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PSEUDOSCALAR MESONS AND INSTANTONS

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A B S T R A C T

We discuss the QCD sum rules for the correlators of pseudoscalar currents with quantum numbers of π , K , η , η' mesons. In the framework of the instanton model of the QCD vacuum suggested earlier by the author, we calculate $SU(3)$ breaking effects such as K , η masses and $\eta\eta'$ mixing, as well as the $U(1)$ breaking effect, the η' mass. We conclude that all these phenomena are very well reproduced by this model.

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1. - INTRODUCTION

The particular topic of this paper is the violation of the $SU(3)$ and $U(1)$ symmetries in the nonet of pseudoscalar mesons π, K, η, η' . This is a long-standing problem of hadronic spectroscopy and, in a sense, the most difficult one. Such particles do not resemble other hadronic multiplets known so far. For example, strange and non-strange components are mixed so strongly that they produce almost ideal $SU(3)$ octet and singlet states.

Let us begin with the consideration of this problem in a somewhat wider context. The understanding of the hadronic spectrum (as well as all other problems of strong interaction physics !) can be reached only if we are able to understand the structure of non-perturbative phenomena in the QCD vacuum. This fundamental object is now studied in two ways. The method of QCD sum rules¹⁾ connects the properties of the vacuum with hadronic phenomenology, while the lattice Monte Carlo calculations try to calculate them from first principles of QCD. Although important information is obtained by both approaches, it is still rather restricted.

An important qualitative observation was made in Ref. 2) : "...hadrons are not all alike". The calculation of masses and couplings of lowest hadrons with different quantum numbers can be successfully made by means of several global parameters of the QCD vacuum, essentially $\langle 0 | (g G_{\mu\nu}^a)^2 | 0 \rangle$ and $\langle 0 | \bar{\psi}\psi | 0 \rangle$, the so-called gluon and quark condensates. However, for spin zero currents it is not so and one should have more detailed information about the structure of QCD vacuum. The most interesting channel of this type is the $J^P = 0^-$ quark current, since the corresponding pseudoscalar mesons are experimentally well known.

In Refs. 3) and 4), we suggested the so-called instanton model of the QCD vacuum. Its important physical property is the fact that rather strong fluctuations are separated by relatively "empty" space-time - the "twinkling" vacuum as it was called. The typical radius of the fluctuations was found to be about $\rho_c \approx 0.3 \div 0.4$ fm, which is relatively small compared with their average separation $R \sim 1$ fm.

Among other results obtained in the framework of such a model, we were able to show that taking into account such small fluctuations in the correlators of the pseudoscalar currents leads to reasonable results for the pion. We now consider other channels and study the $SU(3)$ breaking due to the non-zero mass of a strange quark, $m_s \approx 150$ MeV.

The important rôle of this parameter in non-perturbative QCD (as well as of the light quarks in general) was not previously recognized. Although m_s seems to be numerically small, we stress that non-perturbative phenomena in QCD are changed by an order of unity if one puts $m_s = 0$. This is due to the instanton nature of such phenomena, and the instanton density depends very strongly on quark masses due to the existence of 't Hooft zero modes ⁵⁾. This phenomenon implies that strong fluctuations of the gauge field are possible only if N_f pairs of light quarks are produced (or absorbed). It is important that such a phenomenon, discovered mathematically by 't Hooft, shows up in the real world.

So, on the one hand, the dependence on m_s is non-trivial, and on the other it is well described by the instanton model. We consider this as strong support for the model, although the more sceptical reader may say that SU(3) breaking effects are not very important at all. However, some non-trivial details often play a crucial rôle. As an example somewhat resembling our subject, let us mention the isotopic effect of the superconducting phase transition - a very small, but historically very important.

2. - CHIRAL SYMMETRY BREAKING

This section is also introductory ; it contains known facts to be used later and a discussion of previous works on the subject.

The chiral $U(N_f) = U(1) \otimes SU(N_f)$ symmetry of the massless theory is known to be spontaneously broken by the QCD vacuum. Due to the Goldstone theorem, massless pseudoscalar mesons should therefore exist. The explicit manifestation of the symmetry breakdown is the non-zero average of the $\bar{\psi}\psi$ operator, the so-called quark condensate.

Its value can be connected with the pion mass by means of the PCAC

$$\begin{aligned} \langle 0 | \bar{u}u | 0 \rangle &= \langle 0 | \bar{d}d | 0 \rangle = - \frac{1}{2} f_\pi^2 K \\ K &\equiv \frac{m_\pi^2}{m_u + m_d} \quad ; \quad f_\pi \simeq 133 \text{ MeV} \end{aligned} \quad (1)$$

Note that in the chiral limit $m_u, m_d \rightarrow 0$, $m_\pi^2 \rightarrow 0$ the quantity K remains finite.

Unfortunately, we have now only rough estimates of the values of quark masses [see, for example, the recent discussion in Ref. 1) and references therein]. If we define the renormalization invariant masses as in 1) *)

$$\hat{m} = m(\mu) \left[\ln\left(\frac{\mu}{\Lambda}\right) \right]^{\frac{4}{\beta}} ; \quad \beta \equiv 11 - \frac{2}{3} N_f \quad (2)$$

(where μ is the normalization point) the value of the quark masses is estimated as

$$\hat{m}_u + \hat{m}_d = 10 \div 15 \text{ MeV} \quad (3)$$

The corresponding values of K and $\langle \bar{\psi}\psi \rangle$, normalized in the same way, are

$$K = 1 \div 1.5 \text{ GeV} \quad \langle 0 | \bar{\psi}\psi | 0 \rangle = -1 \div 1.5 \cdot 10^{-2} \text{ GeV}^3 \quad (4)$$

Note the quite large value of K : one may say therefore that the pion is surprisingly heavy !

The chiral symmetry $U(N_f)$ can be decomposed into two separate symmetries, $U(1)$ and $SU(N_f)$. The need to violate the $U(1)$ one was first stressed by Weinberg ⁶⁾, based on the phenomenological observation that the η' meson (the ninth member of the pseudoscalar nonet) is too heavy. The mechanism of its violation was first suggested by 't Hooft ⁵⁾, who discovered that the instanton-generated effective interaction between light quarks does not have this symmetry.

However, the 't Hooft interaction respects $SU(N_f)$ chiral symmetry and therefore one has to invent other mechanisms for its violation. It was suggested by Callan, Dashen and Gross ⁷⁾ that the instanton and anti-instanton together create a strong attraction of quarks and antiquarks, which, if sufficiently strong, will cause the spontaneous breakdown to occur.

This question was considered on a more quantitative level in Ref. 4) in the framework of the instanton model of the vacuum. It has been shown that such a breakdown takes place and the value of $\langle 0 | \bar{\psi}\psi | 0 \rangle$ found are in reasonable agreement with the empirical value.

This result can also be formulated as a generation of the effective quark mass :

*) Or, in other words, normalized at μ_0 such as $\alpha_s(\mu_0) = 0.7$, which is just a convention.

$$m_* \simeq 2\pi g_c (n_c/3)^{1/3} \simeq 200 \text{ MeV} \quad (5)$$

where n_c is the total instanton density, and ρ_c is its typical radius. The numerical value (5) follows from the following values of n_c and ρ_c

$$n_c \simeq 0.8 \cdot 10^{-3} \text{ GeV}^4 \quad g_c \simeq 2 \text{ GeV}^{-1} \quad (6)$$

to be used below. The value of n_c follows directly from the value of $\langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle$ found in Ref. 1), and ρ_c can be considered as the free parameter *) to be phenomenologically determined later.

In Refs. 3) and 4) it was argued that the chiral symmetry is restored at some temperature T_{chiral} which is higher than that of the deconfinement transition. This conclusion is supported by recent results ⁸⁾ of the Monte Carlo lattice calculations with fermions.

3. - QCD SUM RULES FOR PSEUDOSCALAR MESONS

The method of QCD sum rules, developed in Ref. 1), connects in a most natural way the properties of the QCD vacuum and hadrons.

In the case under consideration such a current has the following structure :

$$j = \left[\ln \left(\frac{\mu}{\Lambda} \right) \right]^{-\frac{4}{8}} \sum_{F_1 F_2} C_{F_1 F_2} \left[\bar{\psi}_{F_1} (i\gamma_5) \psi_{F_2} \right]_{\mu} \quad (7)$$

where μ is the normalization point and ψ is the field of some light quark u, d or s . We do not show explicitly the flavour structure, which, for the π^0 channel, should read

$$C_{uu} = -C_{dd} = \frac{1}{\sqrt{2}} \quad (8)$$

Coefficients $C_{F_1 F_2}$ are standard and normalized to unity.

*) In fact, it was estimated in Ref. 4) in some particular model for the "instanton liquid" with the result about 20% smaller.

The important point of Ref. 1) was the consideration of the operator product expansion (OPE) method, including the non-trivial operators. For the π^0 current (8) such a result looks like

$$\begin{aligned} \Pi(q) &\equiv i \int dX e^{iqX} T \{ j(X) j(0) \} = \\ &= \left\{ \frac{3}{8\pi^2} Q^2 \ln \frac{Q^2}{\mu^2} - \frac{1}{2Q^2} (m_u \bar{u}u + m_d \bar{d}d) + \right. \\ &\quad \left. + \frac{\alpha_s}{8\pi Q^2} (G_{\mu\nu}^a)^2 + \frac{1}{Q^4} (O_1 + O_2) + \dots \right\} \left[\ln \left(\frac{\mu}{\Lambda} \right) \right]^{-\frac{8}{b}} \quad (9) \end{aligned}$$

where the four-fermion operators O_1 and O_2 are as follows

$$\begin{aligned} O_1 &= -\pi \alpha_s (\bar{u} \gamma_{\mu\nu} t^a u) (\bar{d} \gamma_{\mu\nu} t^a d) \\ O_2 &= \frac{\pi \alpha_s}{2} \left[(\bar{u} \gamma_{\mu\nu} t^a u)^2 + (\bar{d} \gamma_{\mu\nu} t^a d)^2 \right] + \\ &\quad + \frac{\pi \alpha_s}{3} (\bar{u} \gamma_{\mu} t^a u + \bar{d} \gamma_{\mu} t^a d) \left(\sum_{q,u,d,s} \bar{q} \gamma_{\mu} t^a q \right) \quad (10) \end{aligned}$$

We remind the reader that the calculation of the OPE coefficients is carried out perturbatively, while the vacuum average values of the operators contain the unknown non-perturbative structure of the QCD vacuum.

Similar expressions for vector and axial currents ¹⁾, baryonic currents ⁹⁾, and for charmonium ¹⁾ and upilon ¹⁰⁾ families, mesons ^{11),12)} and baryons ¹¹⁾ with heavy quark, produced a reasonable description of the current correlator, which can be seen from correct values of the masses and couplings of the lowest state found from such expressions.

The standard procedure includes the so-called Borel transformation of the correlator ¹⁾

$$\Pi(\tau) = \lim_{\substack{n \rightarrow \infty \\ Q^2 \rightarrow \infty \\ \tau \equiv \frac{n}{Q^2} = \text{fixed}}} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2} \right)^n \Pi(Q^2) \quad (11)$$

which leads to the following form of dispersion relations

$$\Pi(\tau) = \frac{1}{\pi} \int_0^{\infty} \text{Im} \Pi(s) e^{-s\tau^2} ds \quad (12)$$

The exponential weight in (12) is useful once it suppresses higher states. In all cases mentioned so far the following simple form of the physical spectral density $\text{Im}\Pi(s)$ was assumed

$$\text{Im}\Pi(s) = \pi\lambda^2 \delta(s-m^2) + \theta(s-s_0) \text{Im}\Pi^{\text{pert}}(s) \quad (13)$$

which incorporates its main features : (i) the existence of some lowest resonance with mass m and coupling λ ; (ii) approach to the perturbative result at large s , as demanded by asymptotic freedom. Such a shape of $\text{Im}\Pi$ is close to that observed in e^+e^- annihilation into hadrons.

The last step towards the final formulation of the sum rules is the cancellation of terms singular at $\tau \rightarrow 0$ in both sides of (12), when $\text{Im}\Pi$ is substituted in the form (13). The resulting expression looks like

$$\frac{1}{\pi} \int_0^{s_0} \text{Im}\Pi^{\text{pert}}(s) e^{-s\tau^2} ds + \Pi^{\text{nonpert}}(\tau) = \lambda^2 \exp(-m^2\tau^2) \quad (14)$$

Note that at $\tau \rightarrow \infty$ this becomes the so-called duality relation, connecting the free quark cross-section, integrated over some duality interval s_0 , with the coupling of the resonance.

For the pseudoscalar mesons under consideration, the value of the coupling can be written as

$$\lambda_{\pi} = \langle 0 | j | \pi \rangle = f_{\pi} K \quad (15)$$

where K is defined in (1).

It was noted that the duality interval is always of the order of the distance to the next resonance. In particular, it is so for the ρ meson, nucleon, isobar and other channels where the "primed" resonances are known [see Ref. 11].

For the pseudoscalar channel such "primed" excitations are the $\pi'(1340)$ $K'(1400)$ and presumably $\eta(1420)$ found recently in $\psi \rightarrow \pi + \gamma$ at SLAC. So we assume that for any pseudoscalar channel duality interval, s_0 is of the order of 2 GeV^2 .

The duality relation is as follows

$$\frac{3}{16\pi^2} s_0^2 \left[\ln\left(\frac{\sqrt{s_0}}{\Lambda}\right) \right]^{-\frac{8}{\theta}} \simeq f_\pi^2 K^2 \quad (16)$$

and we see that at $K \approx 1 \text{ GeV}$, $s_0 \approx 2 \text{ GeV}^2$ the right- and left-hand sides are indeed close.

We now come to the central point of this section : let us check whether the right- and left-hand sides of the sum rule (14) also agree for larger values of τ . Using the power corrections (9) in the Borel transformed form

$$\left[\ln \frac{1}{\tau \Lambda} \right]^{-\frac{8}{\theta}} \Pi(\tau) \stackrel{\text{nonpert.}}{=} \frac{1}{32\pi^2} \langle 0 | (g G_{\mu\nu}^a)^2 | 0 \rangle + \tau^2 \langle 0 | O_1 + O_2 | 0 \rangle + \dots \quad (17)$$

with the estimates of the operator averaged as suggested in Ref. 1) :

$$\begin{aligned} \langle 0 | (g G_{\mu\nu}^a)^2 | 0 \rangle &\simeq 0.5 \text{ GeV}^4 \\ \langle 0 | O_2 | 0 \rangle &= \frac{56\pi}{27} \alpha_s \langle 0 | \bar{\psi}\psi | 0 \rangle^2 \\ \langle 0 | O_1 | 0 \rangle &\ll \langle 0 | O_2 | 0 \rangle \end{aligned} \quad (18)$$

one can see that $\Pi^{(\text{power})}$ is too small to compensate the decrease of the perturbative term in (14) with the increase of τ .

Since the pion is massless, it is seen from (14) that $\Pi(\tau)$ should in fact be independent of τ at all its values. This is the contradiction noted in Ref. 2).

This situation is completely different from other cases mentioned above (e.g., that of the ρ meson) in which both sides of the similar sum rules accurately agree up to $\tau \sim 1.5 \text{ GeV}^{-1}$. This means that something important is missing.

4. - INSTANTON CONTRIBUTION TO PION SUM RULES

As discussed in detail in Ref. 4), it is not yet possible to determine the instanton contribution to any process relying only on first principles of QCD. However, a simple model for the instanton density was proposed

$$\frac{dn}{dg} \simeq n_c \delta(g - g_c) \quad (19)$$

containing two parameters n_c, ρ_c . These values can be determined from some data but are approximate, as given in Eq. (6).

The contribution of the individual instanton to the correlate of pseudoscalar currents has the following form :

$$\left[\ln\left(\frac{Q}{\Lambda}\right) \right]^{\frac{8}{\theta}} \Pi^{inst}(Q) = \left| \int dx e^{iqx} \bar{\psi}_0(x) (i\gamma_5) \psi_0(x) \right| \frac{2n_c}{m_*^2} \quad (20)$$

which is obtained directly by the substitution of 't Hooft zero modes $\psi_0(x)$ into the currents and integration over x and the instanton position. In other words, the quark propagator is, in this approximation, equal to

$$S(x, y) = \frac{\psi(x) \psi^+(y)}{-m_*} + O(m_*^0) \quad (21)$$

Corrections to Eq. (20) are of the order of $(m_* \rho_c)^2 \sim 10^{-1}$, so they can be neglected. Notice also that for vector currents due to chirality arguments, the whole effect of the instantons starts from $O(m_*^0)$ order and therefore is much smaller. The smallness of $(m_* \rho_c)$ is therefore the reason why no trace of the direct instantons was observed in other channels.

Another comment is that, due to Eq. (5), m_*^2 is itself the instanton effect and is proportional to n_c . Therefore the instanton contribution (20) depends essentially only on one parameter, the typical instanton size ρ_c .

The Borel transform of the contribution (20) can be made by writing the integral representation for the McDonald functions appearing in Eq. (20) :

$$\left[\ln\left(\frac{1}{\tau\Lambda}\right) \right]^{\frac{8}{\theta}} \Pi^{inst}(\tau) = \frac{2n_c}{\sqrt{2}m_*^2\tau^2} \int_0^\infty \int_0^\infty d\alpha d\beta ch\alpha ch\beta (z^5 t^3 - \frac{3}{2} z^3 t) \exp(-z^2 t^2) ; \quad z \equiv \frac{g_c}{\tau} ; \quad t \equiv \frac{ch\alpha + ch\beta}{2} \quad (22)$$

The corresponding contribution, together with the perturbative one, is shown in Fig. 1. It can be seen that it is really large and dominates over the power corrections for $\tau > 0.5 \text{ GeV}^{-1}$.

Another aspect of the instanton model of the QCD vacuum, also discussed in Ref. 4), deals with the four-fermion operator O_1 in Eq. (10). In contrast with the hypothesis made in Ref. 1) on the dominance of the vacuum intermediate state which predicts the smallness of $\langle 0|O_1|0\rangle$, in the instanton model this quantity is in fact rather large :

$$\langle 0|O_1|0\rangle = \frac{24\alpha_s}{5\pi} \frac{n_c}{m_*^2 g_c^4} \gg \langle 0|O_2|0\rangle \quad (23)$$

Note that such a contribution can be considered as the radiative correction to (22). In the co-ordinate representation, (22) is non-singular at the $x \rightarrow 0$ contribution [see, e.g., the propagator (21)], while the radiative correction develops the logarithmic singularity $\ln x^2$ and therefore shows up among power corrections.

Adding all the contributions to $\Pi(\tau)$ we obtain the curve marked "sum" in Fig. 1, which indeed only slightly depends on τ . It means that the lowest state in this channel is in fact massless, as it should be being due to the Goldstone theorem.

So, the disease of the pion sum rule is cured by the instanton contribution, together with the instanton-induced power correction. At large τ our formulae cannot be applied due to multi-instanton contributions.

The value of K following from this approach is about 0.7 GeV , somewhat smaller than estimated before but still acceptable taking into account the rather poor accuracy of existing estimates of the quark masses.

5. - SU(3) BREAKING IN QCD

The SU(3) symmetry of u, d, s quarks is broken by the strange quark mass $\hat{m}_s \approx 150 \text{ MeV}$, being essentially larger than $\hat{m}_u, \hat{m}_d$. However, the question of how m_s affects the structure of QCD vacuum and hadrons is very non-trivial. This is in fact the central point of the present work.

In many previous considerations it was assumed that m_s is, in a sense, a small parameter of the theory, so all quantities can be expanded in powers of m_s with some small corrections. As we discuss below, this is far from being true in cases when the influence of non-perturbative phenomena is large. This conclusion is rather non-trivial by itself, and we connect it with the fact that main fluctuations in the QCD vacuum have a topological charge and therefore fermion zero modes are related to them.

The first important observation concerning the unexpectedly large rôle of m_s in the QCD vacuum was made in Ref. 2), where the following low-energy theorem was derived :

$$\frac{d \langle 0 | (g G_{\mu\nu}^a)^2 | 0 \rangle}{d m_s} \Big|_{m_s=0} = - \frac{96 \pi^2}{8} \langle 0 | \bar{\psi} \psi | 0 \rangle \quad (24)$$

Substituting the known value of $\langle 0 | \bar{\psi} \psi | 0 \rangle$ into the right-hand side, one can see that the derivative is so large that in fact expansion in m_s up to 150 MeV cannot be trusted. It can be concluded that if m_s is zero, the value of the gluon condensate is about twice as small ²⁾.

On general grounds it was not expected that gluon field fluctuations have so strong a dependence on m_s . However, in the framework of the instanton model, the above conclusion is very natural ⁴⁾.

The instanton density is proportional to the fermion determinant in the instanton field, which contains the factor F

$$F = (m_u + m_*) (m_d + m_*) (m_s + m_*) g^3 \quad (25)$$

This follows from the 't Hooft calculation ⁵⁾, generalized to the case of non-zero m_* due to spontaneous chiral symmetry breaking by Shifman, Vainshtein and Zakharov ¹³⁾. So far as in our model m_* is estimated to be of the same order as m_s , and $\langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle$ is assumed to be generated by instantons, the above conclusion is understood.

We are now going to investigate similar phenomena of SU(3) breaking in the pseudoscalar sector, relying on the same structure (25) of the instanton induced effective Lagrangian.

We start with the current with kaon quantum numbers and discuss the changes in the sum rules considered in the previous section.

In the pion case, the zero modes of u, d quarks were used in the currents so $\Pi^{(inst)}$ was proportional to $(m_u + m_d)$, which was of course included automatically in n_c . In the case of the kaon, we need, say, zero modes of u, s quarks and therefore the instanton contribution (22) becomes smaller by the following amount

$$\zeta_K = \frac{m_u}{m_u + m_s} \simeq 0.6 \quad (26)$$

Also the instanton-induced power correction (23) should be multiplied by ξ_K .

The resulting curves are given in Fig. 2. We see that in contrast with the pion case, the correlator decreases with τ . For the spectrum it implies that in such an approximation ($m_s \neq 0$) the kaon is not massless. Comparing this with the one-resonance fit (see Fig. 2) we really observe good agreement with the experimental value of the kaon mass.

Note that the coupling λ_K is in fact very close to that of the pion

$$\lambda_K \simeq \lambda_\pi \simeq 10^{-1} \text{ GeV}^2 \quad (27)$$

which is in contrast with their masses. This is natural because couplings are connected with small "time" τ at which the perturbative phenomena are dominant. Evidently they depend little on m_s . However, with the increase in τ the non-perturbative phenomena come into play and the π, K correlators become very different at larger τ .

The same calculation can be done for the η meson, only the instanton contribution is now modified by the factor

$$\zeta_\eta = \frac{m_u - 1/3 m_s}{m_u + m_s} \simeq 0.4 \quad (28)$$

The results are displayed in Fig. 3. Again the η mass turns out to be right.

Note that in all cases considered so far, the agreement between the theoretical and the resonance curves holds up to $\tau \sim 1 \div 1.2 \text{ GeV}^{-1}$. This is natural since the non-perturbative effects become, at this point, comparable to the perturbative one, so they should be taken into account more accurately. Firstly, one should iterate them. Secondly, the power OPE corrections surely lose meaning in as far as τ becomes comparable with the instanton size ρ_c . Therefore, they should be summed up in some way.

6. - U(1) BREAKING AND $\eta\eta'$ MIXING

We now come to the two final points of our considerations :

- (i) the U(1) breaking, splitting η' from the SU(3) octet π, K, η ;
- (ii) one more effect due to $m_s \neq 0$, the $\eta'\eta$ mixing.

As we have already mentioned, the large mass of η' was a puzzle ⁶⁾. It was later discovered that the SU(3) singlet axial current is not in fact conserved due to the so-called anomaly relation

$$\partial_\mu \left(\sum_{u,d,s} \bar{\psi} \gamma_\mu \gamma_5 \psi \right) = \frac{3\alpha_s}{8\pi} G_{\mu\nu}^a G_{\sigma\tau}^a \epsilon_{\mu\nu\sigma\tau} \quad (29)$$

so the PCAC-type considerations cannot be applied to η' . This relation implies that η' is mixed with the gluonic world. As discussed in Ref. 2), it really plays an important rôle in the correlator of the gluonic pseudoscalar current [the right-hand side of (29)]. The contribution of the instanton to the correlate of the axial currents was also considered by Geshkenbein and Ioffe ¹⁴⁾. However, as we will show shortly, the consideration of the pseudoscalar (instead of axial) currents makes the calculations much simpler and more transparent.

Calculating the contribution of the instanton zero modes to the correlator of the SU(3) singlet current

$$j_{\eta'} = \frac{i}{\sqrt{3}} (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s) \quad (30)$$

one finds that the instanton-induced effects differ from those for a pion (22) by the factor

$$\xi_{\eta'} = - \frac{2}{3} \left(\frac{m_s + 3m_*}{m_s + m_*} \right) = - 1.4 \quad (31)$$

The minus sign is evident : the π^0 current is $i/\sqrt{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$ so the non-diagonal $\bar{d}d\bar{u}u$ term is of a different sign.

This situation is shown in Fig. 4. Negative correction makes the correlator to fall more quickly, which means higher mass of the particle. We compare the curves with one Gaussian with the η' mass. However the fit to the theoretical curve at $\tau \leq 0.8 \text{ GeV}^{-1}$ produces a somewhat larger value. Recently, the resonance τ was discovered at SLAC with the same quantum number as η' but with a mass $1420 \pm 10 \text{ MeV}$. As one can guess from $\psi \rightarrow \eta' \gamma$ or $\tau \rightarrow \eta' \gamma$ decay rates, such a state

may significantly contribute to the correlator (which is rather unusual for "primed" resonances). It is, of course, also possible that in this channel our approximation becomes invalid at somewhat smaller values of τ as compared with octet particles. So, one can conclude that instantons produce strong short-range attraction in the octet channel, making them lighter, and also strong repulsion in the η' case making it much heavier than "normal" particles like ρ meson.

This is, of course, because the instanton-generated interaction violates the $U(1)$ chiral symmetry explicitly, as was noted long ago by 't Hooft.

Our last point is the $\eta\eta'$ mixing, which is again the $SU(3)$ violating effect due to the non-zero value of m_s . For its consideration we have to consider the non-diagonal correlation between $SU(3)$ singlet and octet currents.

The calculations are made as above, and the corresponding factor of the instanton contribution is now

$$\zeta_{\eta\eta'} = \frac{\sqrt{2}}{3} \left(\frac{m_s}{m_s + m_*} \right) \simeq 0.2 \quad (32)$$

All other contributions like the perturbative term $O(m_s^2)$, the non-perturbative effect due to $m_s \langle 0 | \bar{s}s | 0 \rangle$ and four-fermion operators are also smaller in this case - note the scale in Fig. 5.

We also show in this figure the phenomenological contribution

$$\Pi_{\eta\eta'}(\tau) = 2tg\frac{\theta}{2} \lambda_\eta \lambda_{\eta'} (e^{-m_\eta^2 \tau^2} - e^{-m_{\eta'}^2 \tau^2}) \quad (33)$$

The coupling constants $\lambda_\eta, \lambda_{\eta'}$ are taken from fitted values found above, and the value of the mixing angle $\theta \simeq 12^\circ$ follows from the $SU(3)$ mass formula.

It is seen that the agreement is quite reasonable. The theoretical curve is somewhat steeper as compared with an experimental one. It can either be due to the admixture of higher states such as $1(1420)$, or to the crudeness of the delta function approximation for the instanton density. It is clear that with some distribution over the instanton size the theoretical curve becomes less steep.

We also make a comment on the discussion of $\eta\eta'$ mixing in Ref. 14). The formula for the instanton contribution in the non-diagonal correlator of axial $\eta\eta'$ currents is of the second order in m_s ¹⁴⁾. This is incorrect because

taking into account the chiral symmetry breaking the mixing already occurs in the first order in m_s , as is evident from (32). However, since m_s and m_* are numerically close, the results of Ref. 14) are qualitatively reasonable.

7. - DISCUSSION OF THE RESULTS

The instanton model suggested in Ref. 4) was shown to be able to describe well all phenomena connected with the pseudoscalar nonet of mesons, in particular the SU(3) breaking due to the non-zero strange quark mass and U(1) breaking.

As an important conclusion which results from these considerations let us mention the non-trivial rôle of m_s in non-perturbative QCD in general. It seems to us that this point was not realized before.

In particular, several Monte Carlo calculations on the lattice were reported, made in the approximation in which the virtual quark loops are neglected. In particular, Hamber and Parisi¹⁵⁾ were able to find good values for masses of vector mesons, baryons, etc.. These results are very impressive and they mean that, at least qualitatively, the global features of the QCD vacuum, such as the gluon and quark condensates, follow correctly from the first principles of QCD.

However, they have so far failed to describe the η' mass and mixing phenomena considered in our paper. The disagreement with data is also present for some spin splitting. In Ref. 16), by the same method, it has been shown that the value of the U(1) breaking parameter $A \equiv \int d^4x \langle Q(x)Q(0) \rangle$ (Q being a topological charge) is too small. We think that defects of all these calculations are similar : the lattice spacing a is not small enough to describe properly the small-scale instanton-type fluctuations. Further calculations are needed to improve this and to understand the short-range structure of the QCD vacuum.

REFERENCES

- 1) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Nuclear Phys. B147 (1979) 385.
- 2) V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Nuclear Phys. B191 (1981) 301.
- 3) E.V. Shuryak - Phys.Letters 107B (1981) 103.
- 4) E.V. Shuryak - "The Rôle of Instantons in QCD", Preprints INP 81-118, 18-132, 82-3. Nuclear Phys., to appear.
- 5) G. 't Hooft - Phys.Rev. D14 (1976) 3132.
- 6) S. Weinberg - Phys.Rev. D11 (1975) 3583.
- 7) C. Callan, R. Dashen and D.J. Gross - Phys.Rev. D17 (1978) 2717.
- 8) J. Kogut et al. - Phys.Rev.Letters 48 (1982) 1140.;
J. Engels, H. Satz and F. Karsch - Bielefeld Preprint BI-Tp 82/8 (1982).
- 9) B.L. Ioffe - Nuclear Phys. B188 (1981) 317.
- 10) M.B. Voloshin - Yadern. Fizika 29 (1979) 1368 ;
L.J. Reinders, M.R. Rubinstein and S. Yasaki - Nuclear Phys. B186 (1981) 109.
- 11) E.V. Shuryak - Nuclear Phys. B198 (1982) 83.
- 12) L.J. Reinders, H.R. Rubinstein and S. Yasaki - Phys.Letters 104B (1981) 305.
- 13) M.A. Shifman, V.I. Vainshtein and V.I. Zakharov - Nuclear Phys. 163B (1980) 43.
- 14) B. Geshkenbein and B.L. Ioffe - Nuclear Phys. B166 (1980) 340.
- 15) H. Hamber and G. Parisi - Phys.Rev.Letters 25 (1981) 1792.
- 16) P. Di Vecchia, K. Fabricius, G.C. Rossi and G. Veneziano - Nuclear Phys. B192 (1981) 392.

FIGURE CAPTIONS

- Fig. 1 The correlator of currents with π^0 quantum numbers $\Pi(\tau)$ (GeV^4) as of the Borel parameter τ (GeV^{-1}). The curves marked "Cont", " $\langle G^2 \rangle$ ", " O_1+O_2 " and "Inst" correspond to the perturbative contribution, first and second power corrections and the direct instanton contribution, respectively. The curve marked "sum" is their sum ; it should be compared with the constant (the line marked " π ").
- Fig. 2 The same as in Fig. 1, but for K meson.
- Fig. 3 The same as in Fig. 1, but for η meson.
- Fig. 4 The same as in Fig. 1, but for η' meson.
The negative instanton contribution is shown in the upper part of the picture.
- Fig. 5 Non-diagonal scalar-octet correlator ; all notations are as in Fig. 1.
The curve "sum" should coincide with the " $\eta\eta'$ " one, which is that coming from experiment.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -