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HIERARCHICAL SUPPRESSION OF RADIATIVE QUARK AND LEPTON MASSES
IN SUPERSYMMETRIC GUTS

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ABSTRACT

We show that radiatively generated quark and lepton masses in supersymmetric GUTs are hierarchically suppressed with respect to the corresponding scalar partner masses. As a consequence, in order to account for the observed fermion mass spectrum, one necessarily has to go beyond the minimal models and add new Higgs supermultiplets [45's in SU(5), 126's in SO(10)]. The minimal SO(10) model (10+16+45 Higgses) is not viable since the radiative mass of the right-handed electron-neutrino is at most $m\nu_R \sim m_e (\alpha/\pi)^2$. We also comment on the possible existence of "interlopers" in supersymmetric GUTs.

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Supersymmetric grand unified theories (SUSY-GUTs)¹⁾⁻⁴⁾ show all the attractive features of the usual grand unified models and at the same time provide for a solution to the gauge hierarchy problem. Supersymmetry protects the Weinberg-Salam (WS) doublets from acquiring a large mass and makes the construction of a large hierarchy of symmetry breaking technically possible. At the beginning it was thought^{1),2)} that it was necessary to keep the supersymmetry unbroken down to energies $O(\alpha^{-1}M_W)$ in order to prevent the WS doublets from gaining masses $\gg M_W$. However, it has recently been pointed out^{3),4)} that global supersymmetry may be broken at a very large scale. This may be the case³⁾ if supersymmetry is broken in a gauge singlet sector in such a way that susy-breaking is transmitted to the usual quark-lepton world through several-loop diagrams involving small chiral couplings³⁾. Another large susy-breaking scenario⁴⁾ may be obtained if one breaks supersymmetry at an intermediate scale $\mu^2 \sim M_W M$, where M is the grand unification mass. If the usual quarks and leptons are connected to the susy-breaking sector (which is superheavy) only through gauge interactions, the scalars get masses $\mu^2/M \sim M_W$ as desired. In both schemes, at low energies one is left with an $SU(3) \times SU(2) \times U(1)$ theory with effective soft susy-breaking terms like, for example, s quark and s lepton masses.

The effective terms that one can have at low energies can be very conveniently analyzed in terms of local supersymmetric operators, as was recently emphasized by Susskind and Polchinsky⁵⁾. Only D components of local products of chiral superfields can be generated in perturbation theory. After substituting the appropriate superfield by the F component which is breaking supersymmetry, one is left with the relevant effective term at low energies. Thus, for example, if susy is broken by the F component of a superfield C , the scalar fields of the matter superfields L will, in general, gain radiative masses from an effective operator⁶⁾

$$\frac{g^n}{M^2} (L^\dagger L C C^\dagger)_D \sim g^n \frac{\mu^4}{M^2} (L^\dagger L) \quad (1)$$

where $\mu^2 = F_C$ is the susy-breaking scale and M is the grand unification mass. g^n stands for the coupling constants which, in general, will be present in a given diagram. In Eq. (1) one could also add extra chiral superfields with large vacuum expectation values (v.e.v.) $\sim M$ since that would not change the estimate. In Fig. 1 we show a typical diagram giving rise to scalar masses. Notice that by setting $\mu^2 \sim M_W M$ one gets masses $\sim M_W$, which is the desired order of magnitude for the Higgs mass.

What is the situation regarding quark and lepton masses? In usual GUTs they receive masses from tree-level Yukawa couplings to the WS doublets. Then different Yukawa couplings have to be chosen for each fermion, for example, $G_e \sim 10^{-5}$, $G_b \sim 10^{-1}$. Moreover, it is well known that the $SU(5)$ relation $m(Q=1/3) = m(Q=1)$ does not hold for the lightest generation. The disparity in the Yukawa coupling constants could in fact be understood if the Yukawas for the first two generations (at least) had their origin in higher order graphs involving superheavy particles. This could also explain the failure of the relation $m_e = m_d$ since the radiative terms would be relatively more important for the lightest generations. This scenario for the understanding of fermion masses has recently been studied in some detail by several authors⁶⁾. What we want to point out in this letter is that this radiative mechanism for the understanding of the fermion mass spectrum *does not work in the case of supersymmetric GUTs*. Radiative fermion masses are hierarchically suppressed in susy GUTs. This has other phenomenological implications as we will show later on.

It is easy to understand why radiative quark and lepton masses are hierarchically suppressed in susy GUTs. In Fig. 2 we show possible diagrams giving rise to radiative fermion masses. Let us denote by L and L^C the superfields containing the left-handed fermion and the left-handed antifermion (right-handed fermion), respectively; ϕ will, in general, denote fields with large scalar v.e.v. $\langle \phi \rangle \sim M$, H_W the WS doublet superfield and finally C will be the susy-breaking superfield. Figure 2a would induce an effective operator

$$\frac{g^n}{M^4} (L L^C \phi^+ \phi H_W^+ C^+)_D \sim \frac{g^n M^2}{M^2} (L L^C H_W^+)_F \sim \frac{g^n M^2}{M^2} M_W (\Psi_L \Psi_R) \quad (2a)$$

In the case of Fig. 2b

$$\frac{g^n}{M^5} (L L^C H_W \phi^+ \phi \phi C^+)_D \sim \frac{g^n M^2}{M^2} (L L^C H_W)_F \sim \frac{g^n M^2}{M^2} M_W (\Psi_L \Psi_R) \quad (2b)$$

One can think of many possible diagrams with any number of loops. The important point is that, no matter how complicated the graph may be, it has to have only one susy-breaking $F_C = \mu^2$ insertion, since each fermion already carries one θ with it. Furthermore, it has to have an M_W $SU(2) \times U(1)$ breaking mass insertion since quark and lepton masses have $\Delta I_W = 1/2$. Hence, on dimensional grounds, radiative quark and lepton masses are always of order:

$$m_f \sim g^n \frac{\mu^2}{M^2} M_W \sim g^n \frac{M_W^2}{M} \quad (3)$$

i.e., comparing with Eq. (1) we see that $m_f \sim M_W/M m_\phi$ and radiative fermion masses are hierarchically suppressed. Notice that if one sets $\mu^2/M^2 \sim 1$ in (3) in order to get sizeable fermion masses, then one spoils the relation (1) which keeps the WS doublets light. Notice also that this is true in both the two large susy-breaking schemes mentioned above^{3),4)}. In the EIR type of models³⁾ one simply has $\mu \sim \lambda^m M$, where λ are chiral coupling constants, but still $m_f \sim M_W/M m_\phi$ and hence, if one solves the hierarchy problem (small scalar masses m_ϕ), then the radiative fermion masses are negligible. One could try to build models in which to obtain scalar masses one had to make insertions both of F_C and M_W , getting masses of order $\sim g^n \mu^4/M^4 M_W^2$, in such a way that one could safely choose $\mu^2 = M^2$. However, one can always choose the WS doublet lines and get $\sim g^n \mu^4/M^2$ masses, spoiling the hierarchy.

The smallness of radiative fermion masses then seems unavoidable. Although supersymmetry implies a concept of chirality both for fermions and scalars, the fermions have extra chiral properties due to the fact that they have $\Delta I_W = 1/2$ masses. This has immediate consequences for model building. In order to explain the failure of the relation $m_e = m_d$, one has to go beyond the minimal version of the model and include extra Higgs superfields coupling to quarks and leptons, since radiative masses cannot come to our aid in explaining the disagreement. Thus, for example in $SU(5)$, one has to include at least a 45 and a 126 in $SO(10)$.

Let us now turn to the $\Delta I_W = 0$ fermion masses of the theory. Also, in this case, the fact that the theory has a hierarchy protected by supersymmetry, has important consequences. For example, if L contains a neutral fermion, it may obtain a radiative Majorana mass of order

$$\frac{g^n}{M^n} (L L \phi \dots \phi^+ \dots C^+) \sim g^n \frac{\mu^2}{M} (\Psi \Psi) \quad (4)$$

This is, at most, of the same order of magnitude as the scalar masses in (1), i.e., of order M_W . Consider as an example the right-handed neutrino ν_R in minimal $SO(10)$ in which one only uses 10 + 16 + 45 Higgses. As pointed out by Witten some time ago⁷⁾, although there is no tree level mass for the ν_R

(since there is no 126 Higgs), a radiative Majorana mass may appear in higher orders. A supersymmetric extension of Witten's diagram could be of the form shown in Fig. 3. This yields

$$m_{\nu_R} \sim h \lambda^3 \left(\frac{\alpha}{\mu}\right)^2 \frac{M^2}{M}$$

where h is the usual Yukawa coupling and λ are chiral Higgs coupling constants. If, in order to preserve the hierarchy, one takes $\mu^2 \sim M_W M$, one gets

$$m_{\nu_R} \sim h \lambda^3 \left(\frac{\alpha}{\mu}\right)^2 M_W \sim m_\ell \lambda^3 \left(\frac{\alpha}{\mu}\right)^2 \quad (5)$$

so that each right-handed neutrino is even lighter than the corresponding lepton. This implies neutrino Dirac masses of the same order of magnitude as the lepton (or better u quark) masses, which is certainly incompatible with experiment. Thus, *Witten's mechanism does not work in the susy case* and one necessarily has to introduce a 126 of Higgs fields in order to give a large tree level mass to the right-handed neutrino.

In theories larger than $SU(5)$, there are, in general, more fermions than just the usual quarks and leptons. These extra fermions like, for example, the 10 of $SO(10)$ included in the 27 of E_6 , may acquire large $\Delta I_W = 0$ masses at the grand unification mass. However, it may well be that they do not get tree level masses $\sim M$ but only receive masses in higher orders, as happens with the ν_R in minimal $SO(10)$. These fermions could have masses much smaller than M due to the powers of coupling constants which will appear in their radiative masses^{8),9)}. They would be a sort of "refugees" from the high-energy world, "interlopers", as Georgi calls them⁹⁾. However, in the supersymmetric case, their mass is at most $\sim g^n \mu^2/M \lesssim M_W$, as shown in Eq. (4), so that they may affect the observable phenomenology. Thus, in susy GUTs, one has to be careful in breaking the grand unification group to give tree level masses to all the (real rep.) fermions, otherwise they will be around the low-energy scale M_W . In fact, notice that the Higgsinos, fermionic partners of the WS doublets, are themselves "interlopers" since they do not get tree level masses $\sim M$ and may get radiative masses of at most

$$\frac{g^n}{M^n} (H_u H_d \phi \dots \phi^\dagger \dots C^+) \sim \frac{g^n \mu^2}{M} \psi \psi_0$$

where H_u and H_D are the two WS supermultiplets which are required in these theories. The Higgsinos may also combine with the "winos" to form Dirac particles with $\Delta I_W = 1/2$ masses. One can also have other interlopers transforming, for example, like a $\underline{5} + \overline{\underline{5}}$ of $SU(5)$, which could be found in forthcoming accelerators before the top quark is found. However, one must be careful with these $\underline{5} + \overline{\underline{5}}$ interlopers since they may spoil GIM cancellations⁹⁾ and also their scalar components may give rise to too fast proton decay if they couple to quarks and leptons. Notice also that one cannot have more than $3(\underline{5} + \overline{\underline{5}})$'s since otherwise the $SU(2)$ coupling constant would get $\gtrsim 1$ before the grand unification mass. Also, more than $2(\underline{5} + \overline{\underline{5}})$'s would spoil the prediction for m_b/m_t . Stable singlet interlopers could also be present around M_W , but they may lead to cosmological problems⁹⁾.

Let us finally remark that the hierarchical suppression of radiative fermion masses is also present if supersymmetry breaking proceeds through a vacuum expectation value of a D auxiliary field à la Fayet-Iliopoulos¹⁰⁾. Let us suppose that there exists an unbroken $U(1)$ gauge interaction in which the D component breaks susy at a scale $\mu^2 \sim M_W M$, and that this susy-breaking is only transmitted to the usual quark-lepton world through superheavies⁵⁾. This also means that the quark and leptons are $U(1)'$ singlets. Since $U(1)'$ is unbroken, in the effective theory all the operators must be $U(1)'$ gauge invariant and will depend only on the chiral field strength⁵⁾.

$$W'_\alpha = -\frac{1}{4} \bar{D} \bar{D} D_\alpha V' = \mu^2 \theta_\alpha \quad (6)$$

where D and $\bar{D}$ are the covariant derivatives. From (6) we observe that one needs four W'_α insertions to get a scalar mass term⁵⁾, e.g.,

$$\frac{g^n}{M^m} (L^\dagger L W'^\alpha W'_\alpha W'_\beta W'^\beta \phi \dots \phi^\dagger \dots)_D \sim \frac{g^n \mu^8}{M^6} (L^\dagger L) \quad (7)$$

where L represent some quark and lepton superfields and ϕ are Higgs superfields with large v.e.v.'s in their scalar components. This gives a mass $\sim \mu^4/M^3$. In the case of quarks and leptons, one only needs two W'_α insertions:

$$\frac{g^m}{M^m} (L^c H_w W'_\alpha W'^\alpha \phi \dots \phi^\dagger \dots)_D \sim \frac{g^m \mu^4}{M^4} M_w (\Psi_L \Psi_R) \quad (8)$$

and again one gets a hierarchical suppression M_W/M with respect to the scalar masses in (7). All the phenomenological consequences of this result which we mentioned above remain true in the case of a D term susy-breaking.

Let us summarize the main points of this letter. We have shown that in supersymmetric GUTs with spontaneously broken global supersymmetry, radiative quark and lepton masses are hierarchically suppressed by a factor M_W/M with respect to their scalar partner masses. This has several phenomenological consequences. One has to go beyond the minimal models introducing new Higgses [45's in $SU(5)$, 126's in $SO(10)$] coupling to fermions in order to account for the observed masses of quarks and leptons^{*)}. One cannot blame higher order radiative corrections for explaining the failure of the relation $m_e = m_d$ in the minimal $SU(5)$ and $SO(10)$ models since they are hierarchically suppressed ($\sim M_W^2/M \sim 10^{-2}$ eV). Also, $\Delta I_W = 0$ radiative masses, although not hierarchically suppressed, are at most $\lesssim M_W$. In the case of minimal $SO(10)$, Witten's mechanism for giving a large Majorana mass to the right-handed neutrino does not work since it gives at most masses $\sim (\alpha/\pi)^2 m_\ell$, where m_ℓ is the mass of the corresponding lepton. One is then forced to introduce Higgses in a 126 to give a large tree level mass to ν_R . One has also to be careful when breaking the gauge symmetry at very large energies to give a tree level $\Delta I_W = 0$ to all the unwanted (real rep.) fermions, since radiative masses for them will be at most $\sim M_W$.

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FIGURE CAPTIONS

- Fig. 1 : Diagram contributing to scalar masses.
- Fig. 2 : Diagrams giving $\Delta I_W = 1/2$ masses to quarks and leptons.
- Fig. 3 : Diagram giving a radiative mass to the right-handed neutrino in minimal SO(10).

FIG. 1

FIG. 2

FIG. 3