REF.TH.3388-CERN

N = 1 SUPERSPACE FORMULATION OF N = 2 AND N = 4
SUPER-YANG-MILLS MODELS WITH CENTRAL CHARGE

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ABSTRACT

The component models of $N = 2$ and $N = 4$ supersymmetric Yang-Mills theories of Sohnius, Stelle and West¹⁾ are reformulated in terms of $N = 1$ superfields. The non-supersymmetric constraints are supersymmetrized generalizing the linear multiplet in the presence of the non-Abelian gauge superfield and (in the $N = 4$ case) a doublet of chiral superfields. The extended supersymmetry transformations preserving constraints are explicitly given in terms of $N = 1$ superfields. We are able to introduce the constraints back into the Lagrangian using superfield Lagrange multipliers. The on-shell equivalence of this formulation with the formulation of Fayet²⁾ with one (for $N = 2$) and three (for $N = 4$) chiral superfields is shown. The Abelian $N = 2$ model is worked out to show the connection between full superspace treatment and the $N = 1$ superfield formulation.

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REF.TH.3388-CERN

10 August 1982

1. - INTRODUCTION

The interest in the extended supersymmetric Yang-Mills (SYM) models stems from the belief that they are good candidates for the finite quantum field theory. Recent results³⁾ show that the $N = 4$, i.e., maximally extended SYM theory would be finite to all orders in perturbation theory provided the unconstrained $N = 4$ formulation exists. It was also argued⁴⁾ that even under weaker assumptions, namely that the unconstrained $N = 2$ superspace formulation of the model exists, the finiteness may still be proved. Unfortunately, such a formulation [except for the $N = 2$ Abelian case⁵⁾] is not yet available. For this reason, the search for different formalisms which may help us to understand some of the problems which one encounters in trying to construct the unconstrained extended SYM theories is justified. One of these problems is the presence of the so-called off-shell central charges in the algebra of supersymmetry. In principle, the central charges forbid the unconstrained superfield formulation of the model [unless we use the additional bosonic dimensions⁶⁾ in our superspace, which is rather inconvenient from the physical point of view]. Our attitude towards the off-shell central charges is that they are a specific symptom of the non-closure of the supersymmetry algebra off the mass-shell. If this is so, we shall, in principle, be able to get rid of them by introducing some set of auxiliary fields (or superfields). For instance, in Ref. 4) the $N = 2$ hypermultiplet was constructed using additional superfields to avoid central charge.

The progress in the off-shell formulation of the extended SYM is rather slow. The $N = 2$ and $N = 4$ models were formulated in terms of $N = 1$ superfields²⁾ partially off-shell (i.e., with explicit $N = 1$ supersymmetry). In the component language there are two different off-shell representations of the $N = 2$ model. One is obtained by solving the constraints on the field strengths' superfield and the other, of Sohnius, Stelle and West (SSW), using dimensional reduction by Legendre transformation¹⁾. The latter has a central charge in the supersymmetry algebra and the constraint imposed on the vector field. This constraint in the non-Abelian case apparently has no local solution and, introduced back into the Lagrangian, destroys supersymmetry. Using $N = 1$ superfields we can show how to improve this behaviour, at least preserving $N = 1$ supersymmetry.

The $N = 2$ Abelian SYM model was formulated in terms of a prepotential⁵⁾. We explore this fact in order to show how the $N = 1$ superspace formulation follows from the full superspace treatment. In this way we can enlarge the Fayet formulation by introducing pure gauge superfields to form an "unconstrained" $N = 2$ multiplet of $N = 1$ superfields.

The on-shell $N = 4$ theory was formulated in terms of $N = 1$ superfields by Fayet²⁾. The only existing off-shell formulation is that of SSW¹⁾ with central charge and constraints. We were also able to reformulate this model in the $N = 1$ superspace language using a multiplet consisting of a gauge superfield, a generalized linear superfield and a doublet of chiral superfields. We also show the equivalence of this model with the formulation of Fayet (with a triplet of chiral superfields) via the "grandfather" Lagrangian with superfield Lagrange multipliers.

The paper is organized as follows: in Section 2 we introduce the technique of representing extended supersymmetry on multiplets of $N = 1$ superfields and show the equivalence of this method with the full superspace treatment, using the $N = 2$ model of Mesinchecku⁵⁾ as an example. Section 3 is devoted to a study of the simplest, Abelian version of the $N = 2$ SYM with central charge constructed from $N = 1$ superfields. In Section 4 we introduce the non-Abelian version of the model and in Section 5 we generalize it in the case of $N = 4$ supersymmetry. Section 6 contains some concluding remarks. In Appendix A we list our conventions and some useful formulae and in Appendix B we show the gauge structure underlying the models and stress the relevance of K transformations. Some results of Sections 3 and 4 have already been published as a letter⁸⁾.

2. - THE $N = 1$ FORMULATION OF EXTENDED THEORIES

The lack of the (unconstrained) full superspace formulations of the theories of great interest, such as the extended SYM theories, forces us to investigate other possibilities of constructing such models. Most of the supersymmetric theories were first written down in component fields and then, gradually, translated into the superfield language which is simpler and more powerful, for example in the quantum calculations³⁾. However, the extended theories are resistant to this kind of treatment. The intermediate step, i.e., the formulation in terms of $N = 1$ superfields, thus seems reasonable. For instance, the three-loop finiteness of the $N = 4$ SYM was established using the $N = 1$ superspace Feynman technique⁹⁾. The connection between full superspace and $N = 1$ superspace formulations of a model may be easily established provided that we know the former. For instance, let us assume that a $N = 2$ theory is expressed in terms of a set of superfields $\phi_I(x, \theta, \eta)$, where θ_α and η_α are two sets of Grassmann variables [in the $SU(2)$ covariant notation they are $\theta_i^\alpha, i=1,2$]. We may then expand the superfields with respect to one set of the Grassmann variables, say η_α , the coefficients of the expansion being $N = 1$ superfields.

$$\Phi_I(x, \theta, \eta) = \varphi_I(x, \theta) + \eta^\alpha \Psi_{\alpha I}(x, \theta) + \bar{\eta}_{\dot{\alpha}} \bar{\Psi}_I^{\dot{\alpha}}(x, \theta) + \dots \quad (2.1)$$

This kind of description was used for example in Ref. 10). Here we prefer to use a simpler method in which, instead of listing the components (in our case component superfields) of the power series expansion, we express them as the multiple co-variant derivatives acting on the superfield, taken at $\eta = 0 = \bar{\eta}$. To this end we distinguish the derivatives involving θ_α denoting them by D_α and those involving η_α by d_α . Thus

$$D_{1\alpha} \equiv D_\alpha, \quad D_{2\alpha} \equiv d_\alpha, \quad \bar{D}_{2\dot{\alpha}} \equiv \bar{D}_{\dot{\alpha}}, \quad \bar{D}_{1\dot{\alpha}} \equiv -\bar{d}_{\dot{\alpha}}. \quad (2.2)$$

As an example, let us take the $N = 2$ Abelian SYM model⁵⁾. The prepotential is an $SU(2)$ isovector $V^{ij} = \tau_a^{ij} V^a$, the field strength has the following form $W = \bar{D}^2 U$, where $U = D_{ij} V^{ij}$ (for notational conventions, see Appendix A). The action formula reads

$$S = \frac{1}{2} \int D^4 W^2 \quad (2.3)$$

Let us denote the $SU(2)$ components of V^{ij} in the following way

$$V^{11} = c, \quad V^{22} = -\bar{c}, \quad V^{12} = V^{21} = v. \quad (2.4)$$

We then get

$$U = D_{ij} V^{ij} = D^2 c - d^2 \bar{c} + 2 D^\alpha d_\alpha v, \quad (2.5)$$

$$\begin{aligned} S &= \frac{1}{2} \int D^2 d^2 [(\bar{D}^2 \bar{d}^2 U)(\bar{D}^2 \bar{d}^2 U)] = \\ &= \int D^2 \bar{D}^2 [(d^2 \bar{d}^2 U)(\bar{D}^2 \bar{d}^2 U) + (d^\alpha \bar{d}^2 U) \bar{D}^2 (d_\alpha \bar{d}^2 U)], \end{aligned}$$

or, inserting (2.5),

$$\begin{aligned} S &= \int D^2 \bar{D}^2 [(D^2 d^2 \bar{d}^2 c - d^2 \bar{d}^2 d^2 \bar{c} + 2 D^\alpha d^2 \bar{d}^2 d_\alpha v) \times \\ &\quad \times \bar{D}^2 (D^2 \bar{d}^2 c - \bar{d}^2 d^2 \bar{c} + 2 D^\alpha \bar{d}^2 d_\alpha v) + \\ &\quad + (D^2 d^\alpha \bar{d}^2 c - d^\alpha \bar{d}^2 d^2 \bar{c} + 2 D^\beta d^\alpha \bar{d}^2 d_\beta v) \times \\ &\quad \times \bar{D}^2 (D^2 d_\alpha \bar{d}^2 c - d_\alpha \bar{d}^2 d^2 \bar{c} + 2 D^\beta d_\alpha \bar{d}^2 d_\beta v)]. \end{aligned} \quad (2.6)$$

The relevant components are:

$$A = d^2 \bar{d}^2 c, \quad \bar{A} = \bar{d}^2 d^2 \bar{c}, \quad V = d^\alpha \bar{d}^2 d_\alpha v. \quad (2.7)$$

The fields with more than four derivatives may be expressed as the space-time derivatives of the lower derivative fields, e.g.,

$$\begin{aligned} d^2 \bar{d}^2 d^2 \bar{c} &= -\square d^2 \bar{c}, \\ d^2 \bar{d}^2 d_\alpha v &= -i \partial_{\alpha\dot{\alpha}} \bar{d}^2 \bar{d}^{\dot{\alpha}} v, \\ d^\alpha \bar{d}^2 d^2 \bar{c} &= -i \partial^\alpha_{\dot{\alpha}} \bar{d}^{\dot{\alpha}} d^2 \bar{c}. \end{aligned} \quad (2.8)$$

All the fields with less than four derivatives may be gauged away using the transformations

$$\delta V^{ij} = D^\alpha_k \chi_\alpha^{kij} + h.c. \quad (2.9)$$

with χ_α^{kij} totally symmetric in SU(2) indices. For instance, the v component transforms as follows

$$\delta v = D^\alpha \lambda_{1\alpha} + \bar{D}_{\dot{\alpha}} \bar{\lambda}_1^{\dot{\alpha}} - d^\alpha \lambda_{2\alpha} - \bar{d}_{\dot{\alpha}} \bar{\lambda}_2^{\dot{\alpha}}, \quad (2.10)$$

where

$$\lambda_1 \equiv \chi^{112}, \quad \bar{\lambda}_1 \equiv \bar{\chi}^{212}, \quad \lambda_2 \equiv -\chi^{212}, \quad \bar{\lambda}_2 \equiv \bar{\chi}^{112}.$$

The fields $\lambda_1, \bar{\lambda}_1$ cannot be used to gauge away the whole $\eta = 0$ part of v because of the D derivatives acting upon them. Roughly speaking, they gauge away the "linear" part of v (in fact, even more because λ_1 's are not chiral superfields). The rest of v is gauged by $\lambda_2, \bar{\lambda}_2$ thus leaving the "linear" part of the λ_2 's free (this part is then used to gauge away higher derivative fields). This chain ends up at the "physical" superfields $A, \bar{A}$ and V which transform as follows

$$\delta A = D^\alpha F_\alpha, \quad \delta \bar{A} = \bar{D}_{\dot{\alpha}} \bar{F}^{\dot{\alpha}}, \quad \delta V = D^2 H + \bar{D}^2 \bar{H}, \quad (2.11)$$

where $F_\alpha, \bar{F}^{\dot{\alpha}}, H$ and $\bar{H}$ are constructed from the parts of the gauge parameters that remain free after gauging away the lower derivative fields. The action

(2.6) now reads

$$\begin{aligned} S &= \int D^2 \bar{D}^2 (D^2 A \bar{D}^2 \bar{A} + D^\alpha V \bar{D}^2 D_\alpha V) = \\ &= \int D^2 \bar{D}^2 (D^2 A \bar{D}^2 \bar{A}) + \int D^2 (\bar{D}^2 D^\alpha V \cdot \bar{D}^2 D_\alpha V). \end{aligned} \quad (2.12)$$

We may recognize the superfields $\bar{A}$, A and V as the prepotentials of the chiral, antichiral and gauge field strength superfields, respectively, and the action as the $N = 2$ SYM action first formulated by Fayet²⁾. The use of prepotentials for chiral superfields was also considered in Ref. 3). The knowledge of the full set of pure gauge superfields which complete the Fayet multiplet may be useful in the attempts to find the non-Abelian generalization of the unconstrained full superspace formulation of the model.

3. - $N = 2$ SYM WITH CENTRAL CHARGE. THE ABELIAN CASE

We want to study first the Abelian model which, although trivial in the sense that there are no interactions present, possesses all the features of extended supersymmetry models formulated in terms of $N = 1$ superfields. The multiplet consists of a gauge superfield V and a linear superfield G . The linear superfield is the one which fulfills the constraints

$$D^2 G = 0, \quad \bar{D}^2 G = 0. \quad (3.1)$$

Let us translate (3.1) into the component language. If G is of the form

$$G = \varphi + \theta \Psi + \bar{\theta} \bar{\Psi} + \theta^2 m + \bar{\theta}^2 \bar{m} + \theta \sigma^n \bar{\theta} w_n + \theta^2 \bar{\theta} \bar{\chi} + \bar{\theta}^2 \theta \chi + \theta^2 \bar{\theta}^2 d, \quad (3.2)$$

then the constraints (3.1) pose the following restrictions

$$m = 0 = \bar{m}, \quad \bar{\chi} = -\frac{i}{2} \bar{\partial} \Psi, \quad \chi = -\frac{i}{2} \partial \bar{\Psi}, \quad d = \frac{1}{4} \square \varphi \quad (3.3)$$

$$\partial_n w^n = 0 \quad (3.4)$$

The first set of equations (3.3) may be used to express certain components of G in terms of the others, whereas Eq. (3.4) constrains the vector field to be transversal. Equation (3.4) may be solved by means of an antisymmetric tensor A_{mn} , namely

$$w^n = \epsilon^{nklm} \partial_k A_{lm} = \partial_k {}^* A^{nk} \quad (3.5)$$

which is equivalent to solving (3.1) in terms of a chiral spinor superfield¹¹⁾

$$G = D^\alpha \Psi_\alpha + \bar{D}_{\dot{\alpha}} \bar{\Psi}^{\dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}} \Psi_\alpha = 0 = D_\alpha \bar{\Psi}_{\dot{\alpha}}. \quad (3.6)$$

The action for the $N = 2$ SYM in terms of the multiplet (V, G) is

$$S = -\frac{1}{2} \int D^2 W^\alpha W_\alpha - \int D^2 \bar{D}^2 G^2, \quad (3.7)$$

where $W_\alpha = 2\bar{D}^2 D_\alpha V$ is the gauge field strength. The action (3.7) is explicitly invariant under $N = 1$ supersymmetry as well as under the gauge transformations

$$\delta V = i(\bar{\Lambda} - \Lambda), \quad D_\alpha \bar{\Lambda} = 0 = \bar{D}_\alpha \Lambda. \quad (3.8)$$

Second supersymmetry, along with central charge and internal symmetry transformations, is described by the following formulae

$$\delta V = i G (\epsilon - \bar{\epsilon}), \quad (3.9a)$$

$$\delta G = i D^\alpha (W_\alpha \epsilon) + i \bar{D}_\alpha (\bar{W}^\alpha \bar{\epsilon}), \quad (3.9b)$$

where $\epsilon(\bar{\epsilon})$ is the (anti) chiral x -independent superfield parameter with the following component structure

$$\begin{aligned} \epsilon &= z + \theta \zeta + \theta^2 g, \\ \bar{\epsilon} &= \bar{z} + \bar{\theta} \bar{\zeta} + \bar{\theta}^2 \bar{g}. \end{aligned} \quad (3.10)$$

The (constant) spinors $\zeta_\alpha, \bar{\zeta}_{\dot{\alpha}}$ are the parameters of the second supersymmetry transformations, whereas $(z - \bar{z})$ is the central charge parameter and $g, \bar{g}$ internal $SU(2)/U(1)$ symmetry parameters. To reveal the structure of the central charge transformations let us rewrite (3.9b) in the following form

$$\begin{aligned} \delta G &= i D W \epsilon + i \bar{D} \bar{W} \bar{\epsilon} + i W D \epsilon + i \bar{W} \bar{D} \bar{\epsilon} = \\ &= i D W (\epsilon - \bar{\epsilon}) + i W D \epsilon + i \bar{W} \bar{D} \bar{\epsilon}, \end{aligned} \quad (3.11)$$

where we used the identity $DW = -\bar{D}\bar{W}$. Only the first part of (3.11) involves the central charge parameter. Then, from (3.9) and (3.11) it follows that

$$\delta_{c.c.} V = i G (z - \bar{z}), \quad (3.12a)$$

$$\delta_{c.c.} G = i D W (z - \bar{z}). \quad (3.12b)$$

In the Wess-Zumino gauge, where all the lower components of V are gauged away, the transformation (3.12a) must be followed by the compensating gauge transformation.

It turns out that the result of these two transformations is proportional to the equations of motion. The right-hand side of (3.12b) apparently vanishes on-shell ($DW = 0$ is the equation of motion for V). So (3.12) indeed describe the off-shell central charge. The parameters $g, \bar{g}$ rotate one physical spinor λ_α of V into the other ψ_α of G

$$\delta\psi = -ig\lambda, \quad \delta\lambda = -i\bar{g}\psi. \quad (3.13)$$

They parametrize the coset $SU(2)/U(1)$, where $SU(2)$ is the internal symmetry group associated with $N = 2$ supersymmetry and $U(1)$ is the chiral group associated with the explicit $N = 1$ supersymmetry.

It is easy to show that the constraints (3.1) are preserved by the transformations (3.9). To this end let us note that (3.9b) is of the form (3.6) and $W_\alpha \epsilon$ is a chiral superfield.

It can be directly checked that after choosing the Wess-Zumino gauge, the modified transformation rules obtained from (3.9) (i.e., augmented with the compensating gauge transformations), along with the usual $N = 1$ supersymmetry, give the component transformation laws of Ref. 1). A straightforward calculation shows that they close off-shell modulo constraints (3.3), (3.4) forming the $N = 2$ supersymmetry algebra with an off-shell central charge.

4. - $N = 2$ NON-ABELIAN CASE

In the non-Abelian case the multiplet (V, G) is the same, except that now both superfields are matrices in the adjoint representation of some gauge group. For that reason the covariant gauge field strength now has the form

$$W_\alpha = \bar{D}^2 (e^{-2V} D_\alpha e^{2V}). \quad (4.1)$$

Notice the factor of two in front of V which is put there for convenience because in what follows we shall also use e^V - a square root of the quantity appearing in (4.1). The simplest covariantization of the constraints (3.1) is

$$\mathcal{D}^2 G = 0 = \bar{\mathcal{D}}^2 G, \quad (4.2)$$

where $\mathcal{D}_\alpha = D_\alpha + [A_\alpha, \cdot]$ is a gauge covariant derivative. It is shown in Appendix B that (4.2) is equivalent, in some special (imaginary) gauge, to

$$D^2(e^V G \bar{e}^V) = 0 = \bar{D}^2(\bar{e}^V G e^V). \quad (4.3)$$

Because of the special gauge choice, quantities such as G, e^V transform in a complicated way under the gauge group, whereas e^{2V} and $e^V G \bar{e}^V$ have simple transformation laws

$$\delta e^{2V} = i(\bar{\Lambda} e^{2V} - e^{2V} \Lambda), \quad (4.4a)$$

$$\delta \bar{e}^{2V} = i(\Lambda \bar{e}^{2V} - \bar{e}^{2V} \bar{\Lambda}), \quad (4.4b)$$

$$\delta(e^V G \bar{e}^V) = i[\bar{\Lambda}, e^V G \bar{e}^V], \quad (4.5a)$$

$$\delta(\bar{e}^V G e^V) = i[\Lambda, \bar{e}^V G e^V], \quad (4.5b)$$

[where $\Lambda(\bar{\Lambda})$ are (anti) chiral gauge parameters - matrices from the algebra of the gauge group]. Notice that (4.5) are compatible with (4.4), e.g.,

$$\begin{aligned} \delta(e^V G \bar{e}^V) &= \delta(e^{2V}(\bar{e}^V G e^V)\bar{e}^{2V}) = i\bar{\Lambda} e^V G \bar{e}^V - \\ &- i e^{2V} \Lambda \bar{e}^V G e^V + i e^{2V} \Lambda \bar{e}^V G e^V - i e^V G e^V \Lambda \bar{e}^{2V} + \\ &+ i e^V G e^V \Lambda \bar{e}^{2V} - i e^V G \bar{e}^V \bar{\Lambda} = i[\bar{\Lambda}, e^V G \bar{e}^V]. \end{aligned} \quad (4.6)$$

In the Wess-Zumino gauge ($V = \theta \sigma^m \bar{\theta} V_m + \theta^2 \bar{\theta} \bar{\Lambda} + \bar{\theta}^2 \theta \Lambda + \frac{1}{2} \theta^2 \bar{\theta}^2 D$) the constraints (4.3) read

$$\begin{aligned} m = 0 = \bar{m}, \quad \bar{\chi}^{\dot{\alpha}} &= -\frac{i}{2} \bar{\nabla}^{\dot{\alpha}\alpha} \psi_{\alpha} - [\bar{\lambda}^{\dot{\alpha}}, \varphi], \\ \chi_{\alpha} &= -\frac{i}{2} \nabla_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} + [\lambda_{\alpha}, \varphi], \end{aligned} \quad (4.7)$$

$$d = \frac{1}{4} (\square \varphi - 2i \partial_n [v^n, \varphi] - [v^n, [v_n, \varphi]]),$$

$$\nabla_n w^n - i[D, \varphi] + i\{\lambda^{\alpha}, \psi_{\alpha}\} + i\{\bar{\lambda}_{\dot{\alpha}}, \bar{\psi}^{\dot{\alpha}}\} = 0, \quad (4.8)$$

where gauge covariant derivatives ∇_n act as follows

$$\nabla_n = \partial_n - i[v_n, \cdot]. \quad (4.9)$$

As in the previous case, (4.7) may be used to express the higher components of G in terms of the lower ones (but now with covariantization). However, the generalized transversality condition (4.8) cannot be solved locally as in (3.5). The constraints (4.2) may be instead implemented in the Lagrangian by means of the Lagrange multipliers, losing however the off-shell $N = 2$ supersymmetry. Before showing that, let us first write down the action and transformation rules for the constrained version of the model

$$S = \text{Tr} \int \left\{ -\frac{1}{2} D^2 W^2 - D^2 \bar{D}^2 G^2 \right\}, \quad (4.10)$$

$$\delta e^{2V} = 2i e^V G e^V (\varepsilon - \bar{\varepsilon}), \quad (4.11a)$$

$$\delta(e^V G e^{-V}) = i D^\alpha (e^{2V} W_\alpha e^{-2V} \varepsilon) + i e^{2V} \bar{D}_{\dot{\alpha}} (\bar{e}^{-2V} \bar{W}^{\dot{\alpha}} e^{2V} \bar{\varepsilon}) \bar{e}^{-2V}, \quad (4.11b)$$

where the parameters $\varepsilon, \bar{\varepsilon}$ are specified as in (3.10). The transformation rules for e^{-2V} and $e^{-V} G e^V$ are obtained by noticing that

$$\delta e^{-2V} = -e^{-2V} \delta e^{2V} e^{-2V}, \quad (4.12a)$$

$$\delta(e^{-V} G e^V) = \delta(e^{-2V} (e^V G e^{-V}) e^{2V}). \quad (4.12b)$$

Writing (4.12b) the other way around we can prove the compatibility of the transformation rules. To show the invariance of the constraints let us note that, using the identity (see Appendix B)

$$\bar{D}_{\dot{\alpha}} (e^{-2V} \bar{W}^{\dot{\alpha}} e^{2V}) = -e^{-2V} D^\alpha (e^{2V} W_\alpha e^{-2V}) e^{2V}, \quad (4.13)$$

we may write (4.11b) in the following way

$$\begin{aligned} \delta(e^V G e^{-V}) &= i D^\alpha (e^{2V} W_\alpha e^{-2V}) (\varepsilon - \bar{\varepsilon}) + \\ &+ i e^{2V} W^\alpha e^{-2V} D_\alpha \varepsilon + i \bar{W}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \bar{\varepsilon}. \end{aligned} \quad (4.14)$$

Now, acting with D^2 on (4.14), we obtain

$$D^2 \delta(e^V G e^{-V}) = i D^2 D^\alpha (e^{2V} W_\alpha e^{-2V}) (\varepsilon - \bar{\varepsilon}) + i D^2 (\bar{W}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \bar{\varepsilon}) = 0 \quad (4.15)$$

because $D_{\alpha} \bar{D}_{\alpha} \bar{\epsilon} = \frac{1}{2} \partial_{\alpha\alpha} \bar{\epsilon} - \bar{D}_{\alpha} D_{\alpha} \bar{\epsilon} = 0$. The invariance of the action under (4.11) may be checked by explicit calculation. The arguments of Section 3 about the off-shell closure of the $N = 2$ supersymmetry algebra with central charge also hold in the non-Abelian case.

As we have said, the constraints (4.3) may be implemented in the Lagrangian by means of the chiral superfield Lagrange multipliers. In this way we obtain the "grandfather" action

$$S_{GF} = \text{Tr} \left\{ -\frac{1}{2} D^2 W^2 + D^2 \bar{D}^2 (-G^2 + \sqrt{2} G (e^V \phi e^{-V} + e^{-V} \bar{\phi} e^V)) \right\}. \quad (4.16)$$

The equations of motion resulting from variation with respect to ϕ and $\bar{\phi}$ give the constraints (4.3). On the other hand, variation of G gives

$$G = \frac{1}{\sqrt{2}} (e^V \phi e^{-V} + e^{-V} \bar{\phi} e^V), \quad (4.17)$$

which, inserted back into the action (4.16), produces the other version of the $N = 2$ SYM, namely the action of Fayet²⁾

$$S = \text{Tr} \left\{ -\frac{1}{2} D^2 W^2 + D^2 \bar{D}^2 (\bar{\phi} e^{2V} \phi e^{-2V}) \right\}. \quad (4.18)$$

This shows the classical equivalence of these two versions, one of which possesses and the other does not possess the off-shell central charge.

5. - $N = 4$ NON-ABELIAN SYM

In order to formulate the $N = 4$ theory we must enlarge our multiplet of $N = 1$ superfields by adding a doublet of chiral superfields ϕ^p ($p=1,2$). All the fields belong to the same adjoint representation of the gauge group. The constraints which will be compatible with all the transformation laws are

$$D^2 (e^V G e^{-V}) = -\epsilon^{pq} \bar{\phi}^p \bar{\phi}^q, \quad (5.1a)$$

along with the Hermitean conjugate

$$\bar{D}^2 (e^{-V} G e^V) = \epsilon^{pq} \phi^p \phi^q, \quad (5.1b)$$

where $\epsilon^{pq} = -\epsilon^{qp}$, $\epsilon^{12} = 1$. The action must also contain the kinetic term for chiral fields, so it has the form

$$S = \text{Tr} \int \left\{ -\frac{1}{2} D^2 W^2 + D^2 \bar{D}^2 (\bar{\Phi}^P e^{2V} \Phi^P e^{-2V} - G^2) \right\} \quad (5.2)$$

The gauge group acts on V and G in the same way as before, whereas the chiral superfields transform as follows

$$\delta \Phi^P = i [\Lambda, \Phi^P], \quad \delta \bar{\Phi}^P = i [\bar{\Lambda}, \bar{\Phi}^P]. \quad (5.3)$$

Supersymmetry transformations have now a more complicated structure. Second supersymmetry [the first being realized explicitly in terms of differential generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ (see Appendix A)] acts as follows

$$\delta e^{2V} = 2i e^V G e^V (\varepsilon - \bar{\varepsilon}), \quad (5.4a)$$

$$\begin{aligned} \delta(e^V G e^{-V}) = & i D^\alpha (e^{2V} W_\alpha e^{-2V} \varepsilon) + i e^{2V} \bar{D}_{\dot{\alpha}} (e^{-2V} \bar{W}^{\dot{\alpha}} e^{2V} \bar{\varepsilon}) e^{-2V} - \\ & - i (\varepsilon - \bar{\varepsilon}) (\bar{\Phi}^P e^{2V} \Phi^P e^{-2V} - e^{2V} \Phi^P e^{-2V} \bar{\Phi}^P), \end{aligned} \quad (5.4b)$$

$$\delta \Phi^P = -i \varepsilon^P{}^q \bar{D}^2 (e^{-2V} \bar{\Phi}^q e^{2V} (\varepsilon - \bar{\varepsilon})). \quad (5.5)$$

In (5.5) one can recognize the transformation rules of the hypermultiplet²⁾. The θ independent part of $(\varepsilon - \bar{\varepsilon})$ gives the central charge transformations, e.g.,

$$\delta_{c.c.} \Phi^P = -i \varepsilon^P{}^q \bar{D}^2 (e^{-2V} \bar{\Phi}^q e^{2V}) (z - \bar{z}), \quad (5.6)$$

which are proportional to the equations of motion of $\bar{\Phi}^q$. It is easy to check that the constraints (5.1) are preserved by (5.4) and (5.5). However, to define the full $N = 4$ supersymmetry multiplet, we need two more transformation laws. They are parametrized by a doublet of constant spinors ζ_α^P which, together with parameters z^P and g^P , form a chiral x -independent superfield

$$\eta^P = z^P + \theta \zeta^P + \theta^2 g^P. \quad (5.7)$$

The transformations read

$$\delta e^{2V} = -i\sqrt{2} (\eta^P \bar{\Phi}^P e^{2V} - \bar{\eta}^P e^{2V} \Phi^P), \quad (5.8a)$$

$$\delta(e^V G e^{-V}) = i\sqrt{2} \varepsilon_P{}^q [D^\alpha (e^{2V} \Phi^P e^{-2V} D_\alpha \eta^q) -$$

$$-e^{2V}\bar{D}_{\dot{\alpha}}(e^{-2V}\bar{\Phi}^P e^{2V}\bar{D}^{\dot{\alpha}}\bar{\eta}^Q)\bar{e}^{2V}] + i\sqrt{2}\eta^P[e^V G \bar{e}^V, \bar{\Phi}^P], \quad (5.8b)$$

$$\delta\Phi^P = -i\sqrt{2}[W^{\alpha}D_{\alpha}\eta^P - \epsilon^{PQ}\bar{D}^2(\bar{\eta}^Q e^{-V} G e^V)]. \quad (5.8c)$$

A straightforward calculation shows that those transformations preserve the constraints and form a consistent set [e.g., the equations analogous to (4.12b) hold].

The parameters z^P of (5.7) do not describe additional central charges, but rather field-dependent gauge transformations. Indeed, when we look at the part of (5.8) generated by $z^P, \bar{z}^P$, we recognize the transformations of the type (4.4), (4.5) and (5.3), namely

$$\delta_z e^{2V} = -i\sqrt{2}(z^P \bar{\Phi}^P e^{2V} - e^{2V} \bar{z}^P \Phi^P), \quad (5.9a)$$

$$\delta_z(e^V G e^{-V}) = i\sqrt{2}[e^V G e^{-V}, z^P \bar{\Phi}^P], \quad (5.9b)$$

$$\begin{aligned} \delta_z \Phi^1 &= i\sqrt{2}\epsilon^{12}\bar{D}^2(e^{-V} G e^V)\bar{z}^2 = i\sqrt{2}\bar{z}^2 \epsilon^{rs}\Phi^r \Phi^s = \\ &= i\sqrt{2}[\Phi^1, \bar{z}^2 \Phi^2], \end{aligned} \quad (5.9c)$$

with the superfield

$$\Lambda = -\sqrt{2}\bar{z}^P \Phi^P \quad (5.10)$$

playing the role of a chiral gauge parameter.

The set of parameters g and g^P (the θ^2 components of ϵ and η^P , respectively) parametrize the coset $O(5)/U(2) \simeq Sp(4)/U(2)$. The $Sp(4)$ transformations, being a part of the $N=4$ supersymmetry algebra with central charge, rotate the physical spinors ψ, ψ^P and λ of G, ϕ^P and V , respectively, as well as five physical scalars $\varphi, \varphi^P = 1/\sqrt{2}(\varphi_1^P + i\varphi_2^P)$ of G and ϕ^P and five auxiliary fields $D, F^P = 1/\sqrt{2}(F_1^P + iF_2^P)$ of V and ϕ^P , respectively. The part of this internal symmetry group, namely $U(2)$, acting on the chiral doublet ϕ^P , is realized explicitly

$$\begin{aligned} \Phi^P &\rightarrow A^P{}_Q \Phi^Q, \\ \bar{\Phi}^P &\rightarrow \bar{\Phi}^Q (A^\dagger)_Q{}^P, \quad A \in U(2), \end{aligned} \quad (5.11)$$

the rest, $Sp(4)/U(2)$, being parametrized by g, g^P (i.e., three complex = six real parameters). It must be stressed that the constraints (5.1), action (5.2) and transformation rules (5.4), (5.5) and (5.8), when written down in components using Wess-Zumino gauge, reproduce the constraint, action and transformation rules of Sohnius, Stelle and West¹⁾. The algebra generated by (5.4), (5.5) and (5.8)

closes off-shell modulo constraints (5.1) forming [together with the transformations generated by Q_α and the group $U(2)$ of (5.11)] the representation of $N = 4$ supersymmetry with an off-shell central charge.

Finally, let us note that the constraints (5.1) may be implemented in the action using chiral Lagrange multipliers $\phi^3, \bar{\phi}^3$ (they will form triplets with $\phi^p, \bar{\phi}^p$, hence the index 3). The "grandfather" action reads

$$S_{GF} = \text{Tr} \int \left\{ -\frac{1}{2} D^2 W^2 + D^2 \bar{D}^2 (-G^2 + \sqrt{2} G (e^V \phi^3 e^{-V} + e^{-V} \bar{\phi}^3 e^V) + \bar{\phi}^p e^{2V} \phi^p e^{-2V}) - \sqrt{2} D^2 (\epsilon_{pq} \phi^3 \phi^p \phi^q) + \sqrt{2} \bar{D}^2 (\epsilon_{pq} \bar{\phi}^3 \bar{\phi}^p \bar{\phi}^q) \right\}. \quad (5.12)$$

Variation with respect to $\phi^3, \bar{\phi}^3$ gives the constraints (5.1). On the other hand, variation with respect to G produces the equation of motion

$$G = \frac{1}{\sqrt{2}} (e^V \phi^3 e^{-V} + e^{-V} \bar{\phi}^3 e^V), \quad (5.13)$$

which, introduced back into (5.12), gives the $N = 4$ SYM action of Fayet²⁾ with a triplet of chiral superfields

$$\begin{aligned} \phi^i &= (\phi^p, \phi^3), \quad i = 1, 2, 3, \\ S &= \text{Tr} \int \left\{ -\frac{1}{2} D^2 W^2 + D^2 \bar{D}^2 (\bar{\phi}^i e^{2V} \phi^i e^{-2V}) - \right. \\ &\quad \left. - \frac{\sqrt{2}}{3} D^2 (\epsilon_{ijk} \phi^i \phi^j \phi^k) + \frac{\sqrt{2}}{3} \bar{D}^2 (\epsilon_{ijk} \bar{\phi}^i \bar{\phi}^j \bar{\phi}^k) \right\}. \end{aligned} \quad (5.14)$$

This form of the (on-shell) $N = 4$ SYM action was used in Ref. 9) to prove the three-loop finiteness of the theory.

6. - CONCLUDING REMARKS

The $N = 4$ multiplet may be looked upon as consisting of two $N = 2$ multiplets. In the formalism presented here, the splitting into an $N = 2$ gauge linear (V, G) and hypermultiplet (ϕ^1, ϕ^2) is apparent, e.g., in (5.4) and (5.5) (even more so in the Abelian case because of the fact that there is no mixing between the two submultiplets). However, another splitting - into the gauge-chiral (V, ϕ^1) and $N = 2$ linear (G, ϕ^2) multiplets - is also possible. In the Abelian case, both of them may be represented by $N = 2$ superfields, V^{ij} (Mesinchecku) and L^{ij} (with constraints $D^{(k} L^{ij)} = 0$), respectively. Namely this choice

was considered in Ref. 4). It turns out that such a formulation leads, by suitably relaxing the constraints of the linear multiplet, to an unconstrained theory. We have shown that the $N = 2$ SYM may be constructed by adding a "matter" (i.e., chiral or linear) multiplet to a gauge multiplet. In the case of $N = 4$ theory, the trouble is that the $N = 2$ matter multiplets either have central charge (hypermultiplet) or are constrained (linear multiplet). We have seen in Section 2 that even chirality constraints are solved in terms of prepotentials in the unconstrained formulation. Thus, the crucial problem of our formulation is the possibility of solving constraints. The results of Ref. 4) give hope that this is indeed possible.

Another possibility is given by the "grandfather" Lagrangians (4.16) and (5.12) which may yield the off-shell representations of the extended supersymmetry when augmented with the suitable set of auxiliary superfields. It may be shown that the "grandfather" Lagrangians are not the off-shell extended models because in their case what was called before "a constraint" now becomes an equation of motion. That puts the theory on-shell.

ACKNOWLEDGMENTS

It is a pleasure to thank S. Ferrara, J. Lukierski, H. Nicolai and M. Sohnius for their interest in this work and useful discussions. The author is also indebted to P. Howe who drew our attention to the structure of the gauge group and the relevance of K transformations.

APPENDIX A

In this appendix we list our conventions and some useful formulae. We use the two-component Weyl notation for spinors

$$\psi_\alpha, \quad \bar{\psi}_{\dot{\alpha}} = (\psi_\alpha)^* . \quad (\text{A.1})$$

The indices are raised and lowered by means of an antisymmetric tensor

$$\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}, \quad \epsilon^{12} = 1, \quad (\text{A.2})$$

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}, \quad \epsilon_{12} = -1,$$

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta_\alpha^\gamma, \quad (\text{A.3})$$

$$\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta, \quad \psi_\beta = \epsilon_{\beta\alpha} \psi^\alpha, \quad (\text{A.4})$$

(and the same for dotted indices).

Sigma matrices are defined as follows

$$\sigma^m = (1, \vec{\sigma}), \quad \bar{\sigma}^m = (1, -\vec{\sigma}), \quad (\text{A.5})$$

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\beta}\epsilon^{\alpha\beta} \sigma^m_{\beta\dot{\beta}}, \quad (\text{A.6})$$

where $\vec{\sigma}$ are the usual Pauli matrices.

The products of spinors are defined as follows

$$\psi\chi \equiv \psi^\alpha\chi_\alpha, \quad \bar{\psi}\bar{\chi} \equiv \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}, \quad (\text{A.7})$$

and in particular for the co-ordinates

$$\theta^2 = \theta^\alpha\theta_\alpha, \quad \bar{\theta}^2 = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}. \quad (\text{A.8})$$

The covariant derivatives are

$$D_\alpha = \frac{i}{2}(\partial_\alpha + i\partial_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}), \quad \bar{D}_{\dot{\alpha}} = -\frac{i}{2}(\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha\partial_{\alpha\dot{\alpha}}), \quad (\text{A.9})$$

where

$$\partial_\alpha = \frac{\partial}{\partial \theta^\alpha} , \quad \partial_{\alpha\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^m \partial_m , \quad \partial_m = \frac{\partial}{\partial x^m} . \quad (\text{A.10})$$

D^2 means $D^\alpha D_\alpha$ and $\bar{D}^2 = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}}$. Supersymmetry generators are

$$Q_\alpha = \frac{i}{2} (\partial_\alpha - i \partial_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}) , \quad \bar{Q}_{\dot{\alpha}} = -\frac{i}{2} (\bar{\partial}_{\dot{\alpha}} - i \theta^\alpha \partial_{\alpha\dot{\alpha}}) , \quad (\text{A.11})$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -\frac{i}{2} \partial_{\alpha\dot{\alpha}} , \quad \{D_\alpha, \bar{D}_{\dot{\alpha}}\} = \frac{i}{2} \partial_{\alpha\dot{\alpha}} , \\ \{Q_\alpha, D_\alpha\} = 0 , \quad \{\bar{Q}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 , \quad \{Q_\alpha, \bar{D}_{\dot{\alpha}}\} = 0 , \text{ etc.} \quad (\text{A.12})$$

The $N = 2$ covariant derivatives read

$$D_{\alpha k} = \frac{i}{2} (\partial_{\alpha k} + i \partial_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha} k}) , \quad \bar{D}_{\dot{\alpha}}^k = -\frac{i}{2} (\bar{\partial}_{\dot{\alpha}}^k + i \theta^{\alpha k} \partial_{\alpha\dot{\alpha}}) \quad (\text{A.13})$$

where

$$\partial_{\alpha k} \equiv \frac{\partial}{\partial \theta^{\alpha k}} , \quad \partial_{\dot{\alpha}}^k = \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha} k}} , \quad (\text{A.14})$$

and $\bar{\theta}^{\dot{\alpha} k} = (\theta^{\alpha k})^*$. The $SU(2)$ indices are raised and lowered by means of the antisymmetric metric ϵ_{kj}

$$\theta^{\alpha k} = \epsilon^{kj} \theta^{\alpha j} . \quad (\text{A.15})$$

Pauli matrices with raised indices are symmetric

$$\tau_a^{ij} = \epsilon^{ik} (\tau_a)_k^j , \quad \tau_a^{ij} = \tau_a^{ji} . \quad (\text{A.16})$$

The products of covariant derivatives are denoted by

$$D_{ij} \equiv D^\alpha_i D_{\alpha j} , \quad D^4 \equiv D_{ij} D^{ij} \text{ etc.} \quad (\text{A.17})$$

In action formulae we use covariant derivatives instead of using the integrals over the Grassmann variables (which is completely equivalent).

APPENDIX B

In this appendix the gauge structure of the $N = 1$ SYM is presented in brief and some formulae are written down in a manifestly covariant way. The gauge covariant derivatives have the following form

$$\mathcal{D}_\alpha = D_\alpha + A_\alpha, \quad \bar{\mathcal{D}}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} + \bar{A}_{\dot{\alpha}}, \quad \mathcal{D}_m = \partial_m + A_m. \quad (\text{B.1})$$

They are supposed to fulfil the constraints

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0, \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0, \quad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = \frac{i}{2} \partial_{\alpha\dot{\alpha}}, \quad (\text{B.2})$$

which have the following solution

$$A_\alpha = e^{iU} D_\alpha e^{-iU}, \quad \bar{A}_{\dot{\alpha}} = e^{i\bar{U}} \bar{D}_{\dot{\alpha}} e^{-i\bar{U}}, \quad (\text{B.3})$$

$$A_m = \frac{1}{i} \bar{\sigma}_m^{\dot{\alpha}\alpha} (D_\alpha \bar{A}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} A_\alpha + \{A_\alpha, \bar{A}_{\dot{\alpha}}\}). \quad (\text{B.4})$$

The complex superfield U transforms under gauge transformations in the following way

$$e^{iU} \rightarrow e^{iK} e^{iU} e^{i\bar{\Lambda}}, \quad e^{-i\bar{U}} \rightarrow e^{-i\Lambda} e^{-i\bar{U}} e^{-iK}, \quad (\text{B.5})$$

where $\Lambda(\bar{\Lambda})$ are (anti) chiral superfields and K is a real superfield. It is easy to check that under (B.5)

$$\begin{aligned} A_\alpha &\rightarrow e^{iK} A_\alpha e^{-iK} - i e^{iK} D_\alpha e^{-iK}, \\ \bar{A}_{\dot{\alpha}} &\rightarrow e^{iK} \bar{A}_{\dot{\alpha}} e^{-iK} - i e^{iK} \bar{D}_{\dot{\alpha}} e^{-iK}. \end{aligned} \quad (\text{B.6})$$

The gauge covariant quantity

$$W_\alpha = \bar{D}^2 \{ e^{-i\bar{U}} e^{iU} D_\alpha (e^{-iU} e^{i\bar{U}}) \} \quad (\text{B.7})$$

is associated with the field strength

$$[\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_m] = i F_{\dot{\alpha}m}, \quad (\text{B.8})$$

$$F_{\dot{\alpha}m} \sim W^\alpha \sigma_{m\alpha\dot{\alpha}}, \quad W_\alpha = e^{i\bar{U}} W_\alpha e^{-i\bar{U}}. \quad (\text{B.9})$$

W_α is a chiral superfield, $\bar{D}_\alpha W_\alpha = 0$ and transforms under (B.5) as follows

$$W_\alpha \rightarrow e^{-i\Lambda} W_\alpha e^{i\Lambda}, \quad (B.10)$$

whereas $\mathcal{W}_\alpha$ is covariantly chiral $\bar{D}_\alpha \mathcal{W}_\alpha = 0$ and transforms under K transformations

$$\mathcal{W}_\alpha \rightarrow e^{iK} \mathcal{W}_\alpha e^{-iK}. \quad (B.11)$$

The connection between covariantly chiral and chiral quantities is established by the following formulae

$$\bar{D}_\alpha \Phi_{cov} = 0, \quad \Phi_{cov} = e^{i\bar{U}} \Phi e^{-i\bar{U}}, \quad \bar{D}_\alpha \Phi = 0, \quad (B.12)$$

$$D_\alpha \bar{\Phi}_{cov} = 0, \quad \bar{\Phi}_{cov} = e^{iU} \bar{\Phi} e^{-iU}, \quad D_\alpha \bar{\Phi} = 0, \quad (B.13)$$

because, for example,

$$\begin{aligned} e^{i\bar{U}} \bar{D}_\alpha (e^{-i\bar{U}} \Phi_{cov} e^{i\bar{U}}) e^{-i\bar{U}} &= D_\alpha \Phi_{cov} + [e^{i\bar{U}} \bar{D}_\alpha e^{-i\bar{U}}, \Phi_{cov}] = \\ &= \bar{D}_\alpha \Phi_{cov}, \end{aligned} \quad (B.14)$$

with the field ϕ in the adjoint representation. The covariantly linear superfield fulfils the constraints

$$D^2 G = 0, \quad \bar{D}^2 G = 0, \quad (G - \text{real}) \quad (B.15)$$

which, by the analogous considerations, are equivalent to

$$D^2 (e^{-iU} G e^{iU}) = 0 = \bar{D}^2 (e^{-i\bar{U}} G e^{i\bar{U}}). \quad (B.16)$$

Thus we may define two complex superfields

$$L = e^{-iU} G e^{iU}, \quad \bar{L} = e^{-i\bar{U}} G e^{i\bar{U}} \quad (B.17)$$

with constraints

$$D^2 L = 0, \quad \bar{D}^2 \bar{L} = 0, \quad (B.18)$$

and, because of (B.17) with the "reality" condition,

$$(B.7) \quad \rightarrow \quad W_\alpha = \bar{D}^2 (e^{-2V} D_\alpha e^{2V}) , \quad (4.1)$$

$$(B.16) \quad \rightarrow \quad D^2 (e^V G e^{-V}) = 0 = \bar{D}^2 (e^{-V} G e^V) , \quad (4.3)$$

$$(B.23) \quad \rightarrow \quad D^\alpha (e^{2V} W_\alpha \bar{e}^{2V}) + e^{2V} \bar{D}_{\dot{\alpha}} (e^{-2V} \bar{W}^{\dot{\alpha}} e^{2V}) \bar{e}^{2V} = 0. \quad (4.13)$$

On the other hand, some formulae of the main part of the work may be written down in a general gauge, e.g.,

$$\delta e^{iU} = -2i \varepsilon G e^{iU} , \quad \delta e^{-i\bar{U}} = 2i \bar{\varepsilon} e^{-i\bar{U}} G , \quad (4.11a)$$

$$\delta L = i e^{-iU} (\mathcal{D}^\alpha (W_\alpha \varepsilon) + i \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{W}^{\dot{\alpha}} \bar{\varepsilon})) e^{iU} \quad (4.11b)$$

$$\mathcal{D}^2 G = -\epsilon_{pq} \Phi^p_{cov} \bar{\Phi}^q_{cov} ,$$

$$\delta e^{iU} = -i\sqrt{2} \bar{\eta}^p \Phi^p_{cov} e^{iU} \quad (5.1a)$$

$$\delta e^{-i\bar{U}} = i\sqrt{2} \eta^p e^{-i\bar{U}} \bar{\Phi}^p_{cov}$$

$$\delta L = i\sqrt{2} e^{-iU} \{ \epsilon_{pq} [\mathcal{D}^\alpha (\Phi^p_{cov} D_\alpha \eta^q) - \quad (5.8a)$$

$$- \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{\Phi}_{cov} \bar{D}^{\dot{\alpha}} \bar{\eta}^q)] + [G, \bar{\Phi}_{cov}] \eta^p \} e^{iU} \quad (5.8b)$$

$$\delta \Phi^p = -i\sqrt{2} e^{-i\bar{U}} [W^\alpha D_\alpha \eta^p - \epsilon_{pq} \bar{\mathcal{D}}^2 (G \bar{\eta}^q)] e^{i\bar{U}} \quad (5.8c)$$

$$\bar{L} = e^{-i\bar{U}} e^{iU} L e^{-iU} e^{i\bar{U}}. \quad (B.19)$$

The superfield G transforms under (reality preserving) K transformations

$$G \rightarrow e^{iK} G e^{-iK} \quad (B.20)$$

whereas $L, \bar{L}$ transform with chiral parameters

$$L \rightarrow e^{-i\bar{\Lambda}} L e^{i\bar{\Lambda}}, \quad \bar{L} \rightarrow e^{-i\Lambda} \bar{L} e^{i\Lambda} \quad (B.21)$$

preserving the constraints (B.18).

The field strength superfield W_α fulfils the (Bianchi) identity

$$\mathcal{D}^\alpha W_\alpha + \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = 0 \quad (B.22)$$

which, using the trick analogous to (B.14), may be written down as

$$e^{iU} \mathcal{D}^\alpha (e^{-iU} e^{i\bar{U}} W_\alpha e^{-i\bar{U}} e^{iU}) e^{-iU} + \\ + e^{i\bar{U}} \bar{\mathcal{D}}_{\dot{\alpha}} (e^{-i\bar{U}} e^{iU} \bar{W}^{\dot{\alpha}} e^{-iU} e^{i\bar{U}}) e^{-i\bar{U}} = 0. \quad (B.23)$$

All the formulae in the main part of our paper are written down in a special gauge, which we call "imaginary", because the K transformation was used to gauge away the real part of U . We then denote $U = iV$ where V is real. This choice is compatible with supersymmetry, but not with Λ transformations. Thus, along with the Λ transformation, we have to make the compensating K transformation

$$e^{iU} = e^{-V} \rightarrow e^{iK} e^{-V} e^{i\bar{\Lambda}}, \quad (B.24)$$

where $K = K(\Lambda, \bar{\Lambda}, V)$ is an infinite power series in $\Lambda, \bar{\Lambda}$ and V . In order to avoid the problems associated with K transformations in the imaginary gauge framework, we consider only the quantities which are K invariant, e.g.,

$$e^{2V} = e^{-iU} e^{i\bar{U}}, \quad e^V G e^{-V}, \quad e^{-V} G e^V, \quad \text{etc.} \quad (B.25)$$

As an example, we list some formulae from this appendix translated into the imaginary gauge

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