Ref.TH.3373-CERN

m_b/m_τ IN SUPERSYMMETRIC GUTS: SUPER FLAVOUR-METER

D.V. Nanopoulos
CERN - Geneva

and

D.A. Ross
CERN - Geneva
and
Department of Physics, University of Southampton,
Southampton SO9 5NH, U.K.

A B S T R A C T

We calculate the ratio m_b/m_τ up to two loops in supersymmetric Grand Unified Theories. Agreement between theory and experiment imposes a severe limit on the number of flavours: there should be at most six flavours.

Ref.TH.3373-CERN
29 July 1982

1. The world appears to be made of matter constructed out of the first generation of quarks and leptons only. On the other hand, we know from experiment that there are at least three generations of fermions and thus we are confronted with the so-called "generation puzzle". Clearly it is crucial, before embarking on searches for a theory of flavour, to know the precise number of generations. Interestingly enough, such a question may be answered in the framework of Grand Unified Theories^{1),2)}. It is well known that one of the successful predictions of Grand Unified Theories (GUTs), namely the estimate of the ratio m_b/m_τ ²⁾, will be badly broken if the number of flavours is increased beyond six (or equivalently if the number of generations is increased beyond three)^{3),4)}. However, we know that GUTs suffer from the unpleasant gauge hierarchy problem. One possible (if not the only) way out is the use of supersymmetry⁵⁾. Supersymmetry, because of the so-called "no-renormalization theorem"⁶⁾, solves the gauge hierarchy problem, at least in its technical form. However, the price is high. Every particle of our "low-energy" world now has a supersymmetry (SUSY) partner with masses in the region of 100 - 1000 GeV, otherwise supersymmetry is too badly broken to solve the gauge hierarchy problem. One would have thought that because all these SUSY particles now enter in the evolution of the renormalization group equations, the successes of ordinary GUTs would thereby be destroyed. But it turns out that the values of $\sin^2\theta_w$ and m_b/m_τ (calculated at the one-loop level) are still inside acceptable ranges^{7),8)} while the grand unification scale is postponed by at least two orders of magnitude⁷⁾⁻⁹⁾. Now one naturally wishes to know what happens to the previously "strong" limit on the number of generations^{3),4)} in the framework of SUSY GUTs¹⁰⁾.

In this note we calculate the ratio m_b/m_τ in minimal supersymmetric GUTs at the two loop level. Our results show that even in the framework of SUSY GUTs this limit on the number of flavours still holds and is even strengthened compared with the case of ordinary GUTs (see Tables 1 and 2).

Before getting to the details of our calculation we mention a few words of caution. We once again emphasize why we concentrate our attention on m_b/m_τ only and not on other ratios such as m_s/m_μ or m_d/m_e . In the minimal SU(5) model with a 5 (and $\bar{5}$) of Higgs scalars one gets an "acceptable" value for m_s/m_μ but certainly a "wrong" value for m_d/m_e . We do not believe that this is a problem. The first generation has a current mass of only a few MeV and since there are a number of extra sources for this current mass (apart from the direct $\bar{5}_f 10_f \bar{5}_H$ coupling) which can easily provide a few MeV to all current masses, the relation $m_d = m_e$ at the grand unification scale is badly broken (whereas the relation $m_s = m_\mu$ is only modestly broken and $m_b = m_\tau$ is hardly broken at all) by these contributions of a few MeV.

Therefore, before one can say anything about the masses of the first generation of fermions, all these extra sources must be taken into account. Actually there are models which give very satisfactory results after the masses of the first generation have been treated correctly¹¹⁾. Now we discuss some of the details of the calculation.

2. The one-loop contribution to m_b/m_τ in SUSY GUTs has been calculated previously^{7),8)}. It was found that the anomalous dimension is reduced due to the interaction involving scalar quarks (squarks) and fermion gluons (gluinos). However, the one loop β is reduced even more so that the ratio γ/β is increased. This is almost exactly compensated by the fact that α_{GUM} is larger in SUSY GUTs so that the quantity $(\alpha_{\text{GUM}}/\alpha_{\text{SSB}})^{\gamma/2\beta}$, where α_{SSB} is the value of the strong interaction coupling constant at the supersymmetry breaking scale, is almost unchanged by the introduction of supersymmetric partners to the gluon and quark. In Refs. 7) and 8) only the contributions to the masses due to the SU(3) sector were considered. As is well known, the SU(2)_L and U(1) sectors give contributions to the anomalous dimension of the mass operator which are not gauge invariant and therefore ill defined. However, at the one-loop level the contributions to m_b and m_τ from the SU(2)_L subgroup and the axial vector part of the U(1) interactions are identical, leaving only the vector part of the U(1) interactions which give a (gauge invariant) reduction of 10% to m_b/m_τ , as has been pointed out by Buras, Ellis, Gaillard and Nanopoulos²⁾ (BEGN). This is also almost unaffected by the introduction of supersymmetry.

In order to avoid possible problems of gauge invariance from the treatment of the SU(2) and U(1) sectors at the two-loop level we confine our discussion of the two-loop effects to the SU(3) (strong interactions) sector only^{*)}. We are justified in doing this since the weak interactions only contribute to the evolution of the effective masses for momenta $> 2M_W$, by which time the SU(3) coupling constants are already quite small, so we expect that their contribution at the two-loop level will be negligible.

The two-loop anomalous dimension for the mass operator in ordinary GUTs has been calculated in a previous paper³⁾ and the result confirmed by Tarrach⁴⁾. Here we concentrate on diagrams involving a squark or gluino (or both) in at least

^{*)} It has been pointed out Binétruy and Schücker¹²⁾ that even for the SU(3) sector there is a gauge ambiguity since the "physical" b quark mass is defined to be its effective mass at $q^2 = 4m_b^2$, which is not gauge invariant since it is an off-shell Green's function. However, the difference, for example, between performing the calculation in the Feynman and Landau gauges is negligibly small.

one of the two loops. Since the squark is very massive we have to deal with the problem of calculating an anomalous dimension in the presence of thresholds, so before quoting the result we introduce here a digression on the renormalization prescription ambiguities in this case.

A two-loop diagram which contributes to the anomalous dimension of some operator in which some of the particles in the internal loops have a mass which is larger than the external momenta (i.e., the scale at which the effective anomalous dimension is being calculated) can be treated within the dimensional regularization scheme in one of two different ways.

i) If the massive particle occurs in the external loop the whole of the contribution to the effective anomalous dimension is suppressed for external momenta below this mass. If, on the other hand, the massive particle occurs in the internal loop and the infinity from this loop is subtracted using dimensional regularization (in the M.S. or $\overline{\text{M.S.}}$ scheme) then there will be some remnant of the contribution from this diagram even at momentum scales way below this mass. This is a reflection of the fact that in dimensional regularization there is a finite part left over even when the masses of the particles in the loop are much larger than the external momenta (in contrast to the "mom" scheme in which the whole contribution is subtracted for momenta much smaller than the mass). For diagrams in which there are overlapping divergences there will be some part which is Appelquist-Carrazzone suppressed and some part which is not. This is strictly the correct and consistent way to apply dimensional regularization and it was the method used by us in a previous paper⁴⁾.

ii) Use the "mom" scheme to subtract off infinities arising from loops in which there is a particle whose mass is larger than the external momentum (choosing the subtraction scale to be very much smaller than this mass). In this case all the contributions from a diagram in which there is a massive particle somewhere in one of the loops is Appelquist-Carrazzone suppressed for external momenta below this mass. This is the method we use in this paper since it is much simpler (although we should point out that numerically the difference between the two schemes is negligible).

Each of these two schemes must be applied in conjunction with a definition of the coupling constant which is compatible with that scheme. Thus for scheme (i) the coupling constant must include the finite remnant obtained by considering all the loops containing massive particles after subtraction of the infinities using dimensional regularization. In scheme (ii) the effective coupling constant is calculated by ignoring all loops containing particles whose masses are larger than the momentum scale at which the effective coupling constant is being

calculated (this is what is usually done). When considering the evolution of some quantity from momentum scale q_1 to q_2 , the difference between the effective coupling constants $\alpha(q_1) - \alpha(q_2)$ is renormalization scheme independent. However, the quotient $\alpha(q_1)/\alpha(q_2)$ is affected by the choice of renormalization scheme in order α . When this is raised to the power of $\gamma_0/2\beta_0$, where γ_0 is the one-loop anomalous dimension, the two schemes give rise to a difference which exactly cancels the difference arising from the calculation of the two-loop anomalous dimension in those two schemes.

Returning now to the two loop anomalous dimensions for SUSY GUTs we find, after combining our results with those of Ref. 3), that above the supersymmetry breaking threshold M_{SSB} (defined to be twice the mass of the squarks) the anomalous dimension of the fermion mass operator due to the strong interactions is given (up to order α^2) by

$$\gamma_m = -C_F \left(\frac{\alpha}{\pi}\right) - C_F \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{25}{12} C_A + \frac{C_F}{16} - \frac{2}{3} T_R \right) + \dots \quad (1)$$

where for $SU(3)$, $C_F = 4/3$, $C_A = 3$ and $T_R = 1/2 \times \text{number of flavours}$. The higher order correction to the leading order expression for an effective mass is given by $(1 + (\alpha/\pi)R)$ where

$$R = \frac{\gamma_1}{2\beta_0} - \frac{\beta_1 \gamma_0}{2\beta_0^2} + f_i \quad (2)$$

where β_0 and β_1 are the coefficients of α/π and $(\alpha/\pi)^2$ (respectively) in the expansion for β , γ_0 and γ_1 are the coefficients of α/π and $(\alpha/\pi)^2$ (respectively) in the expansion for γ_m , and f_i is the coefficient of α/π in the finite part of the effective mass, calculated to one-loop in the dimensional regularization scheme. This is $3C_F/4$ above the supersymmetry threshold in the $\overline{MS}$ scheme (compared with $5C_F/4$ for ordinary GUTs). Since we have a threshold at M_{SSB} below which the supersymmetry is broken we must write the full correction factor as

$$1 + \left(\frac{\alpha(2m_b)}{\pi} - \frac{\alpha_{SSB}}{\pi} \right) R_- + \left(\frac{\alpha_{SSB}}{\pi} - \frac{\alpha_{GUT}}{\pi} \right) R_+ \quad (3)$$

where R_+ and R_- are the values for R given by Eq. (2) using the values of $\gamma_0, \gamma_1, \beta_0, \beta_1$ above and below the supersymmetry breaking threshold, respectively. The values of $\gamma_0, \beta_0, \beta_1$ above the supersymmetry breaking threshold are given in Ref. 7). We find in fact that the largest correction above the supersymmetry breaking threshold arises from the term $\beta_1 \gamma_0 / \beta_0^2$. This is because for SUSY GUTs β_0 is much smaller than in ordinary GUTs and β_1 is large for six or more flavours (for five to eight flavours β_1 and β_0 are of opposite sign). Moreover, β_1 and β_2 are very sensitive to the number of flavours (note that $\beta_0 = 0$ for nine flavours) and it is this that gives rise to a very sensitive behaviour for m_b/m_τ on the number of flavours present.

Before presenting our numerical result we mention one complication in principle. At the two-loop level there is one diagram which contains gluinos but no squarks, namely the one for which the quark gives off a gluon whose propagator is renormalized by a gluino loop. The supersymmetric breaking scale M_{SSB} has been defined to be twice the squark mass, whereas it is perfectly possible that the gluino mass is very much smaller¹⁴⁾. In that case this one diagram would contribute all the way down to the b quark mass. However, in practice the contribution from this diagram is sufficiently small so that the extra contributions obtained when it acts from m_b rather than M_{SSB} is not numerically significant.

3. Our numerical results are presented in Tables 1 and 2 in which the ratio m_b/m_τ is shown for several different values of the supersymmetry breaking scale, M_{SSB} . In each case the values in the case of six and eight flavours are shown. The values of $\Lambda_{\overline{M.S.}}$ which have been taken are 150 MeV for Table 1 and 450 MeV for Table 2. The 10% reduction in the ratio m_b/m_τ arising from the $U(1)$ sector has been taken into account. It can be seen from the tables that in the case of six flavours the ratio is within acceptable limits for $\Lambda_{\overline{M.S.}} = 150$ MeV (which is now believed to be the correct value) and the results are a little too high for the larger value of $\Lambda_{\overline{M.S.}}$. On the other hand, in the case of eight flavours we obtain values which are unacceptably large even for $\Lambda_{\overline{M.S.}} = 150$ MeV. Furthermore, it can be seen from the substantial increase in m_b/m_τ as the supersymmetry breaking scale is reduced that (in the case of eight flavours) the interactions of the supersymmetry partners increase the value of m_b/m_τ even further.

Thus we conclude once again that if the successful prediction for m_b/m_τ in GUTs is to be preserved one cannot have more than six flavours and that this limit is further strengthened by the introduction of supersymmetry into GUTs.

Table 1

$\Lambda_{\overline{\text{M.S.}}} = 150 \text{ MeV}$		
$M_{\text{SSB}} \text{ (GeV)}$	$\frac{6 \text{ flavours}}{m_b/m_\tau}$	$\frac{8 \text{ flavours}}{m_b/m_\tau}$
100	3.03	3.80
500	2.93	3.55
1000	2.90	3.48
5000	2.85	3.33

Table 2

$\Lambda_{\overline{\text{M.S.}}} = 450 \text{ MeV}$		
$M_{\text{SSB}} \text{ (GeV)}$	$\frac{6 \text{ flavours}}{m_b/m_\tau}$	$\frac{8 \text{ flavours}}{m_b/m_\tau}$
100	3.79	4.99
500	3.62	4.60
1000	3.56	4.45
5000	3.47	4.21

REFERENCES

- 1) H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438;
H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.
- 2) A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135 (1978) 66.
- 3) D.V. Nanopoulos and D.A. Ross, Nucl. Phys. B157 (1979) 273.
- 4) D.V. Nanopoulos and D.A. Ross, Phys. Lett. B108 (1982) 351.
- 5) Y.A. Gol'fand and E.P. Likhtman, Pisma Zh. Eksp. Teor. Fiz. 13 (1971) ;
D. Volkov and V.P. Akulov, Phys. Lett. 46B (1973) 109;
J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
- 6) J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52;
J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413;
S. Ferrara and O. Piguet, Nucl. Phys. B93 (1975) 261;
M.T. Grisaru, W. Siegel and M. Roček, Nucl. Phys. B159 (1979) 420.
- 7) M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475.
- 8) J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B202 (1982)
- 9) S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681.
- 10) E. Witten, Nucl. Phys. B188 (1981) 513;
S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Zeit. für Phys. C11 (1982) 153.
- 11) J. Ellis and M.K. Gaillard, Phys. Lett. B88 (1979) 315;
R. Barbieri, D.V. Nanopoulos and D. Wyler, Phys. Lett. B103 (1981) 433.
- 12) P. Binétruy and T. Schücker, Nucl. Phys. B178 (1981) 293.
- 13) R. Tarrach, Nucl. Phys. B183 (1981) 384.
- 14) P. Fayet, Phys. Lett. B78 (1978) 417; Proceedings of the CERN Workshop
on "Supersymmetry Versus Experiment", ed. by D.V. Nanopoulos,
A. Savoy-Navarro and C. Tao, CERN preprint TH.3311/EP. 8263 (1982).