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INFLATION CAN SAVE THE GRAVITINO

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A B S T R A C T

Conventional Big Bang nucleosynthesis and baryosynthesis impose severe constraints on the gravitino mass and on the scale of supersymmetry breaking, which are difficult to reconcile with particle physics limits. We show that this "gravitino problem" need not exist if the Universe has experienced even a brief inflationary epoch since cooling below the Planck temperature.

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One of the greatest uncertainties in present applications of supersymmetry ^{1),2)} is the scale d of supersymmetry breaking. Models with $\sqrt{d}$ from $m_W = O(100)$ GeV to the Planck mass $m_P \approx 10^{19}$ GeV have been proposed ³⁾⁻⁸⁾. A related ⁹⁾ puzzle is that of the gravitino mass m_G :

$$m_G \approx \frac{d}{m_P} \quad (1)$$

in models with spontaneous supersymmetry breaking. While particle physics experiments ^{3),10)} and stellar evolution ¹¹⁾ impose constraints on the scale of supersymmetry breaking, some of the most stringent restrictions come from conventional cosmology ^{10),12),13)}. The present mass density of the Universe restricts ¹²⁾ a gravitino with lifetime longer than the age of the Universe to have $m_G < O(1)$ keV (corresponding to $\sqrt{d} < 2 \times 10^6$ GeV) - if the gravitino has a normal cosmological abundance. More massive gravitinos are forbidden ^{10),13)} unless $m_G > O(10^4)$ GeV [corresponding to $\sqrt{d} > O(10^{11})$ GeV] in which case they decay rapidly enough to avoid disturbing conventional Big Bang nucleosynthesis calculations. Indeed, one can derive the even more stringent limit $m_G > O(10^{13})$ GeV [$\sqrt{d} > O(10^{16})$ GeV] from the requirement that gravitino decays do not increase the entropy of the Universe by a factor greater than $O(10^3)$ subsequent to Big Bang baryosynthesis [see also Ref. 13)]. These cosmological limits on m_G and d are not in themselves disastrous, but unless m_G is very small they seem to conflict with a recent ¹⁴⁾ upper bound on m_G deduced from particle physics considerations. In a class of theories with local $N = 1$ supersymmetry spontaneously broken at the tree level it seems ¹⁴⁾ that the gravitino mass should be lighter than all scalar boson masses :

$$m_G < m_H, m_{\tilde{t}}, m_{\tilde{q}}, \dots = O(100) \text{ GeV} \quad (2)$$

corresponding to $\sqrt{d} < O(10^{10})$ GeV, where we use the notation $\tilde{X}$ for the supersymmetric partner of any particle X . Thus supersymmetric theories seem to suffer from a "gravitino problem" analogous to the more familiar "monopole problem" ¹⁵⁾ of grand unified theories (GUTs). This "monopole problem" may be solved along with many others related to the large-scale isotropy, homogeneity and flatness of the Universe if it underwent inflationary de Sitter expansion early in the Big Bang ¹⁶⁾⁻¹⁸⁾.

In this paper we show that the inflationary Universe scenario can also solve the "gravitino problem" and permit any scale of supersymmetry breaking consistent with the particle physics constraints ^{3),10)}. In conventional

cosmology the gravitinos are produced at the Planck epoch when the temperature $T = O(m_P)$. Most of the constraints on m_G arise because the gravitino decay rate is much less than the expansion rate of the Universe until its temperature has fallen way below m_G . By the same token, if the initial density of gravitinos can be substantially suppressed, for example by an epoch of inflationary expansion subsequent to the Planck epoch, then their production rate may also be much less than the expansion rate. Therefore the gravitino density may not be completely re-created once the primordial distribution has been drowned in the entropy generated during inflation ^{*)}. The Figure shows how large an abundance of gravitinos is cosmologically acceptable, as a function of m_G , and shows how many may be produced by some typical mechanisms after inflation. We will see that solving the "gravitino problem" does not require as much inflation as solving the other cosmological problems mentioned earlier (see the Figure). However, the successful achievement of Big Bang baryosynthesis ¹⁹⁾ imposes significant constraints on inflationary scenarios in supersymmetric theories. Since the vacuum energy may vanish when $d \rightarrow 0$, one would prefer a large value of d so as to have sufficient vacuum energy to create the superheavy particles S of mass $m_S \geq 10^{10}$ GeV ²⁰⁾, whose decays are presumed to be responsible for producing the baryon asymmetry. The potential should be quite flat, so as to avoid producing density perturbations which are too large. However, the effective potential $V(\phi)$ should not be too flat, for if $\dot{m}_S/m_S^2 \ll 1$ when the Universe reheats very few of the vital S particles will be created ²¹⁾ as the Universe reheats at the end of the exponential expansion.

We start by discussing the relevant couplings and interaction rates of the gravitino ^{3),10),13)}. It is necessary to consider separately the helicity $\pm \frac{3}{2}$ states of the gravitino which are the $N = 1$ supersymmetric partners of the graviton, and the helicity $\pm \frac{1}{2}$ states which interact like the Goldstino, the spin $\frac{1}{2}$ Goldstone fermion of spontaneously broken supersymmetry which has been eaten to give the gravitino a mass through the superHiggs effect ⁹⁾. The helicity $\pm \frac{3}{2}$ states $G_{\frac{3}{2}}$ have couplings $O(p/m_P)$ to particles X and their supersymmetric partners $\tilde{X}$, where p is a typical momentum in the reaction. The helicity $\pm \frac{1}{2}$ states $G_{\frac{1}{2}}$ have couplings $\Delta m_X^2/d$ where $\Delta m_X^2 \equiv |m_X^2 - m_{\tilde{X}}^2|$ is the supersymmetry breaking difference in $(\text{mass})^2$ in the X supermultiplet, which may be ^{5),6)} much less than d . The ratio of rates of production or decay for the different gravitino helicity states is

$$\frac{\Gamma_{\frac{1}{2}}}{\Gamma_{\frac{3}{2}}} = O\left(\frac{(\Delta m_X^2)^2 m_P^4}{d^4}\right) \quad (3)$$

*) This possibility was also raised by Dimopoulos and Raby ⁸⁾.

In order to "solve" the hierarchy problem in supersymmetric GUTs we require ⁴⁾⁻⁶⁾ small supersymmetry breaking mass splittings within supermultiplets much lighter than the grand unification scale of order 10^{16} GeV :

$$\Delta m_X^2 = O(m_W^2) \quad (4)$$

if $m_X \ll 10^{16}$ GeV. Substituting this estimate into the ratio (3) we find that grosso modo

$$\frac{\Gamma_{1/2}}{\Gamma_{3/2}} \gtrless O(1) \text{ for } \begin{cases} d < O(m_W m_P) \\ d > O(m_W m_P) \end{cases} \quad (5)$$

The particle physics constraint (2) favours the upper branch of Eq. (5), but we will not insist on this point. It is not possible to talk meaningfully of the gravitino helicity when it is non-relativistic. The gravitino will decay as fast as it can, and we follow Weinberg ¹³⁾ in estimating

$$\Gamma_G \approx m_G^3 / m_P^2 \quad (6)$$

Apart from phase space factors, this is the order of magnitude of the decay rate for a gravitino into a lighter photino $\tilde{\gamma}$ plus a pair $X\tilde{X}$ of light conventional particles (e.g., e^+e^-). The gravitino could only decay into $X\tilde{X}$ if $m_G > m_X + m_{\tilde{X}} > \Delta m_X = O(m_W)$, and we see from Eq. (1) that $m_G > m_W$ implies $d > m_W m_P$, in which case Eq. (5) tells us we should estimate the gravitino decay rate using the helicity $\frac{3}{2}$ coupling, which again gives us the estimate (6). According to Eq. (6), a gravitino with mass less than 100 MeV lives longer than the present age of the Universe. Heavier gravitinos may also live this long if there is no lighter supersymmetric particle into which they can decay.

Let us now assume that there has been an inflationary de Sitter expansion ¹⁶⁾⁻¹⁸⁾ subsequent to the Planck epoch during which the scale factor of the Universe has increased by a factor L , resulting in a dilution of the gravitino-to-entropy ratio by a factor $O(1/L^3)$. We will see shortly how large L must be in order to reconcile different gravitino masses with cosmology as shown in the Figure. As mentioned earlier, one might expect that, once primordial gravitinos are suppressed in this way gravitinos would not be copiously re-created, since in conventional cosmology the decay rate (6) is much slower than the expansion rate

$$H \approx \sqrt{\rho / m_P^2} \quad (7)$$

This point is not quite trivial for the following reasons. First, a radiation-dominated Universe with energy density $\rho \sim N_T T^4$, where N_T is the number of light degrees of freedom at the temperature T , expands more slowly than a gravitino-dominated Universe with $\rho \approx m_G T^3$. This in principle affords more opportunity for gravitino production interactions to act. However, it is reasonably clear that most gravitino production must take place at temperatures $T \geq m_G$ before the Universe becomes gravitino dominated. A second remark is that even a small production of gravitinos is potentially disastrous for the subsequent history of the Universe. Big Bang baryosynthesis, nucleosynthesis and the present mass density of the Universe impose the severe upper limits seen in the Figure on Y_G , defined to be the abundance of gravitinos relative to particles of the same mass with a thermal equilibrium distribution ^{*)}, as we shall now see.

If $m_G > O(10^4 \text{ GeV})$ the gravitino decays before nucleosynthesis occurs ^{3),10)} and we need only worry about the baryosynthesis constraint. When gravitinos of relative abundance Y_G decay they produce an increase in entropy density by a factor

$$\frac{S_f}{S_i} = O\left(Y_G^{3/4} \left(\frac{m_P}{m_G}\right)^{1/2}\right) \quad (8)$$

In order not to dilute the standard baryons generated by GUTs, we demand that $S_f/S_i < O(10^{+3})$. This constraint is satisfied for all $m_G > O(10^4) \text{ GeV}$ if $Y_G < O(10^{-6})$: the Figure shows the constraint for all higher values of m_G . If $O(100) \text{ MeV} < m_G < O(10^4) \text{ GeV}$ the gravitino would have decayed between Big Bang nucleosynthesis and the present day ^{**)}. In this case we should demand that the factor (8) of increase in the entropy not be larger than $O(1)$, so that

$$Y_G < O\left(\frac{m_G}{m_P}\right)^{2/3} \quad (9)$$

If $Y_G < O(10^{-14})$ this condition will also be satisfied for all $m_G > O(100) \text{ MeV}$. For m_G less than $O(10^4) \text{ GeV}$ there is also the constraint that the gravitino mass density does not affect the rate of expansion of the Universe during Big Bang nucleosynthesis. This implies that

^{*)} Note that the primordial gravitinos which survive the inflationary phase contribute $\Delta Y_G = O(1/L^3)$.

^{**)} To be on the safe side, we disregard the possibility that the gravitinos decay after nucleosynthesis, but reheat the Universe so that it is undone and then repeated.

$$Y_G < \frac{0.1 \text{ MeV}}{m_G} \quad (10)$$

which is satisfied for all m_G in the relevant range if $Y_G < O(10^{-8})$. Finally we consider the case $m_G < O(100) \text{ MeV}$ where the gravitinos are sufficiently stable to persist until the present day. Heavy gravitinos with a number density suppressed by a factor Y_G will contribute less mass density than gravitinos of normal abundance with mass 1 keV if

$$Y_G < \frac{1 \text{ keV}}{m_G} \quad (11)$$

This condition is satisfied for all the relevant range of m_G if $Y_G < O(10^{-5})$. If the gravitino is the lightest supersymmetric particle, it will be stable and one may need to apply the condition (11) also to heavier gravitinos, but this also presents no problem. The Figure shows a compilation of all the limits on Y_G as a function of m_{G_1} . They can be translated directly into limits on the inflation factor $L \geq Y_G^{-\frac{1}{3}}$. We will now study whether production of gravitinos after inflation may challenge the limits in the Figure.

We start our analysis of gravitino production by studying $2 \leftrightarrow 2$ interactions, and first we look at the annihilation reaction $X\tilde{X} \rightarrow GG$. The cross-section for producing helicity $\pm \frac{3}{2}$ states is a function only of the Planck mass : $\sigma \sim T^2/m_P^4$ for $T \geq m_G$ and hence the production rate $\sum_X n_X \langle \sigma v \rangle \approx N_X T^5/m_P^4$, which is always less than the expansion rate of the Universe $H \sim \frac{1}{2} T^2/m_P$ for all temperatures $T < O(m_P)$. It will produce a relative gravitino abundance

$$Y_G = O(N_X^{\frac{1}{2}} T^3/m_P^3) \quad (12)$$

which is less than $O(10^{-14})$ if the Universe does not reheat above 10^{14} GeV after inflation. The rate for producing light helicity $\pm \frac{1}{2}$ states peaks at $T \sim \Delta m_X \sim m_W$, since the cross-section rises like T^2 at small T and falls as $1/T^2$ for T much larger than $m_{\tilde{X}} \approx \Delta m_X \approx m_W$. The ratio between the production and expansion rates at $T = O(m_W)$ is $O(m_W^2 m_P/d^4)$. This gives Y_G less than $O(10^{-15})$ for $\sqrt{d} > O(10^6) \text{ GeV}$ corresponding to gravitinos weighing more than 1 keV, which are the only ones which could worry us.

We now consider the inelastic $2 \rightarrow 2$ reaction $X + \tilde{\gamma} \rightarrow X + G$. This has a cross-section ^{3),10)}

$$\sigma \sim \begin{cases} \alpha \frac{T^2}{d^2} & \text{for } T \ll m_w \\ \alpha \left(\frac{m_w}{\sqrt{d}}\right)^4 \frac{1}{T^2} & \text{for } T \gg m_w \end{cases} \quad (13)$$

for helicity $\pm \frac{1}{2}$ states which are dominant (5) for $d < m_w m_p$. The production rate Γ for this reaction will also reach a maximum at $T \approx m_w$, at which temperature $\Gamma/H \approx m_p m_w^3/d^2$. This ratio gives a corresponding Y_G which is acceptably small for all m_G , as seen in the Figure.

We now turn to the $1 \leftrightarrow 2$ interactions, starting with the annihilation reaction $X + \tilde{X} \rightarrow G$. We take the rate for this reaction from Eq. (6) multiplied by a Lorentz boost factor $m_G/\sqrt{m_G^2 + T^2}$ and a Boltzmann suppression factor $e^{-m_G/T}$. The rate Γ will clearly peak at $T \sim m_G$ at which temperature $\Gamma/H \approx m_G/m_p$, which is again much less than unity for $m_G \ll m_p$ and gives Y_G acceptably small for all m_G as seen in the Figure. Finally we discuss the production of gravitinos in the decays of other particles $\tilde{X} \rightarrow X + G$. This rate will peak when $T \approx m_{\tilde{X}}$ and yield a number density of gravitinos suppressed relative to that of conventional particles by a factor $\Gamma(\tilde{X} \rightarrow X + G) \times \tau$ where τ is the mean free time between interactions of the $\tilde{X}$ particle. Assuming a total interaction cross-section $\sigma = O(1/m_{\tilde{X}}^2)$ we estimate $\tau = (n\sigma)^{-1} = O(m_{\tilde{X}}^{-1})$. Using the estimates

$$\Gamma(\tilde{X} \rightarrow X + G) = \begin{cases} \Delta m_X \times \left(\frac{m_{\tilde{X}}}{m_p}\right)^2 & \text{for helicity } \pm \frac{3}{2} \\ \Delta m_X \times \left(\frac{\Delta m_X^2}{d}\right)^2 & \text{for helicity } \pm \frac{1}{2} \end{cases} \quad (14)$$

we find that the gravitino number density should be suppressed by a factor

$$Y_G \approx \left[O\left(\frac{m_{\tilde{X}}}{m_p}\right)^2 + O\left(\frac{\Delta m_X^2}{d}\right)^2 \right] \times \left(\frac{\Delta m_X}{m_X}\right) \quad (15)$$

The first term in the expression (15) is clearly negligible for particles weighing $O(m_w)$, but could be significant for particles produced at the time of reheating, such as partners $\tilde{S}$ of those responsible for baryosynthesis (19), (20). These can therefore give

$$Y_G \approx \left(\frac{m_S}{m_p}\right)^2 \times \left(\frac{\Delta m_S}{m_S}\right) \quad (16a)$$

which is rather larger than the annihilation contribution (12) if $\Delta m_S/m_S = O(1)$. However, we expect $\Delta m_S/m_S \ll 1$ [perhaps $\Delta m_S^2 = O(m_W^2) \rightarrow \Delta m_S/m_S = O(m_W^2/m_S^2)$?] so that the source (16a) is unlikely to violate the constraints shown in the

Figure. The second contribution in Eq. (15) is already negligible for $\sqrt{d} > 0(10^6)$ GeV even if $\Delta m_S/m_S = 0(1)$:

$$\chi_g < 0(10^{-16}) \quad (16b)$$

We therefore conclude this analysis of gravitino production subsequent to inflation confident that the constraints shown in the Figure would be respected.

We finish with some general comments on inflation in supersymmetric theories. We note that it is not necessary to have inflation of the Universe during the supersymmetry breaking phase transition at the supersymmetry breaking mass scale $\sqrt{d} \lesssim 10^{10}$ GeV (?). To solve the gravitino problem it would be sufficient to have an inflationary phase transition at any time after the Planck time but before baryosynthesis. The simplest possibility to implement the inflationary Universe scenario is in GUTs with the Coleman-Weinberg mechanism of symmetry breaking ^{17),18),22),23)}. In the context of such theories it is possible to solve the horizon, flatness and monopole problems ^{17),18)} and to obtain a baryon asymmetry which may be one or two orders of magnitude greater than that in the standard scenario ²¹⁾. However, in the simplest versions of these models [e.g., in the SU(5) model] it is difficult to obtain the desired amount of inflation ^{22),23)}. Moreover, the density perturbations which arise after the phase transition in these theories are probably too large, although they have a desirable spectrum for subsequent galaxy formation ^{24),22),25)}. Recently another version of the inflationary Universe scenario was suggested ²⁶⁾, based on the Dimopoulos-Raby supersymmetric model ⁸⁾. In this model it is possible to obtain very large inflation. However, the growth of the supersymmetry breaking scalar field ϕ in this model proceeds extremely slowly. Therefore, it does not seem possible to satisfy the criterion $\dot{m}_S/m_S^2 \gtrsim 1$ ²¹⁾ for producing the superheavy Higgs bosons and fermions in the growing scalar field. As a consequence, it seems impossible to obtain any non-vanishing baryon asymmetry in the context of the Dimopoulos-Raby ⁸⁾ model. For analogous reasons it seems impossible to have any density perturbations after the phase transition in this model. Therefore, we do not know at present of examples of an ultimate theory which would solve all our problems once and for all. Still, one may be rather optimistic about the idea of the inflationary Universe. Indeed, in the Coleman-Weinberg-like models one has a large baryon asymmetry but it is difficult to have large inflation and small density perturbations. On the other hand, in the Dimopoulos-Raby model, one has large inflation but the baryon asymmetry and the density perturbations are presumably too small. So why not try to find something intermediate ?

It is amusing that from the investigation of the phase transition in the Coleman-Weinberg model it was concluded that the inflationary Universe scenario needs something like broken supersymmetry ²³⁾. Now from the results of the present investigation, it seems that supersymmetry also needs inflation. Let us see whether a happy symbiosis can be achieved.

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FIGURE CAPTION

Cosmological constraints on the abundance Y_G of gravitinos relative to the normal cosmological abundance. An inflation factor L would give a primordial gravitino abundance $\Delta Y_G \approx O(1/L^3)$. Also shown in the Figure are some typical gravitino production mechanisms which respect the cosmological constraints.