REF.TH.3381-CERN

RELATIVISTIC CALCULATION OF THE TWO PHOTON
COUPLING OF SCALAR AND TENSOR MESONS

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ABSTRACT

The decay rates of scalar and tensor mesons into two photons are calculated in the Bethe-Salpeter bound state formalism. The suppression of the non-relativistic estimates due to relativistic kinematics is found to be substantial. Production cross-sections in colliding beam experiments are estimated to be in the picobarn range. Also in the relativistic case the helicity 0 production cross-section of tensor mesons is found to be negligible.

^{*)} Supported by the Swedish Natural Research Council (NFR) contract no. F-UR3281-101.

1. - INTRODUCTION

Potential model studies of heavy quarkonium systems have been quite successful in giving the positions of the energy levels. However, very different potentials are able to reproduce the level spacings. For instance, potential models have been constructed which show neither the $1/r$ behaviour expected from QCD, nor the linear confining term indicated from lattice gauge theory studies, yet successfully predict the levels¹⁾.

To further specify the parameters, one also needs some restriction on the behaviour of the wave functions and not just the level structure. For the vector meson ($C=-1$) case this has traditionally been done using the non-relativistic formula which connects the wave function at the origin to the leptonic decay width. It is known, however, that this formula is subject to large relativistic and (to some extent) gluonic corrections²⁾⁻⁴⁾. The main effect of these corrections is to decrease the absolute widths, whereas ratios of widths of different levels remain largely unaffected.

The aim of this paper is to investigate corresponding relativistic effects for the ($C=+1$) P states. For these states, the one-photon decay is C parity forbidden, and the simplest electromagnetic decay is into two photons. For the axial vector (1^{++}) state this is also forbidden by Yang's theorem⁵⁾ and therefore we will only consider the 0^{++} and 2^{++} states here. (The 1^{++} case is worth a separate study, especially concerning the non-vanishing 2γ amplitude with one or both photons virtual, and the singularity of the $e^+e^-\gamma$ decay rate in the non-relativistic limit⁶⁾. The problem of 0^{++} and 2^{++} couplings to virtual photons is also under study⁷⁾.)

Guided by previous studies, e.g. of the pseudoscalar meson decays $P \rightarrow \gamma\gamma$ and $P \rightarrow \ell^+\ell^-\gamma$ ^{8),9)}, we will account for relativistic kinematics by using a Bethe-Salpeter formalism for the bound state. In addition to the decay rates, we will consider the production of 0^{++} and 2^{++} states in $\gamma\gamma$ collisions in the equivalent photon approximation. Finally, we discuss to what extent our results can be applied to the recently measured decays $f \rightarrow 2\gamma$ and $A_2 \rightarrow 2\gamma$.

2. - CALCULATIONS

In the Bethe-Salpeter formalism, the spin J bound state amplitude $\chi_J(P,p)$ satisfies the following equation in the centre-of-mass system:

$$\chi_J(M_J, p) = S_F^{(1)}(p_1) S_F^{(2)}(p_2) \int \frac{d^4 p'}{(2\pi)^4} K(p-p') \chi_J(M_J, p'). \quad (1)$$

Here $K(p-p')$ is the interaction kernel, $S_F^{(i)}(p_i)$ are the full quark propagators, $2p$ is the relative momentum of the two quarks and $p_1 = P/2 + p$, $p_2 = P/2 - p$, $P = (M_J, \vec{0})$.

The helicity amplitudes for the coupling of the bound state to two photons are

$$M_{\lambda_1 \lambda_2}^{(J)} = 4\pi\alpha\sqrt{3}e_q^2 \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left\{ \chi_J(M_J, p) F_{\lambda_1 \lambda_2}(p) \right\}, \quad (2)$$

where, to lowest order in QED and QCD,

$$F_{\lambda_1 \lambda_2}(p) = -i \not{\epsilon}_2^*(\lambda_2) \frac{i}{\not{p} - \not{k}_1 - m_q} (-i \not{\epsilon}_1^*(\lambda_1)) + (1 \leftrightarrow 2). \quad (3)$$

The factor $\sqrt{3}$ accounts for SU(3) colour; λ_1, λ_2 and k_1, k_2 are helicities and momenta of the two photons; e_q and m_q are the electric charge and effective mass of the quark.

To make contact with the Schrödinger equation description of the bound states, we now use an instantaneous kernel $K^I(\vec{p}-\vec{p}')$ and approximate (1) by keeping only the lowest order positive energy projections of the propagators $S_F^{(i)}(p_i)$. This is a good approximation for very heavy quarkonium states and probably adequate for charmonium¹⁰⁾.

The bound state amplitude is then greatly simplified³⁾:

$$\chi_J(M_J, p) = 2\pi\Delta(p_0) \sum_{r,s} \tilde{\Psi}_{rs}^J(\vec{p}) u_r(\vec{p}) \bar{v}_s(-\vec{p}) \sqrt{2M_J} / 2E(\vec{p}), \quad (4)$$

where

$$\Delta(p_0) = \frac{1}{2\pi i} (M_J - 2E) \left(\frac{1}{2}M_J - E + p_0 + i\varepsilon \right)^{-1} \left(\frac{1}{2}M_J - E - p_0 + i\varepsilon \right)^{-1} \quad (5)$$

and $\tilde{\Psi}_{rs}^J(\vec{p})$ is the instantaneous wave function in $\vec{p}$ space. This wave function satisfies the quasi-potential equation^{11),12)}

$$(M_J - 2E) \tilde{\Psi}_{rs}^J(\vec{p}) = \sum_{r',s'} \int \frac{d^3 p'}{(2\pi)^3} V_{rs;r's'}(\vec{p}, \vec{p}') \tilde{\Psi}_{r's'}^J(\vec{p}'). \quad (6)$$

The above equation reduces to the Schrödinger equation in the non-relativistic limit. Inserting (4) into (2) and performing the p^0 integration, we obtain the helicity amplitudes

$$m_{\lambda_1 \lambda_2}^J = 4\pi\alpha\sqrt{3}e_a^2 \int \frac{d^3p}{(2\pi)^3 2E} \sum_{r,s} \tilde{\Psi}_{rs}^J(\vec{p}) \bar{V}_s(-\vec{p}) F_{\lambda_1 \lambda_2}(p) u_r(\vec{p}) \sqrt{2M_J}. \quad (7)$$

For the P wave J^{++} states we have

$$\tilde{\Psi}_{rs}^J(\vec{p}) = \sum_{m,\lambda} \tilde{\phi}_{nP}(|\vec{p}|) \langle 1m1\lambda | JM \rangle \langle rs | 1\lambda \rangle Y_m^1(\theta, \varphi), \quad (8)$$

where $\tilde{\phi}_{nP}(p)$ is the n^{th} radial P wave function in $\vec{p}$ space. Our final expression is then

$$m_{\lambda_1 \lambda_2}^J = 4\pi\alpha\sqrt{3}e_a^2 \sum_{m,\lambda} \int \frac{d^3p}{(2\pi)^3 2E} \tilde{\phi}_{nP}(p) \langle 1m1\lambda | JM \rangle \times \quad (9)$$

$$\times Y_m^1(\theta, \varphi) \langle rs | 1\lambda \rangle \bar{V}_s(-\vec{p}) F_{\lambda_1 \lambda_2}(p) u_r(\vec{p}) \sqrt{2M_J}.$$

3. - DECAY OF O^{++} MESONS

The general amplitude for the decay $O^{++} \rightarrow 2\gamma$ can be written

$$m_{\lambda_1 \lambda_2}^O = 4\pi\alpha f_0 \left\{ 2(k_2 \cdot e_1^*(\lambda_1))(k_1 \cdot e_2^*(\lambda_2)) - 2(k_1 \cdot k_2)(e_1^*(\lambda_1) \cdot e_2^*(\lambda_2)) \right\} \quad (10)$$

which gives the rate

$$\Gamma(O^{++} \rightarrow 2\gamma) = \pi\alpha^2 M_O^3 f_0^2 \quad (11)$$

with

$$f_0 = \frac{1}{4\pi\alpha M_O^2} m_{++}^O = \frac{1}{4\pi\alpha M_O^2} m_{--}^O \quad (12)$$

and

$$m_{+-}^O = m_{-+}^O = 0. \quad (13)$$

Identifying (10) with (9) for $J = 0$, we obtain

$$f_0 = \sqrt{3} e_a^2 \frac{2}{\sqrt{\pi}} M_0^{-5/2} m_a^{-1} \int \frac{d^3 p}{2\pi^2} \left(\frac{m_a}{E} \right)^2 \tilde{\Phi}_{n_F}(p) \left\{ 1 - \frac{EM_0 - 2m_a^2}{2p^2} \left(1 - \frac{E}{p} \ln \left(\frac{E+p}{m_a} \right) \right) \right\}. \quad (14)$$

This gives the rate

$$\begin{aligned} \Gamma(0^{++} \rightarrow 2\gamma) &= 3\alpha^2 e_a^4 \frac{4}{(m_a M_0)^2} \left| \int \frac{d^3 p}{(2\pi)^3} p \tilde{\Phi}_{n_F}(p) \left(\frac{m_a}{E} \right)^2 \right. \\ &\quad \times \left. \left\{ 1 - \frac{EM_0 - 2m_a^2}{2p^2} \left(1 - \frac{E}{p} \ln \left(\frac{E+p}{m_a} \right) \right) \right\} \right|^2. \end{aligned} \quad (15)$$

When trying to extract a "non-relativistic" limit out of this expression, one encounters an ambiguity. Usually, one takes the "static" limit $p \rightarrow 0$ in the expression multiplying the wave function in the integrand and also the weak binding limit $M \rightarrow 2m_Q$, which gives the conventional result^{13),14)}

$$\Gamma^0(0^{++} \rightarrow 2\gamma) = 3 \frac{\alpha^2 e_a^4}{m_a^4} \left| \int \frac{d^3 p}{(2\pi)^3} p \tilde{\Phi}_{n_F}(p) \right|^2. \quad (16)$$

However, in most charmonium models, and certainly for the lighter mesons, the binding energy $E_b = M_0 - 2m_Q$ may be quite substantial. Therefore, a factor $16/M_0^4$ in (16) instead of $1/m_Q^4$ would, for example, give quite a different absolute value for the width, although they should be equivalent in the weak binding limit. When deriving Eq. (15), one finds that M_0 enters through kinematics (the energy of the photons is $M_0/2$), whereas m_Q comes through the quark propagator and spinor factors. Taking the static limit $p \rightarrow 0$ in (15) but keeping the binding energy $\neq 0$, we thus obtain an improved "static" formula

$$\Gamma^{\text{static}}(0^{++} \rightarrow 2\gamma) = \Gamma^0(0^{++} \rightarrow 2\gamma) \left(\frac{M_0 + 4m_a}{3M_0} \right)^2, \quad (17)$$

where Γ^0 is given according to (16). However, although this is an improvement, we still find that the bulk of the corrections in going from (15) to (16) comes from the additional momentum-dependent structure in (15). The numerical results will be discussed in Section 5.

4. - DECAY OF 2^{++} MESONS

For the coupling of a 2^{++} meson to two photons we write the general amplitude in terms of two decay constants f_2 and g_2 :

$$\begin{aligned} m_{\lambda_1 \lambda_2}^{2,M} = & 4\pi\alpha f_2 \left\{ (e_1^*(\lambda_1) \cdot e_2^*(\lambda_2)) k_1 S(M) k_2 + (k_1 \cdot k_2) e_1^*(\lambda_1) S(M) e_2^*(\lambda_2) \right. \\ & \left. - e_1^*(\lambda_1) S(M) k_2 (k_1 \cdot e_2^*(\lambda_2)) - (e_1^*(\lambda_1) \cdot k_2) k_1 S(M) e_2^*(\lambda_2) \right\} \\ & + 4\pi\alpha g_2 \left\{ (e_1^*(\lambda_1) \cdot k_2) (e_2^*(\lambda_2) \cdot k_1) / (k_1 \cdot k_2) - (e_1^*(\lambda_1) \cdot e_2^*(\lambda_2)) \right\} k_1 S(M) k_2, \end{aligned} \quad (18)$$

where $a S(M) b = a_\mu S^{\mu\nu}(M) b_\nu$ and $S^{\mu\nu}(M)$ is the spin-two polarization tensor. The rate is then given by

$$\Gamma(2^{++} \rightarrow 2\gamma) = \frac{\pi\alpha^2}{20} M_2^3 \left\{ f_2^2 + \frac{1}{6} g_2^2 \right\} \quad (19)$$

with

$$f_2 = \frac{1}{2\pi\alpha M_2^2} m_{+,-}^{2,2} \quad (20)$$

$$g_2 = \frac{-\sqrt{6}}{2\pi\alpha M_2^2} m_{+,+}^{2,0}. \quad (21)$$

In our model we obtain

$$f_2 = -\frac{8}{\sqrt{\pi}} e_q^2 M_2^{-5/2} m_q^{-1} \int \frac{d^3 p}{(2\pi)^3} \left(\frac{m_q}{E} \right) p \tilde{\Phi}_{n_p}(p) \left\{ 1 + \frac{3}{4} I(p) \right\}, \quad (22)$$

where

$$\begin{aligned} I(p) = & -1 + \frac{1}{2} \left(1 + \frac{m_q}{E} \right) \frac{E}{p} \ln \left(\frac{E+p}{E-p} \right) + \frac{m_q^2 + 5m_q E}{3p^2} - \frac{m_q E^2}{p^3} \ln \left(\frac{E+p}{E-p} \right) \\ & + \left(1 - \frac{m_q}{E} \right) \frac{E^4}{p^4} - \frac{1}{2} \left(1 - \frac{m_q}{E} \right) \frac{E^5}{p^5} \ln \left(\frac{E+p}{E-p} \right), \end{aligned} \quad (23)$$

and

$$g_2 = -\frac{3}{\sqrt{\pi}} e_q^2 M_2^{-5/2} m_q^{-1} \int \frac{d^3 p}{(2\pi)^3} \left(\frac{m_q}{E} \right) p \tilde{\Phi}_{n_p}(p) \left\{ I_1(p) + \frac{M_2}{E} I_2(p) \right\}, \quad (24)$$

where

$$\begin{aligned} I_1(p) = & \frac{12m_a - 8E}{E} \cdot \frac{E^2}{p^2} + \frac{12(E - m_a)}{E} \cdot \frac{E^4}{p^4} + \\ & + \left(\frac{2m_a}{p} + \frac{6E - 8m_a}{E} \cdot \frac{E^3}{p^3} + \frac{6(m_a - E)}{E} \cdot \frac{E^5}{p^5} \right) \ln \left(\frac{E+p}{E-p} \right) \end{aligned} \quad (25)$$

and

$$I_2(p) = \frac{4E^2}{p^2} + \frac{6(m_a - E)}{E} \cdot \frac{E^4}{p^4} + \left\{ \frac{m_a - 3E}{E} \cdot \frac{E^3}{p^3} + \frac{3(E - m_a)}{E} \cdot \frac{E^5}{p^5} \right\} \ln \left(\frac{E+p}{E-p} \right). \quad (26)$$

This gives for the rate

$$\begin{aligned} \Gamma(2^{++} \rightarrow 2\gamma) = & \frac{16}{5} \frac{e_a^4 \alpha^2}{M_2^2 m_a^2} \left\{ \left| \int \frac{d^3 p}{(2\pi)^3} \left(\frac{m_a}{E} \right) p \tilde{\phi}_{n_P}(p) \left\{ 1 + \frac{3}{4} I(p) \right\} \right|^2 + \right. \\ & \left. + \frac{1}{24} \left| \int \frac{d^3 p}{(2\pi)^3} \left(\frac{m_a}{E} \right) p \tilde{\phi}_{n_P}(p) \frac{3}{4} \left\{ I_1(p) + \frac{M_2}{E} I_2(p) \right\} \right|^2 \right\}. \end{aligned} \quad (27)$$

In the static limit the decay constants become

$$f_2^{\text{static}} = -\frac{8}{\sqrt{\pi}} e_a^2 M_2^{-5/2} m_a^{-1} \int \frac{d^3 p}{(2\pi)^3} p \tilde{\phi}_{n_P}(p), \quad (28)$$

$$g_2^{\text{static}} = \frac{4}{\sqrt{\pi}} e_a^2 M_2^{-5/2} m_a^{-1} \left(\frac{M_2}{m_a} - 2 \right) \int \frac{d^3 p}{(2\pi)^3} p \tilde{\phi}_{n_P}(p). \quad (29)$$

This gives

$$\Gamma^{\text{static}}(2^{++} \rightarrow 2\gamma) = \Gamma^0(2^{++} \rightarrow 2\gamma) \frac{1}{6} \left(1 - 4 \frac{m_a}{M_2} + 28 \left(\frac{m_a}{M_2} \right)^2 \right). \quad (30)$$

and

$$\frac{\Gamma^{\text{static}}(2^{++} \rightarrow 2\gamma)}{\Gamma^{\text{static}}(0^{++} \rightarrow 2\gamma)} = \frac{4}{15} \left(\frac{M_2}{M_0} \right)^3 \frac{3}{2} \frac{28m_a^2 + M_2^2 - 4m_a M_2}{16m_a^2 + M_0^2 + 8m_a M_0}. \quad (31)$$

In the weak binding limit $M_2 \rightarrow M_0 \rightarrow 2m_Q$ we recover the standard result¹³⁾

$$\frac{\Gamma^0(2^{++} \rightarrow 2\gamma)}{\Gamma^0(0^{++} \rightarrow 2\gamma)} = \frac{4}{15}.$$

The full formulas (22) and (24) have a cumbersome appearance. In practice, however, great simplifications can be made. In the expression for $I(p)$, Eq. (23), large cancellations occur and $I(p)$ is very nearly constant (≈ 0) for all values of p . Moreover, as discussed in the next section, g_2 has a numerically small value for heavy mesons (including charmonium), and to a one per cent accuracy one can take

$$f_2 = -\frac{8}{\sqrt{\pi}} e_a^2 M_2^{-5/2} m_a^{-1} A_{nP} \quad (32)$$

$$g_2 = 0, \quad (33)$$

where

$$A_{nP} = \int \frac{d^3p}{(2\pi)^3} \frac{m_a}{E} p \tilde{\Phi}_{nP}(p). \quad (34)$$

To facilitate practical calculations, one may transform this expression into configuration space with the result

$$A_{nP} = \frac{2m_a^3}{\pi} \int dr (r R_{nP}(r)) K_2(m_a r), \quad (35)$$

where $\int |r R_{nP}(r)|^2 dr = 1$; K_2 is a modified Bessel function of the second kind.

Special Cases

In special cases, these integrals may be evaluated explicitly. In the case of the pure Coulomb potential, $V(r) = -\kappa/r$, the result is

$$A_{1P}^{\text{Coulomb}} = \frac{\sqrt{3} (m_a \kappa)^{5/2}}{8\pi} \times \frac{\arctan(\lambda) - \frac{\frac{5}{3}\lambda^3 + \lambda}{(1+\lambda^2)^2}}{(1 - \frac{\kappa^2}{16})^{5/2}}, \quad (36)$$

where $\lambda = \sqrt{16/\kappa^2 - 1}$.

For a harmonic oscillator, $V(r) = 1/4(m_Q \omega^2 r^2)$, the corresponding expression is

$$A_{1P}^{\text{harm. osc.}} = m_Q^2 \left(\frac{m_Q \omega}{2\pi} \right)^{1/4} \times \sqrt{6 \exp\left(\frac{m_Q}{\omega}\right)} W_{-1,1}\left(\frac{m_Q}{\omega}\right), \quad (37)$$

where $W_{-1,1}$ is the Whittaker function. Using the definition of the Whittaker function and the recursion relations of the confluent hypergeometric functions, one can show that

$$W_{-1,1}(z) = \frac{2\sqrt{\pi}}{3} z^{3/2} \left\{ K_0\left(\frac{z}{2}\right) + \left(\frac{1-z}{2}\right) K_1\left(\frac{z}{2}\right) \right\}. \quad (38)$$

Introducing the dimensionless quantity $\rho = \omega/m_Q$, one then gets, for the suppression factor,

$$\varepsilon \equiv \frac{\Gamma(2^{++} \rightarrow 2\gamma)}{\Gamma^{\text{static}}(2^{++} \rightarrow 2\gamma)} = \frac{4}{9\pi} \frac{\exp(1/\rho)}{\rho^5} \left\{ K_0\left(\frac{1}{2\rho}\right) + \left(\frac{\rho-1}{\rho}\right) K_1\left(\frac{1}{2\rho}\right) \right\}^2, \quad (39)$$

where Γ^{static} is related to Γ^0 according to Eq. (30). In the limit of zero binding, i.e., $\rho \rightarrow 0$, the asymptotic properties of the Bessel functions ensure the finite limit $\varepsilon \rightarrow 1$, as it should be. A plot of the suppression factor ε for both the Coulomb and harmonic oscillator case for various potential strengths can be found in Fig. 1.

5. - NUMERICAL TREATMENT

We have calculated the rates given by Eqs. (15) and (27) with $\tilde{\phi}_{1P}(p)$ obtained from solutions to the Schrödinger equation with the potentials given by three different potential models^{1),15),16)}. That this is a good approximation for charmonium (and bottomium) is indicated by the calculation of Ito¹⁰⁾ for the leptonic decay rates of charmonium. For comparison we have also calculated the corresponding static decay rates. In all models we have investigated the contribution from g_2 is on the per cent level or below and hence negligible.

The relativistic effects can be expressed by a suppression factor ε relative to the static widths

$$\Gamma(J^{++} \rightarrow 2\gamma) = \varepsilon \Gamma^{\text{static}}(J^{++} \rightarrow 2\gamma). \quad (40)$$

In Table 1 the results are displayed for the 0^{++} and 2^{++} states χ_0 and χ_2 , respectively in the charmonium system and in Table 2 the corresponding results for the bottomium system. As can be seen from these tables, the suppression factors are of the same order of magnitude as was previously found for the pseudoscalar 0^{-+} charmonium and bottomium states in Ref. 8).

We note that, despite differences in quark masses and potential parameters, all models give approximately the same relativistic values for the decay rates. This is in contrast to the situation for the decay rates $\Gamma(1^{--} \rightarrow e^+e^-)$ and $\Gamma(0^{-+} \rightarrow 2\gamma)$ which, in the static limit, depend on the value of the S wave function at the origin. The static widths for $0^{++} \rightarrow 2\gamma$ and $2^{++} \rightarrow 2\gamma$ on the other hand depend on the value of the derivative of the P wave function at the origin.

The static, zero binding limit decay rates Γ^0 vary substantially due to differences in quark masses and binding energies. The values for Γ^{static} are, however, also all of the same order of magnitude for each process respectively.

One should also consider gluonic corrections to the decay rates. These have been calculated to lowest order in the static limit by Barbieri et al.¹⁷⁾, and found to be large. However, it is difficult to estimate the significance of those calculations, since, as we have seen, the momentum-dependent effects for those decays are quite substantial. In the case of $V \rightarrow e^+e^-$ the gluonic corrections are known to be quite reduced when one goes beyond the static limit³⁾.

6. - PRODUCTION IN e^+e^- BEAMS

Two photon collisions, produced in $e^\pm, e^\pm$ beam experiments have turned up as an interesting tool to determine the decay rates of $C = +1$ even spin mesons into photons. From this point of view, it is interesting to see what could be expected for the χ_0 and χ_2 in charmonium models. From Table 1 we see that the widths $\Gamma(\chi_0 \rightarrow 2\gamma)$ and $\Gamma(\chi_2 \rightarrow 2\gamma)$ are not very sensitive to the choice of potential model. As representative values for these reactions, we have chosen $\sigma(e^\pm e^\pm \rightarrow e^\pm e^\pm \chi_0)$ and $\sigma(e^\pm e^\pm \rightarrow e^\pm e^\pm \chi_2)$ in the equivalent-photon (EP) approximation¹⁸⁾ with the formula

$$\sigma(e^\pm e^\pm \rightarrow e^\pm e^\pm \chi_J) = (4\alpha)^2 (2J+1) \Gamma(\chi_J \rightarrow 2\gamma) M_J^{-3} \left(\ln \frac{E}{m_e}\right)^2 f\left(\frac{M_J}{2E}\right), \quad (41)$$

where

$$f(x) = (2+x^2)^2 \ln(1/x) - (1-x^2)(3+x^2),$$

for several values of the beam energy E . The results are shown in Fig. 2. The cross-section is in the picobarn range, even at the highest energies (~ 100 GeV).

For comparison we have also displayed the cross-section $\sigma(e^+e^- \rightarrow e^+e^-\eta_c)$ calculated from Eq. (41) with the potential of Ref. 16).

The polarized (with respect to the photons) cross-sections satisfy $\sigma_{+-}(\chi_2) = \sigma_{-+}(\chi_2) \approx 2\sigma(\chi_2)$ and $\sigma_{++}(\chi_2) = \sigma_{--}(\chi_2) \approx 0$ since g_2 is negligible.

The accuracy of the EP approximation has been investigated by Brodsky et al.¹⁹⁾. They find that it generally underestimates the cross-section, but becomes successively better at higher energies. The estimate is then expected to be within 30% of the exact value.

7. - LIGHT QUARKONIUM DECAYS

In view of the recently measured widths of η , η' , A_2 and f into two photons by means of e^+e^- colliding beams, we have investigated the two photon decays of the lighter quarkonium systems in our model.

Several authors have tried to extend the non-relativistic potential models to include the light meson systems with remarkably good results for the spectra. One such recent attempt has been made by Bhaduri et al.¹⁵⁾. We have used their potential to calculate the wave functions for the $SU(3)$ nonet states 0^{++} and 2^{++} with ideal mixing. When these wave functions are used for $\tilde{\phi}_{1P}(p)$ in formulas (15) and (27), we get the results shown in Table 3, together with the tentative experimental assignments for these states. The rates for the isovectors are 9/25 times those of the iso-scalars when mass differences are neglected. As can be seen from this table, the relativistic corrections suppress the static values by typically a factor of the order of ten. Also, in this case, the contribution from the helicity 0 amplitude g_2 is found to be negligible. Recent experimental results for these decays range from 2.3 keV to 3.6 keV for the f meson, with a typical error of ± 0.7 keV²⁰⁾. The world average for A_2 is 0.64 ± 0.20 keV²¹⁾, whereas very recent TASSO data indicate a two-photon width for $f'(1515)$ of $0.15 \pm 0.03 \pm 0.04$ keV²¹⁾.

Although the errors are rather large, the experimental widths seem to be a factor 2-3 larger than the full widths in Table 3, whereas the static widths grossly over-estimate them. Thus, although the experimental data indicate a suppression relative to the static widths, our model, with the wave functions from Ref. 15), seem to give too low values for these widths. This should perhaps not be too astonishing, since the non-relativistic wave function used for $\tilde{\phi}_{1P}(p)$ might be inadequate in this case¹⁰⁾. Another source of the discrepancy might be the neglect of the negative energy part of the Bethe-Salpeter amplitude in this case. The earlier-mentioned gluonic corrections could, of course, also be of importance. Previous theoretical estimates have given widely different results for the f width, with values ranging from 0.8 to 28 keV. See, for example, Ref. 22) for a guide to earlier references.

In conclusion, we thus find that the two-photon widths of the 0^{++} and 2^{++} light quarkonium systems are quantitatively not well described at present.

8. - CONCLUSIONS

Our investigations of the suppressions of the non-relativistic estimates of the two-photon decay rates for 0^{++} and 2^{++} heavy quarkonium states due to relativistic kinematic effects show that these suppressions are of the same order of magnitude as for the 0^{-+} pseudoscalar mesons⁸⁾. In particular, we find that the two-photon widths are not very sensitive to the potential model parameters used, indicating that P wave decays do not constitute a good test for such models.

Production cross-sections of the charmonium states χ_0 and χ_2 in e^+e^- colliding beam experiments are estimated to be in the picobarn range. The helicity 0 production cross-section of tensor mesons is found to be negligible.

For the light quarkonium systems our model, with the non-relativistic potential wave functions of Ref. 15), underestimates the experimentally measured widths by a factor of 2-3, whereas the frequently used static, zero-binding-energy limit formulas grossly overestimate them.

	0^{++}					2^{++}				
	$\Gamma(\text{keV})$	$\Gamma^0(\text{keV})$	$\Gamma^{\text{static}}(\text{keV})$	ϵ	$\Gamma(\text{keV})$	$\Gamma^0(\text{keV})$	$\Gamma^{\text{static}}(\text{keV})$	ϵ		
V_I	1.34	6.52	4.19	0.320	0.400	1.74	0.999	0.400		
V_{II}	1.36	2.96	3.35	0.406	0.498	0.789	1.06	0.470		
V_{III}	1.48	3.27	3.51	0.423	0.530	0.872	1.09	0.486		

Table 1

	0^{++}					2^{++}				
	$\Gamma(\text{eV})$	$\Gamma^0(\text{eV})$	$\Gamma^{\text{static}}(\text{eV})$	ϵ	$\Gamma(\text{eV})$	$\Gamma^0(\text{eV})$	$\Gamma^{\text{static}}(\text{eV})$	ϵ		
V_I	24.4	42.1	39.2	0.622	7.41	11.2	10.3	0.719		
V_{II}	27.6	44.3	47.8	0.577	9.35	11.8	13.3	0.703		
V_{III}	26.6	37.1	39.6	0.672	8.61	9.89	10.9	0.790		

Table 2

	0^{++}				2^{++}				
	$\Gamma(\text{keV})$	$\Gamma^0(\text{keV})$	$\Gamma^{\text{static}}(\text{keV})$	ϵ	$\Gamma(\text{keV})$	$\Gamma^0(\text{keV})$	$\Gamma^{\text{static}}(\text{keV})$	ϵ	
$u\bar{u} = d\bar{d}$	I=0	2.00	44.9	21.5	0.093	0.918	12.0	3.88	0.237
	I=1	0.720	16.2	7.75	0.093	0.330	4.30	1.40	0.236
$s\bar{s}$		0.173	1.32	0.979	0.177	0.080	0.352	0.223	0.359

	0^{++}	2^{++}
I = 0	$\epsilon(1300)$	$f(1273)$
I = 1	?	$A_2(1317)$
$s\bar{s}$	$\epsilon'(1550)$	$f'(1515)$

Table 3

Table Captions

Table 1 : Decay widths and suppression factors for χ_0 and χ_2 calculated with three different potential models:

$$\begin{aligned} V_I &= 0.8(1-r/1.25)^2/r \ln(r/1.25), \quad m_Q = 1.20 \text{ GeV}^{16}), \quad V_{II} = \\ &= -0.5203/r + 0.02822 \exp(-r/2.296)/r + 0.1857r - 0.9135 \\ m_Q &= 1.870 \text{ GeV}^{15}), \quad V_{III} = -8.064 + 6.87r^{0.1}, \quad m_Q = 1.80 \text{ GeV}^{1}). \end{aligned}$$

Table 2 : Decay widths and suppression factors for the $1P$ bottomium state, calculated with three different potential models:

$$\begin{aligned} V_I &= 0.8(1-r/1.25)^2/r \ln(r/1.25), \quad m_Q = 4.62 \text{ GeV}^{16}), \quad V_{II} = \\ &= -0.5203/r + 0.003568 \exp(-r/2.296)/r + 0.1857r - 0.9135 \\ m_Q &= 5.259 \text{ GeV}^{15}), \quad V_{III} = -8.064 + 6.87r^{0.1}, \quad m_Q = 5.174 \text{ GeV}^{1}). \end{aligned}$$

Table 3 : Decay widths and suppression factors calculated with the potential of Ref. 15). The quark masses are $m_u = m_d = 337 \text{ MeV}$ and $m_s = 600 \text{ MeV}$. The tentative assignments for these states are shown.

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FIGURE CAPTIONS

Figure 1 : Suppression factors ϵ as a function of the dimensionless variables κ (Coulomb case) and $\rho = \omega/m_Q$ (harmonic oscillator case). The arrows indicate the parameter values needed to reproduce the position of the maximum of $p^2 \tilde{\phi}_{1S}(p)$ for a more realistic charmonium potential.

Figure 2 : Production cross-sections of η_c , χ_0 and χ_2 in e^+e^- collisions.

Fig. 1

Fig. 2