is housed in three separate cryostats. Each cryostat is made of aluminium and composed of an inner ”cold” layer and a concentric outer ”warm” layer. The central solenoid of the inner detector is housed in the barrel cryostat along
with the electromagnetic barrel calorimeter while the two end-cap cryostats house one EMEC, HEC, and one FCal each. The layout of the end-cap cryostats are illustrated in Figure 4.9. The inner layer of the cryostat is kept at approximately 90 K.

![Figure 4.9: The cryostat layout with respect to the calorimeter layout[19].](image)

The electromagnetic calorimeter consists of three components, the barrel component covering $|\eta| < 1.475$ and the end-cap components covering $1.375 < |\eta| < 3.2$. The barrel calorimeter consists of two identical half cylinders joined at $z=0$ with a 4 mm gap, while the end-caps are divided into two coaxial wheel pairs, these inner and outer wheels cover the range $1.375 < |\eta| < 2.5$ and $2.5 < |\eta| < 3.2$ respectively. The accordion design of the LAr/Pb calorimeter ensures full coverage in $\phi$ with no gaps. Figure 4.10 shows a cut out of the electromagnetic calorimeter.

In Figure 4.10 [19] the schematic layout of the barrel region is shown. The structure is divided into three layers in addition to a thin (11 mm) LAr presampler that is placed before the first layer. The presampler is responsible for sampling the elec-
Figure 4.10: A cut out view of the barrel module of the electromagnetic calorimeter, the accordion structure is clearly visible and the origin of the axis is the nominal interaction point[19].

electromagnetic shower before the calorimeter and correcting for the energy losses in the upstream region.

The layout of the end-cap calorimeters is similar, a presampler covers the front face of the end-caps. Each wheel of the end-caps is divided into 8 wedges. The region $1.5 < |\eta| < 2.5$ is known as the precision region and is structured into a three layer system. The front layer is approximately $4.4 X_0$ in thickness and is segmented in strips of $\eta$. The second layer is analogous to its counterpart in the barrel region while the last layer has half the granularity in $\eta$ compared to the previous layer. In the region $|\eta| < 1.5$ of the outer wheel and the region $2.5 < |\eta| < 3.2$ of the inner layer the calorimeter has only two longitudinal layers and is coarser in granularity.
The hadronic calorimetry system in ATLAS consists of the tile calorimeter, the LAr hadronic end-cap calorimeter and parts of the LAr forward calorimeter (which is a combination of electromagnetic and hadronic calorimeters).

![Tile Calorimeter Diagram](image)

Figure 4.11: A schematic view of the tile calorimeter. The z axis represents the radial direction while the dimensions are in cm[19].

The tile calorimeter is a sampling calorimeter that uses steel as the absorber and scintillating plates as the active component. The tile calorimeter covers the region $|\eta| < 1.7$ and consists of three parts as seen in Figure 4.8. Figure 4.11 shows a schematic cut out of the tile calorimeter. The sampling layers are apparent and the readout is from fiber optic cables that feed the signal into photomultiplier tubes. [19].

The hadronic end-cap calorimeters cover the region $1.5 < |\eta| < 3.2$. The HEC
is a copper-LAr sampling calorimeter as seen in Figure 4.12[19]. The HEC is a two wheel device (HEC1 and HEC2), one behind the other, with the rear wheel having a lower granularity than the front wheel. Each wheel contains two longitudinal layers and each wheel is made up of 32 identical wedge modules.

![Diagram of HEC](image)

**Figure 4.12**: The HEC as seen in r-φ (left) and r-z (right), dimensions are in mm[19].

In addition, the forward calorimeters provide coverage in the region $3.1 < |\eta| < 4.9$ approximately 4.7 m from the nominal interaction point. Each of the two FCal’s (on sides A and C) are split into three modules consisting of one electromagnetic module (FCal1) and two hadronic ones (FCal2, FCal3). For FCal1, the absorber is copper while in FCal2 and FCal3, tungsten is the primary bulk material. Figure 4.13 shows
a schematic view of the FCal system.

Figure 4.13: A schematic view of the forward calorimeter[19].

Finally, the region between the barrel calorimeter and the endcap calorimeters warrant special attention. Gaps exist in the region between the physical apparatus of the barrel calorimeters and the endcap calorimeters[19]. Figure 4.14 illustrates the situation. The energy of final state particles that make their way into the gap between the barrel and endcap calorimeters are detected by the extended barrel tile calorimeter and the gap and cryostat scintillators.
Figure 4.14: The transition region between the barrel tile and LAr calorimeters and the endcap tile and LAr calorimeters[19].
4.2.3 Muon Spectrometer

Surrounding the calorimeter is the muon spectrometer, which detects charged particles in the range $|\eta| < 2.7$ that exit the barrel and end-cap calorimeters and measures their momentum. Charged particles that escape the calorimeter are typically muons. The muon system uses large superconducting toroid magnets to determine the momentum of muons based on the magnetic deflection of their tracks. The system consists of precision-tracking chambers divided into eight octants in the barrel region between and on the coils of the barrel toroid magnets. Additional end-cap chambers sandwich the end-cap toroid magnets. Each octant is further divided in $\phi$ into two sections such that there is a small overlap in the azimuthal direction. This is designed so as to minimize gaps. The muon spectrometer determines the overall size of the ATLAS detector.
4.3 ATLAS Trigger and Minimum Bias Events

The trigger system in ATLAS, known as the Trigger and Data Acquisition (TDAQ) system, is responsible for identifying events in the detector, categorizing them based on predetermined physics criteria and feeding them into established data streams. The TDAQ is a three tiered system, named Level 1 (L1), Level 2 (L2) and Event Filter (EF) [19]. At each level, the trigger system refines the choices at the previous level and applies additional selection and filtering as necessary. The L1 triggers can monitor all events during beam collisions, however, when an event is being recorded the trigger is inactive, this is known as dead time. Owing to the quantity and frequency of collisions at ATLAS, the ultimate limit on the rate at which events can be taken is the speed at which data can be written to hard disk.

The L1 trigger makes decisions in less than 2.5 $\mu$s using only partial detector information, it searches for high transverse momentum physics objects of interest: muons, electrons, photons, jets and $\tau$ leptons. In addition, the L1 trigger identifies regions in $\eta$-$\phi$ space that have interesting or unique features with respect to physics based on some predetermined algorithm. These ”regions of interest” (RoI) are passed on to the L2 tier for further processing. Overall, the L1 trigger will bring down the event rate to approximately 75 kHz from the nominal p-p collision rate.

The L2 system uses all the detector data within the RoI to apply further selection criteria to data selected by L1 and reduce the trigger rate to around 3.5 kHz before passing the data to the Event Filter level. At L2, event building occurs, all event data belonging to an event are collected together and gathered under a single identifier before being passed on to the EF level. All particles, momentum, tracks, and vertices that the trigger determines to belong to one event are collected together and tagged as belonging to one event.

The EF reduces the event rate to 200 Hz and separates the events into different
Figure 4.15: The Minimum Bias Trigger Scintillators as seen during assembly, 32 such scintillators are mounted on the end caps of the ATLAS detector.[20] data streams based on predetermined criteria. These can include minimum bias events, cosmic ray events, and various physics objects.

At any level, (typically L1) filtering of events, known as prescaling, can be performed in order to optimize bandwidth usage with respect to luminosity and collision rate. A prescale suppresses a given trigger by a multiplicative factor. Thus, a prescale factor of 10 implies that only 1 in every 10 events for that particular trigger is recorded. In addition, a trigger or combination of triggers can be disabled in any given run thus suppressing all sensitivity to a given physics process.

The minimum bias trigger system at ATLAS is based on signals from a set of minimum bias detector systems. ATLAS is equipped with dedicated machinery for the triggering of minimum bias events.

The Minimum Bias Trigger Scintillators (MBTS) consists of thirty two scintillators
evenly split between the end caps of the ATLAS detector as shown in Figure 4.16 [20]. The MBTS cover the pseudorapidity range $2.1 < |\eta| < 3.8$. The scintillation counters are spatially segmented such that there are two segments in $\eta$ and 8 segments in $\phi$ per side as shown in Figure 4.16. The two ends of the MBTS are labeled A and C, corresponding to the A and C sides of the detector coordinate system.

In addition to the MBTS, ATLAS has several other mechanisms for triggering on minimum bias events:

- The Beam Condition Monitor which provides timing and luminosity measure-
ment capabilities and helps to distinguish between collision and non-collision events also has the capacity to trigger on minimum bias events[20].

- The Zero Degree Calorimeter which is primarily used to detect neutrons and photons that pass down the beam pipe with $|\eta| > 8.3$ in initial data taking provides an additional minimum bias trigger[20].

- The LUCID Cherenkov light detector, which is primarily used to monitor the luminosity during operation of the LHC. LUCID detects inelastic p-p scattering in the forward direction of the detector and can in principle be used in diffractive physics studies[20].

- The inner detector is also designed to be sensitive to minimum bias events. The inner detector has a trigger chain that starts with a random trigger at L1 that is able to reject empty bunch crossings at the L2 and EF level. This trigger chain is sensitive to single, double diffraction and non-diffractive inelastic events[20], [21].

It is expected that the MBTS, as the dedicated minimum bias scintillator, has the greatest sensitivity and efficiency in triggering on minimum bias events. Furthermore, the trigger system will often overlap, for instance an event picked up by an MBTS trigger can also be picked up by the inner detector.

We summarize the possible triggers at the L1 level that can feed the minimum bias stream at the EF level[20] in Table 4.2.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>Detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1_BCM</td>
<td>Beam Condition Monitor</td>
</tr>
<tr>
<td>L1_LUCID</td>
<td>Cherenkov Light Detector</td>
</tr>
<tr>
<td>L1_RDN</td>
<td>Inner Detector</td>
</tr>
<tr>
<td>L1_MBTS</td>
<td>Minimum Bias Trigger Scintillator</td>
</tr>
<tr>
<td>L1_ZDC</td>
<td>Zero Degree Calorimeter</td>
</tr>
</tbody>
</table>

Table 4.2: Triggers feeding the minimum bias stream.

The trigger column of Table 4.2 gives the prefix of the possible triggers. There are significantly more triggers based on the structure of the individual detectors. For
instance, such as L1_MBTS_A and L1_MBTS_B representing hits on the A and C side of the MBTS respectively. Further permutations based on the structure of the detectors exist. For instance, such as L1_MBTS_1 which requires one hit on either A or C or L1_MBTS_2 requiring two hits on either side.
Chapter 5

Missing Transverse Energy

In this chapter, we discuss the definitions, construction, calibration and relevance of Missing Transverse Energy. We begin with a discussion of the definitions of MET in the ATLAS collaboration, followed by a discussion of the relevance of MET with respect to physics analysis in the ATLAS collaboration. We then discuss the reconstruction of MET in the data analysis along with calibration methods. We further discuss the basic characteristics of MET, its resolution and mean and their implications. We close with a discussion of the possibility of an asymmetry in the $\phi$ distribution of the MET.
5.1 Definitions

In a collision between opposing particles, conservation of momentum tells us that the combined initial and final state momenta are equal. In a collider, the transverse component of the momentum\(^1\) is such that \(p_i = p_i' = 0\). However, the total reconstructed transverse momentum will not be exactly zero. Resolution effects and other systematic errors will distort the sum. As well, the detector is not sensitive to all particles - for instance neutrinos. There are also hypothetical particles which may not interact with the detector (recall Chapter 2). The ATLAS calorimeter as discussed in Chapter 4, is responsible for the measurement of the energy of all particles with the exception of neutrinos and muons (which is handled by the muon spectrometer). The ATLAS calorimeter is divided in \(\phi\) and \(\eta\) into individual cells. For highly relativistic particles, energy and momentum are related by \(E^2 = m^2c^4 + p^2c^2 \approx p^2c^2\). Therefore, the energy of a highly relativistic particle is approximately equivalent to the magnitude of its momentum under the convention that \(c = 1\). We formulate the variables \(E_{\text{Miss}}^X\) and \(E_{\text{Miss}}^Y\), which represent the missing energy in the x and y directions respectively, as,

\[
E_{\text{Miss}}^X = - \sum_{i=1}^{N} E_i \sin \theta_i \cos \phi_i, \quad (5.1)
\]

\[
E_{\text{Miss}}^Y = - \sum_{i=1}^{N} E_i \sin \theta_i \sin \phi_i. \quad (5.2)
\]

Where \(N\) is the total number of cells, \(E_i\) is the energy within a single calorimeter cell and \(\theta\) and \(\phi\) are the polar and azimuthal angles that determine the cell position with respect to the nominal origin of the detector. Thus, equations (5.1) and (5.2) represent the missing transverse energy in the x and y directions as sums over the

\(^1\)The longitudinal component, parallel to the beam direction, of the momentum is a less useful quantity due to incomplete \(\eta\) coverage and the fact that the initial momenta of the interacting particles are not well understood.
calorimeter cells. In addition, we formulate the term $E_{T}^{\text{Miss}}$ as,

$$E_{T}^{\text{Miss}} = \sqrt{(E_{X}^{\text{Miss}})^2 + (E_{Y}^{\text{Miss}})^2},$$

(5.3)

and define its orientation with respects to the $x$-$y$ plane by,

$$\phi_{X,Y} = \arctan \left( \frac{E_{Y}^{\text{Miss}}}{E_{X}^{\text{Miss}}} \right).$$

(5.4)

In addition, closely related to the vector sum of momenta, is the scalar sum known as $\sum E_{T}$ and defined as,

$$\sum E_{T} = \sum_{i}^{N} E_{i} \sin \theta_{i}.$$ 

(5.5)

Therefore, we say that $E_{T}^{\text{Miss}}$ is roughly the vector sum of momenta of the final state particles while $\sum E_{T}$ is the scalar sum of the momenta of the final state particles.
5.2 Relevance

MET is a critical observable both with respect to understanding the detector and new physics searches. A variety of issues, both hardware, dead or malfunctioning cells, incomplete azimuthal coverage and software, errors in reconstruction, and energy scale calibration can lead to non-zero MET where there should not be.

Less prosaic, MET is a valuable observable in the search for new physics. In the broadest terms, true MET can indicate the production of particles that do not interact with the detector. For instance, in SUSY theories, dark matter can be explained in terms of electromagnetically neutral weakly interacting particles. These particles may be produced in hadronic interactions. The lightest such particle, called the Lightest Supersymmetric Particle (LSP), is stable in all $R$-parity conserving SUSY models and will be the final product of any SUSY decay chain. Such a particle escapes detection by the calorimeter and could be found by searches using MET techniques, as a total momentum imbalance in the event would result.

The MET resolution determines the precision of the MET measurement. It is, therefore, crucial to the determination of true MET. If the MET for a given event exceeds the $\sigma$ measurement by a certain factor (for example $5\sigma$) it represents a certain probability of a signal for undetected momentum escaping the detector.

As the luminosity observed by the detector increases, the number of minimum bias events per bunch crossing will increase (and therefore the number of pileup events). The collision will then consists of a high transverse momentum event of interest along with a large number of minimum bias events overlaid on top of it. The degree to which MET can be determined to really exist, will then depend in part on the resolution of the MET of the minimum bias events.
5.3 Reconstruction and Calibration

In its simplest form, the MET is built by summing the individual energy contributions from various parts of the detector,

\[ E_{X,Y}^{\text{Miss}} = E_{X,Y}^{\text{Miss,calo}} + E_{X,Y}^{\text{Miss,cryo}} + E_{X,Y}^{\text{Miss,muon}}, \tag{5.6} \]

where the three terms represent contributions to the MET from the calorimeter and corrections for energy loss from the cryostat and muons.

The first and most important term is the contribution from the calorimeter cells. Its construction is identical to that of the general MET definitions as given in equations (5.1), (5.2) - where the summation goes over the calorimeter cells. As we shall see, it is by far the dominant term in minimum bias interactions. Cell level MET, as given in equation (5.6), is built over the geometry \(|\eta| < 4.5\).

Since each individual cell has a certain amount of noise and ATLAS has a large number of cells (a high granularity), the summation is not done over all cells but only over certain cells belonging to clusters.

A topological (Topo) cluster is defined as a three dimensional collection of cells that is constructed starting from a single seed cell chosen by a noise threshold algorithm. A topological cluster is seeded by a cell with an energy deposit above \(|E| > 4\sigma^2\), where \(\sigma\) is the RMS value of the energy distribution for random events taken by the calorimeter\(^2\). Neighboring cells with \(|E| > 2\sigma^2\) are then iteratively added, followed by adding all next neighboring cells to complete the cluster. The summation, \(N\), is then carried over all cells belonging to topological clusters.

A summation by topological clusters yields the MET term at the electromagnetic energy scale, i.e. the calibration for an electron is used. However, because the ATLAS

\(^2\)In random bunch crossings, the average cell energy should be zero.
calorimetry does not give equal response between electromagnetic and hadronic interactions there is a different response between electrons or photons and hadronic objects. A further calibration scheme is usually applied to correct for this fact, as well as compensate for energy loss in dead and malfunctioning cells. This will have the effect of improving the MET mean and resolution.

The simplest calibration schemes are termed “global”, indicating that the same scaling is applied to all topological clusters irrespective of their physical identity. A more nuanced approach is to first associate topological clusters to physics objects and then to accord each object an optimized calibration method. Such an approached is termed a “refined calibration”. There are two global calibration schemes.

Global Cell energy-density Weighting (GCW, CorrTopo) attempts to compensate for the different response of hadrons versus electrons/photons in the detector by assigning appropriate weights to the calorimeter cells with hadronic signals. Hadronic signals are characterized by regions of the detector with low cell energy density\(^3\) and GCW attempts to compensate by scaling those regions up in energy. This will also compensate for dead cells that receive hits. The weights themselves are determined by minimizing the energy fluctuation between reconstructed real jets and jets from Monte Carlo simulation\([22]\), [23].

In Local Cluster Weighting (LCW, LocHadTopo), topological clusters are first categorized as either hadronic or electromagnetic based on their pseudorapidity, energy, depth and cell energy density. Then the clusters are individually weighted three times. Within clusters, calorimeter cells are weighted according to the cluster energy and cell energy density to compensate for the weaker hadronic response in the calorimeter. The clusters themselves are weighted first according to the energy measured around the topological cluster (the nearest neighbors of the cluster) and by the longitudinal

\(^3\)Cell energy density is defined as \(E_{\text{cell}}/V\), where the energy of the cell is divided by the cell volume.
depth of its barycenter, the clusters are also weighted according to their energy and the fractional energy deposited into successive layers of the calorimeter[22]. These weights compensate for deposited energy not contained in the cluster and for energy deposited in dead material. The weights are determined from Monte Carlo simulation of charged and neutral pions[22],[23].

A more sophisticated method is called “refined calibration”. The goal here is to first associate individual calorimeter cells to specific physics objects. An order of precedence exists such that objects are associated first for electrons, then photons, \(\tau\) leptons, jets and finally muons. Each object, along with those clusters not associated to any object, is separately calibrated using an optimal weighting. The resultant missing energy of each object is then added together to form the refined calorimeter term. The formula is thus,

\[
E_{X,Y}^{\text{Miss, calo}} = E_{X,Y}^{\text{Miss, e}} + E_{X,Y}^{\text{Miss, \gamma}} + E_{X,Y}^{\text{Miss, \tau}} + E_{X,Y}^{\text{Miss, jets}} + E_{X,Y}^{\text{Miss, \mu}} + E_{X,Y}^{\text{Miss, CellOut}}.
\] (5.7)

From left to right of equation (5.7), the MET calorimeter terms for the refined calibration are the MET contribution from electrons, photons, hadronically decaying \(\tau\) leptons, jets, energy deposited by muons in the calorimeter and energy from cells that do not form any object.

The next term in equation (5.6) is \(E_{X,Y}^{\text{Miss, cryo}}\), which is a correction factor for energy lost between the Liquid Argon barrel electromagnetic calorimeter and the TileCal barrel hadronic calorimeter. In LCW calibration schemes, a correction factor for the cryostat is already included and therefore \(E_{X,Y}^{\text{Miss, cryo}} = 0\).

In GCW calibration, this correction compensates for the energy loss between the last layer of the electromagnetic calorimeter and the first layer of the hadronic calorimeter. We define,
where the right hand side of (5.8) is defined as,

\[ E_{\text{jet,cryo}}^{X,Y} = - \sum_{\text{jets}} E_{\text{jet,cryo}}^{X,Y}, \] (5.8)

by the definition of equations (5.9) and (5.10), the contribution to the missing energy from equation (5.8) is negligible for minimum bias events. This is because, in minimum bias events we do not expect to see many real jets and equations (5.9) and (5.10) are clearly in reference with respects to jet energy.

The final term, \( E_{\text{Miss,\mu}}^{X,Y} \), is obtained by summing the individual muon momenta in the range \(|\eta| < 2.7\) as detected by the muon spectrometer,

\[ E_{\text{Miss,\mu}}^{X,Y} = - \sum E_{\mu}^{X,Y}, \] (5.11)

where the sum is only taken over muons with matched tracks from the inner detector. This term should not be confused with the muon term from equation (5.7), which represents the small amount of energy lost by the muons as they travel through successive layers of the calorimeter.

Muons are distinguished as either isolated or non-isolated. This classification is determined by their proximity to a jet in a given event. A muon is defined as non-
isolated if it is within \( R = \sqrt{\Delta \eta^2 + \Delta \varphi^2} < 0.3 \) of a jet.

The transverse momentum of isolated muons is determined from combined measurements using the inner detector and the muon spectrometer. In this situation, the energy lost by the muon in the calorimeter, \( E_{X,Y}^{\text{Miss,} \mu} \), is not included in the summation of equation (5.7)[24].

In contrast, it is not possible to separate the energy of non-isolated muons deposited in the calorimeter from the nearby jet energy. Therefore, the measurement of the energy from the muon spectrometer alone is used (with an exception, see [24]). Again, due to the nature of minimum bias events, \( E_{X,Y}^{\text{Miss, muon}} \) does not make a significant contribution to the overall \( E_{X,Y}^{\text{Miss}} \) (as we do not expect any significant muon production in minimum bias events).
5.4 Resolution

We expect in minimum bias events, that there is no preferred orientation to the direction of the missing energy (recall Chapter 2.2). Therefore, we expect, with respect to the central limit theorem, that $E_{\text{Miss}}^X$ and $E_{\text{Miss}}^Y$, given respectively in equations (5.1) and (5.2), are both gaussian distributed with resolution $\sigma$ and mean $\mu$. The statement of no preferred orientation implies that, in principle, $\mu_{X,Y} = 0$. Therefore,

$$P(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\left(x-\bar{x}\right)^2/2\sigma_x^2}, \quad (5.12)$$

$$P(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{-\left(y-\bar{y}\right)^2/2\sigma_y^2}, \quad (5.13)$$

where the equations above represent the probability densities of $E_{\text{Miss}}^X$ and $E_{\text{Miss}}^Y$ respectively. The resolution is given by $\sigma_x$ and $\sigma_y$ while the mean is given by $\bar{x}$ and $\bar{y}$.

The resolution of the detector is uniform in azimuthal angle. Therefore, $\sigma_x = \sigma_y$ and $x$ and $y$ can be taken to be independent variables. The distribution of $E_{\text{Miss}}^T = \sqrt{(E_{\text{Miss}}^X)^2 + (E_{\text{Miss}}^Y)^2}$ is given by,

$$P(x, y) = \frac{1}{2\pi\sigma^2} e^{-\left(x^2+y^2\right)/2\sigma^2}, \quad (5.14)$$

where $\bar{x} = \bar{y} = 0$. Then converting to polar coordinates,

$$dx\,dy = \left| \begin{array}{cc} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{array} \right| \, d\theta \, dr = r \, d\theta \, dr, \quad (5.15)$$

$$P(x, y)\,dA_{x,y} = P(r, \theta)\,dA_{r,\theta}, \quad (5.16)$$
\[ P(r, \theta) = \frac{r}{2\pi \sigma^2} e^{-r^2/2\sigma^2}, \]  

(5.17)

where \( r = E_T^\text{Miss} \) is Rayleigh distributed. Note that,

\[
\int_0^{2\pi} P(r, \theta) \, d\theta = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} = P(r),
\]  

(5.18)

with mean and variance given by,

\[
\bar{r} = \int_0^{\infty} r P(r) \, dr = \frac{1}{\sigma^2} \int_0^{\infty} r^2 e^{-r^2/2\sigma^2} \, dr = \sigma \sqrt{\frac{\pi}{2}},
\]  

(5.19)

\[
\text{Var}(r) = \int_0^{\infty} (r - \bar{r})^2 P(r) \, dr = \frac{1}{\sigma^2} \int_0^{\infty} (r^2 - 2r\bar{r} + \bar{r}^2) r e^{-r^2/2\sigma^2} \, dr,
\]  

(5.20)

\[
= \Gamma(2)2\sigma^2 - \Gamma\left(\frac{3}{2}\right) \bar{r}^2 \sqrt{2\sigma} + \bar{r}^2 = \sigma^2 \left(2 - \frac{\pi}{2}\right),
\]  

(5.21)

and therefore the resolution \( \sigma_R \) is

\[
\sigma_R = \sqrt{\text{Var}(r)} = \sigma \sqrt{2 - \frac{\pi}{2}},
\]  

(5.22)

This leads to the interesting result that the mean of the \( E_T^\text{Miss} \) distribution is proportional to its resolution such that \( \frac{\sigma_R}{\bar{r}} = \sqrt{\frac{4}{\pi} - 1} \).

In order to obtain the stochastic form of the resolution as it is commonly seen, \( \sigma_{X,Y} = A \sqrt{\sum E_T} \), we recall the form of equation (4.12). For a real calorimeter composed of \( N \) number of segmented cells, the resolution is obtained by taking the sum over the resolution of every individual cell (or cluster).

We can express the resolution per cell of the calorimeter as,
\[ \sigma_{\text{cell}}^2 = c^2 + a^2 E_{\text{cell}} + b^2 E_{\text{cell}}^2. \] (5.23)

The total MET resolution is obtained by summing over the variances of all relevant calorimeter cells and projecting into the transverse plane. Therefore,

\[ \sigma_X^2 = \sum_{i=1}^{N} \sin^2 \theta_i \cos^2 \phi_i \sigma_i^2, \] (5.24)

\[ \sigma_Y^2 = \sum_{i=1}^{N} \sin^2 \theta_i \sin^2 \phi_i \sigma_i^2. \] (5.25)

Proceeding now for \( \sigma_X \), as the procedures are identical for \( x \) and \( y \),

\[ \sigma_X^2 = \sum_{i=1}^{N} \cos^2 \phi_i (c_i^2 \sin^2 \theta_i + a_i^2 \sin^2 \theta_i E_i + b_i^2 \sin^2 \theta_i E_i^2), \] (5.26)

and substituting \( E_{Ti} = E_i \sin \theta_i \),

\[ \sigma_X^2 = \sum_{i=1}^{N} \cos^2 \phi_i (c_i^2 \sin^2 \theta_i + a_i^2 \sin \theta_i E_{Ti} + b_i^2 E_{Ti}^2). \] (5.27)

As equation (5.26) is taken over all calorimeter cells, we may rewrite the summation such that,

\[ \sigma_X^2 = \sum_{i}^{N} \sum_{j}^{N} \cos^2 \phi_{ij} (c_{ij}^2 \sin^2 \theta_{ij} + a_{ij}^2 \sin \theta_{ij} E_{Tij} + b_{ij}^2 E_{Tij}^2), \] (5.28)

where we have explicitly divided the summation such that one adds over the orthogonal angles separately, \( i \) is reused as a summation index and \( N\theta N\phi = N \). For instance, rolling the cylindrical calorimeter out into a rectangular series of cells, \( \theta_{ij} \) would represent the polar angle of the calorimeter cell in the \( i \)th column of the \( j \)th row. We
now assume azimuthal symmetry and write,

$$\sigma^2_X = \sum_i (c_i^2 \sin^2 \theta_i + a_i^2 \sin \theta_i E_{Ti} + b_i^2 E_{Ti}^2) \sum_j N_\phi \cos^2 \phi_j. \quad (5.29)$$

Then, using the approximation that $\frac{1}{N} \sum_i f(x_i) \approx \frac{1}{\Delta \Omega} \Omega f(x) \, dx$, we have,

$$\sigma^2_X \approx N_\phi \frac{\int_0^{2\pi} \cos^2 \phi \, d\phi}{2\pi} \sum_i (c_i^2 \sin^2 \theta_i + a_i^2 \sin \theta_i E_{Ti} + b_i^2 E_{Ti}^2), \quad (5.30)$$

now taking the integral over $\phi$ we obtain$^4$,

$$= \frac{N_\phi}{2} \sum_{i=1}^{N_\theta} (c_i^2 \sin^2 \theta_i + a_i^2 \sin \theta_i E_{Ti} + b_i^2 E_{Ti}^2). \quad (5.31)$$

The next step is to assume $c_i$, $a_i$, $b_i$ are independent of angle. In an ideal detector, all cells should have the same response. In addition, we make the assumption that $E_T$ is not a function of $\eta$.

Then, multiplying through by $\frac{N_\theta}{N_\phi}$, we have,

$$\approx \frac{N_\phi}{2} \frac{N_\theta}{2\eta_m} \int_{-\eta_m}^{\eta_m} \sin^2 \theta \, d\eta + a^2 \frac{N_\theta}{2\eta_m} \int_{-\eta_m}^{\eta_m} \sin \theta E_T \, d\eta + b^2 \frac{N_\theta}{2\eta_m} \int_{-\eta_m}^{\eta_m} E_T^2 \, d\eta, \quad (5.32)$$

where $\eta_m$ represents the maximum pseudorapidity coverage of the detector. We can solve for the terms $< \sin \theta >$ and $< \sin^2 \theta >$ such that,

$$\frac{1}{N} \sum_i \sin^2 \theta_i \approx < \sin^2 \theta > = \frac{\int_{-\eta_m}^{\eta_m} \sin^2 \theta \, d\eta}{2\eta_m} = \frac{\tanh \eta_m}{\eta_m} \quad (5.33)$$

$$\frac{1}{N} \sum_i \sin \theta_i \approx < \sin \theta > = \frac{\int_{-\eta_m}^{\eta_m} \sin \theta \, d\eta}{2\eta_m} = \frac{\pi}{2\eta_m}, \quad (5.34)$$

$^4$In solving for $\sigma_Y$ the reader should note $\int_0^{2\pi} \cos^2 \phi \, d\phi = \int_0^{2\pi} \sin^2 \phi \, d\phi = \pi$.
where $d\theta = -\sin \theta d\eta$. Then,

\[
\sigma_X^2 = \frac{N \tanh \eta_m c^2}{2\eta_m} + \frac{NE_T \pi}{4\eta_m} a^2 + \frac{b^2}{2} NE_T^2,
\]

(5.35)

we must make, now, the approximations that $NE_T \approx \sum E_T$ and $NE_T^2 \approx \sum E_T^2$. Then we have,

\[
\sigma_X^2 \approx \frac{N \tanh \eta_m c^2}{2\eta_m} + \frac{\pi a^2}{4\eta_m} \sum E_T + \frac{b^2}{2} \sum E_T^2,
\]

(5.36)

collecting constants together,

\[
\sigma_X = \sqrt{C^2 + A^2 \sum E_T + B^2 \sum E_T^2},
\]

(5.37)

where equation (5.37) is the final form of the MET resolution. With perfect noise suppression, the first term, C, is zero. The last term reflects instrumentation response and only becomes significant for large values of $\sum E_T^2$. The approximate form of the energy resolution is arrived at by neglecting the first and last terms of (5.37),

\[
\sigma_{X,Y} = A \sqrt{\sum E_T}.
\]

(5.38)

Therefore, we have summarized the defining characteristic of the MET, $\sigma$ and parametrized it in terms of the $\sum E_T$.

---

\(^5\)See A.3 of the appendix for this calculation and those of $<\sin \theta>$ and $<\sin^2 \theta>$ above.
5.5 Asymmetry

The term $\phi_{X,Y} = \arctan\left(\frac{E_{X}^{\text{Miss}}}{E_{Y}^{\text{Miss}}}ight)$ is a measure of the difference between $E_{X}^{\text{Miss}}$ and $E_{Y}^{\text{Miss}}$ where none should in principle exist. Therefore, $\phi$ should be distributed as $\frac{dN}{d\phi} \sim$ constant where $N$ is the number of events. A non-constant $\phi$ distribution can be the result of a misalignment of the detector.

Consider the scenario sketched out in Figure 5.1.

![Figure 5.1: A sketch of a possible misalignment of the detector center.](image)

Coordinate system O lies at the real center of the ATLAS detector at coordinate (0,0,0) with the z direction coming out of the page. Coordinate system O’ is defined at the nominal center of the ATLAS detector. O is displaced from O’ by (h,k). In principle, O and O’ should coincide. Let the inscribed circle be of unit distance and define the walls of the calorimeter. Then, $\phi$ and $\phi'$ are defined as:

$$\phi = \arctan\left(\frac{y}{x}\right), \quad (5.39)$$

$$\phi' = \arctan\left(\frac{y'}{x'}\right), \quad (5.40)$$
where prime denotes the O’ system and \((x,y)\) determine the position on the unit circle. The point \((x,y)\) is related to \((x',y')\) by,

\[
(x', y') = (x, y) - (h, k) = (\cos \phi', \sin \phi'),
\]

and

\[
\phi = \arctan \left( \frac{k + \sin \phi'}{h + \cos \phi'} \right).
\]

Whereas the distribution of \(\phi\) is given by \(\frac{dN}{d\phi} \sim \text{constant}\), the distribution where the original is displaced at O’ with respects to O is given by,

\[
\frac{dN}{d\phi'} = \frac{dN}{d\phi} \frac{d\phi}{d\phi'}.
\]

Therefore, the distribution of \(\phi'_{X,Y}\) can be described by,

\[
\frac{dN}{d\phi'} \sim \frac{h \cos \phi' + k \sin \phi' + 1}{h^2 + k^2 + 1 + 2(h \cos \phi' + k \sin \phi')},
\]

where (5.44) is an equation for an asymmetrical distribution of \(\frac{dN}{d\phi'}\). Figure 5.2 shows equation (5.44) for various values of \((h,k)\). The general equation for a circle of arbitrary radius \(r\) is given by,

\[
\frac{dN}{d\phi'} \sim \frac{h' \cos \phi' + k' \sin \phi' + 1}{h'^2 + k'^2 + 1 + 2(h' \cos \phi' + k' \sin \phi')},
\]

where \(h'\) and \(k'\) are given by,

\[
h' = \frac{h}{r}, \quad k' = \frac{k}{r},
\]

and therefore implying that a radially larger calorimeter reduces the effect. As \(r \to \infty\), one recovers \(\frac{dN}{d\phi} \sim \text{constant}\). Equation (5.45) represents a situation where the ATLAS software misconstrues the true origin of the detector. Therefore, during the "recon-
Figure 5.2: This figure shows plots of equation (5.44) for varying values of \( h \) and \( k \), such that: red represents \((h,k)=(0.2,0.5)\), blue \((h,k)=(0.3,-0.1)\), green \((h,k)=(0.01,-0.5)\) and black \((h,k)=(0.001,0.01)\).

The formalism above describes an event with only 1 vertex. In the case of multiple vertices, we may imagine calculating the MET for each individual vertex with respect to the nominal center of the detector. Each vertex being an independent event, it will have its own \( \phi_{X,Y} \) distribution, but because they would be corrected by the reconstruction software to the same nominal center, the values of \((h,k)\) for each one would be the same. Therefore, the combined distribution for the overall event would be the sum of each individual \( \phi_{X,Y} \) distribution, all of which would be in phase with one another.
Chapter 6

Experimental Data and Simulated Events

In this section, we present an overview of the data that were used in the analysis, both real and simulated. As well, we comment on the data preparation method at ATLAS. We give a detailed description of the data and event selection methods used in the analysis. We explain and examine the selection criteria made on the data used in the analysis in terms of data quality, jet quality, and track quality.
6.1 Data Taking at The LHC

As we have discussed in Chapter 4.3, the event rate is reduced to approximately 200 Hz at the EF level. At the EF level, selection takes place that classifies the data on an event by event basis with respect to various physics streams. These are then passed on to output nodes (computers), which write the events to disk. Events are written to predetermined streams, such as electrons, muons, jets, MET, minimum bias based on selection criteria from the L2 and EF trigger levels. In addition to physics streams, there exist calibration and express streams. These are used to monitor detector performance and data quality respectively and follow a different data processing methodology in order to prioritize their usage.

It is at this point that physics reconstruction occurs. By reconstruction, we mean to recreate the final state physics, to create the four vectors and identities of all physics objects. Therefore, from the raw detector information, objects such as MET and physics objects such as electrons, photons, jets are created at this stage (as detailed in Chapter 5 for MET). The reconstructed data are the final data form that is then made available for analysis. They are produced in two "meta-data" types, ESD (Event Summary Data) and AOD (Analysis Object Data), which both contain the full reconstructed events but varying degrees of the original detector data[25]. In ATLAS, reconstruction is handled by ATHENA. ATHENA is a C++ program designed to reconstruct the physics of collision events as observed by the ATLAS detector[25].

At this point, the data are used by combined performance groups. These are dedicated teams of experts, that deal with specific aspects of the physics such as: jets, MET, electrons, and muons. Further data processing is done at the physics group level and specific goal orientated data sets for analysis are produced based on ATLAS needs and objectives (for instance runs with only pile-up events or high
transverse momentum jets). In this analysis, we use the D3PD data format produced by the jet and missing energy combined performance working group (JetEtMissWG). The D3PD format is a data format that eliminates almost all raw detector information and is tailor made for individual physics groups so that objects and details are unique to each working group. The D3PD is a popular format for analysis due to the relative ease in its manipulation, but D3PDs suffer from constant revisions to their data structure and hence have less than perfect documentation[25].

Analysis of D3PDs is performed using ROOT[26]. ROOT is an object orientated analysis tool developed at CERN by Rene Brun and other associates[26] and optimized for large volume data processing. In this dissertation, ROOT is the tool used to perform all analysis.
6.2 Data for Analysis

Data taking at the LHC is divided into periods. Each period consists of an interval of data taking of variable length (typically several weeks). Periods are divided up according to milestones reached with the accelerator or detector; typically new periods coincide with increased luminosity, bunch groups filled or other criteria. Each period is divided into a series of runs. A run is one sustained period of data taking (composed of events) lasting several hours. A run is further divided into lumiblocks, a lumiblock is a period of data taking nominally equal to 120 seconds. A lumiblock is defined to have a single value of average luminosity. Recall equation (3.2), the integrated luminosity is given by the time integral of the instantaneous luminosity over a time interval $\Delta t$. The lumiblock is approximately the time scale parameter of the corresponding Riemann sum of equation (3.2),

$$L_{\text{int}} = \sum_{i=1}^{N} \Delta t_i L_i.$$  (6.1)

Where the lumiblock, $\Delta t_i$, is effectively the time interval over which the luminosity is defined to be "constant".

Data over several periods of LHC beam collisions were analyzed in conjunction with Monte Carlo simulations. The analyzed data are summarized in tables A.5 and A.6 of the appendix. In total, six periods of data, over four months, from mid April to mid August of 2010 were analyzed in addition to one Monte Carlo event sample. The data sets listed in tables A.5 and A.6 consist of minimum bias data that exists in D3PD format. In total approximately $3.67 \text{ pb}^{-1}$ of data are used in this analysis.
6.3 Monte Carlo

The Monte Carlo method is a common tool in science and engineering for simulating complicated processes with many degrees of freedom. A number of Monte Carlo generators exists for simulating minimum bias interactions in the context of collider physics. In terms of the ATLAS experiment, Monte Carlo event generators seek to simulate the physics of the collider[27]. Monte Carlo event generators can be used to make predictions on important physics parameters, such as the cross section of various processes, the expected signature of SUSY or other theoretical particles, evaluate possible backgrounds to important discoveries and in conjunction with detector simulations study the performance of the detector.

PYTHIA is a C++ coded Monte Carlo event generator created at Lund University designed to simulate high-energy physics events[28]. As of writing, its latest iteration is PYTHIA version 8.1. It is a flexible program with many adjustable parameters that can be "tuned" to reproduce existing experimental results. The simulated events we examine in this analysis consists of a dedicated minimum bias run "tuned" to produce large amounts of minimum bias events with in-time pile-up generated using PYTHIA by the Monte Carlo working group at ATLAS.

<table>
<thead>
<tr>
<th>Period</th>
<th>Run</th>
<th>(\int L , dt ) (nb(^{-1}))</th>
<th>Events</th>
<th>(\lambda)</th>
<th>Bunch Spacing (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC</td>
<td>105001</td>
<td>0.082</td>
<td>3988672</td>
<td>2</td>
<td>900</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of basic parameters of the Monte Carlo event sample used for the analysis.

In addition to the event generator, is the detector simulator. The detector simulator simulates the response of the ATLAS detector to the event. In our Monte Carlo, the detector simulator is GEANT4[29]. Table 6.1 summarizes the basic parameters of the Monte Carlo event sample set we used in this analysis. Figure 6.1 illustrates the analogy between data and Monte Carlo. In order to accurately compare Monte Carlo
with data, the raw data format produced in Monte Carlo after the detector simulation must be the same as that of data. In addition, the software and reconstruction methods used for Monte Carlo and data must be the same.

Figure 6.1: A comparison of real life (left) and Monte Carlo simulation (right).
6.4 Event Selection

The data are processed according to a series of selections for run conditions, beam, jet\(^1\), and track quality. These selections are referred to as cuts. A cut is a veto on an event based on a predetermined criteria. For instance, a trivial and nonsensical cut would be to remove all events recorded on a specific date from the analysis.

Table 6.2 summarizes the data selection and cleaning procedures in order of precedence\([22], [30]\). For the rest of this dissertation the term "all cuts" refers to all selection criteria as listed in Table 6.2.

We define the concept of bad and ugly jets for the purposes of event jet cleaning. The specific definitions are given in Table 6.2. Bad jets and ugly jets are terms that define problems with the detector, non-collision events, hardware and software issues based on specific patterns or correlations in the parameter space of jet qualities. Detector issues known as: EM Coherent Noise, HEC Spike and Beam Backgrounds encompass the definition of bad jets. An event with a bad jet is rejected if and only if its transverse momentum at the electromagnetic energy scale exceeds 20 GeV - that is \(p_{T, jet}^{EM} > 20\) GeV. In contrast, events with ugly jets are rejected outright. In addition, to ensure good track quality each event must have at least one vertex with 5 or more tracks and a minimum scalar transverse momentum of 150 MeV \([30]\).

The first cut that is made requires an event to pass a given trigger. As the MBTS is \(~99\%\) efficient in detecting non-single diffractive events, it was chosen as the cut trigger\([20]\). A number of MBTS triggers at L1 are possible. Amongst these are: L1_M BTS\(_1\), L1_M BTS\(_2\), L1_M BTS\(_1\)\(_1\) (recall Chapter 4.3 for trigger logic). L1_M BTS\(_1\) is a good candidate because it is the most inclusive MBTS trigger.

The next cut is to require stable operating conditions within the collider and

\(^1\)In minimum bias events, the vast majority of events should create no jets. It is still, however, possible for the jet reconstruction algorithm to organize specific calorimeter cells into jets. These should not be seen as real jets but nonetheless are treated the same way for selection purposes.
<table>
<thead>
<tr>
<th>Cut</th>
<th>Logic</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger †</td>
<td>L1_MBTS_1==1</td>
<td>T</td>
</tr>
<tr>
<td>Good Runs List †</td>
<td>Pass Good Lumiblocks</td>
<td>GRL</td>
</tr>
<tr>
<td>Timing ‡</td>
<td>$</td>
<td>\Delta t_{LAr}</td>
</tr>
<tr>
<td>EM Coherent Noise ‡</td>
<td>$\text{em}_f &gt; 0.9 &amp;&amp;</td>
<td>Q_{LAr}</td>
</tr>
<tr>
<td>HEC Spike ‡</td>
<td>($n_{90} &lt; 6 &amp;&amp; \text{HEC}_f &gt; 0.8) | (</td>
<td>Q_{LAr}</td>
</tr>
<tr>
<td>Beam Backgrounds ‡</td>
<td>$</td>
<td>t_{jet}</td>
</tr>
<tr>
<td>Ugly Jets ‡</td>
<td>$\text{Corr}_f &gt; 0.5 | \text{tgap}_3 &gt; 0.5$</td>
<td>Ugly Jets</td>
</tr>
<tr>
<td>Track †</td>
<td>$\geq 1 \text{ vertex} &amp;&amp; n_{\text{track}} &gt; 5 &amp;&amp; P_T &gt; 150 \text{ MeV}$</td>
<td>Tracks</td>
</tr>
</tbody>
</table>

Table 6.2: A flow chart of event cleaning and selection given in order of precedence. The logical syntax is in C++ and refers to the parameters as they are named in the D3PDs. The right column represents the legends used to identify the cuts in the analysis plots of Chapter 7. † indicates inclusive logic, ‡ indicates exclusive logic with respect to event selection.
detector. These involve complicated sensors that monitor the beam condition, magnetic fields, and general operating conditions of the LHC and the ATLAS detector. This information is summarized in the Good Runs List (GRL) for any given period. For minimum bias events, the primary items of interest contained within the GRL are stable beam conditions, nominal voltage within the inner-barrel and end-cap detectors, and a lack of major errors in the detector [22]. The GRL vetos events in any given run by machine conditions by removing events that are unsuited for physics analysis. A GRL does this by dividing a run into good and bad lumiblocks. Therefore, the second event selection criterion is to use only good LBs for the analysis.

The calorimeter can also confuse background processes as collision events. Beam-halo, stray cosmic muons and other effects can be recorded as p-p collision events.

Figure 6.2: $\Delta t_{\text{LAr}}$ and $\Delta t_{\text{MBTS}}$ before and after all cuts have been made using period D data. Notice the secondary peaks in $\Delta t_{\text{LAr}}$ and $\Delta t_{\text{MBTS}}$. These peaks suggest background processes taking place during the collision event.
In an interaction originating from a bunch crossing, one expects that the time it takes for the final state particles to reach the end-caps of the detector will not differ substantially. In contrast, events that originate outside of the detector will have signals that tend to have a large timing difference between being sequentially detected by sides A and C of the detector. Therefore, the relative timing difference between sides A and C of the calorimeter for true minimum bias events should be statistically peaked at $\Delta t = 0$, and therefore outliers in the $\Delta t$ distribution represent background processes. The timing cuts are made on the timing difference between signals on the A and C sides of the Minimum Bias Trigger Scintillator and the liquid argon end-cap calorimeter[22]. Figure 6.2 shows the effects of all the cuts on the parameters $\Delta t_{\text{LAr}}$ and $\Delta t_{\text{MBTS}}$.

![Figure 6.2](image1.png)

Figure 6.2: Parameter space of $em_t$ versus $|Q_{\text{LAr}}|$ before cuts (left) and after all cuts (right) using period D data.

The next series of cuts define bad and ugly jets. The jet cleaning selection criteria are applied to anti-$k_t$ algorithm[31] reconstructed jets. These jets in ATLAS are prefixed by the title jet_AntiKt4Topo. Where the prefix defines in order from left to right, the reconstruction algorithm (anti-$k_t$), “cone radius” ($R=0.4$), and cell cluster reconstruction method (topological clusters).
The bad jet selection criteria primarily focus on three effects. The first two bad jet cuts, “EM Coherent Noise”, characterized by the exclusive selections $em_\ell > 0.9 \&\& |Q_{\text{LAr}}| > 0.6$ and “HEC Spike”, characterized by the exclusive selections, $(n_{90} < 6 \&\& HEC_\ell > 0.8) \parallel (|Q_{\text{LAr}}| > 0.3 \&\& HEC_\ell > 0.3) \parallel (HEC_\ell > 1 - |Q_{\text{LAr}}|)$, deal with fake jet signals caused by noise bursts in the detector hardware. A noise burst is a term that is used to describe a situation whereby a large amount of energy appears to be deposited into a few calorimeter cells[22]. In this case, the cuts are meant to deal with noise bursts in the Electromagnetic Calorimeter and the Hadronic Endcap Calorimeter respectively. The effects of the cuts of Table 6.2 are shown for the parameter space of $em_\ell$ vs $Q_{\text{LAr}}$, $HEC_\ell$ vs $Q_{\text{LAr}}$ and $n_{90}$ vs $HEC_\ell$ in Figures 6.3, 6.4, and 6.5.

The term $em_\ell$ is defined as the fraction of the total jet energy that is deposited into only the electromagnetic calorimeter. The term $Q_{\text{LAr}}$ known as the “quality factor”, defines the fraction of LAr cells with a cell Quality factor greater 4000. The cell quality factor, $Q_{\text{cell}}$, is defined as,

$$Q_{\text{cell}} = \sum_{i=1}^{5} (a_{i}^{\text{measured}} - a_{i}^{\text{pred}})^2,$$  \hspace{1cm} (6.2)

where the difference is taken between the time samples of the measured pulse shape and the reference pulse shapes used to reconstruct the energy per cell and then summed over all the time samples (of which there are five). Typically, bad calorimeter cells are represented by large $Q_{\text{cell}}$ factors[22].

$HEC_\ell$, similar to $em_\ell$, is defined as the fraction of the total jet energy that is deposited into the HEC. Bad jets caused by HEC noise bursts are characterized by the appearance of large energy deposits (i.e. large signals but no real energy) in the HEC and a low number of calorimeter cells accounting for a large fraction of the jet energy. The term $n_{90}$ represents the minimum number of calorimeter cells accounting
for at least 90% of the total jet energy. This technique of searching the parameter space of \( \text{HEC}_f \) versus \( n_{90} \) does not work if real jet energy exists in the same place as the HEC burst. In this case, additional searches based on the parameters \( \text{HEC}_f \) and \( Q_{\text{LAr}} \) are used to flag problematic events[22].

The effects of “Beam Background” cuts, defined by the exclusive logic \( |t_{\text{jet}}| > 25 \text{ ns} \) \& \( \text{em}_f < 0.1 \) \& (fsampling_{\text{Max}} > 0.95 \& \& |\eta| < 2 \), are the result of a series of background noise cleaning cuts[30]. Out of time jets are flagged with a timing cut.
The jet time $t_{\text{jet}}$ is defined with respect to the event time of the bunch crossing; a cut on $t_{\text{jet}}$ tries to capture background events that can be reconstructed as jets, but really come from photons produced by cosmic ray muons. The maximum energy fraction in one layer of the end-cap calorimeters, $\text{fracSampling_{Max}}$, is used in conjunction with a cut on the pseudorapidity to tag muons traveling along the beam direction. Finally, a cut on $\text{em}_{f}$ places a lower limit on the energy deposition into the electromagnetic calorimeter. Figure 6.6 shows the effects of the cuts of Table 6.2 on the jet timing parameter, $t_{\text{jet}}$.

Figure 6.6: Effects of all cuts using period D data on the jet timing parameter, $t_{\text{jet}}$.

Ugly jets are defined by jet energy deposition in problematic parts of the ATLAS detector. Ugly jets reconstructed in regions of known malfunctioning or dead cells are tagged by looking at the fraction of jet energy deposited into cells tagged as problematic by the detector cell database. This is identified by the term $\text{Corr}_{f}$. A
second tag is made by looking at the jet energy deposited into the gap between the TileCal barrel and the end-cap and making a cut based on the fraction of the total jet energy deposited into the scintillator located between these detector pieces. This is identified by the term $t_{gap}^3$. This is necessary because the energy response of the gap scintillator is not yet well understood\cite{24}. Figure 6.7 shows the effect of the cuts for ugly jets.

Finally, the track quality cut, is used to select events with good primary vertices. A primary vertex is defined as one resulting from p-p collision, as opposed to a secondary vertex caused by decays of the original final state particles or other spurious effects. It is required that a good primary vertex possesses at least 5 tracks consistent with coming from the same origin with a total minimum scalar transverse momentum of 150 MeV\cite{22}, \cite{30}.

![Figure 6.7: The parameters Corr$_f$ (left) and tgap$_f$ (right) that are used to select ugly jets in period D data.](image)

Table 6.3 summarizes the efficiency of the data selection. Out of 120,454,213 collision events our cuts ultimately select 33,968,840 for a selection efficiency of 28.2%. In addition, there is a discrepancy of an additional 333,076 events between the total number of minimum bias events according to the ATLAS Run Query database\cite{32}.
and the RAW number recovered from the analysis (as indicated by the first row of Table 6.3). In Monte Carlo, out of a total of 3,988,672 events we select 3,729,368 for an efficiency of 93.5%. In Monte Carlo we do not apply jet cleaning cuts[30], events are only selected for trigger, timing and track quality.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Events (Data)</th>
<th>Rejected (Data)</th>
<th>Events (MC)</th>
<th>Rejected (MC)</th>
</tr>
</thead>
<tbody>
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<td>-</td>
<td>3988672</td>
<td>-</td>
</tr>
<tr>
<td>L1_MBTS_1</td>
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<td>52763442</td>
<td>3987831</td>
<td>841</td>
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<td>GRL</td>
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<td>20755203</td>
<td>3987831</td>
<td>-</td>
</tr>
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<td>-</td>
<td>3855603</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.3: Efficiency of Data Selection for collision and simulated events.
Chapter 7

Analysis

This section forms the body of my own work for this dissertation. We first show the results of event selection based on the MET variables as defined in Chapter 5. Next, we show the effects of pile-up on these variables in three different calibration schemes, electromagnetic, global cell weighting and local cluster weighting, before moving on to a discussion of the resolution, mean and azimuthal asymmetry. In all instances, we compare the results to Monte Carlo and attempt to quantify the differences between the collision and simulated events.
7.1 MET Cleaning

Figure 7.1 shows the effect of the data selection on the parameters $E_{T}^{\text{Miss}}$, $\sum E_{T}$, $E_{X}^{\text{Miss}}$ and $E_{Y}^{\text{Miss}}$ for topological clusters at electromagnetic energy scale. The general effect of the data selection is to gradually reduce the tails of the distributions. Figures 7.2 and 7.3 show this process using GCW and LCW weighted topological clusters respectively.

As is evident, screening for bad jets is the most important selection. Screening for bad jets produces the most dramatic effect on the tails of the energy distributions. This is shown in Figures 7.1, 7.2 and 7.3. By carefully examining the cut flow, we can see that the tails of the distributions last appear in the blue line which represents all cuts up to and including MBTS and LAr timing. By tail, we mean the extended, high energy (in GeV) region of the distributions. For instance, the region around 100 GeV in the distribution of $E_{T}^{\text{Miss}}$ before cuts (RAW) would be considered a tail. Removing ugly jets reduces a significant number of events at the centroids of the $E_{X}^{\text{Miss}}$ and $E_{Y}^{\text{Miss}}$ distributions.

For each calibration, we see a second peak on the positive side of the $E_{Y}^{\text{Miss}}$ distribution. By examining the cut flow as indicated by the legends, we can see that it is partially the result of bad lumiblocks and partially the result of bad jets. Examining the bad jet cuts carefully, we find that the effect is eliminated by the Beam Background cuts suggesting that the effect is caused by physical background processes in the detector.
Figure 7.1: The cut flow for $E_{\text{miss}}$, $\Sigma E_T$ (top left), $E_{\text{miss}}^X$ (bottom left), and $E_{\text{miss}}^Y$ (bottom right) according to Table 6.2 with the energy calibration at electromagnetic energy scale.
Figure 7.2: These histograms show the cut flow for $E_{\text{Miss}}$ (top left), $\sum E_T$ (top right), $E_{\text{Miss}}^X$ (bottom left), and $E_{\text{Miss}}^Y$ (bottom right) according to Table 6.2 with GCW weighting.
Figure 7.3: The cut flow for $E_T^{\text{Miss}}$ (top left), $\sum E_T$ (top right), $E_X^{\text{Miss}}$ (bottom left), and $E_Y^{\text{Miss}}$ (bottom right) according to Table 6.2 with LCW weighting.
7.2 MET Response to Pile-up

Figure 7.4 shows the vertex distribution before and after selection in data and Monte Carlo. As we have discussed, the number of vertices as calculated during reconstruction, is an estimate on the number of interactions. Selection eliminates the long tail of the vertex distribution in data. In Figure 7.4 we have sorted the events in data and Monte Carlo by the number of primary vertices in them and histogrammed them. Thus, the $x$ axis represents the number of events with 1 primary vertex in them, with two primary vertices in them and so forth. Therefore, with respect to this dissertation, the term “n vertex events” represents a selection on events with exactly $n$ primary vertices.

![Figure 7.4: Vertex distribution after the selection of Table 6.2 for data (left) and Monte Carlo (right).](image)

We note there is a difference between data and Monte Carlo. In Monte Carlo, the vertex distribution comes from one run. In this run, the mean number of events, $\lambda$, is specifically tuned to $\lambda = 2$. In data, however, what we see is the addition of many vertex distributions from many runs each with its own value of $\lambda$. Thus, while the Monte Carlo vertex distribution is well defined by a poisson distribution with $\lambda \approx 1.9$, the vertex distribution in data is only approximately poissonian.

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The fact that it does not fit to $\lambda = 2$ exactly is likely due to detector effects as simulated by GEANT4.
The missing energy distributions, $E_{\text{Miss}}^T$, $E_{\text{Miss}}^X$, $E_{\text{Miss}}^Y$, $\sum E_T$, are shown with respect to increasing number of pile-up events in Figures 7.5, 7.6 and 7.7 for electromagnetic, GCW and LCW energy scales. Thus, we have sorted the missing energy distributions with respect to the number of primary vertices in each event, in a manner analogous to that of Figure 7.4. These Figures show that the widths of the $E_{\text{Miss}}^X$ and $E_{\text{Miss}}^Y$ distributions increase as a function of pileup. This implies that the resolution is degrading with additional interaction vertices. In $E_{\text{Miss}}^T$ and $\sum E_T$ pile-up tends to move the distributions to higher values in energy.

Figure 7.8 shows the relationship between the average $\sum E_T$ and the number of vertices per event for the different calibration methods overlaid with Monte Carlo. The almost linear response of the average $\sum E_T$ to the number of vertices has important consequences for vertex counting and is directly related to the distributions in Figures 7.5, 7.6 and 7.7. Figure 7.8 explains why the $E_{\text{Miss}}^T$ and $\sum E_T$ distributions shift to higher values in energy with increasing pile-up. On average, each additional primary vertex delivers a proportional value in transverse momentum into the calorimeter. Therefore, a link is established between average transverse energy deposited into the calorimeter and the number of interacting vertices. As ATLAS is expected to eventually produce collisions with large amounts of minimum bias events piled together, at some point the ability to individually distinguish all primary vertices will become degraded and inefficient. A relationship between energy deposition and the number of interacting vertices helps to solve this problem by allowing conversion between the former and the latter.

A linear fit, equation (7.1) is used to describe the relationships in Figure 7.8.

\[ < \sum E_T >= a + b \times \text{Number of Vertices} \tag{7.1} \]

On average, each additional primary vertex adds 29.77±0.02, 43.73±0.03, and 46.10±0.04 GeV
Figure 7.5: $E_T^{\text{Miss}}$ (top left), $\sum E_T$ (top right), $E_X^{\text{Miss}}$ (bottom left) and $E_Y^{\text{Miss}}$ (bottom right) distributions for data (crosses) and Monte Carlo (bars) with the energy calibration at electromagnetic energy scale.
Figure 7.6: $E_{T}^{\text{Miss}}$ (top left), $\sum E_{T}$ (top right), $E_{X}^{\text{Miss}}$ (bottom left) and $E_{Y}^{\text{Miss}}$ (bottom right) distributions for data (crosses) and Monte Carlo (bars) with the energy calibration at GCW energy scale.
Figure 7.7: $E_{\text{Miss}}^T$ (top left), $\sum E_T$ (top right), $E_{\text{Miss}}^X$ (bottom left) and $E_{\text{Miss}}^Y$ (bottom right) distributions for data (crosses) and Monte Carlo (bars) with the energy calibration at LCW energy scale.
Figure 7.8: Average $\sum E_T$ versus number of vertices per event in data and Monte Carlo.
to the $\sum E_T$ of the event with respects to electromagnetic, GCW and LCW calibrated energy scales. The relationship is not exactly linear, as indicated by the large $\chi^2/\text{ndof}$. The degree of non-linearity in the relationship could be interpreted as an indication of a possible mismeasurement on the number of primary vertices. The number of vertices, as calculated by the reconstruction software, is an estimate on the number of collision events, thus, there is, in principle, an error in the number of vertices along the $x$ axis. The degree of non-linearity in the relationship between $\sum E_T$ and the number of vertices may characterizes this.

In addition, we note that the ”calibration constant”, $b$, differs for data and Monte Carlo. Based on Table 7.1, they differ such that Monte Carlo exceeds data by $6.55 \pm 0.01$, $9.11 \pm 0.01$, $8.79 \pm 0.01$ GeV with respect to electromagnetic, GCW and LCW energy scales. This is not yet understood.

<table>
<thead>
<tr>
<th></th>
<th>$a$ [GeV]</th>
<th>$b$ [GeV]/Vertex</th>
<th>$\chi^2$/ndof</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM (Data)</td>
<td>5.55±0.02</td>
<td>29.77±0.02</td>
<td>3934/6</td>
</tr>
<tr>
<td>GCW (Data)</td>
<td>8.57±0.03</td>
<td>43.73±0.03</td>
<td>3624/6</td>
</tr>
<tr>
<td>LCW (Data)</td>
<td>12.18±0.04</td>
<td>46.10±0.03</td>
<td>2834/6</td>
</tr>
<tr>
<td>EM (MC)</td>
<td>-2.04 ±0.04</td>
<td>36.32±0.02</td>
<td>4749/6</td>
</tr>
<tr>
<td>GCW (MC)</td>
<td>-2.35 ±0.06</td>
<td>52.84±0.03</td>
<td>3902/6</td>
</tr>
<tr>
<td>LCW (MC)</td>
<td>1.23±0.06</td>
<td>54.89±0.03</td>
<td>2308/6</td>
</tr>
</tbody>
</table>

Table 7.1: Fit Parameters for equation (7.1).

The MET response can be analyzed by examining the $E_{X,Y}^{\text{Miss}}$ distributions as characterized by,

$$
\frac{dN}{dE_{X,Y}^{\text{Miss}}} = \frac{1}{\sqrt{2\pi}\sigma_{X,Y}^2} e^{-\frac{(E_{X,Y}^{\text{Miss}} - \mu_{X,Y})^2}{2\sigma_{X,Y}^2}},
$$

(7.2)

where the mean ($\mu$) and resolution ($\sigma$) are the parameters of a normal probability distribution. The MET response is parameterized as a function of the scalar total energy deposited into the calorimeter in the transverse direction, $\sum E_T$, as seen in
Figure 7.9: $E_{\text{miss}}$ versus $\sum E_T$ for 1 vertex events at electromagnetic energy scale for data. The resolution and mean correspond to the spread and average of vertical slices of this plot in increments of $\sum E_T$. The color scale is normalized with respect to the number of events in the 2D histogram, such that the bottom (purple) represents 1 event and the top (red) represents the most events.

The distributions of $E_{\text{miss}}^T$, $E_{\text{miss}}^X$, and $E_{\text{miss}}^Y$ parameterized in $\sum E_T$ are shown in Figures 7.10 (Data) and 7.11 (Monte Carlo) for topological clusters at electromagnetic energy scale\(^2\). Combined, these Figures show the relationship between $\sum E_T$ and $E_{\text{miss}}^T$, $E_{\text{miss}}^X$, and $E_{\text{miss}}^Y$ and the effects of pile-up. Figure 7.10 shows that pile-up events tend to occur at higher values in $\sum E_T$, as is characterized by the distributions shifting to the right with increasing number of vertices per event. This is consistent with the description given by equation (7.1). This effect is mirrored in Monte Carlo in Figure 7.11.

We note that the number of events with pile-up falls off drastically in data such that there are significantly less events with 4, 5, and 6 vertices in data than in Monte

\(^2\)See Appendix for GCW and LCW weighted distributions.
Carlo. This will affect the precision of our analysis, as we cannot extend our analysis of the resolution and mean beyond 4 vertices in data due to the large statistical error that it would incur. Additional plots of the form of Figures 7.10 and 7.11, for data and Monte Carlo at GCW and LCW energy scales can be found in the appendix as Figures B.1, B.2, B.3 and B.4.
Figure 7.10: $E_{T}^{{\text{Miss}}}$(top six), $E_{X}^{{\text{Miss}}}$ (middle six), $E_{Y}^{{\text{Miss}}}$ (bottom six) as functions of $\sum E_{T}$ for data. The color scheme is analogous to that of Figure 7.9. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
Figure 7.11: $E_{T}^{\text{Miss}}$ (top six), $E_{X}^{\text{Miss}}$ (middle six), $E_{Y}^{\text{Miss}}$ (bottom six) as functions of $\sum E_{T}$ for Monte Carlo. The color scheme is analogous to that of Figure 7.9. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
7.3 MET Resolution

The MET response is characterized by the quantities given in equation (7.2), the resolution and mean. The mean and resolution are extracted from Figures 7.10 and 7.11 by slicing the sum of the 2D histograms of $E^\text{Miss}_X$ versus $\sum E_T$ and $E^\text{Miss}_Y$ versus $\sum E_T$ along the y axis in segments of the x axis. The cross section of each slice is a Gaussian distributed histogram of $E^\text{Miss}_{X,Y}$ as characterized by equation (7.2) and as seen in Figure 7.12.

![Figure 7.12: A slice of $E^\text{Miss}_X$ (left) and $E^\text{Miss}_Y$ (right) parametrized in $\sum E_T$ of data at the electromagnetic energy scale. We have taken the slices at $\sum E_T = 100$ GeV with a bin width of 1 GeV. These plots contain events with 1 vertex only. The red lines represent fits to gaussian distributions of the form of equation (7.2).](image)

Figure 7.12 shows the results of the gaussian fits of the $\sum E_T$ slices for the resolution, $\sigma_{X,Y}$, for topological clusters at electromagnetic, GCW and LCW energy scales respectively. The error bars represent the statistical errors from the $\chi^2$ fitting algorithm. By forming the ratio between data and Monte Carlo (Figure 7.14), $\frac{\sigma^\text{Data}_{X,Y}}{\sigma^\text{MC}_{X,Y}}$, it is clear that the resolution in data is better than in Monte Carlo, but moving towards unity as $\sum E_T$ increases.

We characterize the resolution in Figure 7.13 using a simplification of equation 3. We comment on the degree to which the notion that $\sigma_X = \sigma_Y$ as a function of $\sum E_T$ is valid in appendix A.2.
Figure 7.13: $\sigma_{X,Y}$ versus $\sum E_T$ in data (left), Monte Carlo (right) at electromagnetic (top), GCW (middle) and LCW (bottom) energy scales.
Figure 7.14: $\frac{\sigma_{\text{Data}}}{\sigma_{X,Y}}$ versus $\sum E_T$ at electromagnetic (top), GCW (middle), and LCW (bottom) energy scales.
where we have neglected the noise term\(^4\) (this is sensible because we expect topological clustering to significantly suppress noise) and require \(A, B > 0\). We show the results of the \(\chi^2\) fits\(^5\) to the resolution in Figure 7.15. In addition, we summarize the numerical results in Tables A.1 and A.2.

The scaling factor, \(A\), varies such that \(A_{\mathrm{EM}} < A_{\mathrm{GCW}} \sim A_{\mathrm{LCW}}\). This would on the surface make \(\sigma_{X,Y}^{\mathrm{EM}}\) favorable over the calibrated resolutions, but this argument is flawed as the effect of the calibration scheme on the function parameter \(\sum E_T\) is not taken into account. Superficially overlaying the resolutions, at the three calibration scales, would show \(\sigma_{X,Y}^{\mathrm{GCW}}\) and \(\sigma_{X,Y}^{\mathrm{LCW}}\) to be worse, since calibration will tend to move the entire resolution to the right towards higher energy. It does not make sense to overlay plots of \(\sigma_{X,Y}\) versus \(\sum E_T\) for different calibrations together, as each plot will have a \(\sum E_T\) at a different scale. To compare the different calibrations, one forms the quantity \(\sum E_T^{\mathrm{EM}} / \sum E_T^{\mathrm{Cal}}\), where Cal is GCW or LCW. By scaling \(\sigma_{X,Y}^{\mathrm{GCW}}\) and \(\sigma_{X,Y}^{\mathrm{LCW}}\) with this quantity, as a function of \(\sum E_T\) at electromagnetic energy scale, the effect of the calibration becomes evident and shows the resolution to be improved using GCW and LCW calibrations. This particular analysis is not a part of the scope of this dissertation and the full details can found in [24].

It appears from the fits and the qualitative forms that the resolution can be approximated to a high degree of accuracy with \(\sigma_{X,Y} = A \sqrt{\sum E_T}\).

\(^4\)The reader must notice that we have interchanged \(\sum E_T^2\) for \((\sum E_T)^2\) with respects to equation (5.37), this is a common parametrization of the constant term and is consistent with ATLAS analysis. A practical reason for this substitution is that the variable \(\sum E_T^2\) is not available in D3PDs. In addition, under the assumption that \(E_T\) is independent of \(\eta\) these two quantities only differ by a constant.

\(^5\)Except for four vertex events, where we have used a log likelihood fit to improve the errors to the fits.
The fact that $\sigma_{X,Y}$ roughly obeys a square root relationship with respect to $\sum E_T$ as described by equation (7.3) has important consequences. The resolutions, of the $x$ and $y$ components of the MET, being the uncertainty in the measurement of MET, are added in quadrature such that for two separate measurements of $E_{X}^{\text{Miss}}$ or $E_{Y}^{\text{Miss}}$ (from two events for instance), we have $E_{X1}^{\text{Miss}} \pm \sigma_1$ and $E_{X2}^{\text{Miss}} \pm \sigma_2$ and the combined value of $E_{X12}^{\text{Miss}}$ is $E_{X1}^{\text{Miss}} + E_{X2}^{\text{Miss}}$ with the error given by $\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2}$. Then, we have,

$$\sigma_{12} = A\sqrt{\sum E_{T1} + \sum E_{T2}}, \quad (7.4)$$

where $\sum E_{T1}$ and $\sum E_{T2}$ represent the values of $\sum E_T$ for events 1 and 2. This is simply the position further along the curve in $\sigma = A\sqrt{\sum E_T}$ at $\sum E_{T12} = \sum E_{T1} + \sum E_{T2}$. Had the resolution of each individual measurement followed any other relationship (for instance $\sigma = A(\sum E_T)^n$, where $n \neq \frac{1}{2}$), the result of equation (7.4) cannot be obtained\(^6\).

The fact that additional vertices also do not change the resolution, has the interesting consequence that events with equal value in $\sum E_T$ are indistinguishable in resolution. This is because, as all p-p events are independent of one another, an event with one vertex with a given $\sum E_T$ will record the same resolution as that of an event with two vertices with the same total $\sum E_T$ of the former event.

\(^6\)Suppose $\sigma = A(\sum E_T)^n$. Now, for two events with $\sum E_{T1}$ and $\sum E_{T2}$, the total resolution is found by $\sigma_{12} = A\sqrt{(\sum E_{T1})^2n + (\sum E_{T2})^2n}$. Now, $\sigma_{12}$ will only correspond to $\sum E_{T12} = \sum E_{T1} + \sum E_{T2}$ if $n = \frac{1}{2}$. For any other value of $n$, the position of $\sigma_{12}$ is not determined by the linear sum of $\sum E_{T1}$ and $\sum E_{T2}$. 
Figure 7.15: Fit parameters A (top) and B (bottom) in data (left) and Monte Carlo (right) for equation (7.3).
7.4 MET Mean

The mean of the MET response is, in principle equal for $E_{\text{Miss}}^{X}$ and $E_{\text{Miss}}^{Y}$ and equal to zero, $\mu_X = \mu_Y = 0$. In a physics analysis, significant deviations away from $\mu_X = \mu_Y = 0$ with respect to the expected uncertainty is the key signal of many beyond the SM searches.

Figures 7.16, 7.17 and 7.18 show the mean of the MET response as a function of $\sum E_T$ for cells in topological clusters at electromagnetic, GCW and LCW energy scales respectively. It is clear from the figures above, that a bias exists in $\mu_Y$ that appears to grow linearly as a function of $\sum E_T$ and is in addition, reproducible in the Monte Carlo. The large error bars at larger values of pile-up for the data plots represent a lack of statistics. We quantify this effect and how it changes with pile-up.

Figure 7.16: $\mu_X$(top), $\mu_Y$(bottom) for data (left) and MC (right) at electromagnetic energy scale.
with a linear approximation. A linear fit to the data in the form of equation (7.5) is made by treating $\mu_X$ and $\mu_Y$ as a function of $\sum E_T$,

$$
\mu_{X,Y} = c + d \sum E_T.
$$

(7.5)

Figure 7.17: $\mu_X$(top), $\mu_Y$(bottom) for data (left) and MC (right) at GCW energy scale.

From Figure 7.19 and the preceeding Figures, we can see that the response in $E_X^{\text{Miss}}$ and $E_Y^{\text{Miss}}$ are both biased as functions of $\sum E_T$ such that the mean of the energy spectrum shifts by an amount on the order of a few MeV for every GeV gained in $\sum E_T$. This effect is considerably worse in $E_Y^{\text{Miss}}$ than in $E_X^{\text{Miss}}$ for both data and Monte Carlo samples. While undesirable, as long as the effect is predictable, a correction along the lines of equation (7.5) can always be made.
Figure 7.18: $\mu_X$ (top), $\mu_Y$ (bottom) for data (left) and MC (right) at LCW energy scale.

Figure 7.19 shows the parameters to the fit of equation (7.5) for $\mu_{X,Y}$ as functions of $\sum E_T$ for the three calibrations. Figure 7.19 quantitatively illustrates that the bias is more significant in $E_{Miss}^Y$ then $E_{Miss}^X$. In addition, Figure 7.19 illustrates that between 1 vertex and $>1$ vertex there is some difference between the slopes of the bias in both data and Monte Carlo as characterized by the parameter $d$. Beyond the first vertex, however, the slopes appear to be more or less consistent with one another. We summarize the numerical results of the fits in Tables A.3 and A.4.
Figure 7.19: The slope (top) and offset (bottom) to a linear fit for $\mu_X$ (X) and $\mu_Y$ (Y) in data (left) and Monte Carlo (right).
7.5 MET Asymmetry

The degree of difference between the response in $E_{X}^{\text{Miss}}$ and $E_{Y}^{\text{Miss}}$ is quantified by forming the term,

$$\phi_{X,Y} = \arctan\left(\frac{E_{Y}^{\text{Miss}}}{E_{X}^{\text{Miss}}}\right).$$  \hspace{1cm} (7.6)

Equation (7.6) is plotted in Figure 7.20 for electromagnetic, GCW and LCW energy scales with respect to pile-up for data and Monte Carlo. In principle, there is no preferred direction in the transverse plane for the missing energy. Therefore, a totally unbiased $\phi_{X,Y}$ plot is uniformly distributed. The sinusoidal like patterns of Figure 7.20 indicate a bias in the MET. Figure 7.20 shows that the effect is slightly more pronounced in Monte Carlo than in data. In addition, the effect is amplified with increasing number of vertices.

Recall from Chapter 5.5 the discussion on asymmetry, we proposed that if the nominal center of the calorimeter is not in fact its true center then an asymmetry could be introduced in the $\phi_{X,Y}$ distribution. The shapes of the $\phi_{X,Y}$ distributions for $>1$ vertex is suggestive of our assertion that for $n$ vertices, the individual $\phi_{X,Y}$ distributions of each vertex add in phase with one another.

From equation (5.41) we can determine the corrected forms of equations (5.1) and (5.2) with respect to the true origin of the detector as,

$$E_{X}^{\text{Miss}} = -\sum_{i=1}^{N} E_{i} \sin \theta_{i} \frac{r \cos \phi_{i} - h}{\sqrt{(r \sin \phi_{i} - k)^2 + (r \cos \phi_{i} - h)^2}},$$ \hspace{1cm} (7.7)

$$E_{Y}^{\text{Miss}} = -\sum_{i=1}^{N} E_{i} \sin \theta_{i} \frac{r \sin \phi_{i} - k}{\sqrt{(r \sin \phi_{i} - k)^2 + (r \cos \phi_{i} - h)^2}},$$ \hspace{1cm} (7.8)

where we have written $\sin \phi'$ and $\cos \phi'$, which are with respect to the azimuthal angle of the nominal center, in terms of $\phi$ the azimuthal angle of the real center of
In order to understand how this effect may introduce a bias in the $\mu_{X,Y}$ as a function of $\sum E_T$, let us consider equations (5.1), (5.2). Here we will proceed for
\[ E_{\text{Miss}}^X = \sum_{i=1}^{N_e} E_i \sin \theta_i \cos \phi_i = -\sum_{i=1}^{N_e} E_{T_i} \cos \phi_i, \]  
\tag{7.9}

where \( N_e \) is the sum over all calorimeter cells\(^7\) for one event. Let us divide the sum over the calorimeter cells in the \( \phi \) and \( \theta \) angles separately,

\[ E_{\text{Miss}}^X = -\sum_{i=1}^{N_\theta} \sum_{j=1}^{N_\phi} E_{T_{ij}} \cos \phi_{ij}, \]  
\tag{7.10}

\[ E_{\text{Miss}}^X = -\sum_{i=1}^{N_\theta} E_{T_i} \sum_{j=1}^{N_\phi} \cos \phi_j, \]  
\tag{7.11}

where we have used the argument of azimuthal symmetry as in equation (5.29) in calculating the resolution. Now we use the fact that \( \frac{1}{\Delta} \int_\Delta f(x) \, dx \approx \frac{1}{N} \sum_i f(x_i) \) to obtain,

\[ E_{\text{Miss}}^X \approx -\frac{N_\phi}{2\pi} \int_0^{2\pi} \cos \phi \, d\phi \sum_{i=1}^{N_\theta} E_{T_i}, \]  
\tag{7.12}

where the term \( \sum_{i=1}^{N_\theta} E_{T_i} \) is proportional to \( \sum E_T \). Therefore, for each event \( E_{\text{Miss}}^X = 0 \) because \( < \cos \phi > = \int_0^{2\pi} \cos \phi \, d\phi = 0 \). However, let us consider the situation where \( \phi \to \phi' \), now the average over all events is given by

\[ < E_{\text{Miss}}^X > = -\frac{1}{N_e} \sum_{j=1}^{N_e} < \cos \phi' > \sum_j E_{T_j}. \]  
\tag{7.13}

This quantity, \( < E_{\text{Miss}}^X > \), is an average taken over all \( N_e \) events in the sample. Here each term in the sum \( (E_{\text{Miss}}^X)_i \neq 0 \) because

\[ < \cos \phi' > = \frac{1}{2\pi} \int_0^{2\pi} \frac{r \cos \phi - h}{\sqrt{(r \cos \phi - h)^2 + (r \cos \phi - h)^2}} \, d\phi \neq 0. \]  
\tag{7.14}

\(^7\) This is a sum over all the calorimeter cells that are a part of the event, which is not necessarily all the cells of the calorimeter.
Imagine now sorting all the terms in (7.13) from lowest to highest in \( \sum E_T \) such that each term in the sum represents one event. The plots shown in Figures 7.16, 7.17 and 7.18 represent this very concept but divided into segments in \( \sum E_T \) (this is the bin width) \(^8\). Therefore, so long as \(< \cos \phi' >\) is a constant and uncorrelated with \( \sum E_T \), the relationship between \(< E_X^{\text{Miss}} > = \mu_X \) and \( \sum E_T \) is linear. This argument similarly applies for \( E_Y^{\text{Miss}} \). The constant of proportionality between \( \mu_{X,Y} \) and \( \sum E_T \) would then be determined by,

\[
d = \frac{1}{2\pi} \int_0^{2\pi} \frac{r \cos \phi - h}{\sqrt{(r \sin \phi - k)^2 + (r \cos \phi - h)^2}} \, d\phi,
\]

(7.15)

for a bin width of unit length. Thus, we can explain why the slope is different for \( E_X^{\text{Miss}} \) and \( E_Y^{\text{Miss}} \) as the terms \(< \cos \phi' >\) and \(< \sin \phi' >\) are different and determined by \( h \) and \( k \). We can fit equation (5.44) to the data to a reasonable degree (see Figure 7.21).

While we do not pretend that the bias can be explained solely based on such a simple proposal, we advance it as a possible way to model the problem. Indeed one can see based on Figure 7.21 that the fit is not perfect as it overestimates the crest and underestimates the trough of the "wave".

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\(^8\)To see this, imagine plotting all the \( j \) individual terms in (7.13) such that \( E_X^{\text{Miss}} \) is on the y axis and \( \sum E_T \) on the x. If one groups the x axis in discrete intervals (as a bin width does), each "strip" in y would then have an average value of \( E_X^{\text{Miss}} \). By averaging each strip in y like this, one obtains plots similar to Figures 7.16, 7.17 and 7.18.
Figure 7.21: A fit of equation (14) (red) onto the $\phi$ distribution of 1 vertex events in data at electromagnetic scale (black).
Chapter 8

Conclusions

In summary, we examined $\int L \, dt = 3.67 \text{ pb}^{-1}$ of events taken at the Large Hadron Collider in 2010. We selected events triggered by the MBTS using the L1\_MBTS\_1 trigger and applied event selection in terms of beam and run conditions, timing, jet quality and track quality. Our selection retained 33,968,840 out of 120,454,213 physics events. In addition, we analyzed $\int L \, dt = 0.082 \text{ nb}^{-1}$ of Monte Carlo data, retaining after selection 3,729,368 out of an original 3,988,672 events. We analyzed the events in the context of the number of measured primary vertices in them to study the effects of in-time pile-up in MET response at electromagnetic, GCW and LCW energy scales.

We found that the relationship between $< \sum E_T >$ and the number of vertices is approximately linear in the form of $< \sum E_T > = a + b \times \text{Number of Vertices}$. We found that Monte Carlo calculations of this relationship for the scaling factor $b$ exceeds that of data by $22.00 \pm 0.04\%$, $20.80 \pm 0.03\%$ and $19.06 \pm 0.02\%$ at electromagnetic, GCW and LCW energy scales respectively. The relationship between $< \sum E_T >$ and pile-up has importance for vertex counting in the ATLAS collaboration when vertex counting becomes inefficient due to excessive in-time pile-up.
We found that the resolution of the MET in minimum bias events obeys the relation $\sigma_{X,Y} = A \sqrt{\sum E_T}$ with a negligible contribution from the constant term. In addition, neither global calibration schemes nor additional vertices significantly changes this relationship. Therefore, we conclude that the MET resolution does not vary as a function of in-time pile-up. In comparing data with Monte Carlo, we found that they differ in $\sigma_{X,Y}^\text{Data}$ such that $\sigma_{X,Y}^\text{Data} < \sigma_{X,Y}^\text{MC}$ as a function of $\sum E_T$ but moves towards unity as $\sum E_T$ increases.

We found a bias in $\mu_{X,Y}$ in both data and Monte Carlo that may be parametrized by $\mu_{X,Y} = c_{X,Y} + d_{X,Y} \sum E_T$. In this case, we did not find any evidence of effects from pile-up beyond the first vertex. Between the first vertex and multiple vertices we did find a variation in the terms $d_X$ and $d_Y$. In terms of $E_{X}^{\text{Miss}}$ and $E_{Y}^{\text{Miss}}$, we found that $d_X > d_Y$ where $d_X < 0$ and $d_Y < 0$. The fact that $\mu_{X,Y} \neq 0$ in minimum bias events is contrary to expectations and most likely represents some sort of systematic error. In order to explain this effect, we examined the degree of asymmetry in the $\phi_{X,Y}$ distribution. We found that the asymmetrical distribution of $\phi_{X,Y}$ can be approximated by the function $\frac{dN}{d\phi'} \sim \frac{h' \cos \phi' + k' \sin \phi' + 1}{h'^2 + k'^2 + 1 + 2(h' \cos \phi' + k' \sin \phi')}$. We advanced the idea that such an asymmetry could result during reconstruction of the missing energy. If the $\phi$ orientation of the individual cell terms is not correctly sourced back to the real center of the detector, a distribution of the form $\frac{dN}{d\phi'} \sim \frac{h' \cos \phi' + k' \sin \phi' + 1}{h'^2 + k'^2 + 1 + 2(h' \cos \phi' + k' \sin \phi')}$ is the natural result. Finally, we explained how this effect could cause a linear bias of the form of $\mu_{X,Y} = c_{X,Y} + d_{X,Y} \sum E_T$ in terms of the $\sum E_T$ and $\mu_{X,Y}$. 
Appendix A

In this section, we provide some additional relevant information. We provide the numerical fit results of our analysis in section A.1, as well we provide some additional exposition on assumptions we have taken advantage of. In addition, we provide the details to a few non-trivial calculations that were used in this thesis. Finally, we provide a summary of the data used in this analysis.
A.1 Fit Parameters

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Table A.1: Summary of fit parameters for the resolution of $E_{X,Y}^{\text{Miss}}$ as functions of $\sum E_T$ in data. This information is cited in Chapter 7.3.

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Table A.2: Summary of fit parameters for the resolution of $E_{X,Y}^{\text{Miss}}$ as functions of $\sum E_T$ in Monte Carlo. This information is cited in Chapter 7.3.
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Table A.3: Summary of fit parameters for the mean of $E_{\text{X,Y}}^{\text{Miss}}$ as functions of $\sum E_T$ in data. This information is cited in Chapter 7.4.
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Table A.4: Summary of fit parameters for the mean of $E_{X,Y}^{\text{Miss}}$ as functions of $\sum E_T$ in Monte Carlo. This information is cited in Chapter 7.4.
A.2 $\sigma_X$ and $\sigma_Y$

Below, we show the ratios $\frac{\sigma_X}{\sigma_Y}$ as a function of $\sum E_T$ with respect to pile-up and different calibration energy scales. This is important, as the fact that $\sigma_X = \sigma_Y$ implies that it is possible to assume $E_T^{\text{Miss}}$ is rayleigh distributed and that it is possible to add the 2D histograms of $E_X^{\text{Miss}}$ versus $\sum E_T$ and $E_Y^{\text{Miss}}$ versus $\sum E_T$ together and form from them the quantity $\sigma_{X,Y}$, as opposed to separately creating the quantities $\sigma_X$ and $\sigma_Y$. 
Figure A.1: $\sigma_X/\sigma_Y$ as a function of $\sum E_T$ for data (right) and Monte Carlo (left) at electromagnetic (top), GCW (middle) and LCW (bottom) energy scales.
A.3 Technical Calculations

The pseudorapidity is defined as,

\[ \eta = -\ln(\tan \frac{\theta}{2}), \quad (A.1) \]

then,

\[ \frac{d\eta}{d\theta} = -\frac{1}{2\tan \frac{\theta}{2} \cos^2 \frac{\theta}{2}}, \quad (A.2) \]

now simplify trigonometrically,

\[ \frac{d\eta}{d\theta} = -\frac{1}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}, \quad (A.3) \]

\[ \frac{d\eta}{d\theta} = -\frac{1}{\sin \theta}, \quad (A.4) \]
Let us see how to calculate the following term,

$$<\sin^2 \theta> = \frac{1}{N} \sum_i^N \sin^2 \theta_i \approx \frac{\int_{-\eta_m}^{\eta_m} \sin^2 \theta d\eta}{2\eta_m}. \tag{A.5}$$

Let $\eta = -\ln(\tan \frac{\theta}{2})$ and $d\theta = -\sin \theta d\eta$ then,

$$\frac{\int_{-\eta_m}^{\eta_m} \sin^2 \theta d\eta}{2\eta_m} = \frac{1}{2\eta_m} \cos \theta \bigg|_{-\eta_m}^{\eta_m}. \tag{A.6}$$

It is easy to show that,

$$\cos \theta = \frac{2}{1 + \tan^2(\theta/2)} - 1, \tag{A.7}$$

and therefore equation (A.6) is,

$$= \frac{1}{2\eta_m} \left( \frac{2}{1 + e^{-2\eta}} - 1 \right) \bigg|_{-\eta_m}^{\eta_m} = \frac{1}{2\eta_m} \left( \frac{1}{1 + e^{-2\eta_m}} - \frac{1}{1 + e^{2\eta_m}} \right). \tag{A.8}$$

We know that,

$$e^{\pm x} = \cosh x \pm \sinh x, \tag{A.9}$$

and that,

$$\cosh^2 x - \sinh^2 x = 1. \tag{A.10}$$

The rest is just careful algebra,

$$\frac{\int_{-\eta_m}^{\eta_m} \sin^2 \theta d\eta}{2\eta_m} = \frac{1}{\eta_m} \left( \frac{2 \sinh 2\eta_m}{1 + 2 \cosh 2\eta_m + \cosh^2 2\eta_m - \sinh^2 2\eta_m} \right), \tag{A.11}$$

$$= \frac{1}{\eta_m} \left( \frac{\sinh 2\eta_m}{1 + \cosh 2\eta_m} \right) = \frac{1}{\eta_m} \left( \frac{2 \sinh \eta_m \cosh \eta_m}{1 + 2 \cosh^2 \eta_m - 1} \right) = \frac{1}{\eta_m} \left( \frac{\sinh \eta_m}{\cosh \eta_m} \right), \tag{A.12}$$

$$= \frac{\tanh \eta_m}{\eta_m} \tag{A.13}$$
Let us see how to calculate the following term,

\[
< \sin \theta > = \frac{1}{N} \sum_{i}^{N} \sin \theta_i \approx \frac{\int_{\eta_m}^{\eta_m} \sin \theta d\eta}{2\eta_m}.
\]  

(A.14)

\[
= -\frac{\theta}{2\eta_m} \bigg|_{\eta_m}^{\eta_m} = \frac{1}{\eta_m} (\arctan e^{-\eta_m} - \arctan e^{\eta_m})
\]  

(A.15)

In order to obtain the desired result, we must realize that,

\[
\tan(\theta - \pi/2) = \cot \theta,
\]  

(A.16)

\[
\tan(\theta - \pi/2) = e^{-\eta_m},
\]  

(A.17)

for some value of \( \theta \) such that \( \cot \theta = e^{-\eta_m} \). Now it is apparent that,

\[
\frac{1}{\eta_m} (\theta - (\theta - \pi/2)) = \frac{\pi}{2\eta_m}.
\]  

(A.18)
## A.4 Data

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Table A.5: Analyzed data, part 1. See part 2 for definitions.
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Table A.6: Analyzed data part 2. Minbias Stream indicates the number of events that belong to the minimum bias stream. Coll. Bunches indicates number of colliding bunches. Bunch trains indicates number of active bunch trains.
Appendix B

B.1 Additional Plots

In this section, we show the plots that are complementary to Figures 7.10 and 7.11. Figures B.1 and B.2 show $E_T^{\text{Miss}}, E_X^{\text{Miss}}, E_Y^{\text{Miss}}$ as functions of $\sum E_T$ at GCW and LCW energy scales in data. Figures B.3 and B.4 show $E_T^{\text{Miss}}, E_X^{\text{Miss}}, E_Y^{\text{Miss}}$ as functions of $\sum E_T$ at GCW and LCW energy scales in Monte Carlo.
Figure B.1: $E_T^{\text{Miss}}$ (top six), $E_X^{\text{Miss}}$ (middle six), $E_Y^{\text{Miss}}$ (bottom six) as functions of $\sum E_T$ in data at GCW energy scale. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
Figure B.2: $E_{T}^{\text{Miss}}$(top six), $E_{X}^{\text{Miss}}$(middle six), $E_{Y}^{\text{Miss}}$(bottom six) as functions of $\sum E_{T}$ in data at LCW energy scale. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
Figure B.3: These plots show \( E_{\text{T}}^{\text{Miss}} \) (top six), \( E_{X}^{\text{Miss}} \) (middle six), \( E_{Y}^{\text{Miss}} \) (bottom six) as functions of \( \sum E_{T} \) in Monte Carlo at GCW energy scale. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
Figure B.4: $E_T^{\text{Miss}}$ (top six), $E_X^{\text{Miss}}$ (middle six), $E_Y^{\text{Miss}}$ (bottom six) as functions of $\sum E_T$ in Monte Carlo at LCW energy scale. The six plots show an increasing number of vertices such that the plots show events with 1 to 6 vertices.
Bibliography


