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CALORIMETRIC MEASUREMENTS OF ^{163}Ho DECAY AS
TOOLS TO DETERMINE THE ELECTRON NEUTRINO MASS

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ABSTRACT

We compute the spectrum of "calorimetric" energy in the electron capture decay of ^{163}Ho . A calorimetric experiment would yield an excellent determination of the (^{163}Ho , ^{163}Dy) mass difference. The proximity of the spectral endpoint to an atomic resonance makes the fraction of events that are sensitive to a non-zero neutrino mass superior in ^{163}Ho decay than in Tritium decay.

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The most restrictive limits on electron antineutrino masses

$$m(\bar{\nu}_e) < 60 \text{ eV (90\% conf.)} \quad [\text{Bergkvist}^{1)}] \quad (1a)$$

$$14 \text{ eV} < m(\bar{\nu}_e) < 46 \text{ eV (99\% conf.)} \quad [\text{Lubimov et al.}^{2)}] \quad (1b)$$

result from the analysis of the shape of the electron energy spectrum³⁾ in ^3H β decay near its maximum energy endpoint. These magnetic spectrometer experiments^{1),2)} have an energy resolution comparable to, or larger than, the energy differences between the various atomic (or molecular) states in which the daughter $^3\text{He}^+$ ion (or ionized molecule) can be left in ^3H decay. Theoretical calculations of the branching ratios into these different final states are needed, since they affect the theoretical shape of the spectrum and thus the experimental $m(\bar{\nu}_e)$ results. This "molecular interplay" problem⁴⁾ can be dealt with either by trusting the theory, or improving the resolution and counting rate to such a level that the contributions of the different decay branches are observable, or performing an entirely different experiment where the problem is absent. J.J. Simpson has chosen the third route and obtains

$$m(\bar{\nu}_e) < 65 \text{ eV (95\% conf.)} \quad [\text{Simpson}^{5)}] \quad (2)$$

The idea is to implant the ^3H source in a $\text{Si}(\text{Li})$ crystal that measures all of the energy released in each decay, but that of the escaping neutrino. The "calorimetric" energy E_c and the neutrino energy E_ν add up to the Q value (mass difference between the ground state ^3H and ^3He atoms, modified by the small difference of their binding energies to the crystal). The value of E_c is the sum of the β ray energy and the de-excitation energy of the level in which the daughter atom (or daughter detector!) is left in the decay. The decay branch and de-excitation pattern chosen by a particular event do not affect the value of E_c : there are no atomic or molecular interplay problems.

In a decay process involving neutrinos, the smaller the energy release, the bigger the fraction of the spectrum that is sensitive to a fixed non-zero neutrino mass. In a calorimetric experiment, moreover, the smaller the Q value the better the energy containment, the smaller the radiation damage. Two nuclides are known to decay with a smaller energy release than ^3H . One of them, ^{163}Ho , decays by atomic electron capture into $^{163}\text{Dy} + \nu_e$. Two experiments have recently measured the lifetime and Q value of this decay, with the results^{*}:

^{*}) In quoting the results [Eqs. (3)] we have assumed that $(Q - E[M_1])^2 \ll m^2(\nu_e)$ where $E[M_1] \approx 2.05 \text{ keV}$ is the M_1 -electron binding energy in Dysprosium.

and its integral, which in an excellent narrow width approximation is:

$$W = \mathcal{N} \sum_H \psi_H^2(0) (Q - E_H) \sqrt{(Q - E_H)^2 - m_\nu^2} \quad (6)$$

The constant $\mathcal{N}$ drops from the normalized spectrum dW/WdE_C of interest to us, and contains the squares of the nuclear matrix element and Fermi coupling strength. The phase space factor in curly brackets in Eq. (5) is nothing but $E_\nu |\vec{p}_\nu|$. The Dy^H atom decays predominantly by electron de-excitations (Coster-Kronig and Auger transitions): the calculated fluorescence yields are⁸⁾ $\omega(M_1) = 1.1 \cdot 10^{-3}$, $\omega(M_2) = 1.8 \cdot 10^{-3}$, $\omega(N_1) = 3.7 \cdot 10^{-5}$, $\omega(N_2) = 7.0 \cdot 10^{-5}$. The Breit-Wigner shapes in Eq. (5) would have been affected by an electron momentum factor p_e from phase space in the dominant Dy^H electron decays, if not because, at these very low energies, $p_e \propto v/c$ is precisely compensated by the Coulomb attraction correction. Finally, Eqs. (5) and (6) are "classical" in that we have neglected interferences between intermediate states with the same quantum numbers (i.e., $H = M_1, N_1, O_1, \dots$) decaying into the same final states; and we have neglected the virtual contributions of $H = K, L$ states. We proceed to give qualitative and quantitative arguments why the "non-classical" corrections are negligible. The $H = K, L$ states in Eq. (4a) lie well above $E_C(\max) = Q$, and are suppressed by large energy denominators. The dominant de-excitations of $H = nS, nP_{1/2}$; $n > 2$ states are Coster-Kronig transitions, $H \rightarrow H'H''e^-$, with one of the final holes in an orbital with the same original n [i.e., $\Gamma(\text{Coster-Kronig})/\Gamma(\text{Auger}) = 16.6, 8.6, 166, 129$; for $M_{1,2}, N_{1,2}$ holes, respectively⁸⁾]. Thus, $(nS, n'S)$ and $(nP_{1/2}, n'P_{1/2})$; $n \neq n'$ interferences between different intermediate states in Eq. (4a) are unlikely, because their decay products tend to be different. We have used the transition matrix elements computed by McGuire⁸⁾ to estimate that the neglected "non-classical" corrections are much smaller than 0.3% (0.1%) at the dominant $N_1(M_1)$ peaks of the spectrum and are at most at the per cent level at the endpoint $E_C \lesssim Q$. The effect of these corrections on the shape of the spectrum a few tens of eVs from the endpoint is entirely negligible.

The full line of Fig. 1a displays dW/WdE_C from Eqs. (5) and (6); for $Q = 2.58$ keV, as in Eqs. (3a) and $m_\nu = 0$. The dashed line in Fig. 1a is the same spectrum smoothed with detector resolution, assumed to be a Gaussian with full width at half maximum $\text{FWHM} = 100$ eV. The M_1 (2.047 keV) and M_2 (1.842 keV) peaks would still be visible with this resolution, but the N_1 (0.416 keV) and N_2 (0.332 keV) peaks merge into one. The O_1 (0.063 keV) and O_2 (0.0026 keV) peaks and a P_1 peak (too narrow to be shown in the figures) become a broad low-energy enhancement. Figure 1b shows two smoothed spectra, both with

FWHM = 100 eV for $Q = 2.58$ keV, as in Eq. (3a) and $Q = 2.3$ keV, as in Eq. (3b). Notice that the ratios between M and N peaks are very sensitive to Q , a fact to be exploited below. The parameters used in these plots are ratios of wave functions at the origin in Ho ($Z = 67$) and full widths of excited states of Dy^H ($Z = 66$). Several calculations of the first set of parameters exist in the literature, and they are quoted in Table 1. The spread between different calculations is generally at the 5% level and would not be very visible in Fig. 1 (except for the O peaks). The widths we have used for the Dy^H states are those calculated by McGuire⁸: $\Gamma(M_1) = 18$ eV, $\Gamma(M_2) = 10$ eV, $\Gamma(N_1) = 15$ eV, $\Gamma(N_2) = 11$ eV. For illustrative purposes, we have adopted the guesses $\Gamma(O_1) = 9$ eV, $\Gamma(O_2) = 2$ eV (the properties of O levels will not affect the physics of interest to us). In a perfect resolution spectrum the widths of the levels would of course play a role and be in fact measurable. Clearly, the sensitivity to the assumed values of Γ_H in an experiment with coarse resolution (FWHM $\gg \Gamma_H$) is very small.

Reference	9)	10)	11)	12)	13)	14)	15)
N_1/M_1	0.233	0.241	0.242	0.230	0.236		
O_1/N_1	0.131	0.147		0.117	0.138		
M_2/M_1	0.0532	0.0542	0.0551			0.0552	0.0543
N_2/N_1	0.0516	0.0526	0.0536				
O_2/O_1	0.05 (our guesstimate, only for illustrative purposes)						

Table 1

Ratios of squared wave function at the nucleus, for $Z = 67$. References 12) and 13) are non-relativistic calculations. The atomic potentials are Hartree-Fock in Refs. 9) and 12); Hartree-Fock-Slater in Refs. 10), 11) and 13); and Thomas-Fermi-Dirac in Refs. 14) and 15).

In Fig. 2 we illustrate a very efficient method to determine $Q(\text{Ho}, \text{Dy})$ in calorimetric experiments. What are shown are ratios of peak heights as functions of Q , as predicted by Eq. (5), convoluted with our assumed Gaussian energy resolution, $\text{FWHM} = 100 \text{ eV}$. Figure 2a shows the ratio of the M_1 peak to the $N_{1,2}$ peak and Fig. 2b the ratio M_1/M_2 . The dashed regions are for $m(\nu_e) = 0$ and their width reflects the spread between the different theoretical calculations of wave function ratios (see Table 1). The ratio M_1/N is much more sensitive to Q than the ratio M_1/M_2 , since the Q value is much closer to the M binding energies than to the N ones. The dashed lines are for $m(\nu_e) = 60 \text{ eV}$ and the wave function ratios of Mann and Waber⁹⁾. Q and m_ν could, in principle, be extracted from a simultaneous measurement of the M_1/N and M_1/M_2 ratios, but, as the figure illustrates, the sensitivity to a non-zero neutrino mass is only relevant if $Q < 2.2 \text{ keV}$ or if the wave function ratios could be determined with high precision either theoretically or experimentally. The ratios of peak heights can be measured with very high statistics. Suppose one arbitrarily takes the spread of computations of wave function ratios (Table 1) to be a measure of theoretical uncertainty. Then, Fig. 2 implies that Q can be measured with a theoretical uncertainty of the order of $\pm 10 \text{ eV}$ ($\pm 5 \text{ eV}$) if Q is in the neighbourhood of 2.58 keV (2.30 keV).

We finally come to the subject of neutrino mass effects near the endpoint $E_c \sim Q$. The first crucial question is one of counting rate. Define a figure of merit g as the fraction of events in the interval $(Q - m_\nu, Q)$ in the normalized spectrum $dW(m_\nu = 0)/dE_c$ of Eqs. (5) and (6).

$$g(Q, m_\nu) = \frac{1}{W(Q)} \int_{Q-m_\nu}^Q dE_c \, dW(Q, m_\nu=0)/dE_c \quad (7)$$

This is a good measure of the difference in fractional counting rates for $m_\nu = 0, m_\nu \neq 0$. The figure of merit in ${}^3\text{H}$ decay is $g \sim 8 \cdot 10^{-9} (m_\nu/30 \text{ eV})^3$. The figure of merit for Ho decay is not quite cubic in neutrino mass, particularly for very low Q values, since the Breit-Wigner matrix element affects the spectral shape. The Ho figures of merit, for a set of representative Q values^{6),7)}, are given in Table 2.

Q(keV)	2.15	2.30	2.48	2.58	2.68
$g(Q, m_\nu = 30 \text{ eV})$	$4.5 \cdot 10^{-6}$	$4.4 \cdot 10^{-7}$	$1.1 \cdot 10^{-7}$	$6.1 \cdot 10^{-8}$	$3.7 \cdot 10^{-8}$
$g(Q, m_\nu = 60 \text{ eV})$	$7.4 \cdot 10^{-5}$	$4.4 \cdot 10^{-6}$	$9.7 \cdot 10^{-7}$	$5.3 \cdot 10^{-7}$	$3.2 \cdot 10^{-7}$

Table 2

Ho figures of merit, from Eqs. (5)-(7)

Notice that the figures of merit in Ho decay are superior to those in ^3H decay. From this point of view, and for $m_\nu \sim 30 \text{ eV}$ and $Q = 2.58$, as in Eq. (3a) Ho is superior to ^3H by an order of magnitude; for $Q = 2.30$, as in Eq. (3b), Ho is superior to ^3H by a factor of ~ 50 . If the implantation of Ho into a solid state detector is as successful as the implantation of ^3H , this is good news.

In Fig. 3 we have displayed the traditional "Kurie" plots: $(dW/dE_c)^{1/2}$ versus E_c . Figure 3a is for perfect resolution; in Fig. 3b theory is convoluted with a Gaussian with FWHM = 100 eV. Both figures show curves for $m_\nu = 0,60 \text{ eV}$ and $Q = 2.3, 2.58 \text{ keV}$. Since the Q value can, in principle, be determined with high precision from the peak ratios discussed before, and the figures of merit are favourable, it is not excluded that an electron neutrino mass at the 30 eV level can be detected in this kind of experiment. It need not be emphasized that here, resolution is the main concern. Since $Q(\text{Ho}) \ll Q(^3\text{H})$, the final word on resolution, one way or another, may not have been said⁵⁾.

A limiting factor of calorimetric experiments is "pileup": two single events can accidentally occur within the same time resolution window, and register the sum of their energies. The time resolution, Δt , is limited below by the charge collection time, of the order of 6 μs in Simpson's⁵⁾ relatively huge detector (80mm² $\times$ 5mm). Let N and $T_{1/2}$ be the number and half-life of ^{163}Ho atoms. The fraction of accidental double coincidences per single event is $N\Delta t \ln 2 / T_{1/2}$. The energy spectrum of accidentals, normalized in the same way, is:

$$\frac{dR}{dE_c} = \frac{N\Delta t \ln 2}{T_{1/2}} \frac{1}{a} \frac{da}{dE_c} \quad (8a)$$

$$\frac{1}{a} \frac{da}{dE_c} = \frac{1}{W^2} \int dE_1 dE_2 \frac{dW}{dE_1} \frac{dW}{dE_2} \delta(E_c - E_1 - E_2) \quad (8b)$$

with dW/WdE the single spectrum of Eqs. (5) and (6). In Fig. 4a we show da/adE_C for $Q = 2.58$ keV, both with ideal resolution and smoothed with a Gaussian (FWHM = 100 eV). Figure 4b is a comparison of smoothed spectra for $Q = 2.58$ keV and 2.3 keV. The pile-up spectrum peaks at $E_C = E_H + E_{H'}$, with $H = M_{1,2}, N_{1,2}, O_{1,2}$ (the P_1 contribution is negligible). For the Q values of Eqs. (3), the endpoint of singles spectrum is close to the $M+N$ peaks of the doubles spectrum. To illustrate the relevance of accidentals, we compute the number of atoms $N(Q)$ for which single and accidental events are equally numerous in the region sensitive to the neutrino mass. For an experiment with $FWHM > m_\nu$ (but not much bigger) one can see from Fig. 3b that the m_ν sensitive region roughly extends from $Q - 2(FWHM)$ to $Q + FWHM$. For $\Delta t = 1\mu s$ and $FWHM = 100$ eV, $N(Q = 2.58 \text{ keV}) = 6.8 \cdot 10^{13}$ and $N(Q = 2.3 \text{ keV}) = 6.1 \cdot 10^{15}$, a huge number. The number of single events in this m_ν sensitive region would be $2.9 \cdot 10^5$ per year at $Q = 2.58$ keV, and $3.1 \cdot 10^{10}$ per year at $Q = 2.3$ keV. With the use of the figures of merit of Table 2, we conclude that the first number of events/year is barely enough to see the effect of $m_\nu = 60$ eV, while the second number is more than enough to measure $m_\nu = 30$ eV. The limitation for the unfavourable Q values (close to a large accidental peak) may be much less serious than it sounds, since the accidental background can be computed from the observed singles spectrum and/or measured with great accuracy by artificially increasing the time gate Δt and/or tamed with other pile-up rejection methods.

To summarize, an assured outcome of an ^{163}Ho calorimeter experiment would be a very good and very welcome determination of the (Ho,Dy) Q value, or, more precisely, the ratios of phase space factors $\nu(Q - E_H)[(Q - E_H)^2 - m_\nu^2]^{1/2}$ for $H = M_1, M_2$ and N levels. Because of atomic physics uncertainties, these ratios are convincingly sensitive to a ~ 30 eV neutrino mass only for Q values in the lower range of the KEK experiment⁷⁾. The endpoint of the Ho decay spectrum is resonance-enhanced by its proximity to the M_1 binding energy. Consequently, and for a Ho calorimeter detector performing in all respects as well as Simpson's Tritium detector, the favourable figure of merit would make the endpoint analysis superior for Holmium than for Tritium. This statement must be qualified by the limitation implied by accidental coincidences, a violently varying function of time resolution, experimental ingenuity and plain good luck (the precise value of Q). We believe that this kind of experiment should be given one or more tries.

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FIGURE CAPTIONS

- Fig. 1 : Normalized energy spectra for $m_\nu = 0$.
- Fig. 2 : Ratio of peak heights with Gaussian resolution (FWHM = 100 eV), for
 $m_\nu = 0$ and $m_\nu = 60$ eV.
- Fig. 3 : "Kurie" plots near to the spectral endpoint.
- Fig. 4 : "Pile-up" normalized spectra of double accidental coincidences.

Fig. 1

Fig. 2

Fig. 3

Fig. 4