Ref.TH.3367-CERN

IDENTICAL PARTICLE INTERFERENCE IN D^+ MESON LIFETIME

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A B S T R A C T

A calculation is presented of the contribution to the D^+ non-leptonic decay rate of the interference between the active and spectator $\bar{d}$ antiquarks, relating this effect to the shape of the electron energy spectrum in D semi-leptonic decay. Predictions for the D^+ lifetime and semi-leptonic branching ratio are also derived and discussed.

Ref.TH.3367-CERN

23 July 1982

The observed difference in the lifetimes of the D^0 and D^+ charmed mesons ¹⁾ directly indicates a departure from the parton approach where one identifies the hadron decay width with that of a free heavy quark. This departure from the parton model is naturally attributed to the fact that the c quark mass is not sufficiently heavy and pre-asymptotic effects are still important in charm decay. Two main mechanisms have been proposed to explain the lifetime difference between D^0 and D^+ . One is the W exchange between constituents accompanied by gluon emission : the so-called annihilation mechanism ²⁾. This effect can substantially decrease the D^0 lifetime, while the D^+ rate is only affected at the Cabibbo suppressed level. The second mechanism is the destructive interference, due to Fermi statistics, between the spectator antiquark $\bar{d}$ in D^+ decay and the identical $\bar{d}$ arising in the final state from non-leptonic c decay ^{3),4)}. This interference increases the D^+ lifetime, while the D^0 is totally unaffected.

In this note we consider the D^+ inclusive decay rate. In particular we present a detailed evaluation of the interference contribution which is considerably more accurate than the existing estimates. This calculation is performed following a treatment of bound state effects which was introduced in Ref. 5). In that paper, the effect of the charmed quark bound state motion on the charged lepton spectrum in semi-leptonic D decay was studied. The information on the momentum distribution of the c quark in the D meson derived from the existing data on the lepton spectrum is used here to estimate the interference term. The small contribution to the total rate of D^+ from the Cabibbo forbidden annihilation term can be phenomenologically related to the ratio of D^+ and F^+ lifetimes. Thus the analysis presented here, together with that of Ref. 5), leads to a set of predictions for the D^+ lifetime τ^+ and its semi-leptonic branching ratio B_{SL}^+ .

Our results indicate that the interference term was considerably overestimated in the simplified calculations so far performed ⁴⁾. Once the momentum dependence of the matrix elements and of phase space is taken into account, the resulting interference contribution is rather small. For realistic values of the parameters, it amounts at most to 20% of the normal term and it is probably much smaller (Fig. 1). Correspondingly for the D^+ lifetime we find values in the range (Fig. 2) :

$$\cos^2 \gamma \tau^+ = (8 \pm 4) \cdot 10^{-13} \text{ sec} \quad (1)$$

where $\cos y$ is the $c \rightarrow s_c$ coupling in the Kobayashi-Maskawa mixing matrix ⁶⁾ in the notation of Ref. 7). The semi-leptonic branching ratio of D^+ , which provides an important test for the validity of the QCD short-distance modification of the basic weak non-leptonic Hamiltonian ⁸⁾, is predicted to be (Fig. 3) :

$$B_{SL}^+ = (14.5 \pm 3.5) \% \quad (2)$$

These predictions are consistent with the present experimental results. The average semi-leptonic branching ratio of D^0 and D^+ is given by ⁹⁾ $(\bar{B}_{SL})_{\text{exp}} = (8.2 \pm 1.2)\%$ and the lifetimes ¹⁰⁾ are $(\tau^0)_{\text{exp}} = (2.4^{+0.7}_{-0.5})10^{-13}$ sec, $(\tau^+)_{\text{exp}} = (6.9^{+3}_{-2})10^{-13}$ sec.

Our starting point is a picture of the D^+ meson decay as shown in Fig. 4a,b, where the interference term is given by the diagram 4b. The spectator antiquark is taken as an on-shell particle of mass m_{sp} and momentum $\vec{q}$, while the heavy quark is treated as a virtual particle of invariant mass W :

$$W^2 = M_D^2 + m_{sp}^2 - 2M_D \sqrt{\vec{q}^2 + m_{sp}^2} \quad (3)$$

where M_D is the D^+ mass. The structure of the diagram 4a is identical to that of the diagram describing the D^+ semi-leptonic rate considered in Ref. 5). In that case the semi-leptonic rate was expressed as a convolution of the probability of finding the c quark in the meson with momentum $\vec{q}$ and mass W and the decay rate in flight of the c quark of mass W :

$$\Gamma_{SL} = \int d^3q |\phi(q)|^2 \Gamma_{SL}^Q(\vec{q}, W) \quad (4)$$

with

$$\int d^3q |\phi(q)|^2 = 1 \quad (5)$$

We adopt an identical description for the contribution of Fig. 4a to the non-leptonic D^+ rate, in terms of the same $|\phi(q)|^2$. The relation between $|\phi(q)|^2$ and the meson vertex form factor is unambiguous if one identifies the physical c quark mass with W . In fact, the meson vertex is in general a superposition of pseudoscalar and pseudovector couplings which coincide for on-shell quarks. To the extent that this identification is valid, one can then use the same vertex in the evaluation of diagram 4b and express the interference contribution in terms of a double convolution involving $\phi(\vec{q}_1)\phi^*(\vec{q}_4)$, $\vec{q}_1$ and $\vec{q}_4$ being the momenta of the interfering $\bar{d}$ antiquarks. The form of the kernel

arising from Fig. 4b is then completely specified up to terms of order m_{SP}/M_D or p_F/M_D , p_F being the typical momentum of the bound c quark in the D^+ meson, which are not important for a sufficiently damped wave function $\phi(q)$.

The effective Hamiltonian for charm changing non-leptonic transitions, including the leading QCD corrections ⁸⁾, is given by :

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left\{ \frac{1}{2} (c_+ + c_-) \bar{c} \gamma_\mu (1 - \gamma_5) s \bar{d} \gamma^\mu (1 - \gamma_5) u + \right. \\ \left. + \frac{1}{2} (c_+ - c_-) \bar{c} \gamma_\mu (1 - \gamma_5) u \bar{d} \gamma^\mu (1 - \gamma_5) s \right\} \quad (6)$$

where mixing angles are omitted for simplicity. The total amplitude for D^+ decay is then obtained in the form :

$$A^+ = \frac{1}{2} (c_+ + c_-) [A(q_1, q_2, q_3, q_4) - A(q_4, q_2, q_3, q_1)] + \\ + \frac{1}{2} (c_- - c_+) [A(q_1, q_3, q_2, q_4) - A(q_4, q_3, q_2, q_1)] \quad (7)$$

where q_1 and q_4 are the momenta of the active and spectator $\bar{d}$ antiquarks, respectively, while q_2 and q_3 refer to the final u and s quarks. The total non-leptonic decay rate of D^+ is then given by :

$$\Gamma_{NL} = \frac{1}{2} \frac{1}{2M_D} \int \frac{d^3 q_1}{(2\pi)^3 2E_1} \dots \frac{d^3 q_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(P - \sum_{i=1}^4 q_i) \sum_{\substack{\text{spin} \\ \text{colour}}} |A^+|^2 \quad (8)$$

where the factor of $\frac{1}{2}$ arises from the presence of two identical particles in the final state.

The calculation procedure outlined above amounts to take the following expression for the amplitude $A_1 \equiv A(q_1, q_2, q_3, q_4)$:

$$A_1 = \frac{-iG_F}{[6E_{q_4}(1 + \frac{m_{SP} W_4}{E_4 M_D})]^{1/2}} \bar{u}(q_1) \gamma_\mu (1 - \gamma_5) v(q_2) \bar{u}(q_4) (\not{P} - \not{q}_4 + \not{W}_4) \gamma^\mu (1 - \gamma_5) v(q_3) \phi(q_4) \quad (9)$$

where $\phi(q_4)$ is normalized as in Eq. (5) and $E_W = \sqrt{q^2 + W^2}$. This form of A_1 reduces to that of Ref. 4) in the appropriate limit $E_4 \simeq m_{SP}$, $W = M_D$.

In the evaluation of the total rate from Eqs. (7)-(9), we differ from Ref. 4) in that we treat all matrix elements and the phase space integrations exactly, apart from the neglect of terms of order m_{SP}/M_D and p_F/M_D in the interference contribution, which are beyond reach anyway as observed above.

The final result is given by :

$$\Gamma_{NL}^+ = \Gamma_U + \Gamma_I \quad (10)$$

$$\Gamma_U = (2c_+^2 + c_-^2) J_{2L} \int d^3 q_u |\phi(q_u)|^2 \left(\frac{W_u}{E_{W_u}} \right) \frac{G_F W_u^5}{192 \pi^3} g\left(\frac{m_s}{W_u}, \frac{m_{sp}}{W_u}, \frac{m_{sf}}{W_u}\right) \quad (11)$$

$$\Gamma_I = (2c_+^2 - c_-^2) \int \frac{d^3 q_1 d^3 q_4}{(2\pi)^3} \phi(q_1) \phi^*(q_4) \left(\frac{W_1 W_4}{E_{W_1} E_{W_4}} \right)^{1/2} \frac{G_F^2}{\pi} \frac{(M_{23}^2 - m_s^2)^2}{M_{23}^2} \frac{m_{sp}^2}{E_1 E_4} \frac{M_D}{\sqrt{W_1 W_4}} \cdot \frac{2 M_D}{\left[\left(M_D + \frac{m_{sp} W_1}{E_1} \right) \left(M_D + \frac{m_{sp} W_4}{E_4} \right) \right]^{1/2}} \frac{1}{4} \left[1 + \frac{2 E_1 W_1}{m_{sp} M_D} + \frac{W_1 W_4}{M_D^2} \frac{E_1 E_4 - \vec{q}_1 \cdot \vec{q}_4}{m_{sp}^2} \right] \quad (12)$$

where m_s is the strange quark mass, $M_{23}^2 = (P - q_1 - q_4)^2$ is the invariant mass squared of the u, s system, W_1 and W_4 are functions of $\vec{q}_1$ and $\vec{q}_4$, respectively, defined in Eq. (3), and W/E_W is the Lorentz factor for the decay in flight. $g(x_a, x_b, x_c)$ is the three-body phase space factor given by ¹¹⁾ :

$$g(x_a, x_b, x_c) = 12 \int_{(x_a+x_b)^2}^{(1-x_c)^2} \frac{dy}{y} (y - x_a^2 - x_b^2)(1+x_c^2-y) \left\{ [y - (x_a - x_b)^2] \cdot [y - (x_a + x_b)^2] \right\} \cdot \left[(1+x_c^2-y) \left[(1-x_c^2)-y \right] \right\}^{1/2} \quad (13)$$

J_{2L} is the next-to-leading QCD correction as evaluated in Ref. 12) for massless final quarks. The neglect of masses in J_{2L} is venial as one is dropping a correction to a small correction. In the appropriate limit, i.e., $m_s = 0$, $E \approx m_{sp}$, $W = M_D$, $M_{23}^2 = M_D^2$, Γ_I in Eq. (12) is immediately seen to reproduce the non-relativistic approximation of Ref. 4).

For a numerical evaluation of Γ_{NL}^+ we take a Gaussian form for $\phi(q)$:

$$\phi(q) = (\sqrt{\pi} p_F)^{-3/2} \exp\left(-\frac{\vec{q}^2}{p_F^2}\right) \quad (14)$$

as in our previous analysis ⁵⁾ of the charged lepton spectrum in D semi-leptonic decays. The range of p_F indicated by the data was found there to be $p_F \lesssim 150$ MeV. To be on the safe side, we present here our results for $p_F \lesssim 300$ MeV. Note that in our treatment of bound state effects, once p_F is given, the effective mass W_{eff} of the decaying quark is fixed. For the normal

term Γ_U in Eq. (11), W_{eff} is roughly given by the value of W at $q = p_F$ where $q^2 |\phi(q)|^2$ reaches its maximum. This gives $W_{\text{eff}} \sim 1.55 \text{ GeV}$ for $m_{SP} = 0.15 \text{ GeV}$ and $p_F = 0.25 \text{ GeV}$ ($M_D = 1.87 \text{ GeV}$ throughout this note). Thus the wave function spread reproduces a Γ_U term comparable to that obtained from the decay of a free c quark of mass close to $\frac{1}{2}M_{J/\psi}$. On the other hand the function $\phi(q_1)\phi^*(q_4) \cdot q_1^2 \cdot q_4^2$, which enters in Γ_I , has its maximum at a larger momentum: $q_1 = q_4 = \sqrt{2}p_F$. Correspondingly for the angular average of M_{23}^2 one finds $(M_{23}^2/M_D^2)_{\text{eff}} \approx 0.4$. The remaining factors in Eq. (12) besides $((M_{23}^2 - m_S^2)^2/M_{23}^2)$, taken altogether, do not deviate much from unity in the region of the peak. Thus the depletion of the interference term with respect to its non-relativistic estimate can be mainly attributed to the reduction of the u and s quark phase space due to the total momentum carried away by the two $\bar{d}$ (and to a smaller extent to the strange quark mass).

These crude estimates are fully supported by a numerical evaluation of Eqs. (11) and (12). All our results are presented in the following as functions of p_F for different values of Λ_{QCD} and of final quark masses. For Λ_{QCD} we took the values $\Lambda_{\text{QCD}} = 100, 250, 400 \text{ MeV}$, corresponding to $\alpha_s(M_D) = 0.20, 0.28, 0.36$, respectively, derived from the two-loop improved form of the running coupling for four flavours¹³⁾. As for quark masses, they were chosen having in mind that the final state of D^+ decay is mainly made up of one K plus π 's^{*)}. To display the sensitivity on quark masses we took two sets of masses (with $m_u = m_d = m_{SP}$), i.e., $m_S = 300 \text{ MeV}$, $m_{SP} = 150 \text{ MeV}$ and $m_S = 400 \text{ MeV}$, $m_{SP} = 100 \text{ MeV}$.

In Fig. 1 our results on the ratio $R = -\Gamma_I/\Gamma_U$ are shown. For values of p_F as preferred by the charged lepton spectrum in semi-leptonic D decay, i.e., $p_F \lesssim 150 \text{ MeV}$, $R \lesssim 3\div 6\%$ and reaches $9\div 21\%$ for the maximum value $p_F \sim 300 \text{ MeV}$ allowed by the semi-leptonic data. We thus conclude that the interference mechanism although contributing in the right direction, can only play a marginal rôle in explaining a lifetime difference between D^+ and D^0 by a factor of $2\div 3$ as observed.

To proceed with the calculation of the total D^+ rate we consider the semi-leptonic rate, which is given by

$$\Gamma_{SL}^+ = \int d^3q |\phi(q)|^2 \left(\frac{W}{E_W} \right) \frac{G_F^2 W^5}{192 \pi^3} g\left(\frac{m_S}{W}, 0, 0\right) \left[1 - \frac{2\alpha_s}{3\pi} \left(\pi^2 - \frac{25}{4} \right) \right] \quad (15)$$

*) For a different point of view, see Ref. 15) where higher values of constituent quark masses were adopted.

where, as in the case of J_{2L} in Eq. (11), the QCD correction term ¹⁴⁾ is included in the massless limit.

Mixing angles have been omitted in Eqs. (11), (12), (15) for Γ_{NL}^+ and Γ_{SL}^+ . When Cabibbo suppressed modes from quark decay are also included and phase space distortions are neglected, one only needs to multiply Γ_{SL}^+ by $(1-|U_{cb}|^2)$ and Γ_{NL}^+ by $(1-|U_{cb}|^2)(1-|U_{ub}|^2)$, where U is the KM mixing matrix ⁶⁾. In the notations of Ref. 7), $U_{ub} = \sin\beta$ and $U_{cb} = \cos\beta\sin\gamma \approx \sin\gamma$. The annihilation contribution to D^+ Cabibbo suppressed modes can be estimated by observing that the annihilation term is identical for D^+ and F^+ decays when mass effects are neglected and the mixing angles are taken separately into account. We therefore adopt the following form for the total D^+ rate :

$$\Gamma^+ = \cos^2\gamma \left\{ 2\Gamma_{SL}^+ + \Gamma_{NL}^+ \left[1 + t_g^2 \theta_c \left(\frac{\tau_{D^+}}{\tau_{F^+}} - 1 \right) \right] \right\} \quad (16)$$

and we shall take in the following $((\tau_{D^+}/\tau_{F^+})-1) \sim 2$.

The quantity $\cos^2\gamma\tau^+$ is plotted in Fig. 2. For $p_F \lesssim 150$ MeV one finds $\cos^2\gamma\tau^+ \sim (4\div 7)10^{-13}$ sec, while for $p_F \lesssim 300$ MeV one obtains Eq. (1). Given the experimental value for the D^+ lifetime, as quoted before in this note, one is at present unable to derive a significant bound on $\cos^2\gamma$ even if we take $p_F \lesssim 150$ MeV. The semi-leptonic branching ratio B_{SL}^+ is shown in Fig. 3. B_{SL}^+ depends little on p_F and its value is mainly determined by α_s . For $\alpha_s(M_D) \approx 0.28$, $B_{SL}^+ \approx (15\pm 1.5)\%$, while as $\alpha_s(M_D)$ is varied between 0.20 and 0.36 the range in Eq. (2) is obtained.

Finally, we observe that the interference term is completely negligible in b meson decay, its relative size being below the 1% level.

ACKNOWLEDGEMENTS

We gratefully acknowledge the hospitality of the CERN Theoretical Physics Division, where this work was done.

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FIGURE CAPTIONS

Fig. 1 The ratio $R = -\Gamma_I/\Gamma_U$ of the interference and the direct term in D^+ non-leptonic decay, as a function of p_F for different values of $\alpha_S(M_D)$ and of final quark masses. The upper curve in each shaded band is for $m_S = 300$ MeV, $m_{SP} = 150$ MeV, while the lower curve corresponds to $m_S = 400$ MeV, $m_{SP} = 100$ MeV.

Fig. 2 $\cos^2\gamma\tau^+$ as a function of p_F for different values of $\alpha_S(M_D)$ and of final quark masses. The upper curve in each shaded band is for $m_S = 400$ MeV, $m_{SP} = 100$ MeV, while the lower curve corresponds to $m_S = 300$ MeV, $m_{SP} = 150$ MeV.

Fig. 3 The semi-leptonic branching ratio B_{SL}^+ of D^+ as a function of p_F for different values of $\alpha_S(M_D)$ and of final quark masses. The upper curve in each shaded band is for $m_S = 300$ MeV, $m_{SP} = 150$ MeV, while the lower curve corresponds to $m_S = 400$ MeV, $m_{SP} = 100$ MeV.

Fig. 4 Diagrams for the D^+ meson non-leptonic decay :
(a) direct term, (b) interference term.

Fig. 1

Fig. 2

Fig. 3

Fig. 4