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ON HOT SU(N) GLUON MATTER

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A B S T R A C T

The thermodynamic properties of hot gluon matter are discussed in the context of an effective interaction model, lattice gauge theory and perturbative QCD for SU(2) and SU(3). Above the deconfinement transition there is good agreement between the models. In order to have agreement, we find that the strength of the gluon-gluon interaction and the lattice parameter must be related by $K \simeq 2.9 \times 10^3 \Lambda_L^2$ for SU(3) and $K \simeq 0.53 \times 10^3 \Lambda_L^2$ for SU(2).

The study of nuclear matter at very high densities and temperatures provides us with a means for understanding the transition from the hadronic state to a deconfined quark-gluon plasma state. This transition has been considered in the early Universe (very high temperatures) ¹⁾⁻³⁾, neutron stars (very high densities) ⁴⁾⁻⁵⁾ and high energy heavy ion collisions (possibly both) ^{6),7)}. Recent advances in Monte Carlo calculations on the lattice show explicitly, within a single framework, the existence of a phase transition ^{8),9)}. While other methods have been employed to describe the properties of hot dense matter, they suffer in not being able to explicitly show a phase transition. Perturbative QCD, for example, cannot reliably be extended into the strong coupling region where confinement occurs and the effective interaction model ²⁾ shows a transition only by construction. It is useful, therefore, and the aim of this letter is to compare the results of each of these models and to discuss the relationship between them.

Before making this comparison we will briefly describe each of these models for an $SU(N)$ Yang-Mills gas. Let us begin therefore with the effective interaction model.

As the effective gluon-gluon interaction, we take

$$U_g = a_N K n_g^{-1/3} \quad (1)$$

where n_g is the number density of gluons, $a_N = (2N^2)/N^2 - 1$, and K is a somewhat arbitrary constant. If one takes the average interparticle separation as $\langle r \rangle \sim n_g^{-1/3}$ the potential (1) appears linear in $\langle r \rangle$ and for heavy quarks it is tempting to associate K with the string tension $1/2\pi\alpha' \simeq 0.18 \text{ GeV}^2$ (α' is the slope of the Regge trajectories). Colour screening factors have been suppressed in Eq. (1). K must really be viewed as a product of screening and the string tension where the effect of screening may allow values of K as low as 0.01 GeV^2 . In addition, one expects that screening will be temperature dependent $\sim 1/T$ and this dependence is realized in the association $n_g^{-1/3} \sim 1/T$.

Given the form of the potential (1) one can then calculate the energy density and pressure of the gluon gas. The number density of gluons is given by

$$n_g = \frac{d_g}{2\pi^2} \int_0^\infty \frac{k^2 dk}{e^{\frac{k \cdot U}{T}} - 1} \quad (2)$$

where $d_g = 2(N^2 - 1)$. Since U_g is a function of n_g , Eq. (2) must be solved self-consistently. The pressure and energy density are then given by ²⁾

$$p = \frac{d_g}{6\pi^2} \int_0^\infty \frac{k^4 dk}{(k + U_g)(e^{\frac{k+U_g}{T}} - 1)} - f(n_g) \quad (3.a)$$

$$\mathcal{E} = \frac{d_g}{2\pi^2} \int_0^\infty \frac{(k + U_g) k^2 dk}{e^{\frac{k+U_g}{T}} - 1} + f(n_g) \quad (3.b)$$

where $f(n_g) = \frac{1}{2} a_N K n_g^{2/3}$. In the Figure we show p and $\delta \equiv \epsilon/3 - p$ as a function of $T/K^{1/2}$ (curves a and b) for SU(3). The transition temperature is found to be ^{*)}

$$T_c \approx 1.48 K^{1/2} \quad (4.a)$$

For SU(2) a similar relationship is found

$$T_c \approx 1.87 K^{1/2} \quad (4.b)$$

For the remainder of the discussion we shall assume that the thermodynamical quantities such as the thermodynamical potential $\Omega(T) = -p$ for SU(N) gluon matter at high temperature is well represented by Monte Carlo calculations ⁹⁾, i.e., that $\Omega(T) \approx \Omega_{MC}(T)$. Such calculations have been performed in the absence of quarks for $N=2$ ⁸⁾ and $N=3$ ⁹⁾ and indicate a deconfinement transition temperature $T_c \approx 43 \Lambda_L$ for SU(2) and $T_c \approx 80 \Lambda_L$ for SU(3). The Monte Carlo data clearly show that the energy density approaches the Stefan-Boltzmann limit rather rapidly. Roughly speaking, one can say that the limit is reached to within 10% around $T \sim 2T_c$. One can see in the Monte Carlo data that the pressure curve exhibits more structure than the curve for the energy density. In the Figure we plot Monte Carlo results for p and δ (curves c and d).

*) The structure of the solutions for $n_g(T)$ in the effective interaction model the following: above T_0^g there are three solutions, the physical solution, a local maximum and a zero pressure solution ($n_g \approx 0$); below T_0^g the only solution is the zero pressure solution, ¹⁰⁾. In general $T_c \gtrsim T_0^g$.

In order to get some feeling for the transition temperature and energy density near the transition, one must know Λ_L or Λ_{mom} . These cannot be obtained in a unique manner, but a reasonable estimate can be obtained by considering e^+e^- experimental data ¹¹⁾ on the one hand, and fits to hadronic masses ¹²⁾ on the other. The ITEP group study ^{11),13)} indicates that $\Lambda_{\text{mom}} \sim 250$ MeV and hadronic mass fits give a similar value. One can relate Λ_L and Λ_{mom} by ¹⁴⁾

$$\Lambda_{\text{mom}} \approx 83.5 \Lambda_L, \text{ for } SU(3) \quad (5.a)$$

and

$$\Lambda_{\text{mom}} \approx 57.5 \Lambda_L, \text{ for } SU(2) \quad (5.b)$$

This would indicate that the deconfinement transition temperature [for $SU(3)$] in the absence of fermions is $T_c \simeq \Lambda_{\text{mom}} \sim 250$ MeV ⁹⁾. Needless to say there are no experimental data relevant for Λ_{mom} in $SU(2)$.

In perturbation theory the thermodynamical potential $\Omega(T)$ has been calculated only up to $O(g^3)$ ^{1),15)}. The plasmon term of order $g^3 T^4$ overestimates the pressure greatly and the higher order plasmon contribution should therefore be computed. A fit to $SU(2)$ and $SU(3)$ Monte Carlo data indicates that the total $O(g^4 T^4)$ contribution to $\Omega_{\text{pert}}(T)$ is large and positive ¹⁶⁾ and therefore corrects the "naïve" plasmon term.

Of the non-perturbative corrections only the effect of dilute instantons has been computed ^{16),17)}. The instantons are greatly suppressed by thermal fluctuations and it is found ¹⁷⁾ that the instanton contribution in the dilute gas approximation (DGA) is

$$\Omega_{\text{inst}}^{\text{DGA}} \simeq i(T) \Lambda_{\text{mom}}^4 \quad (6)$$

where $i(T_c) \simeq -0.03$ and the magnitude of $i(T)$ decreases rapidly with increasing temperature.

It is quite clear that neither the perturbative model nor the effective interaction model can probe the vacuum structure of the $SU(N)$ gauge theory. The work of the ITEP group ^{11),13)} and also Monte Carlo calculations ¹⁸⁾ indicate that

$$\Omega_{vac}^{SU(3)} \simeq - (250 \text{ MeV})^4 \quad (7)$$

The variation of this quantity with temperature is not known. Calculations of the thermodynamic properties of the Savvidy vacuum ansatz ¹⁹⁾ indicate, however, a temperature dependence ²⁰⁾, but the results are not conclusive.

Other non-perturbative effects, plausibly related to the vacuum structure and to confinement effects must likewise be suppressed by thermal fluctuations and it is therefore likely that

$$|\Omega_{non-pert}(T > T_c)| \ll T_c^4 \quad (8)$$

A fit of $\Omega_{pert}^{(n \leq 4)}$ (n denotes order in g) to Monte Carlo data ¹⁶⁾ indicates that perturbation theory gives a reasonable approximation for $T \gtrsim 1.3 T_c \sim \sim 300 \text{ MeV}$. Our expression for $p_{pert}(T)$ is ¹⁶⁾

$$P(T) \simeq \frac{(N^2-1)\pi^2 T^4}{45} \left\{ 1 - \frac{5}{4} \frac{N_g^2}{4\pi^2} + \frac{10}{13} \left(\frac{N_g^2}{4\pi^2} \right)^{3/2} - \frac{5}{4} a_4 \left(\frac{N_g^2}{4\pi^2} \right)^2 \right\} \quad (9.a)$$

with

$$\frac{N_g^2}{4\pi^2} \simeq \frac{6}{11 \ln 4T/\Lambda_{mom}} - \frac{306}{1331} \cdot \frac{\ln \ln 4T/\Lambda_{mom}}{(\ln 4T/\Lambda_{mom})^2}, \quad a_4 \simeq 7.3 \quad (9.b)$$

The pressure is plotted as a function of $83.5 T/\Lambda_{mom}$ for $\Lambda_{mom} = 250 \text{ MeV}$ in the Figure (curve e).

Regarding these perturbative results, a cautious remark is in order. The magnitude of the $O(g^4 T^4)$ term cannot be trusted until at least the $O(g^4 T^4)$ correction term in the plasmon has been computed ²¹⁾.

Having briefly discussed each of the three models describing hot gluon matter, we can compare the results of the effective interaction model, the lattice model and perturbative results. The perturbative model, of course, cannot say anything for temperatures $T \simeq T_c$. As we have seen, T_c is given in terms of $K^{\frac{1}{2}}$ or Λ_L for the effective interaction and lattice models, respectively. One can then relate these two and find

$$K \simeq 2.9 \times 10^3 \Lambda_L^2, \quad N=3 \quad (10.a)$$

$$K \simeq 0.53 \times 10^3 \Lambda_L^2, \quad N=2 \quad (10.b)$$

In the Figure we have plotted both $p(T)$ and $\delta(T)$ for each of the three models. The Monte Carlo curves are taken from Ref. 9) and the effective interaction model results are derived using the method of Ref. 2) for gluons only. As one can see, there is reasonable agreement between the models. For temperatures $T \geq 300 \Lambda_L$ all three models give essentially identical results. For lower temperatures the effective interaction model shows a larger departure from the Stefan-Boltzmann behaviour (compare curves b and d). Furthermore for $T \lesssim 120 \Lambda_L$ $p(T)$ in the lattice gauge model shows more structure and drops more sharply.

We can speculate as to these different behaviours. To fully account for the effective gluon-gluon interaction a Coulomb-like term should be added to the potential of Eq. (1),

$$U_c \sim \frac{-\alpha_s}{\langle r \rangle} \sim -\alpha_s n_g^{1/3} \quad (11)$$

For high temperatures, asymptotic freedom tells us that the coupling α_s becomes very small and the effect of this term (as well as the one included) is negligible. For very low temperatures n_g becomes small and this term is negligible compared with the one included. For intermediate temperatures the Coulomb term might be comparable to the one included. Since it appears with a relative minus sign its effect would be to diminish the magnitude of the potential and hence being both p and ϵ up towards the free gas limit, thus reducing δ as well. This idea has not yet been tested, but gives an intuitive feeling for the difference in the results.

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FIGURE CAPTION

The pressure p/T^4 as a function of T/Λ_L (or equivalently) $T/18.5 K^{\frac{1}{2}}$ for the effective interaction model (a), lattice gauge model (c) and perturbative QCD (e). The quantity $(\delta/T^4) = (\epsilon/3-p)/T^4$ for the effective interaction and lattice gauge models (curves b and d) respectively. For perturbative QCD δ has been fit to curve (d) and thus is omitted here. Note that the kinks in curves (a) and (b) are only due to the change in scale at $T=200 \Lambda_L$.