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COMPOSITE GRAVITY AND COMPOSITE SUPERGRAVITY
FROM NON-LINEAR REALIZATIONS OF SUPERSYMMETRY

J. Lukierski ^{*)}
CERN - Geneva

A B S T R A C T

Using non-linear realizations of $OSp(N;4)$, we define composite vierbeins ($N \geq 1$), composite $SL(2,c)$ curvatures ($N \geq 1$) and composite achtbein superfields ($N \geq 2$). Composite gravity and composite supergravity models are proposed. We show that one can construct, from one set of fundamental Goldstone fields, the composite gravity as well as the composite Yang-Mills theory describing internal symmetries.

^{*)} On leave of absence from the Institute for Theoretical Physics, University of Wrocław, 50-205 Wrocław, Ul. Cybulskiego 36, Poland.

1. In the field theoretical models of elementary particle interactions with a sequence of symmetry breakings, one idea seems to be attractive: that at shorter distances, described by the dynamics with larger symmetry groups, the elementary objects become composite. According to this point of view, for example, the standard $SU(3) \times SU(2) \times U(1)$ model with elementary gauge fields is an effective theory describing physics only at sufficiently large distances. The composite structure of $SU(3) \times SU(2) \times U(1)$ gauge potentials would, however, be revealed if we passed on, for example, to the description of distances of the order of Planck length.

The composite gauge potentials describing the internal symmetry group H can be constructed from the σ fields which take values in the internal symmetry group G [see Refs. 1)-10)]. If we assume that $(H, K = G/H)$ form a symmetric Riemannian pair, the local H group transformations describe the gauge freedom, and the gauge invariant modes are described by Goldstone fields with values in $K = G/H$. These subconstituent σ fields in $D = 4$ are subcanonical and form the set of elementary objects in the composite Yang-Mills (YM) gauge theory for the group H . The known examples of such a construction are the composite $U(n) \times U(m)$ potentials obtained from $U(n+m)/U(n) \times U(m)$ σ fields³⁾⁻⁶⁾, $Sp(1)$ composite gauge fields in $HP(n)$ σ models^{7),8)}, or $SU(8)$ gauge fields constructed by $E_7/SU(8)$ σ fields¹¹⁾ in the scalar sector of $N = 8$ supergravity. The preliminary results indicating in which sense one can understand the quantized conventional YM theory with elementary gauge potentials as an approximation at low energies were also obtained [see, e.g., Ref. 12)].

The aim of this paper is to extend the above framework to supergroups and to σ fields, taking values in supercosets. The physical supergroups $\tilde{G}^*)$ [we choose further $\tilde{G} = OSp(N;4)$ for $D = 4$] have two commuting bosonic subgroups G_1 and G_2 describing space-time (gravity) and internal symmetries (YM) respectively; the presence of the fermionic sector $\tilde{G}/G_1 \times G_2$ implies the supersymmetric unification. There are several advantages in using elementary subconstituent Goldstone fields which take values in the supercosets of $\tilde{G}$; for example, one can

*) We call there physical supergroups those which unify space-time and internal symmetries [for $D = 4$ these supergroups are $OSp(4;N)$ with its contractions to the graded Poincaré group, with or without central charges¹³⁾, and $SU(2,2;N)$]. We do not consider here graded Lie groups describing only internal symmetries [see, e.g., Refs. 14) and 15)], which for $D = 4$ seem to violate the correct relation between spin and statistics.

- a) geometrize fundamental fermionic subconstituent fields as Goldstone spinors^{*)}. It should be mentioned here that several authors have recently constructed composite gauge fields^{19),20)} as well as composite gravity²¹⁾⁻²⁴⁾ from fundamental fermionic fields^{**)};
- b) obtain composite gauge models in the geometric sector (composite gravity) and internal sector (composite YM theory) from the same set of fundamental subconstituent fields. Such models also turn out to be invariant under global non-linear $OSp(4;N)$ supersymmetry transformations;
- c) construct composite achtbein superfields and subsequently the composite supergravity model invariant under $N = 1$ local superco-ordinate transformations, with fundamental Goldstone superfields breaking global $OSp(N;4)$ supersymmetry ($N \geq 2$) to $OSp(1;4)$ (we choose further $N = 2$).

In this paper we use the technique of non-linear realizations of supergroups applied firstly in supersymmetric models by Volkov, Akulov and Soroka^{26),27)}, and further considered by many other authors [see, e.g., Refs. 28)-35)]. We shall present here the formulae for the composite vierbeins and composite achtbein superfields for minimal choices of the non-linearly realized supergroup, and only at the end of the paper we shall briefly discuss the unification of gravity and YM theories for $OSp(N;4)$ with arbitrary N , as well as some important general questions. The presentation with more details will be given in future publications.

2. We introduce the fundamental fermionic fields in composite gravity as Goldstone fermions taking values in $\tilde{G} = OSp(1;4)$. We split the supergroup $OSp(1;4)$ in the following way:

- a) spin sector $H = SL(2,c)$, describing local spin transformations;
- b) translational sector $M = Sp(4)/SL(2,c)$, describing the supergroup parameters related to the space-time translations^{***)} by the choice of the Goldstone vector field $v^a(x)$;
- c) Goldstone sector $S = OSp(1;4)/Sp(4)$ describing the fundamental Majorana spinor $\psi^\alpha(x)$.

^{*)} For example, the two-dimensional Weyl spinor parametrizes $OSp(1,2;c)/Sp(2;c)$, and the four component Majorana spinor describes $OSp(1;4)/Sp(4)$ [see, e.g., Refs. 16)-18)].

^{**) When this work was completed we learnt that A. D'Adda had proposed a model of composite supergravity²⁵⁾ based on a different approach.}

^{***)} The Goldstone vector field in the $O(3,2)$ gauge theory of gravity was discussed in Refs. 32) and 33).

The fields $v^a(x)$, $\psi^\alpha(x)$ parametrize the supercoset $\tilde{K} = \text{OSp}(1;4)/\text{SL}(2,c)$ as follows:

$$\tilde{K} = \exp(v^a(x)P_a) \exp(\psi^\alpha(x)Q_\alpha) \quad (1)$$

where we call $v^a(x)$ co-ordinate Goldstones, and ψ^α is a fundamental Goldstone spinor. The $\text{SL}(2,c)$ generators M_{ab} , P_a and Q_α satisfy the suitably rescaled $\text{OSp}(1;4)$ superalgebra

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= -\frac{1}{2}(\gamma^a C)_{\alpha\beta} P_a + \frac{1}{R}(\sigma^{ab} C)_{\alpha\beta} M_{ab} \\ [P_a, Q_\alpha] &= \frac{1}{R}(\gamma^a)_{\alpha\beta} Q_\beta \\ [M_{ab}, Q_\alpha] &= (\sigma_{ab})_{\alpha\beta} Q_\beta \end{aligned} \quad (2a)$$

and

$$[P_a, P_b] = -\frac{1}{R^2} M_{ab} \quad (2b)$$

Let us define the $\text{OSp}(1;4)$ superalgebra-valued Cartan one-forms on $\tilde{K}$ which describe the composite connections $\tilde{A}_\mu$

$$\begin{aligned} \tilde{A}_\mu &= \tilde{K}^{-1} \frac{\partial}{\partial x^\mu} \tilde{K} = \Omega_\mu + E_\mu + F_\mu = \\ &= \omega_\mu^{ab} M_{ab} + e_\mu^a P_a + \Phi_\mu^\alpha Q_\alpha \end{aligned} \quad (3)$$

Because $\tilde{A}_\mu$ are pure $\text{OSp}(1;4)$ gauge fields, i.e.,

$$\frac{\partial \tilde{A}_\mu}{\partial x^\nu} - \frac{\partial \tilde{A}_\nu}{\partial x^\mu} = [\tilde{A}_\mu, \tilde{A}_\nu] \quad (4)$$

one obtains

$$\begin{aligned} R_{\mu\nu}^{ab} &\stackrel{\text{df}}{=} \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \omega_{[\mu}^a \omega_{\nu]}^b = \\ &= \frac{1}{R^2} e_{[\mu}^a e_{\nu]}^b - \frac{1}{R} \Phi_{[\mu}^\alpha (\sigma^{ab} C)_{\alpha\beta} \Phi_{\nu]}^\beta \end{aligned} \quad (5a)$$

$$T_{\mu\nu}^a \stackrel{df}{=} \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \omega_\mu^a{}_b e_\nu^b = \frac{1}{2} \Phi_{[\mu}^\alpha (\gamma^a)_{\alpha\beta} \Phi_{\nu]}^\beta \quad (5b)$$

We see therefore that in our model the condition $T^a = 0$ characterizing the geometry of pure Einstein gravity is modified. One can say that we obtain a particular version of Einstein-Cartan geometry, with $R^{ab} \neq 0$ and $T^a \neq 0$.

In order to obtain composite gravity models one can take any of the proposed actions for gravity (Einstein, Weyl, cosmological term, torsion square) and substitute composite formulae, from (3) and (5). In particular, the composite Einstein action can be written down as follows:

$$\begin{aligned} S_E &= \frac{1}{\kappa^2} \int d^4x \epsilon_{abcd} \epsilon^{\mu\nu\sigma\tau} e_\mu^a e_\nu^b R_{\sigma\tau}^{ab} = \\ &= \frac{1}{\kappa^2} \left(\frac{4!}{R^2} \int d^4x \det(e_\mu^a) - \frac{1}{R} \int d^4x \cdot \right. \\ &\quad \left. \epsilon_{abcd} \epsilon^{\mu\nu\sigma\tau} e_\mu^a e_\nu^b \Phi_{[\mu}^\alpha (\gamma^{ab})_{\alpha\beta} \Phi_{\nu]}^\beta \right) \end{aligned} \quad (6)$$

where the composite vierbein e_μ^a and composite spinor connection Φ_μ^α can be calculated from Eqs. (2) and (3). One gets

$$\begin{aligned} e_\mu^a &= \frac{\partial V^a}{\partial x_\mu} - \frac{1}{2} \left(1 - \frac{\bar{\Psi}\Psi}{2R} \right) \left(\Phi_\mu^\alpha (\gamma^a)_{\alpha\beta} \Psi^\beta - \right. \\ &\quad \left. - \bar{\Psi}^\alpha (\gamma^a)_{\alpha\beta} \Phi_\mu^\beta \right) + O\left(\frac{1}{R^2}\right) \end{aligned} \quad (7a)$$

$$\begin{aligned} \Phi_\mu^\alpha &= \left(1 - \frac{\bar{\Psi}\Psi}{R} \right) \frac{\partial \Psi^\alpha}{\partial x_\mu} - \frac{1}{R} \left(\frac{\partial \bar{\Psi}}{\partial x_\mu} \Psi \right) \Psi^\alpha - \\ &\quad - \frac{1}{R} \frac{\partial V^a}{\partial x_\mu} (\bar{\Psi} \gamma_a)^\alpha + O\left(\frac{1}{R^2}\right) \end{aligned} \quad (7b)$$

In terms of composite fields (7), one can also express the topological charges: the Euler index Q_E and Pontriagin index Q_P . In particular, we get

$$Q_E = c \int d^4x \epsilon_{abcd} \epsilon^{\mu\nu\sigma\tau} R_{\mu\nu}^{ab} R_{\sigma\tau}^{cd} =$$

$$= c \int d^4x \left(\frac{4!}{R^4} \det(e_\mu^a) - \frac{2}{R^2} \epsilon_{abcd} \epsilon^{\mu\nu\sigma\tau} e_\mu^a e_\nu^b \right). \quad (8)$$

$$\cdot \bar{\Phi}_\rho^\alpha (\sigma^{cd})_{\alpha\beta} \Phi_\tau^\beta + \frac{1}{R^2} \bar{\Phi}_\mu^\alpha (\sigma^{ab})_{\alpha\beta} \Phi_\nu^\beta \cdot \bar{\Phi}_\rho^\gamma (\sigma^{cd})_{\gamma\delta} \Phi_\tau^\delta$$

Performing the Fierz rearrangement one can show that the last term in the right-hand side of (9) vanishes. Because Q_E is the integral over the four divergence the term $e \wedge e \wedge \Phi \wedge \Phi$ can be expressed modulo boundary terms by $e \wedge e \wedge e \wedge e$. We thus obtain from (7) the action for composite gravity described solely by the cosmological term with composite vierbeins, as in the formulation of Amati and Veneziano^{(23), (24)}.

Such a composite model has several interesting features, for example:

a) because the $OSp(4,1)$ curvatures vanish, the general co-ordinate transformation can be expressed as a particularly chosen field-dependent local $OSp(1;4)$ transformation [see, e.g., Ref. 36)];

b) if we assume the relation $\langle e_\mu^a \rangle = \partial V^a / \partial x_\mu$, the different choices of $V^a(x)$ describe different vacuum solutions.

If we put $V^a = \delta_\mu^a x^\mu$, such a choice is preserved only under the transformations generated by the physical Poincaré algebra, with Lorentz generators described by the sum of orbital and spin angular momenta [i.e., the $O(3,1)$ parameters describing the orthogonal rotations in tangent bundle should be identified with $SL(2, \mathbb{C})$ parameters of the spin sector of $OSp(1;4)$]. If $R \rightarrow \infty$ from (7a) we obtain

$$e_\mu^a = \delta_\mu^a - \frac{1}{2} \left(\frac{\partial \bar{\Psi}^\alpha}{\partial x_\mu} (\gamma^a)_{\alpha\beta} \Psi^\beta - \bar{\Psi}^\alpha (\gamma^a)_{\alpha\beta} \frac{\partial \Psi^\beta}{\partial x_\mu} \right) \quad (9)$$

in accordance with the Volkov-Akulov choice of non-linear realizations of super-Poincaré symmetry^{(26), (27)};

c) one can introduce in the coset space $\tilde{K}$ [see Eq. (1)] a second dimensionless contraction parameter f , corresponding to the rescaling

$$P_a \rightarrow P_a \quad M_{ab} \rightarrow M_{ab} \quad Q_\alpha \rightarrow f Q_\alpha \quad (10)$$

whose inverse has a geometric meaning of the curvature in the fermionic sector of our internal superspace.

3. We shall describe $N = 1$ composite supergravity by the standard superspace action³⁷⁾

$$S = \int d^4x d^4\theta \, s \det(E_M^A) \quad (11)$$

where E_M^A is the 8×8 supermatrix of composite achtbein superfields, obtained from the non-linear realization of $OSp(2;4)$. We again split $OSp(2;4)$ supergroup into three sectors:

- a) spin sector $H = SL(2, c)$, describing local spin transformations;
- b) translational sector $\tilde{M} = OSp(1;4)/SL(2, c)$, parametrized by vector and spinor superfields $V^a(x, \theta)$, $\eta^\alpha(x, \theta)$;
- c) Goldstone sector $\tilde{S} = OSp(2;4)/OSp(1;4)$, described by four spinor superfields $\psi^\alpha(x, \theta)$ and the scalar superfield $t(x, \theta)$. The five-component superspinor (ψ^α, t) describes the fundamental linear representation of $OSp(1;4)$ [see, e.g., Refs. 16) and 18)].

We parametrize the coset $OSp(1;4)/SL(2, c)$ by the translational and the "true" Goldstone superfields

$$\begin{aligned} \tilde{K} = & \exp(v^a(x, \theta) P_a + \eta^\alpha(x, \theta) Q_\alpha) \cdot \\ & \cdot \exp(\psi^\alpha(x, \theta) S_\alpha + t(x, \theta) T) \end{aligned} \quad (14)$$

where the generators (M_{ab}, P_a, Q_α) describe $OSp(1;4)$ subalgebra (2), which is extended to $OSp(2;4)$ by four real supercharges S_α and the $O(2)$ Abelian generator T as follows [see, e.g., Ref. 38)]^{*)}

$$\begin{aligned} \{S_\alpha, S_\beta\} &= -\frac{1}{2} (\gamma^a C)_{\alpha\beta} P_a + \frac{1}{R} (\sigma^{ab} C)_{\alpha\beta} M_{ab} \\ \{S_\alpha, Q_\beta\} &= -C_{\alpha\beta} \frac{1}{2R} T \end{aligned} \quad (15)$$

^{*)} For simplicity we introduce only one contraction parameter; in general one can have three of them.

$$\begin{aligned}
 [P_a, S_\alpha] &= \frac{1}{R} (\gamma^a)_{\alpha\beta} S_\beta & [P_a, T] &= 0 \\
 [M_{ab}, S_\alpha] &= (\sigma^{ab})_{\alpha\beta} S_\beta & [M_{ab}, T] &= 0 \\
 [T, Q_\alpha] &= S_\alpha & [T, S_\alpha] &= -Q_\alpha
 \end{aligned} \tag{15}$$

The composite one-forms are defined as

$$\begin{aligned}
 \tilde{A} = \tilde{K}^{-1} d\tilde{K} &= \omega^{ab} M_{ab} + E^a P_a + E^\alpha Q_\alpha + \\
 &+ \Phi^\alpha S_\alpha + U \cdot T
 \end{aligned} \tag{16}$$

where

$$E^a = dv^a - \frac{1}{2} (\bar{\eta} \gamma^a d\eta - d\bar{\eta} \gamma^a \eta) - \frac{1}{2} (\bar{\Psi} \gamma^a d\Psi - d\bar{\Psi} \gamma^a \Psi) + O\left(\frac{1}{R}\right) \tag{17}$$

$$E^\alpha = d\eta^\alpha - (t d\psi^\alpha - dt \psi^\alpha) + O\left(\frac{1}{R}\right)$$

If we use the formula for v^a

$$\begin{aligned}
 dv^a &= \frac{\partial v^a}{\partial x^\mu} dx^\mu + \frac{\partial v^a}{\partial \theta^\alpha} d\theta^\alpha = \\
 &= \frac{\partial v^a}{\partial x_\mu} dz^\mu + D_\alpha v^a d\theta^\alpha
 \end{aligned} \tag{18}$$

where $dz_\mu = dx_\mu - \frac{1}{4} \bar{\theta} \gamma_\mu d\theta$, $D_\alpha = \partial/\partial \theta^\alpha - (\frac{1}{4} \bar{\theta} \gamma_\mu) \partial/\partial x_\mu$, and analogously for other differentials in (17), we can write

$$\begin{aligned}
 E^a &= E_\mu^a dz^\mu + E_\alpha^a d\theta^\alpha \\
 E^\alpha &= E_\mu^\alpha dz^\mu + E_\beta^\alpha d\theta^\beta
 \end{aligned} \tag{19a}$$

where

$$\begin{aligned}
 E_\mu^a &= \frac{\partial v^a}{\partial x_\mu} - \frac{1}{2} \left(\bar{\eta} \gamma^a \frac{\partial \eta}{\partial x_\mu} - \frac{\partial \bar{\eta}}{\partial x_\mu} \gamma^a \eta \right) - \\
 &\quad - \frac{1}{2} \left(\bar{\Psi} \gamma^a \frac{\partial \Psi}{\partial x_\mu} - \frac{\partial \bar{\Psi}}{\partial x_\mu} \gamma^a \Psi \right) + O\left(\frac{1}{R}\right)
 \end{aligned} \tag{19b}$$

$$\begin{aligned}
 E_\beta^a &= D_\beta V^a - \frac{1}{2} (\bar{\eta} \gamma^a D_\beta \eta - (D_\beta \bar{\eta}) \gamma^a \eta) - \\
 &\quad - \frac{1}{2} (\bar{\Psi} \gamma^a D_\beta \Psi - (D_\beta \bar{\Psi}) \gamma^a \Psi) + O\left(\frac{1}{R}\right) \\
 E_\mu^\alpha &= \frac{\partial \eta^\alpha}{\partial x_\mu} - \left(t \frac{\partial \psi^\alpha}{\partial x_\mu} - \frac{\partial t}{\partial x_\mu} \cdot \psi^\alpha \right) + O\left(\frac{1}{R}\right) \\
 E_\beta^\alpha &= D_\beta \eta^\alpha - \left(t D_\beta \psi^\alpha - D_\beta t \cdot \psi^\alpha \right) + O\left(\frac{1}{R}\right)
 \end{aligned} \tag{19b} \text{ cont.}$$

The formulae (19) describe composite achtbein superfields which we insert in the supergravity action (9).

If we choose the local superco-ordinate frame

$$V^a(x, \theta) = \delta_\mu^a x^\mu \quad \eta^\alpha(x, \theta) = \delta_\beta^\alpha \theta^\beta \tag{20}$$

and put $\psi^\alpha = t = 0$, we get from (19)

$$E^{(0)A}{}_B = \begin{pmatrix} \delta_\mu^a & 0 \\ \frac{1}{4} (\bar{\theta} \gamma^a)_\beta & \delta_\beta^\alpha \end{pmatrix} \tag{21}$$

i.e., we obtain the "flat" achtbein describing rigid Salam-Strathdee superspace with constant torsion, [see, e.g., Ref. 36)]. In fact, the "flat" achtbein (21) and correct flat super Poincaré limit were introduced indirectly by the change of the "naïve" canonical cobasis $(dx_\mu, d\theta_\alpha) \rightarrow dY_K = (dz_\mu, d\theta_\alpha)$ and corresponding dual basis $(\partial/\partial x_\mu, \partial/\partial \theta_\alpha) \rightarrow \partial/\partial Y_K = (\partial/\partial x_\mu, D_\alpha)$.

In order to compare the geometry of our model with conventional supergravity, one can write the curvature and torsions in the bosonic and fermionic sector of superspace. In particular, the torsion equations do have the form^{*)}

$$\begin{aligned}
 T_{KL}^a &= \frac{\partial}{\partial Y_{<K}} E_{>L}^a - \omega_{<K}^a{}^b E_{>L}^b = \\
 &= \frac{1}{2} E_K^\alpha (\gamma^a C)_{\alpha\beta} E_L^\beta + \frac{1}{2} \Phi_K^\alpha (\gamma^a C)_{\alpha\beta} \Phi_L^\beta
 \end{aligned} \tag{22}$$

^{*)} By $\langle K \dots L \rangle$ we denote graded antisymmetrization.

$$\begin{aligned} T_{KL}^{\alpha} &= \frac{\partial}{\partial Y_{<K}} E_L^{\alpha} - \omega_{<K}^{ab} (\sigma_{ab})^{\alpha}_{\beta} E_L^{\beta} = \\ &= \frac{1}{R} E_{<K}^a (\gamma_a C)^{\alpha}_{\beta} E_L^{\beta} + \frac{1}{R} E_{<K}^a (\gamma_a C)^{\alpha}_{\beta} \Phi_L^{\beta} \end{aligned} \quad (22)$$

The conventional constraints on torsion in supergravity are modified, e.g.,

$$2 T_{\gamma\delta}^a = (\gamma^a C)_{\gamma\delta} \rightsquigarrow (\gamma^a C)_{\gamma\delta} + \Phi_{\gamma}^{\alpha} (\gamma^a C)_{\alpha\beta} \Phi_{\delta}^{\beta} \quad (23)$$

where $\Phi^{\alpha} = \Phi_{\mu}^{\alpha} E^{\mu} + \Phi_{\gamma}^{\alpha} E^{\gamma}$. The change (23) is analogous to the modification of the $T^a = 0$ condition in Einstein gravity [see (5b)].

4. We would like to add the following remarks:

a) in general, the decomposition of a physical supergroup $\tilde{G}$ yielding the composite gravity with spin group H_1 , as well as the composite YM theory with locally gauged internal symmetry H_2 , is obtained from the following supercoset decomposition

$$\tilde{K} = \frac{\tilde{G}}{H_1 \times H_2} = \frac{G_1}{H_1} \times \frac{G_2}{H_2} \times \frac{\tilde{G}}{H_1 \times H_2} \quad (24)$$

where

- G_1 describes the geometric sector of $\tilde{G}$, with the coset G_1/H_1 parametrized by co-ordinate Goldstones;
- G_2 describes the internal symmetry sector of $\tilde{G}$, with the coset G_2/H_2 parametrized by the scalar Goldstones;
- $\tilde{G}/G_1 \times G_2$ is parametrized by fermionic Goldstones;
- both pairs (G_1, H_1) and (G_2, H_2) are symmetric.

The choice of $\tilde{K}$ described by the product of symmetric cosets permits us to construct the actions invariant under global (non-linear) $\tilde{G}$ -supersymmetry in terms of $H_1 \times H_2$ invariants [see, e.g., Ref. 29)].

For $OSp(N;4)$ one can make the choices

- $\alpha)$ $H_1 = SL(2, c)$, $H_2 = O(N)$ (only fermionic Goldstones);
- $\beta)$ $H_1 = SL(2, c)$, $H_2 = U(k)$ (scalar and fermionic Goldstones, $N = 2k$).

The H_2 curvatures obtained from Cartan-Maurer equations for $OSp(N;4)$ provide the composite YM field strengths.

b) The composite formulae for vierbeins and gauge fields proposed in this paper describe only particular modes of all possible vierbeins and gauge potentials, which occur, for example, in Feynman path quantization of gravity or YM theory. In conventional σ models, with fundamental Lie group-valued scalar Goldstone fields, there are rigorous theorems determining the family of Goldstone fields providing all sufficiently smooth potentials³⁹⁾; we are not aware of any generalization of these theorems to supergroups.

c) In the composite gravity and supergravity models presented above one can introduce elementary $SL(2,c)$ spin connections by introducing $SL(2,c)$ covariantized Cartan forms²⁷⁾

$$\tilde{K}^{-1} dK \longrightarrow \tilde{K}^{-1} (d + \Omega) \tilde{K} \quad (25)$$

where Ω describes elementary $SL(2,c)$ gauge fields. In such a model one can construct the composite spin connections only as the solution of algebraic field equations, in the second order formalism.

d) One can, in principle, also obtain composite vierbeins and composite $SL(2,c)$ curvatures expressed in terms of bosonic subconstituents - by assuming that the "genuine" Goldstone sector S is purely bosonic, i.e., by replacing the supergroup $\tilde{G}$ by the "conventional" Lie group G . The group G should have again three sectors, describing spin, translations and "true" Goldstones. An example for $D = 4$ is provided by the following decomposition of G

$$G = O(3,1) \times \frac{O(3,2)}{O(3,1)} \times \frac{G}{O(3,2)} \quad (26)$$

The minimal choice is $O(4;2)$ (it is not the conformal group!), and the composite vierbein is constructed out of fundamental four vector and fundamental scalar Goldstone fields.

e) It should be mentioned that our approach can be incorporated as a particular construction in the general framework of supergroup manifold formalism [see, e.g., Ref. 40)]. We consider here the Goldstone sector on a suitably restricted supergroup manifold [see (24)]. Our composite connections are the solutions of the constraints on the supercoset $\tilde{K}$ stating that $\tilde{G}$ -curvatures vanish [see, e.g., (4)].

f) The composite nature of the potentials is a property of classical theory. By introducing suitable Lagrange multipliers and passing to Feynman quantization via functional integrals, one can perform the integrations over the subconstituent field and obtain the effective theory of composites described by "primary" fields, in the spirit of the papers²⁰⁾⁻²⁴⁾. In composite gravity and $N = 1$ supergravity the effective action can be derived only in the presence of a finite cut-off Λ . It is only in extended composite supergravity that one can hope to obtain the cancellation of divergences in the limit $\Lambda \rightarrow \infty$.

Finally we would like to mention that this paper grew out from the author's efforts to make physically interesting the supersymmetric σ models in which we supersymmetrize the manifold of field values⁴¹⁾⁻⁴³⁾. It appears that in $D = 4$ such generalized σ models provide consistent theories with general co-ordinate invariance ($\tilde{G}/H$ valued σ fields) or general superco-ordinate invariance ($\tilde{G}/H$ valued σ superfields). In such a way we obtain composite gravity and composite supergravity models. Composite YM theory and composite supersymmetric YM theory in $D = 4$ ⁴⁴⁾⁻⁴⁶⁾ can be constructed respectively from σ fields and σ superfields taking values in the "conventional" internal symmetry group G .

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