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SOFT FERMIONIC MODES IN CHIRAL- AND SUPER- SYMMETRIES

AT POSITIVE TEMPERATURE

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A B S T R A C T

The conformally invariant Wess-Zumino model is studied at various levels of approximation at positive temperature. It is found that the bosons acquire a mass of order gT , whereas due to chiral symmetry the fermion exhibits the soft dispersion relation $\omega = \frac{1}{3}k$ for momenta much less than g^2T . Weak-coupling but non-perturbative methods are needed to obtain this result.

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Supersymmetry has some remarkable consequences ¹⁾. The vacuum energy density is identically zero, particles of different spin but belonging to the same supermultiplet have identical masses, and more generally there are a number of non-renormalization theorems. Various of these properties are aesthetically pleasing, and in addition may help to solve the grand unified hierarchy problem, explain the smallness of the cosmological constant, and possibly even lead to a unification of particle and gravitational physics.

However, it is clear that we do not live in a supersymmetric world. The usual scenarios envisage the spontaneous breaking of supersymmetry at very high temperatures in the early Universe ²⁾. Thus it is imperative to develop an understanding of supersymmetry at positive temperature.

It is believed that positive temperature vitiates many of the results obtained in supersymmetric theories at zero temperature ³⁾. For example, fermions and bosons acquire different masses at positive temperature. The reasons for this are clear. First, they have different single-particle thermal distribution functions since they obey different statistics ^{3),4)}. Second, particles of different spin have different Lorentz structures for their self-energies, and these are influenced differently by the presence of a preferred rest frame, the centre-of-mass of the many-body system.

The purpose of this note is to study the simplest supersymmetric field theoretical model in four dimensions at positive temperature under various levels of approximation. Most of the results obtained so far on supersymmetry at positive temperature ^{3),5)} depend upon the applicability of the mean field approximation, which unfortunately can sometimes lead to false conclusions concerning the single-particle excitation spectra. For example, when a global $U(1)$ symmetry is spontaneously broken at positive temperature the mean field approximation alone does not yield a zero frequency excitation at zero momentum. It is necessary to use a more sophisticated approximation technique to be in accord with Goldstone's theorem ⁶⁾. Nevertheless the outcome of the present study is not obvious since supersymmetry is fundamentally different from a global $U(1)$ symmetry. Indeed it will turn out that the most novel result obtained here is a consequence of the chiral symmetry of the model Lagrangian.

We will study the conformally-invariant version of the Wess-Zumino model ⁷⁾ with the Lagrangian

$$\mathcal{L} = \partial_\mu \Phi \partial^\mu \Phi + \frac{i}{2} \bar{\psi} \not{\partial} \psi - \frac{1}{2} \bar{\psi} \left[\frac{1}{2} (1 + \gamma_5) W'' + \frac{1}{2} (1 - \gamma_5) W^{*''} \right] \psi - W' W^{*'} \quad (1)$$

The superpotential is

$$W(\Phi) = \frac{1}{3!} g \Phi^3. \quad (2)$$

Here ψ is a Majorana spinor and Φ is a complex scalar field. We will assume that $g \ll 1$. This is the only parameter in the Lagrangian and it is dimensionless. Therefore the phenomenon of dimensional transmutation will take place. According to the renormalization group the coupling constant will be replaced by an effective coupling $g(T/\Lambda)$ which depends on the ratio of the temperature T to the intrinsic energy scale Λ . In lowest order $g^2(T/\Lambda) \sim 16\pi^2/3 \ln(\Lambda/T)$. Assuming $g \ll 1$ is equivalent to assuming that $T \ll \Lambda$. This is the opposite of asymptotically free QCD.

We may express Φ in terms of a scalar field A and a pseudoscalar field B by writing $\Phi = (A+iB)/\sqrt{2}$. The free field behaviour of Φ is obtained by setting $g = 0$. Then the dispersion relation for all of the fields is $\omega = k$, and of course this is not changed by temperature effects.

The mean field approximation ³⁾ consists of dividing the boson fields into classical condensed components and fluctuating components, $A = A_{cl} + A_{fl}$, $B = B_{cl} + B_{fl}$. Usually the condensation of pseudoscalar fields is disfavoured because of parity. For simplicity we will assume $B_{cl} = 0$ although the analysis may be carried through otherwise with only minor complications.

The fluctuating components and the spinor field are treated as free fields whose masses can be read off from the Lagrangian, Eq. (1) :

$$\begin{aligned} \overline{m}_A^2 &= \frac{3}{4} g^2 A_{cl}^2, \\ \overline{m}_B^2 &= \frac{1}{4} g^2 A_{cl}^2, \\ \overline{m}_\psi^2 &= \frac{1}{2} g^2 A_{cl}^2. \end{aligned} \quad (3)$$

Notice that these mean field (or tree) masses satisfy the equation

$$\sum_J (-1)^{2J} (2J+1) \text{Tr } M_J^2(A_{cl}) = \overline{m}_A^2 + \overline{m}_B^2 - 2\overline{m}_\psi^2 = 0. \quad (4)$$

for any value of A_{cl} . This means that in the computation of the thermodynamic (or effective) potential at zero or positive temperature the coefficient of the quadratically divergent term is zero. In consequence any breaking of supersymmetry by temperature is said to be a soft breaking⁸⁾.

The classical field is determined self-consistently by minimizing the thermodynamic potential at fixed T . A high temperature expansion,

$$\Omega = -\frac{\pi^2}{24} T^4 + \frac{1}{16} g^2 T^2 A_{cl}^2 + \frac{1}{16} g^2 A_{cl}^4 + \dots, \quad (5)$$

is sufficient to show that $A_{cl} = 0$ for all T . This is obvious since there are no tachyonic mass terms in the Lagrangian. Furthermore, the non-negative energy condition of supersymmetry forbids a Coleman-Weinberg⁹⁾ type of radiative symmetry breaking at $T = 0$; and so it will not occur at $T > 0$ either. This may easily be checked in one-loop approximation. Hence all fields remain massless with the dispersion relation $\omega = k$.

The mean field approximation does not contain the effects of thermal fluctuations on the particle masses. The semi-classical approximation^{10),6)} takes into account at least some of the thermal fluctuations. The classical equations of motion are

$$\partial^2 A = -\frac{1}{4} g^2 (A^3 + A B^2) - \frac{1}{2\sqrt{2}} g \bar{\Psi} \Psi, \quad (6)$$

etc. Take the ensemble average of these equations and linearize them by self-consistent factorization, e.g., $\langle A^3 \rangle \rightarrow 3\langle A^2 \rangle \langle A \rangle$. Then the boson masses become

$$\begin{aligned} m_A^{*2} &= \frac{1}{4} g^2 (3 \langle A^2 \rangle + \langle B^2 \rangle), \\ m_B^{*2} &= \frac{1}{4} g^2 (3 \langle B^2 \rangle + \langle A^2 \rangle). \end{aligned} \quad (7)$$

In general these are coupled integral equations for the masses, but since $g \ll 1$ we may evaluate them perturbatively as

$$m_A^{*2} = m_B^{*2} = g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p} \frac{1}{\exp(p/T) - 1} = \frac{1}{12} g^2 T^2. \quad (8)$$

In the fermion sector $\langle \bar{\Psi} \Psi \rangle = \langle \bar{\Psi} \gamma_5 \Psi \rangle = 0$ and the fermion is massless.

In diagrammatic language the semi-classical approximation corresponds to a retention of only tadpole diagrams.

The perturbative one-loop self-energy diagrams for the bosons and the fermion are shown in the Figure. These are readily evaluated using the temperature Green function approach wherein the energy components are discrete, being $2\pi nT$ for bosons and $(2n+1)\pi T$ for fermions. To obtain the excitation spectra one analytically continues to continuous energies in Minkowski space and locates the poles of the propagators. [This point was not appreciated in Ref. 3).]

The A and B fields have identical propagators

$$D(k_0, k) = (k_0^2 - k^2 - \Pi(k_0, k))^{-1}$$

where

$$\begin{aligned} \Pi(k_0, k) = & \frac{1}{12} g^2 T^2 + \frac{1}{2\pi^2} g^2 \int_0^\infty \frac{dp p}{\exp(p/T) - 1} \left\{ 1 + \right. \\ & \left. + \frac{k_0^2 - k^2}{8 p k} \ln \left| \frac{k_0^2 - (2p+k)^2}{k_0^2 - (2p-k)^2} \right| \right\}. \end{aligned} \quad (9)$$

This is written in Minkowski space with $k = |\vec{k}|$. With certain configurations of momenta there is also a small imaginary part which will not concern us here. Notice that $\pi(0,0) = \frac{1}{8}g^2T^2$ whether we take $k_0 = 0$ first and then $k \rightarrow 0$, or vice versa. Notice also that $\pi(k_0=k) = \pi(0,0)$ for any value of k . Therefore, to lowest order in g , the dispersion relation for bosons is

$$\omega^2 - k^2 = m^2(T) = \frac{1}{8} g^2 T^2. \quad (10)$$

This is not affected by the vacuum contribution to the boson self-energy or by its imaginary part, neither of which are included in Eq. (9). The first term on the right side of Eq. (9) comes from the tadpole diagrams in the Figure. We see that there is an additional contribution to the boson mass, coming from the fermion loop, which is not contained in the mass predicted by the semi-classical approximation, Eq. (8).

The fermion propagator is

$$S(k_0, \vec{k}) = (k_\mu \gamma^\mu - \Sigma_\mu(k_0, \vec{k}) \gamma^\mu)^{-1}.$$

The fermion self-energy Σ_μ can be expressed in terms of integrals similar to those in Eq. (9). They are quite lengthy, however, and for our purposes it is sufficient to evaluate them in the limit $k_0, k \ll T$ ¹¹⁾.

$$\Sigma_0(k_0, \vec{k}) = \frac{1}{64} g^2 \frac{T^2}{k} \ln \left| \frac{k_0 + k}{k_0 - k} \right|, \quad (11)$$

$$\vec{\Sigma}(k_0, \vec{k}) = \frac{1}{32} g^2 \vec{k} \frac{T^2}{k^2} \left\{ \frac{k_0}{2k} \ln \left| \frac{k_0 + k}{k_0 - k} \right| - 1 \right\}. \quad (12)$$

The interesting limits of these expressions are : $\Sigma_0(0, k) = 0$, $\Sigma_0(k_0, 0) = g^2 T^2 / 32 k_0$, $\Sigma_0(k_0 = k)$ not defined ; $\vec{\Sigma}(0, \vec{k}) = -g^2 \vec{k} T^2 / 32 k^2$, $\vec{\Sigma}(k_0, 0) = 0$, $\vec{\Sigma}(k_0 = k)$ not defined. The fermion mass may be defined as the location of the pole in the limit $k = 0$. This occurs when $k_0 - \Sigma_0(k_0, 0) = 0$ so that

$$m_\psi^2(T) = \frac{1}{32} g^2 T^2. \quad (13)$$

More generally the low momentum expansion for the fermion dispersion relation is ¹¹⁾

$$\omega = m_\psi(T) + \frac{1}{3} k + \frac{k^2}{3 m_\psi(T)} + \dots \quad (14)$$

In this one-loop perturbative expansion not only does the fermion develop a mass different from the boson mass but the functional forms of the dispersion relations are also different. There is still a chiral symmetry in the sense that the fermion self-energy does not have a contribution proportional to the unit matrix and that the fermion propagator is an odd function of the four-momentum.

It is the fundamental structure of the naive perturbation expansion which prevents the appearance of a zero frequency fermion at zero momentum. The temperature dependent part of Σ must go as a positive power of T , and of course Σ must be an odd function of k_μ . Therefore it is understandable that, in the limit $k \ll k_0 \ll T$, $\vec{\Sigma} \rightarrow 0$ (because of rotational symmetry) and $\Sigma_0 \rightarrow T^2/k_0$ (because of dimensional reasons). The last limit is quite significant and is seen in practice in Eqs. (11) and (13).

Now we realize that there is a new energy scale in the problem, gT , which may negate the absence of a soft fermion in perturbation theory. The bosons with which the fermion interacts are not the massless bosons implied by the free field limit of the Lagrangian, rather they are the massive dressed bosons which obey the dispersion relation of Eq. (10). These dressed bosons ought to be used in the internal lines in the Figure (c). They may provide an infra-red cut-off which would drastically alter the infra-red limit of the fermion self-energy.

This is equivalent to a resummation or a re-ordering of the perturbation expansion. (This need not be done for the boson self-energies to lowest order in g since there the limit $k_0, k \rightarrow 0$ is smooth and finite.)

This approximation may be thought of as a weak-coupling but self-consistent solution of the two-point Schwinger-Dyson equations. The resulting fermion self-energy may be expressed in terms of integrals similar to that in Eq. (9). When $k_0, k \ll g^2 T$ they take the interesting forms

$$\Sigma_0 = -\frac{\pi^2}{16} g^2 \left(\frac{T}{m(T)} \right)^4 k_0 = -\left(\frac{2\pi}{g} \right)^2 k_0, \quad (15)$$

$$\vec{\Sigma} = \frac{1}{3} \left(\frac{2\pi}{g} \right)^2 \vec{k}. \quad (16)$$

The limit $k_0, k \rightarrow 0$ is now smooth and finite. Despite the appearance of $1/g^2$ the results are in some sense small since $\Sigma \ll T$ in this limit. The resulting dispersion relation is

$$\omega = \frac{1}{3} k \quad (17)$$

when $k \ll g^2 T$. This soft mode carries the quantum numbers of a Majorana particle, and may be thought of as a bound state of the fermion and the A and B fields or, better still, a long-wavelength low-frequency collective oscillation of those fields. Obviously such a result cannot be obtained in a naive perturbation expansion.

This segment of the dispersion relation has some analogy with spin waves in ordinary low temperature physics. The form of the dispersion relation, Eq. (17), ensures that these are stable, elementary oscillations which cannot decay into any combination of themselves and the bosons. One may check mathematically that there is no imaginary part of Σ in this limit.

In the limit $k \gtrsim gT$ the dispersion relation is the same as that obtained with massless bosons. Notice the appearance of two new energy scales, gT and $g^2 T$, in addition to T . It is easy to see that the origin of this soft fermionic mode lies in the chiral symmetry and not the supersymmetry of $\mathcal{L}$. First we note that supersymmetry is not broken in the conventional sense. Thermal averages of the boson fields vanish. So does the thermal average ⁴⁾ of

$$\delta \Psi = \frac{1}{4} \epsilon \left\{ \frac{g}{\sqrt{2}} (A^2 - B^2 + 2AB\gamma_5) - i \not{D} (A + B\gamma_5) \right\}, \quad (18)$$

since the scalar and pseudoscalar fields have the same dispersion relation even when $T > 0$. Thus there is no reason to expect a Goldstino^{3),12)}. Second we note that these same results, namely Eqs. (15) and (16) (apart from inconsequential numerical factors) and Eq. (17), would be obtained in most any unbroken chiral theory involving massless scalar and pseudoscalar bosons. Consider, for instance, a massless scalar field and a massless Dirac field with only a Yukawa interaction.

The methods and results of this paper ought to be applied to more realistic chiral or supersymmetric models of the Universe.

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FIGURE CAPTION

The one-loop self-energy corrections to (a) the scalar field A , (b) the pseudoscalar field B , and (c) the fermion field ψ . The curly lines, dashed lines, and solid lines represent, respectively, the A , B , and ψ fields.

$$\frac{3}{4} \text{ (wavy line with a scalloped circle) } + \frac{1}{4} \text{ (wavy line with a dashed circle) } - \frac{1}{4} \text{ (wavy line with a solid circle) }$$

(a)

$$\frac{3}{4} \text{ (dashed line with a solid circle) } + \frac{1}{4} \text{ (dashed line with a scalloped circle) } - \frac{1}{4} \text{ (dashed line with a solid circle) }$$

(b)

$$\frac{1}{4} \text{ (solid line with a scalloped semi-circle) } + \frac{1}{4} \text{ (solid line with a dashed semi-circle) }$$

(c)