Ref.TH.3363-CERN

ELECTRONS AS ACCELERATED THERMOMETERS

J.S. Bell and J.M. Leinaas
CERN - Geneva

A B S T R A C T

The possibility of using accelerated electrons to exhibit the quantum field theoretic relation between acceleration and temperature is considered. In principle, the depolarization of electrons in a magnetic field could be used to give the temperature reading. The effect is examined for linearly accelerated electrons, but the result is that the relevant orders of magnitude are too small for real experiments in linear accelerators. For electrons in storage rings sufficiently large accelerations can be obtained, and the residual depolarization which has been found theoretically and experimentally is shown to be an effect closely related to the thermal effect of linearly accelerated electrons.

Ref.TH.3363-CERN
21 July 1982

1. - INTRODUCTION

An idealized thermometer which is accelerated through vacuum, will register a non-zero temperature, as if vacuum were hot¹⁾. For a relativistically uniform acceleration a , the temperature which is registered is given by

$$kT = \frac{\hbar a}{2\pi c} \quad (1)$$

This effect was first discussed by Unruh²⁾ and Davies³⁾, who related it to the Hawking effect of thermal radiation from black holes⁴⁾.

That the relation between thermometer reading and acceleration given above refers only to an idealized thermometer is obviously true. The performance of a real thermometer depends on the particular elastic properties of the materials of which it happens to be made and on just where the accelerating forces are applied. Thus, the mercury column of a standard thermometer might be compressed and the glass might be stretched so that the temperature seems to fall rather than to rise. Moreover, these sordid effects will be, like the temperature effect itself, of first order in the acceleration. Therefore, they cannot be made negligible by making the acceleration small.

For temperatures of the order of 1°K , the corresponding acceleration in Eq. (1) is enormous on any ordinary scale. Thus, 1°K corresponds to

$$a = 2.4 \cdot 10^{20} \text{ m/s}^2 \quad (2)$$

For comparison, the acceleration of a proton due to the electric field of an electron at a distance d of one ångström is

$$\frac{e^2}{m_p d^2} = 1.4 \cdot 10^{19} \text{ m/s}^2 \quad (3)$$

It is clear that such accelerations cannot be applied to macroscopic bodies without destroying them, or to atomic systems without greatly distorting them.

We will not rule out the possibility that one might be able to detect thermal effects of acceleration for temperatures which are much smaller than 1°K , even if the discrimination of thermal effects due to ordinary temperature then becomes an increasingly large problem. However, the considerations above suggest the interest of using elementary particles as thermometers, since these are "robust" systems⁵⁾ even on the scale mentioned.

In particular an electron is relatively free from structure, and therefore difficult to distort, it is also relatively light and therefore easy to accelerate. As a measure of robustness one may consider the "natural unit" of acceleration defined by the electron mass

$$\frac{mc^3}{\hbar} = 2.3 \cdot 10^{29} \text{ m/s}^2 \quad (4)$$

This is very much bigger than the acceleration of Eq. (2). Moreover, one may consider using the electron as a sensitive thermometer, simply by subjecting it to a magnetic field. In a field B of 10^4 gauss, the splitting of the "spin up" and "spin down" levels is

$$\frac{e\hbar B}{mc} = 1.2 \cdot 10^{-4} \text{ eV} = 1.3^\circ \text{K} \quad (5)$$

As regards ease of acceleration, note that for an electron the acceleration (2) corresponds to an electric field E of about

$$E \approx 1400 \text{ MV/m} \quad (6)$$

This is rather large, for example compared with the 7 MV/m of the SLAC linac. However, such fields are readily generated, in the rest frame of the particle, by motion through a magnetic field. For motion with $\gamma = (1-\beta^2)^{-1/2} \approx 10$, across the field B of 10^4 gauss, the induced electric field is

$$E = \gamma \beta B \approx 3000 \text{ MV/m} \quad (7)$$

For ultra relativistic electrons in a storage ring, the acceleration in the rest frame is given by

$$a = \frac{\gamma^2 c^2}{R} \quad (8)$$

with R as the radius of curvature of the particle trajectory. At LEP energies, with $\gamma \approx 10^5$ and bending radius $R = 3.1$ km, the acceleration is $a \approx 2.9 \cdot 10^{23} \text{ m/s}^2$, corresponding to a temperature $T \approx 1200^\circ \text{K}$. For existing accelerator rings, like SPEAR at Stanford, similar values for the acceleration are obtained.

The figures given above suggest that it may not be impossible to see thermal effects due to acceleration in particle accelerators. One may in fact raise the question whether this effect has not already been observed in electron storage rings. It is known theoretically⁶⁾ and experimentally⁷⁾ that circulating electrons become polarized, but not fully so. Could the residual depolarization be regarded as a thermal effect associated with the centripetal acceleration? We consider this question in the second part of the paper (Section 5), and answer with a qualified affirmative. The effect is of the same kind as the thermal effect associated with linear acceleration, although in detail it is quite different.

Before discussing the effect of acceleration for circulating electrons, we examine in Section 2-4 spin excitations for linearly accelerated particles. This is a more academic case, but it is instructive to see how the thermal effect appears in this case. In this connection we discuss a complication due to the Thomas precession of the spin. The discussion is based on a semi-classical approach, where the interaction with the radiation field gives rise to a small perturbation of the classical motion of the particle.

2. - LINEAR ACCELERATION

Consider a charged particle of spin one-half, mass m and magnetic moment μ moving in the z direction. Suppose it to be subject to external electric and magnetic fields E_0 and B_0 , both in the z direction, and to the radiation field. If we assume the particle to have no initial velocity perpendicular to the z direction, then the unperturbed, classical, path of the particle is given by

$$\begin{aligned}x &= y = 0 \\z &= \frac{c^2}{a} \cosh \frac{a}{c} \tau \\t &= \frac{c}{a} \sinh \frac{a}{c} \tau\end{aligned}\tag{9}$$

where τ is the proper time of the particle. The (rest frame) acceleration a is

$$a = \frac{e}{m} E_0\tag{10}$$

with e as the electric charge of the particle.

Following the Thomas prescription (see for example Jackson⁸⁾ or Bell⁸⁾ for specifying the spin of a moving particle, Derbenev and Kondratenko⁹⁾ and Jackson¹⁰⁾ have written the effective spin-field interaction Hamiltonian as

$$H = -\vec{\sigma} \cdot \left\{ \gamma^{-1} \mu \vec{B}' + (\gamma+1)^{-1} \frac{e\hbar}{2mc} \vec{\beta} \times \vec{E}' \right\} \quad (11)$$

where

$$\vec{\beta} = \frac{\vec{v}}{c}, \quad \gamma = (1 - \beta^2)^{-1/2}$$

The components of $\vec{\sigma}$ are the Pauli matrices, and $\vec{E}'$ and $\vec{B}'$ are electromagnetic fields in a system in which the particle is momentarily at rest,

$$\begin{aligned} \vec{E}' &= \gamma \left\{ \vec{E} + \vec{\beta} \times \vec{B} - \frac{\gamma}{\gamma+1} \vec{\beta} \vec{\beta} \cdot \vec{E} \right\} \\ \vec{B}' &= \gamma \left\{ \vec{B} - \vec{\beta} \times \vec{E} - \frac{\gamma}{\gamma+1} \vec{\beta} \vec{\beta} \cdot \vec{B} \right\} \end{aligned} \quad (12)$$

The second term in Eq. (11) has a simple interpretation. With the use of the classical equation of motion it can be written as¹⁰⁾

$$H_T = \frac{\hbar}{2} \vec{\omega}_T \cdot \vec{\sigma} \quad (13)$$

where $\vec{\omega}_T$ gives the Thomas precession of the particle spin,

$$\vec{\omega}_T = \frac{\gamma^2}{\gamma+1} \frac{d\vec{\beta}}{dt} \times \vec{\beta} \quad (14)$$

For the classical path (9) the Thomas precession vanishes, and the second term of the Hamiltonian (11) therefore will give contributions only when the fluctuations of the particle path are taken into account. For the moment we will omit this term, assuming that the particle is forced to follow the unperturbed path. The particle spin, on the other hand, is assumed to interact with the external fields E_0 and B_0 as well as with the radiation field.

When only the fields E_0 and B_0 are considered, the time evolution operator can be written in the simple form

$$U(\tau, \tau') = \exp \left\{ \frac{i}{\hbar} (\tau - \tau') \mu B_0 \sigma_z \right\} \quad (15)$$

If we therefore change to τ as time parameter [corresponding to multiplying the Hamiltonian (11) with the factor γ] the spin has two (quasi) stationary states, defined by $\sigma_z = \pm 1$, with an energy splitting

$$\Delta = 2|\mu||B_c| \quad (16)$$

The radiation field causes transitions between the two spin states. First order, time dependent perturbation theory gives for the transition probabilities

$$|c_{\pm}(\tau, \tau_2)|^2 = \frac{\mu^2}{\hbar^2} \int_{\tau_1}^{\tau_2} \int_{\tau_1}^{\tau_2} d\tau d\tau' \exp\left\{\pm \frac{i}{\hbar} \Delta(\tau - \tau')\right\} G(\tau - i\varepsilon, \tau') \quad (17)$$

where c_+ corresponds to transitions up (in spin energy), c_- to transitions down, and $G(\tau, \tau')$ is the following correlation function of the radiation field,

$$G(\tau, \tau') = \langle B_{\pm}^{\dagger}(\vec{r}(\tau), t(\tau)) B_{\pm}^{\dagger}(\vec{r}(\tau'), t(\tau')) \rangle \quad (18)$$

with

$$B_{\pm}^{\dagger} = B_x^{\dagger} \pm i B_y^{\dagger} \quad (19)$$

From the electromagnetic free field Wightman function (Gaussian units)

$$\begin{aligned} \langle 0 | F_{\mu\nu}(x) F_{\rho\sigma}(x') | 0 \rangle = & \\ 4 \frac{\hbar c}{\pi} (x-x')^{-6} \{ (x-x')^2 (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) - 2 [(x-x')_{\mu} (x-x')_{\rho} g_{\nu\sigma} & (20) \\ - (x-x')_{\nu} (x-x')_{\rho} g_{\mu\sigma} - (x-x')_{\mu} (x-x')_{\sigma} g_{\nu\rho} + (x-x')_{\nu} (x-x')_{\sigma} g_{\mu\rho}] \} \end{aligned}$$

we find

$$\begin{aligned} G(\tau, \tau') &= \frac{\hbar a^4}{2\pi c^7} \left\{ \sinh \left[\frac{a}{2c} (\tau - \tau') \right] \right\}^{-4} \\ &\equiv f(s-s') \end{aligned} \quad (21)$$

where

$$S = \frac{a}{c} \tau \quad (22)$$

Then, omitting the trivial integration over $\frac{1}{2}(\tau+\tau')$ in Eq. (17), we obtain for the transition probabilities per unit time

$$|C_{\pm}|^2 = \frac{\mu^2 c}{\hbar^2 a} \int_{-\infty}^{+\infty} ds \exp\left(\mp i s \frac{c\Delta}{a\hbar}\right) f(s-i\varepsilon) \quad (23)$$

The function $f(s)$ has the periodicity property

$$f(s+2\pi i) = f(s)$$

and also

$$f(s) \rightarrow 0 \text{ when } \text{Re } s \rightarrow \pm \infty$$

Therefore the integral (23) can easily be found by completing the contour of integration with the path returning along the line $\text{Im } s = 2\pi$. The integral along the closed contour gives

$$\left(1 - e^{\pm 2\pi \frac{c\Delta}{a\hbar}}\right) |C_{\pm}|^2 = \mp \frac{8}{3} \frac{\mu^2 \Delta}{\hbar^4 c^3} \left(\Delta^2 + \frac{a^2 \hbar^2}{c^2}\right) \quad (24)$$

Consequently the ratio between up and down transitions is given by

$$|C_+|^2 / |C_-|^2 = - \frac{1 - \exp(-2\pi \frac{c\Delta}{a\hbar})}{1 - \exp(2\pi \frac{c\Delta}{a\hbar})} = e^{-2\pi \frac{c\Delta}{a\hbar}} \quad (25)$$

Since this defines the equilibrium ratio of populations of the upper and lower states, the equilibrium distribution over the levels indeed has a thermal character and is in accordance with the temperature formula (1). It is, however, worth noting, with Boyer¹¹⁾, that the correlation function (21) is not identical to the corresponding thermal Green's function of the system, which would be relevant for the system embedded at rest in a heat bath of temperature $T = \hbar a / 2\pi k c$,

$$G_T(t-t') = \left(\frac{2\pi}{\hbar c}\right)^3 (kT)^4 \left\{ \left(\sinh \left[\frac{\pi kT}{\hbar} (t-t') \right] \right)^{-4} + \frac{2}{3} \sinh \left[\frac{\pi kT}{\hbar} (t-t') \right] \right\}^{-2} \quad (26)$$

This means that the transition probabilities between the levels are not identical for the two cases, although the equilibrium distribution is the same.

3. - THOMAS PRECESSION

When we tentatively include the second term of the Hamiltonian (11) in the discussion, an interesting complication arises. The correlation functions $G(\tau, \tau')$ of Eq. (17) no longer work out nicely as functions of $\tau - \tau'$. The dependence on $\tau + \tau'$ means that somehow one moment of proper time is distinguished from another. Naïvely, this seems to be in contradiction with the fact that the whole process is invariant under Lorentz boosts in the direction of the motion.

The problem can be understood in the following way. The Thomas prescription for the spin direction is not Lorentz invariant. In the Hamiltonian (11) the spin is defined by boosting to rest in the lab frame, i.e., the rest frame of the particle at $\tau = 0$. As long as the particle strictly follows the classical path (moves in the z direction) no change of the spin vector is introduced by making a transition from the lab frame to another frame moving in the z direction. However, when the Thomas term of the Hamiltonian is included, this means that one allows for transverse motions of the particle induced by the vacuum fluctuations of the field. A change of frame by a boost in the z direction will now introduce a rotation of the spin. Therefore, since the spin is no longer invariant under boosts in the z direction, it is not surprising that one moment of proper time does not seem equivalent to another.

The distinction of one moment of time from the other can be avoided by a redefinition of the spin three-vector. Instead of choosing a fixed frame along the path (the lab frame) we can refer the spin motion to a time dependent frame, or rather a family of inertial frames associated one with each point of the particle path. We define these frames by boosting the lab frame along the z direction, such that the unperturbed particle would be momentarily at rest in the new frame.

The new spin vector $\vec{S}'$ is related to the old one by a rotation which is small in so far the perturbation of the motion is small,

$$\vec{S}' = \vec{S} - \frac{\gamma^2}{\gamma+1} (\delta\vec{\beta} \times \vec{\beta}) \times \vec{S} \quad (27)$$

where $\delta\vec{\beta}$ gives the perturbation of the particle velocity. This expression is derived in Appendix A. In Appendix B we find that the Thomas equation for $\vec{S}$ is replaced by the following equation of motion for $\vec{S}'$

$$\frac{d\vec{S}'}{d\tau} = \vec{\Omega}' \times \vec{S}' \quad (28)$$

where τ is the proper time of the unperturbed particle and the vector $\vec{\Omega}'$, to first order in the perturbations, is given by

$$\vec{\Omega}' = \frac{e}{2mc} \left\{ -g \vec{B}' + (g-2) \frac{\Delta \vec{v}}{c} \times \vec{E}_0 - g \frac{e}{mc^2} \vec{B}_0 \cdot \vec{E}_0 \cdot \Delta \vec{x} \right\} \quad (29)$$

Here $\vec{B}'$ denotes the magnetic field in the rest frame of the (unperturbed) particle, including both the external field $\vec{B}_0$ and the radiation field, $\Delta \vec{x}$ and $\Delta \vec{v}$ are the perturbations of the position and velocity of the particle in the same frame, and g is the gyromagnetic factor, related to the magnetic moment μ by

$$\mu = \frac{e\hbar}{4mc} g \quad (30)$$

Since Eq. (29) only refers to quantities measured in the rest frame of the unperturbed particle, the expression for $\vec{\Omega}'$ explicitly shows that the motion of $\vec{S}'$ is independent of any preferred reference point along the particle path.

The two last terms of Eq. (29) can be interpreted as recoil terms in the sense that they depend on fluctuations of the particle path induced by interactions with the radiation field. The last of these terms is of no importance for spin flips. The first of the two terms, on the other hand, gives contributions to the transitions between the two spin states. Moreover, this term, although a "recoil" term, is not in fact of higher order in $1/m$, for a fixed acceleration, than the perturbation of the $\vec{B}'$ term. This is because

$$E_0 = \frac{m}{e} a$$

is of the order m . To allow for these recoil effects would be to allow for the fact that even the electron, in quantum electrodynamics, is an extended system. It is our intention to treat extended systems systematically elsewhere. Here we will be content with noting that for $g = 2$ the awkward term is absent. For electrons, with $g \approx 2$, we tentatively suppose that the semi-classical treatment is adequate and that only the first term in Eq. (29) is important.

4. - ORDERS OF MAGNITUDE

To examine the relevance of the above discussion for real experiments, we look at a case where the accelerating field has the value

$$E_0 = 10 \text{ MV/m} \quad (31)$$

(roughly as in the Stanford linac). The acceleration of an electron in this field is

$$a = \frac{e}{m} E_0 = 1.7 \cdot 10^{17} \text{ m/s}^2 \quad (32)$$

with an equivalent temperature

$$T = 0.7 \cdot 10^{-3} \text{ }^\circ\text{K} \quad (33)$$

For the acceleration to give a substantial effect on the polarization of the electrons, the energy splitting of the spin states should be of the order

$$\Delta = 2|\mu||B_0| \approx \frac{\hbar a}{2\pi c} = \frac{1}{2\pi} \frac{e\hbar}{mc} E_0 \quad (34)$$

With $\mu \approx e\hbar/2mc$, this means

$$B_0 \approx \frac{1}{2\pi} E_0 \approx 50 \text{ gauss} \quad (35)$$

The values of the fields given above do not create any problem. However, the small value of the temperature (33) indicates that it would be very difficult to see the thermal effect of the acceleration on the background of other depolarizing effects. These effects include ordinary temperature, which for $T \approx 300 \text{ }^\circ\text{K}$ would be larger than the temperature (33) by more than the factor 10^5 . (One should, on the other hand, be aware of the fact that the effect of ordinary temperature is reduced for large particle velocities.)

However, a much more serious problem is the time scale of the process. Assuming $\Delta = a\hbar/c$ (and $g=2$), we find from Eq. (24) that the decay rate of the upper into the lower state is

$$\Gamma \approx \frac{4}{3} \alpha \frac{\Delta^3}{m^2 c^4 \hbar} \quad (36)$$

For the values of the fields given above, this means that the time needed for the system to reach equilibrium would be of the order

$$\Delta\tau \approx \tau^{-1} \approx 3 \cdot 10^{18} \text{ s} \quad (37)$$

Since this time in fact exceeds the age of the Universe, it must be admitted that an experiment along these lines does not look very realistic. This becomes even clearer when we realize that in the lab frame the time (37) corresponds to

$$\Delta t \approx \frac{c}{2a} \exp\left(\frac{a}{c} \Delta\tau\right) \approx \exp(10^{28}) \text{ s} \quad (38)$$

One should, however, note that these times are strongly reduced when the acceleration is increased. In fact, for circulating electrons, where stronger accelerating fields can be obtained, the picture is completely different, as we shall see in the next section.

5. - CIRCULATING ELECTRONS

We now consider the motion of a charged spin one-half particle in a circular orbit perpendicular to a uniform magnetic field $\vec{B}_0$. The magnetic and electric fields in an instantaneously co-moving frame are

$$\vec{B}'_0 = \gamma \vec{B}_0, \quad \vec{E}'_0 = \gamma \vec{\beta} \times \vec{B}_0 \quad (39)$$

The proper acceleration of the particle therefore is

$$a = \frac{e}{m} E'_0 = \beta \frac{e}{m} B'_0 \quad (40)$$

Consequently, a naïve application of Eq. (1) would give

$$kT = \frac{a\hbar}{2\pi c} = \frac{1}{2\pi} \beta \frac{e\hbar}{mc} B'_0 \quad (41)$$

For a particle with magnetic moment $\mu = (e\hbar/4mc)g$, the energy splitting of the spin states, in the rest frame, is

$$\Delta = \frac{e\hbar}{2mc} |g| |B'_0| \quad (42)$$

This gives

$$\frac{\Delta}{kT} = \frac{1}{\beta} \pi |g| \quad (43)$$

For ultra-relativistic particles ($\beta \approx 1$) the temperature formula, therefore, predicts the following ratio between the population of the upper and lower spin levels

$$p_+/p_- = e^{-\pi |g|} \quad (44)$$

with the corresponding polarization

$$P = \frac{1 - e^{-\pi g}}{1 + e^{-\pi g}} \quad (45)$$

As mentioned in the Introduction, there exist calculations of the effect of spin flip radiation for the polarization of circulating electrons. In the work by Jackson¹⁰⁾, this is done in a semi-classical way by use of the spin Hamiltonian (11). One should note that in the case of circular motion no re-definition of the spin vector is needed for the calculations of the transition probabilities, since the process is already stationary as seen from the laboratory frame. Moreover in this frame it is justified [according to Refs. 9) and 10)] to neglect quantum and recoil effects for γ large [but not so large that the acceleration approaches that of Eq. (4)].

The calculations of Refs. 9) and 10) give the following expression for the polarization

$$P = F_2(a) / [F_1(a) e^{-\sqrt{2}|a|} + \frac{a}{|a|} F_2(a)] \quad (46)$$

where $a = \frac{1}{2}(g-2)$ and the functions $F_1(a)$ and $F_2(a)$ are given by

$$\begin{aligned} F_1(a) = & \left(1 + \frac{41}{45}a - \frac{23}{18}a^2 - \frac{8}{15}a^3 - \frac{14}{15}a^4 \right) \\ & - \frac{8}{5\sqrt{3}} \frac{a}{|a|} \left(1 + \frac{11}{12}a - \frac{17}{12}a^2 - \frac{13}{24}a^3 + a^4 \right) \end{aligned} \quad (47)$$

and

$$F_2(a) = \frac{8}{5\sqrt{3}} \left(1 + \frac{14}{3}a + 8a^2 + \frac{23}{3}a^3 + \frac{10}{3}a^4 + \frac{2}{3}a^5 \right) \quad (48)$$

This result is represented graphically in Fig. 1, where the polarization is shown as a function of g . Comparison with the curve obtained from the naïve formula (45) shows that there are clear similarities in the qualitative features of the two curves. However, there are also important differences. The curve obtained by the detailed calculation is shifted relative to the one found by use of the temperature formula, and it also differs in some of the details. In particular, if we look at the asymptotic behaviour for large $|g|$, the formula (46) gives

$$P_+/P_- \approx \left(\frac{1516}{|g|} - \frac{1500}{g} \right) e^{-\sqrt{3}|g-2|} \quad (49)$$

The form of the exponent here suggests a temperature about twice as big as the one given by Eq. (45). On the other hand, the complicated dependence on g in Eq. (46) clearly shows that the polarization does not unambiguously correspond to a particular temperature.

Some of the difference between the two expressions for the polarization, can be explained as due to the fact that the particles are accelerated in a circle rather than along a straight line. This can be seen already in a simpler case¹²⁾. Consider a spin zero particle with one excited state, due to some internal degree of freedom, coupled to a massless scalar field $\phi(x)$ which induces transitions between the two levels. The transition probability in the Born approximation is

$$|C_{\pm}|^2 = \frac{f^2}{\hbar^2} \iint d\tau d\tau' e^{\mp \frac{i}{\hbar} \Delta(\tau-\tau')} G(\tau-i\varepsilon, \tau') \quad (50)$$

where f is the coupling constant, Δ is the energy splitting of the two levels and $G(\tau, \tau')$ is the correlation function

$$G(\tau, \tau') = \langle \phi(\vec{r}(\tau), t(\tau)) \phi(\vec{r}(\tau'), t(\tau')) \rangle \quad (51)$$

$[\vec{r}(\tau), t(\tau)]$ defines the classical space-time trajectory which the particle is assumed to follow.

For the scalar field we have

$$\langle \phi(x) \phi(x') \rangle = \frac{\hbar c}{4\pi^2} \{ (\vec{r} - \vec{r}')^2 - c^2 (t - t')^2 \}^{-1} \quad (52)$$

and this gives for the linearly accelerated particle

$$G_c(\tau, \tau') = - \frac{a^2 \hbar}{16\pi^2 c^3} \{ \sinh \left[\frac{a}{2c} (\tau - \tau') \right] \}^{-2} \quad (53)$$

With this correlation function one obtains a thermal distribution over the levels, in the same way as described in Section 2. For circular motion the form of the correlation function is somewhat more complicated

$$G_c(\tau, \tau') = - \frac{\hbar}{4\pi^2 c} \{ \gamma^2 (\tau - \tau')^2 - (2 \frac{c}{a} \beta^2 \gamma^2)^2 \left(\sin \left[\frac{a}{2c} \frac{\tau - \tau'}{\beta \gamma} \right] \right)^2 \}^{-1} \quad (54)$$

and the equilibrium distribution over the levels cannot be found in the same easy way as for the linear acceleration. However, some of the difference between the two cases can be seen by expanding the denominators of the correlation functions in powers of $(\tau - \tau')$

$$G_c(\tau, \tau') = - \frac{\hbar}{4\pi^2 c} (\tau - \tau')^{-2} \left(1 + \frac{1}{12} \left[\frac{a}{c} (\tau - \tau') \right]^2 + \frac{1}{360} \left[\frac{a}{c} (\tau - \tau') \right]^4 + \dots \right)^{-1} \quad (55)$$

$$G_c(\tau, \tau') = - \frac{\hbar}{4\pi^2 c} (\tau - \tau')^{-2} \left(1 + \frac{1}{12} \left[\frac{a}{c} (\tau - \tau') \right]^2 - \frac{1}{360 \beta^2 \gamma^2} \left[\frac{a}{c} (\tau - \tau') \right]^4 + \dots \right)^{-1} \quad (56)$$

Note that Eq. (56) contains an extra parameter compared to Eq. (55), namely β or γ . Of course, no velocity relative to the laboratory would appear in the linear acceleration case, by Lorentz invariance. For circular motion, on the other hand, the velocity does appear as an additional parameter, and this corresponds to the fact that the radius of the circle can be varied independently of the acceleration.

The circular case simplifies for large γ . Assuming the energy splitting to be not too small ($\Delta \geq (1/\gamma)(a\hbar/c)$), we can then approximate the correlation function by

$$G_c(\tau, \tau') = -\frac{\hbar}{4\pi^2 c} (\tau - \tau')^{-2} \left\{ 1 + \frac{1}{12} \left[\frac{a}{c} (\tau - \tau') \right]^2 \right\}^{-1} \quad (57)$$

The integral (50) can now be computed by completing the integration path in the complex plane. It gives for the transition probabilities per unit time

$$|C_+|^2 = \frac{f^2}{8\sqrt{3}\pi\hbar} \frac{a}{c^2} e^{-\sqrt{12} \frac{\Delta c}{a\hbar}}$$

$$|C_-|^2 = \frac{f^2}{8\sqrt{3}\pi\hbar} \frac{a}{c^2} \left(e^{-\sqrt{12} \frac{\Delta c}{a\hbar}} + 4\sqrt{3} \frac{\Delta c}{a\hbar} \right) \quad (58)$$

For $\Delta \gg (\hbar/c)a$, the equilibrium population of the upper level relative to the lower is then

$$\rho_+/\rho_- = |C_+|^2/|C_-|^2 \approx \frac{1}{4\sqrt{3}} \frac{a\hbar}{\Delta c} e^{-2\sqrt{3} \frac{\Delta c}{a\hbar}} \quad (59)$$

This is to be compared with the ratio for linear acceleration

$$\rho_+/\rho_- = e^{-2\pi \frac{\Delta c}{a\hbar}} \quad (60)$$

The difference between Eqs. (60) and (59) is quite similar to the difference between Eqs. (44) and (49). In both cases the exponential function for the circular motion indicates a temperature which is higher by a factor $\pi/\sqrt{3} \approx 1.8$ than the temperature for linear acceleration. On the other hand, if we, in order to make a closer comparison, define

$$g = \frac{2c}{a\hbar} (E_b - E_a) \quad (61)$$

where E_a and E_b are the two energy levels of the scalar particle ($\Delta = |E_a - E_b|$), and introduce the "polarization"

$$P = \frac{g}{|g|} \frac{|p_-|^2 - |p_+|^2}{|p_-|^2 + |p_+|^2} \quad (62)$$

we find from Eq. (55) that this is numerically quite close to the polarization found by the naïve temperature formula (45) (see Fig. 1). This point can be illustrated also in another way, by evaluating the effective temperature, defined by

$$kT_{\text{eff}} = \Delta [-\ln(P_+/P_-)]^{-1} \quad (63)$$

For $g \lesssim 2$, T_{eff} is much closer to the temperature $T = \hbar/2\pi k c$ than to its asymptotic value $(\pi/\sqrt{3})T$ (see Fig. 2).

The polarization curve for the spin one-half particle in the electromagnetic field, shows a much larger deviation from the curve obtained by the naïve temperature formula. The larger difference, which mainly consists of a displacement of curve by approximately 1.2 units along the g axis, indicates that there are important complications peculiar to this system, such as the Thomas precession. In fact, such a complication is suggested already by the form of the unperturbed Hamiltonian. The polarization formula (45) is based on using the energy splitting Δ defined in the rest frame of the particle [Eq. (42)]. This expression for Δ is proportional to $|g|$. On the other hand, the energy splitting defined by the unperturbed part of the Hamiltonian (11) is (up to corrections of the order $1/\gamma$) proportional to $|g-2|$, due to the presence of the Thomas term. Therefore, if this expression for Δ were used (corrected by a factor γ for the effect of time dilatation), the polarization curve would be shifted relative to the original one, but with two units along the g axis, rather than the 1.2 units found in the more detailed calculation of the polarization effect.

6. - POLARIZATION MEASUREMENTS

The characteristic time for building up the polarization of relativistic circulating electrons ($g=2$) is according to Ref. 6) given by the formula

$$\Delta t = \frac{8}{5\sqrt{3}} \frac{m^2 c^2 R^3}{e^2 \hbar \gamma^5} \quad (64)$$

where R is the radius of the circle. In terms of the energy splitting Δ of Eq. (42) this can be written as

$$\Delta t = \frac{8}{5\sqrt{3}} \alpha^{-1} \gamma \frac{m^2 c^4 \hbar}{\Delta^3} \quad (65)$$

The corresponding expression for the proper time, $\Delta\tau$, is apart from an unimportant numerical factor, identical to what was found in Section 4 for the linearly accelerated particles [see Eq. (36)]. However, the larger value for the acceleration obtained in the case of circular motion, gives a value of Δt which is completely different from what previously was found for the linear acceleration. Introducing numbers relevant at LEP energies, $R = 3.1$ km and $\gamma = 10^5$ we find $\Delta t \approx 2\hbar$ [for a more serious discussion of polarization at LEP see, e.g., Ref. 13)]. Due to the time dilatation factor the time is even smaller for smaller accelerators. For example, in the storage ring SPEAR at Stanford a beam of 4 GeV electrons would be polarized in 10 min.¹⁰⁾ One should also note that (for fixed radius) the time Δt is strongly reduced with larger values of γ .

There do in fact exist measurements of the polarization effects discussed above. In a recent experiment at SPEAR, the maximum polarization has been measured as a function of γ for energies around 3.6 GeV. The experimental data indicate several values of γ with resonant depolarization. However, away from these points the values of maximum polarization are in very good agreement with the theoretical value $P = 0.924$. Also the build-up time of the polarization, which has been measured for several values of γ , is in very good agreement with the theoretical prediction.

6. - CONCLUSIONS

We have examined in this paper the possibility of using an accelerated spin one-half particle in an external magnetic field as a "thermometer", to measure the thermal properties of the vacuum fluctuations in an accelerated frame. For particles accelerated linearly in an external electric field, the thermal effect is reproduced when the particles are assumed strictly to follow the classical path. However, when realistic values of the fields are assumed, the transition time between the levels is found to be extremely long, and this makes it completely unrealistic to suggest an experiment along these lines.

In the case of circular acceleration, much larger accelerations can easily be obtained. Existing calculations of the polarization of circulating electrons show that for large γ , a maximum polarization of $P \approx 0.92$ is obtained. This corresponds to a population of the upper spin level relative to the lower one of 4%. Measurements of the polarization of circulating electrons have found such an effect, with the value of the polarization in good agreement with the predicted value⁷⁾.

From the calculations of the polarization effect it is clear that it is of the same nature as the thermal effect found for linearly accelerated particles. In both cases, the population of the upper spin level is caused by interactions with the vacuum fluctuations of the radiation field. However, the numerical results for the polarization, in the case of circular motion, show substantial differences from the ones obtained by a naïve interpretation of the polarization as due to a thermal effect. The discrepancy can in part be explained by the fact that the particles follow a circular orbit rather than being accelerated linearly, and the effect of the Thomas precession is an additional complication.

APPENDIX A

ALTERNATIVE SPIN

Let $(s_0, \vec{s})$ be the components, in the lab frame, of the BMT spin four-vector⁸⁾ of a particle of lab velocity $\vec{w}$. Then the Thomas spin $\vec{S}$ is defined by

$$\begin{pmatrix} 0 \\ \vec{S} \end{pmatrix} = B(-\vec{w}) \begin{pmatrix} s_0 \\ \vec{s} \end{pmatrix} \quad (\text{A.1})$$

where $B(-\vec{w})$ is a Lorentz boost bringing the particle to rest. Written out in detail it gives

$$\begin{aligned} \gamma (s_0 - \vec{\beta} \cdot \vec{s}) &= 0 \\ \vec{s} + \frac{\gamma^2}{\gamma+1} \vec{\beta} \vec{\beta} \cdot \vec{s} - \gamma \vec{\beta} s_0 &= \vec{S} \end{aligned} \quad (\text{A.2})$$

where $\vec{\beta} = \vec{w}/c$ and $\gamma = (1-\beta^2)^{-1/2}$.

The same procedure applied in another inertial frame, where the BMT components are $(s'_0, \vec{s}')$ and the velocity $\vec{w}'$, leads to a different vector $\vec{S}'$,

$$\begin{pmatrix} 0 \\ \vec{S}' \end{pmatrix} = B(-\vec{w}') \begin{pmatrix} s'_0 \\ \vec{s}' \end{pmatrix} \quad (\text{A.3})$$

If the second frame is obtained from the first by a boost with velocity $\vec{v}$, we have

$$\begin{pmatrix} s'_0 \\ \vec{s}' \end{pmatrix} = B(-\vec{v}) \begin{pmatrix} s_0 \\ \vec{s} \end{pmatrix} \quad (\text{A.4})$$

whence

$$\begin{pmatrix} 0 \\ \vec{S}' \end{pmatrix} = B(-\vec{w}') B(-\vec{v}) B(\vec{w}) \begin{pmatrix} 0 \\ \vec{S} \end{pmatrix} \quad (\text{A.5})$$

In what follows the second frame will be one in which a hypothetical, unperturbed particle would be momentarily at rest. The velocities

$$\begin{aligned}\delta\vec{U} &= \vec{\omega} - \vec{U} \\ \Delta\vec{U} &= \vec{\omega}'\end{aligned}\tag{A.6}$$

then are both assumed to be small. The rotation arising from the combination of boosts in Eq. (A.5) is one familiar from the discussion of the Thomas precession,

$$\vec{S}' = \vec{S} - \delta\vec{\phi} \times \vec{S}\tag{A.7}$$

$$\delta\vec{\phi} = \frac{\gamma^2}{\gamma+1} \delta\vec{\beta} \times \vec{\beta}\tag{A.8}$$

with $\delta\vec{\beta} = \delta\vec{v}/c$. The quantities $\Delta\vec{v}$ and $\delta\vec{v}$ are related by

$$\Delta\vec{U} = \gamma\delta\vec{U} + \frac{\gamma^3}{\gamma+1} \frac{1}{c^2} \vec{U} \vec{U} \cdot \delta\vec{U}\tag{A.9}$$

whence Eq. (A.7) can also be written as

$$\delta\vec{\phi} = \frac{\gamma}{\gamma+1} \frac{1}{c^2} \Delta\vec{U} \times \vec{U}\tag{A.10}$$

In what follows we will also use Eq. (A.7) in matrix form

$$S' = S - \delta\phi S = RS\tag{A.11}$$

where S and S' are column matrices and $\delta\phi$ is the appropriate 3×3 anti-symmetric matrix.

The orientation of the second frame above was defined by a rotation free boost from the lab frame. If we orient it in some other way, we will have instead

$$S' = R_0^{-1} (\mathbb{I} - \delta\phi) S\tag{A.12}$$

where R_0 is some rotation matrix. In particular R_0 can be chosen as a function of time, so that, as observed from the moving frame, the transformation to the frame used a short time later, is a rotation free boost. Then the Thomas precession of $\vec{S}'$ is cancelled - to zero order in $\delta\vec{v}$.

APPENDIX B

PRECESSION OF $\vec{S}$

The Thomas equation for the evolution of $\vec{S}$ can be written as

$$d\vec{S} = d\vec{\Theta} \times \vec{S} \quad (\text{B.1})$$

with

$$d\vec{\Theta} = d\vec{\Theta}_M + d\vec{\Theta}_T \quad (\text{B.2})$$

where $d\vec{\Theta}_M$ describes the part of the precession which depends on the magnetic moment μ of the particle and $d\vec{\Theta}_T$ gives the Thomas precession. We have

$$\begin{aligned} d\vec{\Theta}_M &= -2\bar{\mu} \vec{B}' d\tau \\ &= -2\bar{\mu} \gamma \left\{ \vec{B} - \vec{\beta} \times \vec{E} - \frac{\gamma}{\gamma+1} \vec{\beta} \vec{\beta} \cdot \vec{B} \right\} d\tau \end{aligned} \quad (\text{B.3})$$

where $\vec{B}'$ is the magnetic field in the rest frame and $\bar{\mu}$ is defined by

$$\bar{\mu} = \frac{\mu}{\hbar} = \frac{e}{4mc} g$$

$\vec{E}$ and $\vec{B}$ are the electromagnetic fields in the lab frame and $d\tau$ is the change in proper time along the particle path. The Thomas precession is given by

$$d\vec{\Theta}_T = \frac{\gamma^2}{\gamma+1} d\vec{\beta} \times \vec{\beta} \quad (\text{B.4})$$

When the expressions above refer to the external fields $\vec{E}_0$ and $\vec{B}_0$ and to the unperturbed particle path, we write the precession vector $d\vec{\Theta}$ as

$$\begin{aligned} d\vec{\Theta}_0 &= d\vec{\Theta}_{0M} + d\vec{\Theta}_{0T} \\ &= -2\bar{\mu} \vec{B}_0' d\tau + \frac{\gamma^2}{\gamma+1} d\vec{\beta} \times \vec{\beta} \end{aligned} \quad (\text{B.5})$$

We now consider the perturbation of $d\vec{\theta}$, denoted by $\delta d\vec{\theta}$, caused by fluctuations in the fields, $\delta\vec{E}$, $\delta\vec{B}$, and the corresponding perturbations $\delta\vec{x}$, $\delta\vec{v}$ of the particle path. From Eqs. (B.3) and (B.4) we obtain after some manipulation (to first order in the perturbations)

$$\begin{aligned} \delta d\vec{\theta}_M = & -2\bar{\mu} (\vec{B}_0' \delta d\tau + \delta\vec{B}' d\tau) \\ & + 2\bar{\mu} (\gamma \delta\vec{\beta} \times \vec{E}_0' + \frac{\gamma^3}{\gamma+1} \vec{\beta} \cdot \delta\vec{\beta} \vec{\beta} \times \vec{E}_0' \\ & + \frac{\gamma^2}{\gamma+1} [\delta\vec{\beta} \vec{\beta} \cdot \vec{B}_0' - \vec{\beta} \delta\vec{\beta} \cdot \vec{B}_0']) d\tau \end{aligned} \quad (B.6)$$

(where $\delta\vec{B}'$ is the radiation field in the momentary rest frame of the unperturbed particle) and

$$\begin{aligned} \delta d\vec{\theta}_T = & \frac{\gamma^4(\gamma+2)}{(\gamma+1)^2} \vec{\beta} \cdot \delta\vec{\beta} d\vec{\beta} \times \vec{\beta} + \frac{\gamma^2}{\gamma+1} d\delta\vec{\beta} \times \vec{\beta} \\ & + \frac{\gamma^2}{\gamma+1} d\vec{\beta} \times \delta\vec{\beta} \end{aligned} \quad (B.7)$$

The equation of motion for $\vec{S}$, which is obtained from Eqs. (A.12) and (B.1), has the following matrix form

$$dS' = R_c^{-1} \{ d\theta - dR_c R_c^{-1} - d\delta\varphi - [\delta\varphi, d\theta] \} R_c S' \quad (B.8)$$

Consequently the vector $d\vec{\theta}'$, which determines the precession of $\vec{S}'$, has the unperturbed part

$$d\theta_0' = R_c^{-1} (d\theta_0 - dR_c R_c^{-1}) R_c \quad (B.9)$$

and the perturbation is given by

$$\delta d\theta' = R_c^{-1} (\delta d\theta - d\delta\varphi - [\delta\varphi, d\theta_0']) R_c \quad (B.10)$$

In the following we assume R_0 to be chosen to cancel the Thomas term of the unperturbed motion. We then have, in vector notation

$$d\vec{\Theta}'_0 = R_0^{-1} (-2\bar{\mu} \vec{B}'_0 d\tau) \quad (B.11)$$

and

$$\delta d\vec{\Theta}' = R_0^{-1} (\delta d\vec{\Theta} - d\delta\vec{\varphi} - \delta\vec{\varphi} \times d\vec{\Theta}_0) \quad (B.12)$$

By use of Eqs. (A.8), (B.6) and (B.7) we now find

$$\begin{aligned} \delta d\vec{\Theta}'_m &= R_0^{-1} (\delta d\vec{\Theta}_m - \delta\vec{\varphi} \times d\vec{\Theta}_{0m}) \\ &= -2\bar{\mu} R_0^{-1} \{ \vec{B}'_0 \delta d\tau + \delta\vec{B}'_0 d\tau - \gamma \delta\vec{\beta} \times \vec{E}'_0 d\tau \\ &\quad - \frac{\gamma^3}{\gamma+1} \vec{\beta} \cdot \delta\vec{\beta} \vec{\beta} \times \vec{E}'_0 d\tau \} \end{aligned} \quad (B.13)$$

In terms of $\Delta\vec{v}$, this can be written as

$$\delta d\vec{\Theta}'_m = -2\bar{\mu} R_0^{-1} \{ \vec{B}'_0 \delta d\tau + \delta\vec{B}'_0 d\tau - \frac{1}{c} \Delta\vec{v} \times \vec{E}'_0 d\tau \} \quad (B.14)$$

The expression which is found for $\delta d\vec{\Theta}'_T$ is

$$\begin{aligned} \delta d\vec{\Theta}'_T &= R_0^{-1} (\delta d\vec{\Theta}_T - d\delta\vec{\varphi} - \delta\vec{\varphi} \times d\vec{\Theta}_{0T}) \\ &= R_0^{-1} (\gamma^3 d\vec{\beta} \times \delta\vec{\beta} - \frac{\gamma^4}{\gamma+1} \vec{\beta} d\vec{\beta} \cdot (\delta\vec{\beta} \times \vec{\beta})) \end{aligned} \quad (B.15)$$

The acceleration of the unperturbed particle, measured in the inertial rest frame, is given by

$$\vec{a} d\tau = \gamma d\vec{v} + \frac{\gamma^3}{\gamma+1} \frac{1}{c^2} \vec{v} \vec{v} \cdot d\vec{v} \quad (B.16)$$

In terms of $\vec{a}$ and $\Delta\vec{v}$, Eq. (B.15) gets the simple form

$$\delta d\vec{\Theta}'_T = \frac{1}{c^2} \mathcal{R}_0^{-1} (\vec{a} \times \Delta\vec{v}) d\tau \quad (\text{B.17})$$

The variation $\delta d\tau$ in proper time is determined by

$$c^2 (\delta d\tau) d\tau = (\delta dx^\mu) dx_\mu \quad (\text{B.18})$$

where δx^μ satisfies the condition

$$\delta x^\mu dx_\mu = 0 \quad (\text{B.19})$$

This gives

$$\begin{aligned} \delta d\tau &= - \frac{1}{c^2} \delta x^\mu \frac{d^2 x_\mu}{d\tau^2} d\tau \\ &= \frac{1}{c^2} \Delta\vec{x} \cdot \vec{a} d\tau \end{aligned} \quad (\text{B.20})$$

where $\Delta\vec{x}$ is the perturbation of the particle path measured in the rest frame of the unperturbed motion.

If we now introduce the angular velocity

$$\vec{\Omega}' = \frac{d\vec{\Theta}'}{d\tau} \quad (\text{B.21})$$

such that the spin equation reads

$$\frac{d}{d\tau} \vec{S}' = \vec{\Omega}' \times \vec{S}' \quad (\text{B.22})$$

and collect all the terms from Eqs. (B.11), (B.14) and (B.17), we obtain for $\vec{\Omega}'$,

$$\begin{aligned} \vec{\Omega}' &= \mathcal{R}_0^{-1} \left\{ -2\vec{\mu} (\vec{B}_0' + \delta\vec{B}' - \frac{1}{c} \Delta\vec{v} \times \vec{E}_0' \right. \\ &\quad \left. + \frac{1}{c^2} \vec{B}_0' \Delta\vec{x} \cdot \vec{a}) + \frac{1}{c^2} \vec{a} \times \Delta\vec{v} \right\} \end{aligned} \quad (\text{B.23})$$

If we further assume the classical equation of motion to be satisfied for the unperturbed motion,

$$\vec{a} = \frac{e}{m} \vec{E}_0' \quad (\text{B.24})$$

we get

$$\begin{aligned} \vec{\Omega}' = & -\frac{e}{2mc} R_0^{-1} \left\{ g(\vec{B}_0' + \delta \vec{B}') + (g-2) \vec{E}_0' \times \frac{\Delta \vec{v}}{c} \right. \\ & \left. + g \frac{e}{mc^2} \vec{B}_0' \vec{E}_0' \cdot \Delta \vec{x} \right\} \end{aligned} \quad (\text{B.25})$$

For the case of linear acceleration, we have $R_0 = I$, $\vec{B}_0' = \vec{B}_0$ and $\vec{E}_0' = \vec{E}_0$. The expression can then be written as

$$\begin{aligned} \vec{\Omega}' = & -\frac{e}{2mc} \left\{ g(\vec{B}_0 + \delta \vec{B}') + \frac{1}{c} (g-2) \vec{E}_0 \times \Delta \vec{v} \right. \\ & \left. + g \frac{e}{mc^2} \vec{B}_0 \vec{E}_0 \cdot \Delta \vec{x} \right\} \end{aligned} \quad (\text{B.26})$$

For the case of circular motion in a magnetic field, we have

$$\vec{B}_0' = \gamma \vec{B}_0, \quad \vec{E}_0' = \gamma \vec{\beta} \times \vec{B}_0 \quad (\text{B.27})$$

$\vec{\Omega}'$ can then be written as

$$\begin{aligned} \vec{\Omega}' = & -\frac{e}{2mc} R_0^{-1} \left\{ g(\gamma \vec{B}_0 + \delta \vec{B}') + \frac{\gamma}{c} (g-2) \vec{\beta} \vec{B}_0 \cdot \Delta \vec{v} \right. \\ & \left. + \frac{\gamma}{c} (g-2) \vec{B}_0 \vec{\beta} \cdot \Delta \vec{v} - g \frac{e}{mc^2} \vec{B}_0 \vec{B}_0 \cdot (\vec{\beta} \times \Delta \vec{x}) \right\} \end{aligned} \quad (\text{B.28})$$

REFERENCES

- 1) For a review see, for example,
N.D. Birell and P.C.W. Davies, "Quantum fields in curved space", Cambridge,
Cambridge University Press, (1982).
- 2) W.G. Unruh, Phys. Rev. D14 (1976) 870.
- 3) P.C.W. Davies, J. Phys. A8 (1975) 609.
- 4) S. Hawking, Nature 248 (1974) 30 and Comm. Math. Phys. 43 (1975) 199.
- 5) D.W. Sciama, P. Candelas and D. Deutsch, Adv. in Phys. 30 (1981) 327.
- 6) A.A. Sokolov and I.M. Ternov, Dokl. Akad. Nauk. SSR 153 (1963) 1052 [Sov.
Phys. Dokl. 7 (1964) 1203].
- 7) J.R. Johnson, R. Prepost, D.E. Wiser, J.J. Murray, R.F. Schwitters and
C.K. Sinclair, preprint SLAC-PUB-2903 (April 1982).
- 8) J.D. Jackson, "Classical Electrodynamics", New York, Wiley (1962);
J.S. Bell, "Polarized Particles for Accelerator Physicists", CERN yellow
report 75-11 (September 1975).
- 9) Ya.S. Derbenev and A.M. Kondratenko, Zh. Eksp. Teor. Fiz. 64 (1973) 1918
[Sov. Phys. JETP 37 (1973) 968].
- 10) J.D. Jackson, Rev. Mod. Phys. 48 (1976) 417.
- 11) T.H. Boyer, Phys. Rev. D21 (1980) 2137.
- 12) J.R. Letaw and J.D. Pfautsch, Phys. Rev. D22 (1980) 345.
- 13) A.M. Kondratenko and V.W. Montague, CERN-ISR-TH/80-38 (August 1980).

FIGURE CAPTIONS

- Fig. 1 : Polarization of circulating particles as a function of the magnetic moment. A gives the polarization of spin one-half particles calculated according to Refs. 9) and 10), B is the corresponding curve obtained from a temperature interpretation of the spin excitations and C gives the "polarization" obtained from the excitation of circulating scalar particles, as explained in the text.
- Fig. 2 : The effective temperature for circulating scalar particles. T_{eff} is shown as a function of the energy splitting, measured in terms of g . It is shown on the scale where the temperature $T = \hbar a / 2\pi k c$, with a as the acceleration, is put equal to unity. T_{as} is the asymptotic value of T_{eff} for $|g| \rightarrow \infty$.

Fig.1

Fig. 2