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LOW-ENERGY n-p CAPTURE AS A PROBE OF TWO-NUCLEON DYNAMICS

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Abstract

We examine the sensitivity of the cross section and polarization observables in radiative neutron-proton capture to the uncertainty in the two-nucleon potential at short and intermediate range. These uncertainties are modelled by unitary phase-equivalent transformations of the Hamiltonian. We find that the photon linear polarization is very sensitive to the resulting changes in the wave functions. The neutron analyzing power is rather insensitive to these changes and could play a role in identifying effects not included in a potential model description.

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1. Introduction

Radiative neutron capture by protons and the time reversed reaction, deuteron photodisintegration, are a basic testing ground of models for the low-energy nucleon-nucleon interaction. Potential model calculations describe these reactions quite well although some discrepancies do occur¹⁻⁸). These may be indicating that nucleon-nucleon potentials derived phenomenologically or by fitting meson exchanges to nucleon-nucleon scattering data require some modification, for example, in the tensor force that determines the deuteron D-state components^{3,4,5}), or that there are important effects at work, even at low energy, which are completely outside the realm of potentials models. Examples of the latter are mesonic exchange currents²), isobar contributions⁵) and finite size effects due to the quark structure of the nucleons¹). To distinguish among these various possibilities a more systematic approach, both experimental and theoretical, than has been taken up till now is required. This paper is intended as a first step in that direction.

We examine neutron radiative capture at energies below pion production threshold. Working in a non-relativistic potential model we calculate cross sections and various polarization observables for a set of unitarily phase-equivalent interactions. The goals of this calculation are to determine the relative sensitivity of various n+p + γ observables to changes in the nucleon-nucleon potential in a region where the potentials are not well determined and to examine how the changes in different observables are correlated. With this information we have a more rational basis on which to suggest stringent tests of potential models for radiative capture.

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The starting point of the calculation is the Reid soft-core nucleon-nucleon interaction. Other potential models give very similar results for cross sections^{1,7)} so we restrict our calculation to one potential. The calculation of radiative capture is, by now, standard and we use the formalism established by Partovi⁸⁾. We use a phase-equivalent unitary transformation similar to that of Coester et al.⁹⁾. Our version has the desirable feature that the strength of the transformation and the range over which it changes the wave functions can be varied independently.

In the past such transformations have been restricted to a very short range, usually less than 1 fm⁹⁻¹¹⁾. This was done with the prejudice that at intermediate ranges the meson exchange picture of nucleon-nucleon forces was still valid¹²⁾. Now, however, the notion that the finite-size composite structure of the nucleons can play a role in determining the effective nucleon-nucleon interaction at distances comparable to the geometrical size of the nucleon is taken more seriously¹³⁾, and so we relax the short distance constraint on the unitary transformation. It is also desirable to localize the effect of the transformation. This allows us to learn something about which spatial separations are important in determining the reaction amplitudes. We concentrate on transformations whose maximum effect on the wave functions is between one and two fermis.

In addition to neutron radiative capture we also calculate the deuteron elastic form factors. This provides a good global indication of the magnitude of the transformation and thus helps establish limits for "reasonable" transformation parameters.

2. Unitary transformation of the two-nucleon wave function

In this section we describe the phase-equivalent unitary transformations used in our calculations. The transformed wave function is defined⁷⁾ by

$$\tilde{\psi}(\vec{r}) = \int d\vec{r}' U(\vec{r}, \vec{r}') \psi(\vec{r}') , \quad (1)$$

where U is unitary, so the norm of the wave function is invariant. The corresponding Hamiltonian is

$$\tilde{H} = U H U^\dagger , \quad (2)$$

where $\tilde{H}$ will be Hermitian if H is Hermitian. For spin-independent transformations we can write:

$$U(\vec{r}, \vec{r}') = 1 - \sum_{\ell m} \Lambda_\ell(r, r') Y_{\ell m}(\hat{r}) Y_{\ell m}^*(\hat{r}') , \quad (3)$$

so that

$$\tilde{u}_\ell^\dagger(r) = u_\ell^\dagger(r) - r \int_0^\infty dr' r' \Lambda_\ell(r, r') u_\ell^\dagger(r') , \quad (4)$$

where

$$\psi_{J\ell s}^M(\vec{r}) = \sum_{m_\ell, m_s} C(\ell s J | m_\ell m_s M) \chi_{m_s} Y_{\ell m_\ell}(\hat{r}) \frac{u_\ell^\dagger(r)}{r} , \quad (5)$$

and a similar equation for $\tilde{\psi}(\vec{r})$. If one expands $\Lambda_\ell(r, r')$ in a separable series:

$$\Lambda_\ell(r, r') = \sum_{i,j} a_{ij}^\ell g_i^\ell(r) g_j^\ell(r') , \quad (6)$$

then one can ensure phase-equivalence by making $g_j^\ell(r)$ short ranged.

Unitarity leads to the condition

$$a_{ij}^\ell + a_{ji}^{\ell*} = \sum_m a_{im}^\ell a_{jm}^{\ell*} , \quad (7)$$

where we assumed that the g_i^ℓ are orthonormal, i.e.

$$\int_0^\infty dx x^2 g_i^\ell(x) g_j^\ell(x)^* = \delta_{ij} . \quad (8)$$

Below the pion threshold Λ_ℓ is real. In the rank-two case [considered by Coester et al.⁹⁾] one can satisfy the unitarity equations by

$$(I) \quad a_{11} = a_{22} = 1 - \cos \alpha$$

$$a_{12} = -a_{21} = \sin \alpha$$

or by

$$(II) \quad a_{11} = 1 - \cos \alpha$$

$$a_{22} = 1 + \cos \alpha$$

We use the set I as it allows the transformation to be varied continuously from the identity. This transformation has no counterpart in the rank-one case, in contrast to transformation II, which reduces to rank-one if $\alpha = 0$. This is the reason for the absence of continuous transformations (in the sense discussed above) in the rank-one case^{11, 14, 15}. Since it is desirable to localize the effect of the transformation we have used Gaussian form factors of the following form:

$$g_1^k(r) = \sqrt{N_1} \exp(-(r-c)^2/2b^2), \quad (9)$$

$$g_2^k(r) = \sqrt{N_2} (1 - nr^2) \exp(-(r-c)^2/2b^2), \quad (10)$$

which are independent of k . Strictly speaking the form factors should behave like $O(r^k)$ for $r \rightarrow 0$. For the intermediate range transformation ($c = 1.3$ fm, $b = 0.35$ fm) used in most of our calculations this requirement is not of practical importance. For the short range ($c = 0.3$ fm, $b = 0.3$ fm) test case discussed in sect. 4 we have included r^k factors in g_1^k and g_2^k to guarantee correct small r behaviour. In all cases the normalization constants N_1 and the quantity n are easily determined from the orthonormality condition eq. (8). The strength of the transformation is given by the parameter α . The range of the transformation obviously depends on b and c ; close examination reveals that the maximum effect on the wave function is around $b + c$.

Figs. 1 and 2 show the deuteron S- and D-state wave function for the

Reid soft core potential along with the transformed wave functions for $c = 1.3$ fm, $b = 0.35$ fm and $\alpha = \pi/18$. To get an idea of how drastically the wave functions are being changed we also calculate the deuteron elastic form factors¹⁶ $A(Q)$ and $B(Q)$ shown in fig. 3. In calculating the form factors it is assumed that interaction currents²⁰ induced by the non-locality of the potential are purely longitudinal. This corresponds to Hellier's maximal choice²¹ for the interaction current and it allows us to calculate the form factors with transformed wave functions and no additional two-body operators. This is in line with what had previously been done by Coester and Ostebee¹⁰. We see from the figures that transformations with negative α very soon lead to poor results for $B(Q)$ but a certain amount of depression of the wave function at intermediate ranges, which occurs for positive α , can be tolerated.

3. Calculation of $n\pi\pi + \gamma\pi\pi$ observables

The calculation of $n\pi\pi + \gamma\pi\pi$ is done within the non-relativistic potential framework. The formalism is standard and will not be repeated here. It is given in detail in the papers of Parçovi⁸ and of de Swart²². In this section we discuss some definitions of observables and the approximations made in the present calculation.

The Reid soft core potential is used as the standard untransformed interaction. Where needed, Day's generalization²³ of the Reid potential is used for higher partial waves.

All quantities are calculated for the centre-of-mass (c.m.) frame. Kinematics are defined so that 0° means the photon is emitted in the original neutron beam direction. All of our results for the analyzing power A_y are for polarized neutrons. The Madison Convention²⁴ is followed with the positive y -direction defined along $\hat{p} \times \hat{k}$ where $\hat{p}$ is the

initial neutron momentum and $\vec{k}$ is the photon momentum. We also calculated the photon linear polarization $\Sigma(\theta)$ which is the cross section for photons with linear polarization in the reaction plane minus the cross section for photons with polarization perpendicular to the reaction plane divided by the sum. This definition is the same as de Swart²² and opposite in sign to Gorbenko et al.²⁵.

We use E1, E2 and M1 multipoles in the calculation. However, those pieces which constitute retardation corrections to the electric multipole amplitude have been neglected. Consider the multiple expansion⁸ of the photon wave function

$$\begin{aligned} \vec{e}_\mu e^{i\vec{k}\cdot\vec{r}} = & \sum_{L,M} D_{M\mu} \left\{ -\left(\frac{2\pi(2L+1)}{L(L+1)}\right)^{1/2} i^{L+1} \frac{1}{k} \vec{\nabla} \left[\left(1 + r \frac{d}{dr}\right) j_L(kr) y_M^L(r) \right] \right. \\ & - \left(\frac{2\pi(2L+1)}{L(L+1)} \right)^{1/2} i^{L+1} k \vec{r} j_L(kr) y_M^L(r) \\ & \left. - i \left(\frac{2\pi(2L+1)}{L(L+1)} \right)^{1/2} i^L j_L(kr) \left[\vec{r} y_M^L(r) \right] \right\}. \end{aligned} \quad (11)$$

The first term is a total divergence and using Siegert's theorem gives the usual electric multipoles in terms of the charge density operator²⁵. This is true even in the presence of two-body interaction currents which we would expect to be present since our transformed nucleon-nucleon interactions are explicitly non-local. The second term gives contributions to the amplitude which are $O(k/M)$ compared to the leading Siegert terms. We neglect these contributions as they would involve the two-body interaction current explicitly. Finally, the last term of (11) couples to the transverse magnetization current. We retain only the isovector M1 contribution. This is done to keep the calculation a tractable size. Just as in the calculation of the form factor it is assumed that there are no transverse two-body currents induced by the non-local interaction.

The approximation above is almost the same as Partovi's approximation C (except we include all isovector M1 terms). We recognize that at the highest energy considered this approximation introduces a certain amount of error, particularly in the shape of the differential cross section [see Partovi⁸, table III]. Since all matrix elements depend linearly on the same radial integrals we expect that the qualitative behaviour of the observables with respect to phase-equivalent transformations would not be changed even if all $O(k/M)$ corrections were included. One should be cautious, however, about comparing our results directly with experimental data.

4. Results

We have calculated $n+p \rightarrow \gamma+d$ cross sections and other observables at energies up to 240 MeV. For a given energy-independent transformation the effect on the cross section increases monotonically with energy. In general it is found that changes in the analyzing power are very small for all transformations while the photon linear polarization seems to be the observable most sensitive to phase-equivalent transformation. In addition we have examined the $n-p$ spin correlation coefficient and deuteron tensor polarization. These quantities do not have any particularly interesting behaviour under unitary transformation (i.e., they are neither the most nor the least sensitive) and will not be discussed further.

Fig. 4 shows the $n+p \rightarrow \gamma+d$ total cross section as a function of energy for the untransformed Reid soft core interaction and transformed wave functions for $c = 1.3$ fm, $b = 0.35$ fm and $a = \pm\pi/18$. Positive a leads to a depression of the wave function at intermediate distances and a decrease in the cross section. Fig. 5 shows the effect on the angular

distribution at neutron lab energy of 200 Mev. The change in the differential cross section with respect to the Reid soft core result seems to be fairly uniform as a function of angle.

The neutron analyzing power at 160, 200 and 240 Mev is given in Fig. 6. The changes induced by the phase-equivalent transformation are quite small, less than 0.02 in absolute magnitude. In contrast the photon linear polarization is very sensitive to the transformation. This is shown in Fig. 7 along with some data from Gorbunov et al.²⁵. The variations of $\Sigma(\theta)$ are significant even compared with the large uncertainties of the data. Obviously any scheme designed to reduce the cross section, for example, to obtain agreement with the 0° deuteron photodisintegration data^{3,4}), should be checked to see that it does not have a deleterious effect on $\Sigma(\theta)$.

So far we have assumed that the uncertainty in the intermediate-range behaviour of the wave function can be reflected by an energy and state-independent transformation. One could, however, well imagine that the uncertainty coming from the suppression of non-nuclear degrees of freedom is both energy and state dependent. In order to assess the possible consequences of such a dependence we have also determined the observables if only the bound state or the continuum wave function is transformed.

Results for the neutron analyzing power are shown in fig. 8. A_y is somewhat sensitive to the individual transformations but the effects almost cancel when both bound and continuum states are transformed. In contrast the changes in $\Sigma(\theta)$ have the same sign under transformations in the bound and continuum channels, explaining the greater sensitivity of this observable. The cross section seems to be affected mainly by changes in the bound state wave function.

Finally we note the effect of making the transformation of shorter range. As the range is decreased (e.g., by decreasing c) the strength parameter a must be increased to keep the change in the radiative capture cross section the same. For example, with $c = 0.3$ fm, $b = 0.3$ fm we need $a = \pi/2$ to have the same change in cross section as given by our intermediate long-range transformation with $a = \pi/18$. The S-state wave function corresponding to the short-ranged transformation is shown in fig. 9, while fig. 10 gives the elastic form factors $A(Q)$ and $B(Q)$. The strong short-range transformation has a rather violent effect on the form factor at large momentum transfer. This reinforces the suggestion of Hadjimichael and Saylor¹) that in low-energy photodisintegration and radiative capture any discrepancies are due to intermediate-range effects rather than to short-range effects which would have to be very strong.

5. Conclusions

This paper was motivated by the question of what could be learned by comparing potential model calculations for low-energy photodisintegration or radiative capture with data. To this end we investigated the sensitivity of the cross section and polarization observables to energy-independent phase-equivalent unitary transformations of the strong interaction Hamiltonian. Not surprisingly, not all observables are equally sensitive to the transformation. For a wide range of energies and transformation parameters the photon linear polarization seems to be most sensitively dependent on the two-nucleon wave function in the interaction region while the analyzing power is almost independent of the transformation.

If a potential model description of low-energy radiative capture is valid, then a precise measurement of $\Sigma(\theta)$ provides a stringent test of

the potential. Our calculations indicate that $\Sigma(\theta)$ is the best observable for discriminating among potentials with different off-shell behaviour. On the other hand the analyzing power changes very little under phase-equivalent transformations. If discrepancies between calculations and experiment arise in this variable they would have to be attributed to effects not included in the potential model description, for example, mesonic and isobar effects or more exotic phenomena associated with nucleon structure. In this sense the analyzing power is the best observable in which to test the validity of the potential model description.

We stress that the above conclusions are based on calculations where the wave functions in all bound and continuum channels were changed by the same unitary transformation. Naturally, other transformations are possible, as well. In particular, one could imagine a transformation which would only change the asymptotic D/S ratio of the bound-state deuteron wave function. Such a change will affect the radiative capture cross section quite strongly according to recent studies³⁾. However, the separable transformations considered in this work and previous studies⁹⁻¹¹⁾ do not allow for such a possibility as it would imply an unacceptable modification of the long-range behaviour of the nucleon-nucleon potential.

Our conclusion concerning the analyzing power in radiative capture should be contrasted to that of Bohannon²⁷⁾ who investigated the effects of phase-equivalent transformations in proton-proton bremsstrahlung. Bohannon found that the unitary transformation had a significant effect on the analyzing power of bremsstrahlung, but his transformations differ from ours in two important ways. First his transformations are much more drastic than ours [see fig. 4 of ref. 27)] and secondly he only makes the

transformation in one partial wave. Our calculations show that there can be important cancellations which greatly reduce the sensitivity of the analyzing power when the transformation is made in all channels.

In summary our results indicate that all models constructed to describe radiative capture total or differential cross section data should be further tested by investigating additional observables and other electromagnetic reactions involving the two-nucleon system.

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Figure Captions

1. Deuteron S-state wave function: ——— untransformed Reid soft core
wave function; ----- transformed wave function with $c = 1.3$ fm, $b =$
 0.35 fm, $\alpha = \pi/18$; ---- same with $\alpha = -\pi/18$.
2. Deuteron D-state wave function. For meaning of curves see caption
fig. 1.
3. Deuteron electric form factor $A(Q)$ and magnetic form factor $B(Q)$ for
the Reid soft core (——) and the transformed wave functions with
 $c = 1.3$ fm, $b = 0.35$ fm and $\alpha = \pi/18$ (-----), respectively $\alpha = -\pi/18$
(----). The standard dipole proton form factor $G_E^p = (1+Q^2/18.2)^{-2}$
has been used whereas the neutron form factor G_E^n is taken from
Galster et al. [ref. 17)]. Data for $A(Q)$ are from Elias et al.
[ref. 18)] and for $B(Q)$ from Buchanan and Yearian [ref. 19)].
4. Total radiative capture cross section as a function of neutron
laboratory kinetic energy T_n . For meaning of curves see fig. 1.
5. Differential cross section for neutron laboratory kinetic energy of
200 MeV. For meaning of curves see fig. 1.
6. Neutron analyzing power $A_y(\theta)$ as function of angle for three dif-
ferent energies. For meaning of curves see fig. 1.

7. Photon linear polarization $\Sigma(\theta)$ as a function of angle for three energies. Data from Gorbenco et al. [ref. 25)]. Meaning of curves same as in fig. 1.

8. Neutron analyzing power A_y for $T_n = 240$ MeV if only the continuum (-----) or only the bound state wave function (-----) is transformed ($c = 1.3$ fm, $b = 0.35$ fm, $\alpha = \pi/18$). The two transformations tend to cancel, as is also apparent by comparing this figure with the overall results in fig. 6.

9. Untransformed (—) and transformed S-state deuteron wave functions with a short-range transformation with $c = 0.3$ fm, $b = 0.3$ fm and $\alpha = \pi/2$ (-----).

10. Effect of a short-range transformation on the deuteron elastic form factors $A(Q)$ and $B(Q)$. Meaning of the curves is the same as in fig. 9.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10