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STUDY OF BARYON AND LEPTON VIOLATING TWO-PROTON ANNIHILATION
IN GAUGE THEORIES

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A B S T R A C T

The baryon and lepton violation process $(A, Z) \rightarrow (A-2, Z-2) + e^+ + e^+$ is investigated within the framework of two gauge models recently proposed. One is an $SU_L(2) \times U(1) \times SU_C(3)$ model extended to include two isospin triplets which couple to quarks and leptons. The second is an extension of the minimal GUT $SU(5)$ model to include a 45-plet and a 50-plet of Higgs scalars. With a reasonable choice of the parameters of these models and a proper treatment of nuclear physics, mean lifetimes of 10^{33} yr may be expected.

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Recent developments in gauge theories have rekindled interest in the old question of baryon ^{1),2)} (B) and lepton ^{3),4)} (L) number non-conservation. Most of such investigations have hitherto focused on the decay of individual protons and double β decay. Recently, however, processes like neutron-antineutron, hydrogen-antihydrogen oscillations, and double proton decay, have also been considered. In the present paper, we will focus on the experimentally attractive B-L conserving process

$$(A, Z) \rightarrow (A-2, Z-2) + e^+ + e^+ \quad (1)$$

(A similar analysis applies to the $\mu^+\mu^+$ production.) Bounds on the amplitudes for such a process have been given by Feinberg, Goldhaber and Steigman ⁵⁾. Two gauge models which may give rise to reaction (1) have recently been proposed by Mohapatra and Senjanović ⁶⁾ and by Arnellos and Marciano ⁷⁾, which gave periods for hydrogen-antihydrogen oscillations in the optimistic range of 10^{13} to 10^{15} yr. Among other things, we shall examine whether process (1) may be suppressed from nuclear physics effects even if it is not suppressed by gauge couplings and masses.

Since the gauge models which are going to serve as input in our work have been described elsewhere ^{6),7)} in considerable detail, we will only give their essential features here.

A. - The Mohapatra-Senjanović model

This is an $SU_2(L) \times U(1) \times SU_3(C)$ model extended to include two isospin triplets of Higgs particles $\Delta_\ell(3, 2, 1)$ and $\Delta_q(3, -\frac{2}{3}, \frac{2}{3})$, which couple to leptons and quarks with Yukawa couplings f_ℓ and f_q , respectively, with their $SU(2)$, $U(1)$ and $SU(3)$ quantum numbers indicated in parentheses. It is worth mentioning that this model possesses a discrete symmetry which guarantees the stability of the proton without the need of introducing superheavy masses. The Feynman diagram contributing to process (1), which is exhibited in Fig. 1, leads to an effective interaction of the form

$$H_{eff} = \Lambda \kappa \frac{1}{m_p^2} \bar{p}_L^c p_L \bar{e}_L^c e_L + h.c. \quad (2)$$

The quantity Λ contains the parameters of the gauge model and is given by

$$\Lambda = \frac{\lambda f_q^3 f_\ell m_p^6}{m_{\Delta_q}^6 (m_{\Delta_\ell^+})^2} \quad (3a)$$

while the parameter κ depends on the wave function at the origin of the two annihilating quarks

$$\kappa = \left\{ \frac{|\psi(0)|^2}{m_p^3} \right\}^2 \quad (3b)$$

For numerical purposes, we will adopt the values ⁶⁾ $\lambda = 1$, $f_q \approx 10^{-3}$, $f_\ell \approx 10^{-2}$, $m_{\Delta_q} = m_{\Delta_\ell} = 100$ GeV and the value $|\psi(0)|^2 = 8 \times 10^{-3} \text{ GeV}^3$ which is commonly used in standard proton decay calculations ⁸⁾. We thus get

$$\Lambda \sim 10^{-27} \quad \kappa \sim 10^{-4} \quad (4)$$

which must be compared with the value of $(\Lambda\kappa) \sim 0.5 \times 10^{-28}$ previously used ⁶⁾ [which corresponds to $m_{\Delta_{\ell}} = 15$ GeV, $|\psi(0)|^4 = 10^{-3} (\text{GeV})^6$].

B. - The Arnellos-Marciano model ⁷⁾

This is an extension of the minimal SU(5) grand unification model ⁹⁾ which accommodates colour sextets, weak isospin triplets and singlets mediating quark-antiquark transitions, and a colourless double charged isospin singlet which can decay into two positrons. In the SU(5) language, this amounts to adding a 50-plet and a 45-plet to the minimal structure. Out of this plethora of Higgs of interest to us are the $\phi_6(1, -\frac{2}{3}, \bar{6})$ member of the 45-plet and the $\phi_8(1, -4, 1)$, $\phi_{11}(1, \frac{8}{3}, \bar{6})$ and $\phi_{12}(3, -\frac{2}{3}, \bar{6})$ members of the 50-plet (see Fig. 2). One must specify the masses of these particles and the two h_{45} and h_{50} Yukawa couplings to the 5 $\times$ 10 and 10 $\times$ 10 combinations of fermions, respectively. For our numerical estimates, we will adopt the choice of Ref. 7), i.e., $(h_{50}/m_{12}) \approx 2 \times 10^{-7} \text{ GeV}^{-1}$, $(h_{50}/m_{11}) \approx 5 \times 10^{-6} \text{ GeV}$, $h_{45} \approx 10^{-4}$, $m_6 = m_8 = 100$ GeV. (This choice does not affect proton stability.) Then from the pertinent diagrams shown in Fig. 2, we derive the same Hamiltonian as above with effective couplings

$$\Lambda = \frac{h_{50}^4 m_p^8}{m_8^2 m_{11}^2 m_{12}} \approx 10^{-37} \text{ (fig. 2a)}, \quad \Lambda = \frac{h_{45}^2 h_{50}^2 m_p^8}{m_6^4 m_8^2 m_{12}} \approx 10^{-31} \text{ (fig. 2b)} \quad (5)$$

We notice that the diagram of Fig. 2a is negligible in comparison with that of Fig. 2b and both are smaller than those of the previous model.

With the effective interaction of Eq. (2), we can sum over lepton spins and transform the hadronic operator to co-ordinate space to get, for the amplitude squared :

$$|M|^2 = \frac{1}{8} (\Lambda\kappa)^2 \frac{1}{m_p^4} |\langle f | \Omega | i \rangle|^2 \frac{E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2}{m_e^2} \quad (6)$$

where $E_i, \vec{p}_i$, $i = 1, 2$ denote the energies and momenta of the emitted leptons and $\langle f | \Omega | i \rangle$ is the nuclear matrix element of the operator

$$\Omega = \sum_{i \neq j} \tau(i) \tau(j) \delta(\vec{x}_i - \vec{x}_j) \quad (7)$$

where i and j label the nucleons. We thus see that not only we have the attractive experimental feature of two positrons emitted with a summed energy $2m_p$ but with a distinct angular distribution of the form $1 - \hat{p}_1 \cdot \hat{p}_2$. (The positrons are relativistic so $E_L \approx |\vec{p}_L|$, $i = 1, 2$. Furthermore, no distortions of their plane wave functions are necessary.) The width for process (1) is easily found to be

$$\Gamma = \frac{1}{4} \frac{(1\kappa)^2}{(2\pi)^3} m_p |\langle f | \Omega | i \rangle|^2 f_0(\epsilon_0) \quad (8)$$

$$f_0(\epsilon_0) = \frac{1}{2m_p^5} \int_0^{\Delta_0} p_1 dE_1 \int_0^{\Delta_0} p_2 dE_2 \delta(E_1 + E_2 - \Delta_0) \int_{-1}^1 (\vec{E}_1 \vec{E}_2 \cdot \hat{p}_1 \hat{p}_2) d\xi \quad (9)$$

where $\epsilon_0 = \Delta_0/m_p$, Δ_0 = available energy, $\xi = \hat{p}_1 \cdot \hat{p}_2$. For relativistic positrons $f(\epsilon_0) = \epsilon_0^5/30 \approx 1.1$.

Before proceeding further, we notice the unpleasant feature of a δ function interaction in Eq. (7) which is a remnant of the fact that we treated the protons as pointlike particles. This is unpleasant because, in the presence of short-range correlations, it will make the nuclear matrix element small and not reliably calculable. Indeed if one writes, for the two-nucleon wave function

$$\psi_{cor}(\vec{x}_i, \vec{x}_j) = [1 - c(|\vec{x}_i - \vec{x}_j|)] \psi_{unc}(\vec{x}_i, \vec{x}_j) \quad (10)$$

and uses simple but fairly realistic correlation functions of the type ^{10), 11)}

$$c(r) = e^{-ar^2} (1 - br^4), \quad a = 1.1 \text{ fm}^{-2} \quad b = .68 \text{ fm}^{-2} \quad (11)$$

or

$$c(r) = \begin{cases} 1 & r < r_c \\ 0 & r > r_c \end{cases} \quad r_c = \text{hard core radius} \approx 0.4 \text{ fm} \quad (12)$$

one finds that the nuclear matrix element vanishes. This situation is not catastrophic, however, since we know that nucleons have a size which, for our purposes, can be taken into account by giving them a form factor. This form factor is adequately represented by a dipole shape ^{11), 12)}

$$F(q^2) = \frac{1}{(1 - q^2/M_A^2)^2}, \quad M_A \approx 0.85 \text{ GeV} \quad (13)$$

By introducing such a form factor into the relevant Fourier integral, we find that Ω becomes

$$\Omega = \sum_{i \neq j} \tau_{-}(i) \tau_{-}(j) \frac{M_A^3}{64\pi} e^{-\frac{1}{2} M_A |\vec{x}_i - \vec{x}_j|} \quad (14)$$

This operator is of finite range. We shall see later that its matrix elements are non-vanishing in the presence of short-range correlations and do not sensitively depend on the exact form of such correlations. The introduction of the form factor of Eq. (13) also modifies the phase space function by introducing inside the integrals the factor

$$F(\vec{Q}^2) = \frac{1}{(1 + \vec{Q}^2/M_A^2)^2}, \quad \vec{Q}^2 = \vec{p}_1^2 + \vec{p}_2^2 + 2\vec{p}_1 \cdot \vec{p}_2 \quad (15)$$

The effect of this modification is small [$f_0(\epsilon_0)$ is reduced by a factor of $I_8(\epsilon_0) \approx 1.5$].

In order to estimate the lifetime for process (1), the nuclear matrix elements must be evaluated. For the experimentally interesting transitions to discrete final states, the uniform nuclear density approximation previously employed ⁵⁾ is not adequate since only protons near the nuclear surface are going to participate. We thus have to resort to the nuclear shell model. We then write

$$\langle f | \Omega | i \rangle = \langle \psi_{nf} | \psi_{ni} \rangle \sum_{\alpha} \langle (z-2)f; \alpha | \{ z_i \} \rangle \langle 0 | \Omega | \alpha \rangle \quad (16)$$

where ψ_{ni} and ψ_{nf} are the neutron wave functions of the initial and final states and $|\alpha\rangle$ is a short-hand notation for the possible two-proton configurations which, in the harmonic oscillator basis, takes the form

$$|\alpha\rangle = |[n\alpha l_{\alpha} l_{\alpha}; n'_{\alpha} l'_{\alpha} l'_{\alpha}] JM\rangle \quad (17)$$

The expression $\langle (z-2)f; \alpha | \{ z_i \} \rangle$ is a numerical coefficient (cfp) which expresses the initial proton wave function as a properly antisymmetrized product of the configuration $|\alpha\rangle$ and the final proton wave function $|(z-2)f\rangle$. The essential physics is contained in the matrix element $\langle 0 | \Omega | \alpha \rangle$ which takes the form

$$\langle 0 | \Omega | \alpha \rangle = \delta_{J_0} \delta_{J_2 J_2'} \delta_{l_2 l_2'} \sqrt{\frac{2J_2+1}{2}} I_{n_\alpha n_\alpha' l_\alpha} \quad (18)$$

where $I_{n_\alpha n_\alpha' l_\alpha} = \langle n_\alpha l_\alpha | \Omega | n_\alpha' l_\alpha \rangle$. With a δ function interaction ($M_A = \infty$) we get $\delta_{n_\alpha n_\alpha'}$ or zero without or with short-range correlations, respectively. In the case of the operator of Eq. (14) ($M_A < \infty$) these matrix elements cannot be given analytically. We find that the off-diagonal matrix elements ($n_\alpha \neq n_\alpha'$) are less than 20% of the corresponding diagonal ones. Furthermore, the presence of configurations with $n_\alpha \neq n_\alpha'$ in the ground states of relevant nuclei will be small because of their energies. We can thus completely neglect their contribution. Typical diagonal matrix elements are given in the Table. We notice that they do not sensitively depend on the correlation used and that short-range correlations do not reduce them by more than 50% of the maximum δ interaction value.

We are now in position to estimate the mean lifetime for process (1) from the relation

$$\tau = 2 \times 10^{-29} \text{ yr} \frac{1}{(\Lambda \kappa)^2} \frac{1}{|\langle f | \Omega | i \rangle|^2} I_0(\xi_0) \quad (19)$$

where $I_0 = 1$ and 1.5 for $M_A = \infty$ and 0.85 GeV, respectively. For purposes of general orientation, we can assume an average nuclear matrix element of 0.5. Then from the existing experimental limit ^{5), 13)} $T_{1/2} > 2 \times 10^{23} \text{ yr}$ we extract $(\Lambda \kappa) \leq 10^{-26}$. Taking $\kappa \sim 10^{-4}$ we find $\Lambda \leq 10^{-22}$ which is clearly met by both models described above. Clearly, there is need for improving the experimental bound.

From the above discussion, we see that the time scale of the Mohapatra-Senjanović model is 10^{33} yr (10^{28} yr if their own parameters are adopted) and 10^{40} yr for the Arnellos-Marciano model. Assuming that these estimates will not discourage the experimentalists we will present estimates for some nuclear systems which we can simply treat theoretically. Initial systems with two protons outside the closed shell or final nuclei with two-proton holes are preferred. Also we would prefer orbitals which have as large as possible angular momentum [see Eq. (18)]. We do not, however, expect dramatic nucleus dependent changes.

We focus specifically on this system because experiments involving large Ni quantities are currently planned to detect low energy ($\sim 0.08 \text{ MeV}$) positrons ¹⁴⁾ originating from the lepton violating ${}^{58}_{30}\text{Ni} \rightarrow {}^{56}_{28}\text{Fe} + e^+ + e^+$ decay.

Admittedly the lifetime for the latter process is much shorter ¹⁵⁾ ($T_{1/2} \approx 10^{29}$ yr), but the background problems are much more formidable. We thus suggest that the experimentalists also look for the high-energy positrons originating from process (1). The initial nucleus can be thought of as two neutrons outside the $^{56}_{28}\text{Ni}$ closed shell, while the final nucleus can be thought of as two neutron particles and two proton holes in the $1f_{7/2}$ shell. If C_0 is the amplitude of the $J = 0$ proton hole pair, we get

$$\langle \psi_{nf} | \psi_{ni} \rangle \langle (z-r)f; \alpha | \{ z_i \} \rangle = C_0$$

Even though it is not very difficult to calculate C_0 directly, we will take the value $C_0 = 0.8$ already available for a similar system ¹⁶⁾. We thus find $\langle f | \Omega | i \rangle = 1.6, 0.9, 1.0$ and 1.1 for δ interaction and no form factor, form factor and correlation of Eq. (11), form factor and correlation of Eq. (12), and form factor but no correlation, respectively. The corresponding lifetimes are 8×10^{32} yr, 4×10^{33} yr, 3×10^{33} yr and 2×10^{33} yr. These mean lifetimes must be decreased by a factor of 3×10^5 if the parameters of Ref. 6) are used but they should be increased by a factor of 10^7 if the model of Ref. 7) is used.

Here the two-proton $J = 0$ component in the initial state has an amplitude ¹¹⁾ of $C_0 = 0.85$. Thus the results are the same as above.

Here $C_0 \approx 1.0$ and everything else is the same as above so the predicted lifetimes are those of i) multiplied by 1.5.

Both the initial and final nuclei are believed to have neutrons in closed shells. The final system consists of protons in closed shells while the initial system is characterized by proton wave function of the form

$$\psi_p = a p_{1/2}^2 + \sqrt{1 - |a|^2} g_{7/2}^2$$

Phenomenological as well as realistic interactions predict similar values for a . Using the interaction of Goekler and Serduke ¹⁷⁾, which describes other data very well, we obtain $a = -0.81$ and thus $\langle f | \Omega | i \rangle = -0.81 I_{221} + 0.58 \sqrt{s} I_{1,1,4}$. The two orbitals subtract from each other and yield

$\langle f|\Omega|i\rangle = 0.50, 0.18, 0.23$ and 0.24 (in the above order), i.e., mean lifetimes 8×10^{33} , 8.0×10^{34} , 8.0×10^{34} and 7.0×10^{34} yr, respectively.

With the final proton wave function given above, the neutron wave function is unchanged and the proton wave function of the initial state is obtained by using the same interaction as above, i.e.,

$$\psi_p = .93 \, p_{\frac{1}{2}}^2 \, g_{\frac{1}{2}}^2 + .37 \, g_{\frac{1}{2}}^4$$

We obtain $\langle f|\Omega|i\rangle = 1.38, 0.64, 0.77$ and 0.80 which yield lifetimes 1.0×10^{33} , 7.0×10^{33} , 5.0×10^{33} , 5.0×10^{33} yr. A somewhat smaller rate is expected for transitions to the first excited 0^+ state.

In conclusion, we can say that the rates for process (1) depend on nuclear parameters which can be fairly accurately calculated. The ME given above will enter regardless of the gauge model. Gauge theory models permit lifetimes of 10^{33} yr which may be observable in the planned $\beta\beta^+$ experiments. We should not forget, however, that our predictions depend on large powers of the masses and couplings involved in the Higgs sector (16^{th} and 8^{th} powers, respectively). So even small changes in those parameters can drastically modify our predictions.

$n_{\alpha}^{\ell_{\alpha}}$	$1s$	$1p$	$1d$	$2s$	$1f$	$2p$	$1g$	$2d$	$3s$	$1h$	$2f$	$3p$	$1i$	$2g$	$3d$	$4s$
$J_{n_{\alpha}^{\ell_{\alpha}}}$																
C_1	0.943	0.651	0.591	0.603	0.536	0.553	0.489	0.509	0.518	0.446	0.469	0.482	0.407	0.431	0.448	0.455
	0.820	0.751	0.689	0.700	0.634	0.650	0.585	0.605	0.613	0.541	0.563	0.576	0.500	0.525	0.541	0.651
C_2	0.703	0.628	0.561	0.576	0.503	0.524	0.483	0.476	0.487	0.436	0.434	0.450	0.366	0.422	0.428	0.423
	0.805	0.727	0.659	0.673	0.600	0.620	0.547	0.571	0.582	0.500	0.528	0.543	0.459	0.487	0.507	0.515
	0.861	0.783	0.715	0.730	0.655	0.678	0.603	0.627	0.638	0.556	0.583	0.599	0.514	0.542	0.562	0.570

TABLE - The matrix elements $\langle n_{\alpha}^{\ell_{\alpha}} | \Omega | n_{\alpha}^{\ell_{\alpha}} \rangle$ for the indicated harmonic oscillator orbits for two values of $\hbar\omega = 41 \text{ A}^{-1/3}$. C_0 (no correlations) and C_1, C_2 with correlations are given by Eqs. (11) and (12), respectively. These integrals should be compared to those of a δ interaction which are 1 and 0 for the case of no correlation and correlation, respectively.

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FIGURE CAPTIONS

- Fig. 1 A diagram contributing to the $p + p \rightarrow e^+ + e^+$ decay in the framework of the $SU_L(2) \times U(1) \times SU_C(3)$ model of Ref. 6). For notation, see text.
- Fig. 2 The same process as in Fig. 1, but in the framework of the $SU(5)$ model of Ref. 7).

- Figure 1 -

- Figure 2 -