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MORE ABOUT
BARYON NUMBER VIOLATION CATALYZED BY GRAND UNIFIED MONOPOLES

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ABSTRACT

We investigate the large rates for $\Delta B \neq 0$ processes catalyzed by grand unified monopoles (GUMs). We demonstrate that a Dirac monopole in the exact low energy colour $SU(3) \times U(1)$ electromagnetism group can have large $\Delta B \neq 0$ cross-sections. Therefore there are no factors of $\exp(-8\pi^2/g^2)$, $1/m_X^2$ or $1/m_W^2$ in the rates for such reactions. Regularizing the Dirac monopole in a specific grand unified theory (GUT) enables one to discuss flavour mixing. We find there is none in simple $SU(5)$ and $SO(10)$ GUTs, and give the low energy effective $\Delta B \neq 0$ interaction for more than one fermion generation. We estimate the $\Delta B \neq 0$ GUM-baryon cross-section to be $O(10^{-3} \text{ to } 10^{-5})/\beta \text{ GeV}^{-2}$ where β is the GUM velocity. We discuss how proton decay experiments could look for GUMs, including the expected hierarchy of "decay" modes: $B \rightarrow e^+\pi \gg B \rightarrow \mu^+K \gg \gg e^+\pi\mu^+\mu^-$, $\mu^+Ke^+e^-$; $B \neq e^+K$, $\mu^+\pi$, $\bar{\nu}_X$ and the possibility of seeing a chain of baryon "decays" in a large detector. Astrophysical restrictions on the GUM flux and $\Delta B \neq 0$ cross-sections are studied, and it is found that neutron stars can provide significant but not fatal constraints for β between 10^{-3} and 10^{-4} . In the unlikely event that GUMs were trapped in the Sun there could be an observable flux of $O(100)$ MeV neutrinos at the Earth's surface. We speculate on the possibility of trapping GUMs and utilizing the energy released if they catalyze baryon "decays" in large quantities.

1. - INTRODUCTION

The possible existence of isolated magnetic charges fascinated physicists even before Maxwell's equations came to incarnate the possible symmetry between electric and magnetic charges. Interest in magnetic monopoles was stimulated by Dirac's pioneering work¹⁾ on their properties in quantum electrodynamics, demonstrating in particular that free magnetic charges h should be quantized: $eh/4\pi = n/2$. Dirac's magnetic monopole had to be accompanied by a string and a free monopole is not a finite-energy solution of the equations of electrodynamics but has to be put in as an external source. However, 't Hooft and Polyakov²⁾ realized that finite-energy monopole solutions would exist in any gauge theory with a semi-simple group broken spontaneously down to an exact low energy gauge group containing the electromagnetic $U(1)$ group. For a while it seemed that the electroweak interactions might be unified into such a semi-simple group, but this possibility has now receded because of the phenomenological success of the Glashow-Salam-Weinberg model³⁾ based on an $SU(2) \times U(1)$ group. On the other hand, the idea of unification into a semi-simple group has been revived in grand unified theories (GUTs) of the strong, weak and electromagnetic interactions⁴⁾. In the simplest examples of such theories based on the $SU(5)$ ⁵⁾ and $SO(10)$ ⁶⁾ groups the GUT symmetry is broken at the mass scale of heavy vector bosons X , which violate baryon-number conservation and are estimated to have masses m_X of order 10^{14} to 10^{16} GeV. Since 't Hooft-Polyakov monopoles have masses $O(m_X/\alpha_X)$ and $\alpha_X \approx 1/41$ to $1/20$ in such GUTs, they should have superheavy masses $O(10^{16}$ to $10^{17})$ GeV. The theory of such grand unified monopoles (GUMs) has been studied extensively. There are stringent phenomenological limits on their abundance and flux both from direct experimental searches⁷⁾ and from astrophysical considerations⁸⁾⁻¹⁴⁾. These seem to pose serious problems for conventional GUTs in the normal Big Bang cosmology¹⁰⁾. This has been one of the motivations for considering an inflationary epoch in the very early Universe which would have suppressed the monopole abundance far below the phenomenological limits¹⁵⁾.

Since GUTs violate baryon-number (B) conservation at the grand unification scale $m_X \gtrsim O(10^{14})$ GeV, it has long been apparent that GUM interactions could also be expected to violate B ¹⁶⁾. It had generally been thought that their B violating cross-sections would be geometrical of order $1/m_X^2$, and hence not of practical significance. However, Rubakov^{17),18)} argued some time ago, that the B violating cross-sections of GUMs might be comparable with those of conventional strong interactions. He argued that the electric and magnetic fields around a GUM would generate via the axial anomaly a B violating condensate of fermion fields f extending out to their Compton wavelength $1/m_f \gg 1/m_X$.

Rubakov's work^{17),18)} was largely ignored until quite recently. A few months ago Wilczek¹⁹⁾ suggested that $\Delta B \neq 0$ GUM cross-sections might be $O(m_W^{-n})$ and be large at energies $E \sim m_W$. Phenomenological interest in GUMs was regenerated by the report²⁰⁾ of an upper limit on their flux $F(< 7 \times 10^{-10} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1})$ which also described a candidate monopole event. Meanwhile, Callan²¹⁾ re-examined the condensation of fermions around a GUM and revived the suggestion of Rubakov^{17),18)} that $\Delta B \neq 0$ GUM cross-sections might be as large as strong interaction cross-sections.

Three of us (ENO) were encouraged by this heady combination of possibly large GUM fluxes and $\Delta B \neq 0$ cross-sections to re-examine GUM phenomenology in a previous paper²²⁾. Parametrizing $\Delta B \neq 0$ cross-sections in the form

$$\sigma_{\text{GUMB} \neq 0} = \begin{cases} \frac{1}{v} \frac{\sigma_0}{E_f^2} & (E_f \geq 1 \text{ GeV}) \\ \frac{1}{v} \frac{\sigma_0}{(1 \text{ GeV})^2} & (E_f \leq 1 \text{ GeV}) \end{cases} \quad (1.1)$$

where v is the relative velocity of the GUM and a target fermion of energy E_f , we showed that GUMs would not degenerate the baryon asymmetry of the Universe if $\sigma_0 < O(10^{+6})$. We also showed that the lower limit of 3×10^{30} years on the baryon lifetime implies an upper limit

$$F < O(2 \times 10^{-12}) (\beta/\sigma_0) \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1} \quad (1.2)$$

on the GUM flux. Cabrera's candidate event²⁰⁾ would therefore be difficult to interpret as a GUM unless $\sigma_0 \leq O(10^{-5})$, whereas forthcoming baryon decay experiments could be sensitive to baryon number violation catalyzed by GUMs if $\sigma_0 = O(1)$ and the GUM flux is close to other phenomenological limits. We discussed the "decays"

$$p, n \rightarrow e^+ + \text{pions}, \quad p, n \rightarrow e^+ + \text{pions} + \mu^+ \mu^- \quad (1.3)$$

as possible signatures for $\Delta B \neq 0$ catalysis by GUMs. In this paper we discuss in more detail some of the theoretical and phenomenological issues associated with GUMs and their $\Delta B \neq 0$ cross-sections.

In Section 2 we discuss the forms of fermion condensates around GUMs. At radii $r \gg 1/m_X$ an 't Hooft-Polyakov monopole²⁾ looks much like an Abelian Dirac monopole¹⁾. We show how the analysis of Rubakov¹⁸⁾ can easily be adapted to the case of a Dirac monopole associated with an Abelian subgroup of the exact

low-energy colour $SU(3) \times U(1)$ electromagnetic gauge group, showing that it also can be surrounded by a $\Delta B \neq 0$ fermion condensate of radius $r = O(1/m_F)$. This enables us to discuss the phenomenology of $\Delta B \neq 0$ GUM cross-sections in arbitrary GUTs. Also, since the weak interaction scale m_W never enters into the Dirac formulation, we see that no factors of order $1/m_W$ need appear in $\Delta B \neq 0$ GUM cross-sections. We also discuss the forms of possible condensates for m_F large and small, as required in our analyses of $\Delta B \neq 0$ GUM interactions at different energies.

In Section 3 we discuss the forms and phenomenology of these interactions. We first show that generalized Cabibbo-Kobayashi-Maskawa mixing angles²³⁾ do not appear to complicate matters in most GUTs. We then estimate the magnitude of the cross-sections (1.1). Although no factors of $O(1/m_W)$ or $1/m_X$ appear the over-all factor σ_0 in (1.1) seems likely to be quite small:

$$\sigma_0 = O(10^{-3} \text{ to } 10^{-5}) ? \quad (1.4)$$

It should be emphasized immediately that the estimate (1.4) is not as precise as conventional GUT calculations of the baryon decay rate for a given value of m_X , and could easily be in error by orders of magnitude in either direction. We then predict the hierarchy of nucleon "decay" modes catalyzed by GUMs:

$$p, n \rightarrow e^+ \pi \gg \mu^+ K \gg e^+ \pi \mu^+ \mu^-, \mu^+ K e^+ e^- \quad (1.5a)$$

$$\approx 1 : O(10^{-1} \text{ to } 10^{-3}) : O(10^{-5})$$

with no "decays"

$$p, n \not\rightarrow \bar{\nu} + X, e^+ K, \mu^+ \pi, \text{ etc.} \quad (1.5b)$$

We discuss phenomenological signatures of GUM catalyzed baryon "decays" in forthcoming experiments, such as a large transverse three-momentum "kick" $O(300)\text{MeV}$ but a small energy "kick" $\Delta E = O(\beta) \times 1 \text{ GeV}$. We emphasize the desirability of mounting baryon decay detectors and monopole search experiments²⁴⁾ together to look for coincidences. We also suggest looking for "chains" of baryon "decays" as a GUM crosses a baryon decay detector. These can only be seen if one minimizes the "dead time" after an event during which the detector cannot record another one.

In Section 4 we discuss astrophysical constraints on the flux F and $\Delta B \neq 0$ cross-sections σ_0 of GUMs. Neutron stars can be expected to capture GUMs if they are sufficiently slow and not too heavy, and the catalyzed baryon "decays" may then release excessive energy from the neutron star. Properties of known neutron stars do not give a significant constraint:

$$F\sigma_0 < O(10^{-7})\beta^2 \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1} \quad (1.6)$$

Better constraints can be obtained by considering upper limits on the X-ray background from the computed astrophysical abundance of undetected neutron stars¹⁴⁾. However, when one takes proper account of the thermalization of the baryon "decay" products, and if $\sigma_0 = O(10^{-3} \text{ to } 10^{-5})$ these constraints are not strict enough to rule out a flux close to the experimental limits⁷⁾ on GUMs with velocities $\beta \gtrsim O(10^{-3})$. Furthermore, depending on their masses, GUMs with velocities greater than $O(10^{-3})$ may be able to pass straight through a neutron star without losing sufficient energy to be captured. When GUMs have velocities $< O(10^{-4})$ one must take into account the dynamics of the solar system¹³⁾ containing the neutron star and one obtains weaker limits on the possible GUM flux. These astrophysical constraints do not rule out the experimental observation of baryon "decays" catalyzed by GUMs, especially if $\beta = O(10^{-5} \text{ to } 10^{-4})$. We argue that the Sun is unlikely to capture GUMs. However, if it does there might be an observable spectrum of ν_e and ν_μ with energies $O(100)\text{MeV}$ emanating from the Sun¹⁷⁾.

Finally, in Section 5 we summarize the prospects for looking for GUMs and their large $\Delta B \neq 0$ cross-sections. We also develop our earlier speculation²²⁾ about using GUMs to solve the "energy crisis" by exploiting the GUM's ability to release 1 GeV of energy every time it catalyzes baryon "decay". Maybe GUTs and GUMs will be useful as well as fundamental.

2. - THEORY OF MONOPOLES AND FERMIONS

The monopoles of 't Hooft and Polyakov²⁾ are an unavoidable feature of all GUTs with a semi-simple unifying group G . These monopoles may have been created abundantly in the very early Universe thereby posing some interesting puzzles¹⁰⁾ if one wants to combine GUTs with standard cosmological models¹⁵⁾. Recently, an important new development in monopole (dyon) physics has taken place^{17)-19),21)}. Rubakov^{17),18)} showed that GUMs will induce baryon number violating processes and in particular should catalyze baryon decay. These issues are of obvious phenomenological interest. In this section we discuss some theoretical aspects which are relevant for the phenomenology we discuss in the following sections. In subsection 2.1 we recall Rubakov's result^{17),18),21)}. As his analysis was made in the limit of vanishing classical monopole size his result can be interpreted entirely within the unbroken subgroups. We reformulate his results purely in terms of Dirac monopoles associated with an Abelian subgroup of the low energy $SU(3) \times U(1)$ gauge symmetry. This is our starting point in Subsection 2.2 where

we study Rubakov's results^{17),18)} in the low-energy $SU(3)^C \times U(1)^{EM}$ effective theory. We are led to the (model independent) conclusions that:

- i) in any sensible unification scheme only very particular (including baryon number violating) monopole induced processes are possible;
- ii) Apparently, the weak $SU(2)$ breaking scale m_W does not come into play.

In Subsection 2.3 we consider the effects to be expected in realistic models with many (not necessarily massless) left-handed multiplets coupled to the monopole.

2.1 Rubakov's result^{17),18)}

Classical considerations led to the discovery of finite energy monopole²⁾ and dyon²⁵⁾ solutions in gauge theories spontaneously broken down to $U(1) \times K$ where K is any group. The following observations can be made:

- 1) The mass of these objects is typically

$$m_D \sim \frac{m_X}{\alpha} \sqrt{h^2 + q^2} \left[1 + O(m_H/m_X) \right] \quad (2.1)$$

where h is the $U(1)$ magnetic charge and q the $U(1)$ electric charge. As the mass of the charged states increases proportionally to m_X one expects the dyons to be unstable towards decay into the pure monopole and light charged particles if they are present in the theory²⁶⁾.

- 2) The total conserved angular momentum J of a monopole (dyon) with some charged field is given by

$$\underline{J} = \underline{L} + \underline{I} + \underline{S} \quad (2.2)$$

where $\underline{L}$ is the orbital part, $\underline{I}$ refers to the generators of some relevant $SU(2)$ subgroup of G and $\underline{S}$ to the intrinsic spin of the charged field. So one may have fermionic states in a theory with bosons (e.g., $S = 0, L = 0, |\underline{I}| = 1/2$)²⁷⁾.

- 3) Studies of the Dirac equation in the presence of a (Dirac) monopole field revealed that in the $J = 0$ channel (in fact only in the $J = 0$ channel) one may have helicity flipping amplitudes.

- 4) The previous point is closely related to the anomalous non-conservation of chirality in the presence of dyons due to the non-vanishing of $\underline{E} \cdot \underline{B}$ which appears in the anomaly

$$\partial_t Q_5 \propto \int d^3x (\underline{E} \cdot \underline{B}) \quad (2.3)$$

In fact one may show that in the monopole sector there exist dyon-like configurations which allow for the correct number of $J = 0$ fermionic zero modes as to saturate the anomaly^{17)-19),21)}. The results we mentioned so far were obtained from studies of the response of fermions to a fixed external monopole or dyon field. The authors of Refs. 18) and 21) have gone one step further by studying the coupled system of spherically symmetric $L = 0$ radial electric fields E_r with the $J = 0$ fermionic modes in the monopole background field. For massless fermions this system is exactly soluble, and it leads to a picture of the monopole which changes radically if light fermions are present. As higher J modes are expected to decouple there should be some mass gap of order m_X , so one has a rather clear understanding of the low-lying states and dynamics of the monopoles and dyons in the presence of fermions. We briefly summarize the calculation in the $SU(2)/U(1)$ theory with two massless left-handed doublets.

In the limit where the scale $O(m_X^{-1}, m_H^{-1})$ of the monopole goes to zero, the 't Hooft-Polyakov monopole solution²⁾ looks just like a Dirac monopole¹⁾ associated with an Abelian subgroup generated by the τ^3 of the original $SU(2)$ group^{*}). In the gauge $A_0 = 0$ taking the Abelian limit one considers the following configuration:

$$\left. \begin{aligned} A_0 &= 0 \\ A_i &= \Omega A_i^m \Omega^{-1} - i (\partial_i \Omega) \Omega^{-1} \end{aligned} \right\} \quad (2.4)$$

with

$$A_i^m = A_i^D \left(\frac{\tau_3}{2} \right), \quad A^D = \frac{\underline{\hat{r}} \times \underline{\hat{z}}}{r(1 - \underline{\hat{r}} \cdot \underline{\hat{z}})} \quad (2.5)$$

the usual Dirac monopole potential. Furthermore, the radial electric fields are parametrized by

$$\Omega = e^{i\tau_3 \omega(r,t)}, \quad \omega(0,t) = 0 \quad (2.6)$$

The fermionic part of the Lagrangian reduces to a variant of the massless Schwinger model which can be solved exactly [i.e., for any $\omega(r,t)$]. Rubakov's solution can be written entirely in terms of τ_3 in the Abelian limit. The fermionic determinant can be evaluated explicitly and one is left with an effective boson Lagrangian of the form

^{*}) Our analysis is related to that of Ref. 18) by an 't Hooft gauge transformation.

$$S_{\text{eff}}(\sigma) = \frac{2}{\pi} \int dr dt \left[\frac{1}{2} (\partial_r \sigma)^2 + \frac{1}{2} (\partial_t \sigma)^2 + \frac{8\pi^2}{e^2} r^2 (\nabla^2 \sigma)^2 \right] \quad (2.7)$$

where the field σ is given in terms of ω by

$$\sigma \equiv \int [\Delta^{-1}(r-r', t-t') + \Delta^{-1}(r+r', t-t')] \omega' dr' dt' \quad (2.8)$$

with Δ^{-1} the two-dimensional massless scalar propagator

$$\Delta^{-1}(r, t) = \frac{1}{4\pi} \log \mu (r^2 + t^2) \quad (2.9)$$

where μ is some arbitrary scale.

In (2.7) the first two terms arise from the fermion determinant whereas the last term is just the contribution from the electric field. Again since this Lagrangian is quadratic in σ it can be solved exactly. The propagator is of the form

$$K(r, r'; t, t') = -\frac{\pi}{2} \left[\Delta^{-1}(r-r', t-t') + \Delta^{-1}(r+r', t-t') - \tilde{K}(r, r'; t, t') \right] \quad (2.10)$$

where

$$\tilde{K}(r, r'; t, t') = -\frac{1}{2\pi} Q_\nu \left[\frac{(r'-r)^2 + (t'-t)^2}{2rr'} + 1 \right] \quad (2.11)$$

is a Legendre function with

$$\nu = \frac{1}{2} \left[\left(1 + \frac{e^2}{\pi^2} \right)^{\frac{1}{2}} - 1 \right] = \frac{e^2}{4\pi^2} + O(e^4) \quad (2.12)$$

One is now in a position to calculate the chiral condensate, i.e., the expectation value of

$$F \equiv \epsilon_{\alpha\beta} \epsilon_{ij} \chi_{\alpha i}^{(1)} \chi_{\beta j}^{(2)} \quad (2.13)$$

(α, β Lorentz indices, i, j group indices) in the monopole state. This is most easily done by considering

$$\begin{aligned} \lim_{t-t' \rightarrow \infty} \langle M | F(x) F(x') | M \rangle &= \\ &= \frac{1}{(4\pi)^2 r^2 r'^2} \text{Tr} \left[G(x, x' | \omega=0) G^\dagger(x, x' | \omega=0) \right] \times \\ &\times \int d\sigma \exp \left[-S_{\text{eff}}(\sigma) + 2\sigma(x) - 2\sigma(x') \right] \end{aligned} \quad (2.14)$$

where $G(x, x' | \omega=0)$ is the appropriate free fermion propagator in two-dimensions is

$$G(x, x' | \omega=0) = G_0^{(2)}(r-r', t-t') + G_0^{(2)}(r+r', t+t') \quad (2.15)$$

with

$$G_0^{(2)}(r, t) = \frac{1}{2\pi} \frac{\tau_3 t + \tau_1 r}{r^2 + t^2} \quad (2.16)$$

The first factor in (2.14) comes from normalization. All dependence on ω is now contained in the integral over σ which simply yields

$$\exp \left[-4K(x, x') + 2K(x, x) + 2K(x', x') \right] \quad (2.17)$$

Using the cluster decomposition property and the asymptotic properties of K , Rubakov¹⁸⁾ finds

$$\langle M | F(x) | M \rangle = \frac{1}{4\pi^2 r^3} 2^{1/4} \left[1 + O(e^2) \right] \quad (2.18)$$

Apparently, the monopole is surrounded by a fermion antifermion condensate which drops only as r^{-3} ! It should be noted that there are no factors of $e^{-m_X/\alpha}$, no factor $1/m_X^2$ and moreover no dependence on the coupling e . The first point is related with the fact that in the monopole sector there exist gauge field configurations [i.e., certain functions $\Omega(r,t)$ in (2.4)] which change chirality by a definite amount q , and have the corresponding number of fermionic zero modes, but whose action is vanishing^{17),18)}. The second point is to be expected since the calculation is basically Abelian and does not involve m_X in any way. The last point reflecting the existence of a non-vanishing zeroth order effect is not too surprising since we consider the coupling of the fermion to the monopole which involves the quantity $eh/2 = 2\pi$ which is a pure number.

2.2 Monopoles in the effective $SU(3) \times U(1)$ theory

In the previous section we emphasized the basic Abelian nature of Rubakov's calculations. The things that one retains from the non-Abelian structure after taking the Abelian limit are:

- i) boundary conditions on the fields which eliminate certain ambiguities²⁸⁾ which were present in the study of the Dirac Hamiltonian in the Dirac monopole field pur sang;
- ii) the natural appearance of different fields which couple with equal but opposite strength to the Abelian monopole configuration [for example by contracting $SU(2)$ to the $U(1)$ subgroup generated by τ_3].

Now instead of directly jumping to particular GUTs we may first consider the possibilities that arise from introducing Dirac type monopoles in the interaction part of the low energy colour $SU(3) \times U(1)$ electromagnetic effective Lagrangian for the first generation fermions:

$$\mathcal{L} = g [\bar{u} \not{A} u + \bar{d} \not{A} d] + e \left[\frac{2}{3} \bar{u} \not{A} u - \frac{1}{3} \bar{d} \not{A} d - \bar{e} \not{A} e \right] \quad (2.19)$$

where G_μ represents the gluons and A_μ the photon. As we are interested in Abelian monopoles we may restrict ourselves to the diagonal generators only. More specifically we will consider the (λ_8, Q) subspace where we have

$$\mathcal{L}' = \bar{u} (g \not{A}^8 + \frac{2}{3} e \not{A}) u + \bar{d} (g \not{A}^8 - \frac{1}{3} e \not{A}) d - \bar{e} \not{A} e \quad (2.20)$$

Now suppose we try to put in a $U(1)^{EM}$ Dirac monopole with magnetic charge

$$eh_{em} = 2\pi \quad (2.21)$$

so that

$$eA = 2\pi A^D \quad (2.22)$$

with A^D given by (2.5). This is not an acceptable monopole since it does not satisfy the appropriate quantization condition as the fractionally charged quarks would be able to detect its string¹⁾. We can add some colour magnetic charge to resolve this difficulty, for example

$$gh_c = 2\pi \begin{pmatrix} \frac{1}{3} & & 0 \\ & \frac{1}{3} & \\ 0 & & -\frac{2}{3} \end{pmatrix} \quad (2.23)$$

so that

$$gA^8 = (gh_c) A^D \quad (2.24)$$

Writing the interactions in terms of left-handed fields only we have the following couplings to the standard Dirac monopole configuration:

$$\begin{aligned} \mathcal{L}'' = 2\pi \bigg[& \bar{u}_{1L} A^D u_{1L} - \bar{u}_{1L}^c A^D u_{1L}^c \\ & + \bar{u}_{2L} A^D u_{2L} - \bar{u}_{2L}^c A^D u_{2L}^c \\ & - \bar{d}_{3L} A^D d_{3L} + \bar{d}_{3L}^c A^D d_{3L}^c \\ & - \bar{e}_L A^D e_L + \bar{e}_L^c A^D e_L^c \bigg] \end{aligned} \quad (2.25)$$

At this point we may assemble any two left-handed fields which couple with opposite strength into a would-be pair and consider their possible condensates à la Rubakov^{17),18)}. Many assignments of the first generation fermions (2.25) are a priori possible, and we will see how the requirement that the $SU(3) \times U(1)$ Dirac monopole can be embedded in a GUT provides boundary conditions at short distances $O(1/m_X)$ which resolve the ambiguity.

For example, there is the following assignment^{*)}

$$\begin{pmatrix} u_1 \\ u_1^c \end{pmatrix}_L \quad \begin{pmatrix} u_2 \\ u_2^c \end{pmatrix}_L \quad \begin{pmatrix} d_3^c \\ d_3 \end{pmatrix}_L \quad \begin{pmatrix} e^+ \\ e^- \end{pmatrix}_L \quad (2.26)$$

which would imply a condensate of the form

$$\langle M | (u_1 u_2^c + u_1^c u_2) (d_3^c e^- + d_3 e^+) | M \rangle \neq 0 \quad (2.27)$$

This condensate would induce interactions which would violate ordinary electromagnetic charge conservation, as is already obvious from the fact that the charge differences in the would-be pairs (2.26) are not equal. This basically implies that any unification scheme which would produce the assignment (2.26) either would contain X bosons which do not have a definite charge (which is evidently not possible), or else the $SU(2)$ subgroup is the diagonal subgroup of the product of some mutually commuting regular $SU(2)$ subgroups (for the example given one would need three of them and this possibility cannot be realized in simple GUTs). Analogous arguments based on charge can be used to exclude many other pair assignments, leaving only the possibilities

$$\begin{pmatrix} u_2^c \\ u_1 \end{pmatrix}_L \quad \begin{pmatrix} u_1^c \\ u_2 \end{pmatrix}_L \quad \begin{pmatrix} d_3 \\ e^+ \end{pmatrix}_L \quad \begin{pmatrix} e^- \\ d_3^c \end{pmatrix}_L \quad (2.28a)$$

and

$$\begin{pmatrix} u_2^c \\ u_2 \end{pmatrix}_L \quad \begin{pmatrix} u_1^c \\ u_1 \end{pmatrix}_L \quad \begin{pmatrix} d_3 \\ e^+ \end{pmatrix}_L \quad \begin{pmatrix} e^- \\ d_3^c \end{pmatrix}_L \quad (2.28b)$$

However, the alternative (2.28b) cannot be realized in any GUT, since it is not possible to make these pairs into doublets of an $SU(2)$ whose raising and lowering generators carry definite colour. The possible condensates associated with the unique remaining choice (2.28a) are

^{*)} Here and elsewhere in this section we neglect possible phase differences between the upper and lower members of the pairs. We return to them in Section 3.

$$(u_2^c u_2 + u_1 u_1^c)(d_3 d_3^c + e^+ e^-) + (\text{herm. conj.}) \quad (2.29a)$$

$$(u_2^c e^+ + u_1 d_3)(u_1^c d_3^c + u_2 e^-) + (\text{herm. conj.}) \quad (2.29b)$$

$$(u_2^c d_3^c + u_1 e^-)(u_1^c e^+ + u_2 d_3) + (\text{herm. conj.}) \quad (2.29c)$$

which conserve electromagnetic charge as well as B-L. As the monopole state $|M\rangle$ itself is not $SU(3)^C$ invariant one does not expect that the condensate is. Note also that the first condensate is like a mass term, allowing for the usual helicity flip amplitudes. The other condensates induce baryon number violating processes.

In the standard $SU(5)$ model⁵⁾ our example has a very simple interpretation, as we have just taken the charge of a fundamental monopole and decomposed it into its $U(1)^{EM}$ and colour components:

$$g_s h = 2\pi \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = 2\pi \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & -1 & 0 \end{bmatrix} + 2\pi \begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \end{bmatrix} \quad (2.30)$$

The fundamental monopole of $SU(5)/SU(3) \times U(1)$ sits in the regular $SU(2)$ subgroups spanned by the pair of indices $(i,4)$ with $(i = 1,2,3)$. These are of course $SU(3)^C$ gauge equivalent configurations.

The first generation fermions have the following $SU(5)$ assignments (neglecting flavour mixing):

$$\underline{5}: \psi_{La} \equiv \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_{eL} \end{pmatrix} \quad \underline{10}: \chi_L^{ab} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L \quad (2.31)$$

It is clear that the pairs (2.28a) find natural homes as doublets of the $SU(2)$ group generated by

$$\underline{I} \equiv \frac{1}{2} \text{diag}(0, 0, \pm, 0) \quad (2.32)$$

corresponding to the stable 't Hooft-Polyakov monopole²⁾ mentioned above. We therefore retain (2.28a) as the only possible doublets of an $SU(2)$ subgroup of any full GUT group.

What this exercise has taught us is that:

- i) the allowed baryon number violation induced by GUMs can be understood within the framework of the low energy $SU(3)^C \times U(1)^{EM}$ theory;
- ii) the mass of the weak interaction boson m_W does not come into play and does not (2.18) control the magnitude of the condensates (2.29).

The weak interaction scale may only enter the discussion via the fermion masses, i.e., in conjunction with the small Yukawa couplings. This will be discussed in Subsection 2.3.

Some questions may be posed concerning the viability of the ideas we have just presented. For example, one may wonder whether at very low energies the fundamental $SU(5)$ monopole is well described by the Dirac monopole potential, because the monopole has long range colour magnetic fields which are presumably screened by the QCD vacuum. We have refrained from considering this dynamical complication. One could, of course, consider GUMs which are colour singlets, for example, with a charge

$$g_s h = 2\pi \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.33)$$

An analysis similar to Rubakov's could be carried out. However, this GUM is expected to decay into the previously considered fundamental monopoles and is hence not expected to be phenomenologically relevant.

2.3 Towards realistic models

We saw that in the standard $SU(5)$ model⁵⁾ each generation yields four left-handed doublets under a $SU(2)$ subgroup corresponding to a fundamental monopole. It implied the existence of three fermionic condensates around the monopole. What we want to consider next is the situation which arises if one has $2n$ left-handed doublets. Again if the fermions are assumed to be massless one can

exactly calculate the condensate. Returning to Rubakov's calculation¹⁸⁾ one easily sees that in the effective boson Lagrangian $S_{\text{eff}}(\sigma)$ in (2.7) the part $[(\partial_t \sigma)^2 + (\partial_r \sigma)^2]$ coming from the fermion determinants gets a factor n . In other words one has that

$$S_{\text{eff}}^{(n)}(\sigma, e^2) = n S_{\text{eff}}^{(1)}(\sigma, n e^2) \quad (2.34)$$

Calculating the condensate of the $2n$ fermion fields

$$\langle M | F_x^{(1)}(x) F_x^{(2)}(x) \dots F_x^{(n)}(x) | M \rangle \equiv f^{2n}(x) \quad (2.35)$$

in an analogous fashion to that described by (2.14) one easily verifies that

$$f^{2n}(x) = \left[\frac{1}{4\pi^2 r^3 2^{1/4}} \right]^n [1 + O(n e^2)] \quad (2.36)$$

The total number of condensates K ($\kappa = 1, 2, \dots, K$) equals

$$K = \frac{(2n)!}{2^n n!} \quad (2.37)$$

For the application of these ideas to realistic GUTs one has to take into account somehow the effect of the fermion masses on the condensates. Callan²¹⁾ has shown that the massive fermion case (two doublets) reduces to a system of two Sine-Gordon fields coupled by some Coulomb potential. The fermionic degrees of freedom are described by the (anti)solitons of the reduced model. Though the model is not exactly soluble it provides an excellent qualitative picture of the physics. The major conclusion on the ground state of the system is again that its net electric charge is zero but that the condensate only extends up to the Compton wavelength $1/m_F$ of the fermion. For distances large compared with the Compton wavelength one expects the condensate to be exponentially suppressed

$$\langle M | F(x) | M \rangle \approx e^{-m_F r}$$

Let us now consider the situation where we have four doublets $\psi^{(r)}$. We assume the mass terms in the Lagrangian to be of the form

$$\mathcal{L}_m = m_1 F_2^{(1)} + m_2 F_1^{(2)} + (\text{herm. conj.}) \quad (2.38)$$

with

$$\left. \begin{aligned} F_i^{(1)} &= \epsilon_{\alpha\beta} \epsilon_{ij} \chi_{\alpha i}^{(1)} \chi_{\beta j}^{(2)} \\ F_i^{(2)} &= \epsilon_{\alpha\beta} \epsilon_{ij} \chi_{\alpha i}^{(3)} \chi_{\beta j}^{(4)} \end{aligned} \right\} \quad (2.39)$$

Close to the monopole ($m_i r \ll 1$, $i = 1, 2$) the masses should play no role and one expects all the condensates

$$\langle M | F_x^{(1)} F_x^{(2)} | M \rangle = O(1) \quad (x = 1, 2, 3) \quad (2.40)$$

However, two-fermion condensates are also possible, but these will be suppressed by mass factors

$$\langle F_i^{(1)} \rangle \propto m_2, \quad \langle F_i^{(2)} \rangle \propto m_1 \quad (2.41)$$

That this will be case can be seen if one expands in the fermion mass, i.e.,:

$$\begin{aligned} \langle M, m | F_i^{(1)}(x) F_i^{(2)\dagger}(x) | M, m \rangle &\simeq \\ &\simeq m_2^2 \int d^4 y \int d^4 y' \langle M, 0 | F_i^{(1)}(x) F_i^{(2)}(y) F_i^{(1)\dagger}(x') F_i^{(2)\dagger}(y') | M, 0 \rangle \end{aligned} \quad (2.42)$$

The calculation of the exact coefficients is (not surprisingly) plagued by the appearance of various ultra-violet divergences, which would have to be removed by an appropriate renormalization procedure. Combining the remarks we have made so far one expects the following mass dependences in a range where $1/m_2 \ll r \ll 1/m_1$:

$$\left. \begin{aligned} \langle M | F_i^{(1)}(x) F_i^{(2)}(x) | M \rangle &\propto e^{-m_2 r} \\ \langle M | F_i^{(1)}(x) | M \rangle &\propto O(1) \\ \langle M | F_i^{(2)}(x) | M \rangle &\propto m_1 e^{-m_2 r} \end{aligned} \right\} \quad (2.43)$$

A sketch of the resulting multiple core structure of the GUM and its condensates is shown in Fig. 1. This analysis suggests that for monopole induced proton decay one should only consider condensates involving the light fermions, i.e., u, d, s, l and μ , in other words one expects only six fermion condensates and four fermion condensates [suppressed by $m_\mu(m_s)$] to be relevant.

A final remark concerns finite temperature effects. We would expect the condensates to dissolve at radii $r > 1/T$ so that the monopole only has a "geometric" size. For this reason we expect GUM-fermion cross-sections to fall $\propto 1/T^2$ at high temperatures, as in the very early Universe. In this case there is little chance of GUMs washing out the baryon number of the Universe, as has already been remarked in Ref. 22).

3. - IMPLICATIONS FOR EXPERIMENT

In this section we develop the phenomenological consequences of the analysis of Section 2 for experimental searches for GUMs using present and future baryon decay experiments. First we discuss generalized Cabibbo-Kobayashi-Maskawa flavour mixing²³⁾, then the order of magnitude of the $\Delta B \neq 0$ GUM-hadron cross-section $\sigma_{\text{GUMB} \neq 0}$, then hazard some guesses about the branching ratios of baryon "decays" catalyzed by GUMs, and finally discuss the capabilities of specific baryon decay experiments for seeing the $\Delta B \neq 0$ interactions of GUMs.

3.1 Flavour mixing

We saw in Section 2 that even a Dirac monopole sitting in an Abelian subgroup of the exact low-energy $SU(3) \times U(1)$ gauge group could in principle have $\Delta B \neq 0$ interactions. We also saw that the unique assignment of doublets which could be accommodated in a GUT was that of (2.28a). Furthermore, this possibility was in fact realized in the minimal $SU(5)$ GUT (2.31) and (2.32) and hence in any GUT containing $SU(5)$. Our arguments so far do not constrain the possible phase differences between members of the doublets (2.28a) which we introduce in the form

$$\begin{pmatrix} u_1 \\ u_2 e^{i\alpha} \end{pmatrix}_L, \begin{pmatrix} u_2 \\ -u_1 e^{i\beta} \end{pmatrix}_L, \begin{pmatrix} d_3^c \\ e^- e^{i\gamma} \end{pmatrix}_L, \begin{pmatrix} e^+ \\ -d_3 e^{i\delta} \end{pmatrix}_L \quad (3.1)$$

with arbitrary phases $\alpha, \beta, \gamma, \delta$. Also we have not yet discussed flavour mixing between different fermion generations, and we now address this topic. We recall that in simple $SU(5)$ ⁵⁾ and $SO(10)$ ⁶⁾ GUTs it is possible to choose a mass eigenstate basis for N_G generations of $(\underline{5} + \underline{10})$ fermions in which^{23),29)}: (A) all elements of the $\underline{5}$ are mass eigenstates and they have no relative mixing angles or phases, (B) there is a Kobayashi-Maskawa³⁰⁾ mixing matrix between mass eigenstates coming from the first four rows and columns and the last row and column of the $\underline{10}$ representations of fermions, and (C) mass eigenstates from the first three rows and columns of the $\underline{10}$ representations are further rotated relative to the fourth row and column by a diagonal phase matrix²³⁾. The property

(A) means that the $(d_3^c, e^-)_L$ doublet in (3.1) is not affected by flavour mixing but is joined by $(s_3^c, \mu^-)_L$, $(b_3^c, \tau^-)_L$ doublets, and the phase $\delta = 0$. The property (B) has similar implications for the $(e^+, d_3)_L$ doublet in (3.1) and implies $\gamma = 0$. The property (C) means that the first two doublets in (3.1) do not mix with their analogues from higher generations. However, the lower components u_{iL}^c , c_{iL}^c and t_{iL}^c ($i=1,2$) should be rotated relative to the upper components u_{iL} , c_{iL} , t_{iL} by phase factors $\alpha = \beta$ which are unknown and practically unobservable since they do not affect interaction rates. We therefore focus on the $\Delta B = 1$ condensates obtainable from the first generation doublets (3.1). They are

$$(u_{2L}^c d_{3L}^c)(u_{1L}^c e_L^+) + (u_{1L}^c d_{3L}^c)(u_{2L}^c e_L^+) + (\text{herm. conj.}) \quad (3.2a)$$

$$(u_{1L} \bar{e}_L)(u_{2L} d_{3L}) + (u_{2L} \bar{e}_L)(u_{1L} d_{3L}) + (\text{herm. conj.}) \quad (3.2b)$$

where we have used parentheses () to symbolize contraction on I SU(2) and spin indices and we expect no Cabibbo-Kobayashi-Maskawa mixing factors. We notice that the operators (3.2a) and (3.2b) have the same form as the operators O^3 and O^5 in the standard decomposition³¹⁾ of $|\Delta B| = 1$ operators of dimension six, with the differences that they are not SU(2) invariant - and have no neutrino fields - and not SU(3) invariant - to be anticipated since the monopole is also not SU(3) invariant. The familiar form of the $|\Delta B| = 1$ operators (3.2) will facilitate our calculation of $\sigma_{\text{GUMB} \neq 0}$ in the next Subsection.

3.2 Estimate of the $\Delta B = 1$ GUM-nucleon cross-section

We estimate $\sigma_{\text{GUMB} \neq 0}$ by relating it to the conventional nucleon decay rate calculation^{29),32),33)}, keeping track of all factors of $(2\pi)^n$ and assuming that all unknown strong interaction parameters with dimensions of mass are between m_B and $O(300)\text{MeV} \approx m_B/3 \approx m_q$ where m_q is an effective quark mass inside the baryon B. We consider the GUM as an external fixed source so that energy is conserved in the GUM rest frame, implying a transfer of energy $\Delta E = \beta \times O(1) \text{ GeV}$ in the nucleon rest frame. However, the three-momentum transfer $|q|$ from the GUM could well be $O(m_q) = O(300)\text{MeV}$. We expect on the basis of dimensional analysis and analogy with conventional calculations of e.g., Primakoff production from a heavy nucleus, that the cross-section for $B + M \rightarrow l^+ +$
+ mesons (m) + M takes the form

$$\sigma_{\text{GUMB} \neq 0} = \frac{O(1)}{v} \frac{\hat{\Gamma}(B \rightarrow l^+ + m)}{m_q^3} \quad (3.3)$$

In this formula, v is the relative velocity between the GUM and the B which becomes β for slow GUMs striking stationary baryonic targets. This familiar flux factor is not¹⁷⁾ cancelled by any other factor in the cross-section since neither the amplitude nor the final state phase space vanish at threshold^{*)}. The quantity $\hat{\Gamma}$ is the baryon decay rate computed as for a conventional super-heavy X vector boson exchange but with the following replacement in the amplitude

$$\frac{g^2}{2\sqrt{2}m_X^2} \rightarrow \frac{1}{m_q^2} \quad (3.4)$$

The initial factors of $g^2/2\sqrt{2}$ come from the conventional effective Lagrangian for B decay, while the factors of $(1/2\pi)^2$ come from the prefactor (2.34) of the four-fermion condensates (3.2) derived in Section 2, and the $1/m_q^2$ factor in Eq. (3.4) is guessed by dimensional analysis. We take as a central value for the nucleon decay rate in conventional GUTs

$$\left. \begin{aligned} \Gamma(B \rightarrow l^+ + m) &= 1/10^{29} \text{ years} \\ \text{for } m_X &= 2.25 \times 10^{14} \text{ GeV}, \quad \frac{g^2}{4\pi} = \frac{1}{41} \end{aligned} \right\} \quad (3.5)$$

From Eqs. (3.4) and (3.5) we estimate

$$\hat{\Gamma}(B \rightarrow l^+ + m) = \frac{O(5)}{(2\pi)^8} \text{ GeV} \quad (3.6)$$

and hence substituting into the expression (3.3) for $\sigma_{\text{GUMB} \neq 0}$ we find that if we parametrize

$$\sigma_{\text{GUMB} \neq 0} \equiv \frac{\sigma_0}{\beta \times 1 \text{ GeV}^2} \quad (3.7)$$

where σ_0 is a dimensionless reduced cross-section, then

$$\sigma_0 \approx O(10^{-4}) \quad (3.8)$$

We tend to regard this value for σ_0 as approximate, since some of dimensional factors that we have taken to be $O(m_q = m_B/3)$ might actually turn out to be $O(m_B)$ or $O(\Lambda_{\text{QCD}} \approx 150 \text{ MeV})$. For example, the $\Delta B = 1$ four-fermion condensate

^{*)} It has been suggested³⁴⁾ recently that the GUM-proton cross-section might be larger than the GUM-neutron cross-section because of capture effects. However, this is only likely to apply to protons which are essentially free, which ours will not be.

is presumably cut off by a product of $\exp(-m_f|\underline{x}|)$ factors at large $|\underline{x}|$, and taking $m_f = m_q$ three times gives a combined factor $\exp(-m_B|\underline{x}|)$. If we replace two of the factors of m_q by m_B in the amplitude substitution (3.4) then the estimate (3.8) is reduced by an order of magnitude, while replacing m_q by Λ_{QCD} can easily give an order of magnitude greater. Thus a possible representation of the uncertainty may be

$$\sigma_0 = O(10^{-3} \text{ to } 10^{-5}) \quad (3.9)$$

However, within our uncertainties σ_0 could easily be somewhat larger or smaller than the range (3.9). For example, our estimate (3.4) of the baryon decay rate corresponding to the chosen reference value of m_X could easily be an order of magnitude wrong. This caveat should be borne in mind although in subsequent parts of this paper we will use the range (3.9) for the dimensionless reduced cross-section σ_0 introduced in Eq. (3.7)

3.3 Hierarchy of catalyzed baryon "decay" modes

In Subsection 3.1 we discussed generalized Cabibbo-Kobayashi-Maskawa mixing and verified that the condensates (3.2) should exist around GUMs. We then used these condensates with the normalizations derived in Section 2 to estimate $\sigma_{\text{GUMB} \neq 0}$. In this Subsection we look more carefully at the full gamut of condensates to be expected around GUMs and estimate the resulting branching ratios for the different baryon "decay" modes they may catalyze.

We saw in Section 2 that condensates containing heavy fermions f are suppressed by factors $O(e^{-m_f|\underline{x}|})$ at large radii $|\underline{x}|$. Therefore, we do not expect condensates involving t, b, c quarks and/or the τ lepton, or any heavier fermions, to be strong at the distance scales $|\underline{x}| \gtrsim O(1)\text{GeV}^{-1}$ which dominate $\sigma_{\text{GUMB} \neq 0}$. On the other hand, condensates involving s quarks and/or μ leptons might be important in this distance range, as well as the first generation $\underline{I}$ doublets (3.1). We also recall that condensates which do not involve all the fermions "operational" on a given distance scale can also exist, but suppressed by factors $O(m_f|\underline{x}|)$ where m_f is here the mass of one of the fermions omitted from the condensate.

What interesting $\Delta B \neq 0$ condensates can we make out of the available doublets

$$\begin{pmatrix} u_1 \\ u_2^c \end{pmatrix}_L, \begin{pmatrix} u_2 \\ -u_1^c \end{pmatrix}_L, \begin{pmatrix} d_3^c \\ e^- \end{pmatrix}_L, \begin{pmatrix} e^+ \\ -d_3 \end{pmatrix}_L, \begin{pmatrix} s_3^c \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \mu^+ \\ -s_3 \end{pmatrix}_L \quad (3.10)$$

By omitting the two s and μ doublets we can get the condensates (3.2):

$$\left. \begin{array}{c} (u_2^c d_3^c)_L (u_1^c e^+)_L \\ \text{or} \\ (u_1 e^-)_L (u_2 d_3)_L \end{array} \right\} \times \left\{ \begin{array}{c} m_s \\ \text{or} \\ m_\mu \end{array} \right. + (\text{herm. conj.}) \quad (3.11)$$

where we have exhibited explicitly the mass factors corresponding to the s and the μ . Correspondingly by omitting the two d and e doublets we can get the condensates

$$\left. \begin{array}{c} (u_2^c s_3^c)_L (u_1^c \mu^+)_L \\ \text{or} \\ (u_1 \mu^-)_L (u_2 s_3)_L \end{array} \right\} \times \left\{ \begin{array}{c} m_d \\ \text{or} \\ m_e \end{array} \right. + (\text{herm. conj.}) \quad (3.12)$$

We have omitted from Eqs. (3.11) and (3.12) any mass factors for the heavier generations: these are expected to be the same for the two sets of condensates. Whereas the condensates (3.11) give "decays" $B \rightarrow e^+ + \text{pions}$, the condensates (3.12) give "decays" $B \rightarrow \mu^+ + \text{strange particles}$. We therefore expect

$$\Gamma(B \rightarrow e^+ \pi) : \Gamma(B \rightarrow \mu^+ K) \approx 1 : \left(\frac{m_d}{m_s} \right)^2 \quad (3.13)$$

The ratio in (3.13) depends whether we put in short distance current algebra masses $((m_d/m_s)^2 = 0(1/400))$ or long distance constituent mass $((m_d/m_s)^2 = 0(1/2))$. Taking into account the kinematic factors which suppress $B \rightarrow \mu^+ K$ relative to $e^+ \pi$ even in conventional GUTs³²⁾, we guess

$$\Gamma(B \rightarrow e^+ \pi) \gg \Gamma(B \rightarrow \mu^+ K) \approx 1 : 0(10^{-1} \text{ to } 10^{-3}) \quad (3.14)$$

We now turn to the more interesting six-fermion condensates. It is a simple exercise to enumerate those with $|\Delta B| = 1$ and the final states they can lead to:

$$\left. \begin{array}{lll} e^+ \pi & \text{e.g. from} & (ud)(ue)(\bar{s}s) \\ \mu^+ K & \text{e.g. from} & (ud)(us)(\bar{\mu}\bar{d}) \\ e^+ \pi \mu^+ \mu^- & \text{e.g. from} & (ud)(ue)(\mu^+ \mu^-) \text{ (ref. 17, 18)} \\ \mu^+ K e^+ e^- & \text{e.g. from} & (ue)(us)(e^+ \mu^-) \end{array} \right\} \quad (3.15)$$

The symbols π and K in Eqs. (3.13)-(3.15) should be understood somewhat figuratively. They may represent 0, 1, 2 or more body meson systems with the same quantum numbers. In particular, in the case of proton "decay" catalyzed by a GUM one could obtain from (3.15) "decays" such as

$$p \rightarrow e^+ \mu^+ \bar{\mu} \quad (3.16)$$

with no accompanying mesons. Even more strikingly, since energy is essentially conserved in GUM catalysis, but not three-momentum, one could also have

$$p \rightarrow e^+ \quad (3.17)$$

giving a single final state electron with energy 940 MeV. However, this "decay" would require a momentum transfer of $O(1)$ GeV which one might expect to be suppressed by form factor effects. No such suppression need occur for the "decay" (3.16). On the other hand, a simple count of factors of 2π suggests

$$\frac{\Gamma(B \rightarrow e^+ \mu^+ \bar{\mu})}{\Gamma(B \rightarrow e^+ \pi)} = O\left(\left(\frac{1}{4\pi^2}\right)^2 \times \left(\frac{1}{2\pi}\right)^2\right) = O\left(\frac{1}{2\pi}\right)^6 = O(10^{-5}) \quad (3.18)$$

where the factors come from the relative magnitudes of the six and four-fermion condensates and from phase space, respectively. While small, the ratio (3.18) is still much larger than what one might expect from conventional higher order electroweak diagrams in the absence of an $l^+ l^-$ condensate. To summarize this Subsection, we expect the following hierarchy of baryon "decays" catalyzed by GUMs:

$$\left. \begin{aligned} p, n &\rightarrow e^+ \pi : \mu^+ K : e^+ \mu^+ \bar{\mu} \dots, \mu^+ K e^+ e^- \\ &\approx 1 : O(10^{-1} \text{ to } 10^{-3}) : O(10^{-5}) \\ p, n &\not\rightarrow \bar{\nu} X, e^+ K, \mu^+ \pi \end{aligned} \right\} \quad (3.19)$$

This is distinctively different from any other model for baryon decay known to us.

3.4 Experimental considerations

In this Subsection we develop some considerations about the experimental signatures of baryon "decays" catalyzed by GUMs, in particular the possibility of a "chain" of baryon "decays" along the path of a GUM and the desirability of designing the electronics of a baryon decay experiment in such a way that they

do not go "dead" after the first event and miss the rest of the "chain". It is clearly desirable that every baryon decay detector be equipped with a conventional monopole detector (ionization or coil) to look for coincidences between baryon "decays" and the passages of monopoles.

As far as individual baryon "decay" events are concerned, we only recall that in addition to the characteristic hierarchy (3.19) one should also be on the look-out for "decay" events which do not appear to conserve three-momentum. We saw in Subsection 3.2 that there is no energy transfer from the GUM in the limit where it is very non-relativistic and infinitely heavy. It is simple to verify that for a GUM with velocity β striking a stationary nucleon the allowed energy transfer

$$\Delta E = O(\beta) \times 1 \text{ GeV} \quad (3.20)$$

which is probably undetectably small for GUMs at the Earth's surface: $\beta \ll 0.1$? (See, however, Section 4 for astrophysical situations where relativistic GUMs may be relevant.) We also saw in Subsection 3.2 that a three-momentum transfer $|q| = O(300) \text{ MeV}$ from the GUM is to be expected. This may be confused with Fermi momentum in experiments whose nucleons sit in heavy nuclei, but the effect could be more easily detectable in experiments using light nuclei such as the water Cerenkov detectors. Any detector should be able to see the extreme case (3.17) of $p \rightarrow e^+$ with a momentum of 940 MeV.

On the basis of the form (1.1), (3.7) of the interaction cross-section we expect a mean free path λ between successive $\Delta B \neq 0$ GUM-baryon collisions of order

$$\lambda \approx \frac{43m}{\rho} \left(\frac{\beta}{\sigma_0} \right) \quad (3.21)$$

in material of density $\rho \text{ gm/cm}^3$, corresponding to a mean free time if $\rho = 1$

$$\tau = \frac{\lambda}{\beta} = \frac{0.14}{\sigma_0} \text{ } \mu\text{sec} \quad (3.22)$$

which is independent of the velocity β of the monopole. If we insert into (3.21) and (3.22) the estimate (3.9) that $\sigma_0 = O(10^{-3} \text{ to } 10^{-5})$ we find that

$$\lambda = O(4300 \text{ to } 0.43) \text{ m for } \beta = O(10^{-5} \text{ to } 10^{-3}) \quad (3.23a)$$

$$\tau = O(0.14 \text{ to } 14) \text{ msec} \quad (3.23b)$$

These estimates are moderately encouraging for the search for "chains" of baryon "decays" - which would look far more promising if σ_0 had been $O(1)$ - but we should recall that there are still considerable uncertainties in the estimate (3.9) of σ_0 and hence of the mean free path (3.23a). We see from Eq. (3.21) that there is a good likelihood of a second "decay" event occurring inside a given detector of linear dimension d metres if

$$\beta < \sigma_0 \left(\frac{d}{43m} \right) \rho \quad (3.24)$$

Several baryon decay detectors²⁴⁾ have the characteristic that they record everything that happens within a short period after being triggered. This live time is typically a few microseconds in order to pick up stopping μ decays. Then typical experiments have a dead time of order a millisecond during which they put the event and related happenings on tape²⁴⁾. Unfortunately, we see from Eq. (3.23b) that it is quite likely that a second "decay" could occur within this dead time. It is therefore desirable for GUM hunting to arrange the electronics of a baryon decay experiment so that one retains at least some information about what happens during the dead time. This point is made graphically in Fig. 2 which portrays the sensitivity of some experiments (NUSEX and IBM) to baryon "decays" catalyzed by GUMs. We see that a dead time of one millisecond is particularly unfortunate for GUMs in the range of velocities $\beta = O(10^{-5}$ to 10^{-3}) favoured by astrophysical considerations. We recall that retaining information from this dead period is also useful for hunting bursts of neutrinos from collapsing supernovae.

4. - ASTROPHYSICAL CONSTRAINTS

It is well known that astrophysics and cosmology can provide very tight constraints on the properties of elementary particles. Here we are concerned with the constraints which can be applied to the abundance (or flux) and interactions of GUMs. The existing constraints derived from limits on the mass density of the Universe^{10),11)} and galaxy and from magnetic fields in the galaxy^{8),9),11),12)} provide very strong upper limits to the allowed flux of monopoles. Because of the excitement about the baryon number violating interactions associated with GUMs it is worth while to examine what additional constraints can be placed on their flux in combination with their baryon non-conserving cross-sections. We will begin with a brief review of the previous flux limits, then consider what limits may be derived from neutron stars [see also Ref. 14)] and finally what effects (if any) may be seen in the Sun.

4.1 Previous Limits

As was stated above, some of the most powerful constraints on the allowed monopole flux come from limits on the mass density of Universe¹⁰⁾. If the monopole velocity β is greater than 10^{-3} , they will not be confined to the galaxy but rather they will form a uniform extragalactic sea of density say n_m . The energy density contained by monopoles $\rho_m = mn_m$ where m is the monopole mass. We will consider m in the range $10^{16} - 10^{19}$ GeV and hereafter our units for GUT masses will always be GeV. Although standard SU(5)⁵⁾ prefers the lower end of this range, monopole masses in supersymmetric GUTs may be much higher³⁵⁾. The energy density ρ_m is then constrained by

$$\rho_m = m n_m < \Omega \rho_c \quad (4.1)$$

where the critical density $\rho_c = 1.88 \times 10^{-29} h_0^2 \text{ g cm}^{-3}$, the present value of the Hubble parameter is given by $H_0 = 100 h_0 \text{ km Mpc}^{-1} \text{ s}^{-1}$ and Ω is the ratio of the total mass density to the critical density. Conservative upper limits on the cosmological parameters are $\Omega < 2$ and $h_0 < 1$. Thus

$$n_m \lesssim \frac{2.1 \times 10^{-5}}{m} \text{ cm}^{-3} \quad (4.2)$$

which can be translated into a flux limit:

$$F = \frac{n_m \beta}{4\pi} \lesssim 5 \times 10^4 \frac{\beta}{m} \text{ cm}^{-2} \text{ sec}^{-1} \text{ sr}^{-1} \quad (4.3)$$

A comparable limit to (4.3) can be derived from considering the effects of monopoles passing through the galactic magnetic field^{8),9),11),12)}. The argument leading to this limit is rather simple. As monopoles pass in and out of the galaxy, they will on average gain energy at the expense of existing magnetic fields. In order for the fields to remain intact, they cannot be depleted faster than the timescale for regeneration of the magnetic fields. This led Parker⁸⁾ to his original limit $F < 10^{-16} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. Recently, this limit was re-examined taking into account the monopole mass and it was found¹²⁾ that

$$F \lesssim 10^{-26} m \beta^2 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.4)$$

which for $m = 10^{16}$ GeV and $\beta \leq 5 \times 10^{-2}$ is slightly better than the constraint (4.3), while for $\beta \geq 5 \times 10^{-2}$ it is slightly worse.

For low velocity monopoles, $\beta < 10^{-3}$, neither limit is applicable since monopoles are bound to the galaxy. Instead one can derive a somewhat weaker limit based on the upper limit on the mass density in the galaxy. The amount of matter contained in the galaxy³⁶⁾ which is inferred by galactic rotation curves is $M_G \approx 1.4 \times 10^{12} M_\odot$ out to a distance $R_G \sim 30$ Kpc implying that

$$\rho_g \lesssim 9 \times 10^{-25} \text{ g cm}^{-3} \quad (4.5)$$

or a flux limit given by

$$F < 10^9 \frac{\beta}{m} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.6)$$

for $10^{-4} < \beta < 10^{-3}$. For $\beta < 10^{-4}$ one would expect monopoles to be confined to solar systems and there have been up to the present time no solid astrophysical (or experimental) limits on the monopole flux.

3.2 Limits from Neutron Stars

Let us now consider what limits can be placed on the flux of GUMs due to their baryon number violating interactions in neutron stars [see also Ref. 14]. For the following discussion we will consider a neutron star of mass M given in solar mass units ($1M_\odot \approx 2 \times 10^{33} \text{ g}$) at a density n in units of nuclear density ($n_0 = 0.17 \text{ fm}^{-3}$). The basic quantities of interest are the following. The mean free path λ for a GUM traversing a star of density n is [scaled from Eq. (3.21)]

$$\lambda = 1.5 \times 10^{-11} \beta / \sigma_0 n \text{ cm} \quad (4.7)$$

where σ_0 is the cross-section coefficient of Eqs. (3.11) and (3.13). The rate of baryon number violating interactions per GUM trapped in the star is

$$\Gamma = 2 \times 10^{21} n \sigma_0 \text{ s}^{-1} \quad (4.8)$$

The rate at which GUMs are captured by the star (assuming they are stopped with 100% efficiency) is

$$\Gamma_c = 4\pi^2 R_N^2 C F \text{ s}^{-1} \quad (4.9)$$

where R_N is the neutron star radius

$$R_N = 12 \times 10^6 \left(\frac{M}{n} \right)^{1/3} \text{ cm} \quad (4.10)$$

and C is a general relativistic correction to the neutron star radius¹⁴⁾

$$\begin{aligned} C &= (1 + 2MG/\beta^2 R_N) / (1 - 2MG/R_N) \\ &\approx (4 + M^{2/3} n^{1/3} \beta^{-2}) / (4 - M^{2/3} n^{1/3}) \end{aligned} \quad (4.11)$$

Finally, the number of GUMs contained in the neutron star is given by

$$N_m = \Gamma_c t_N \quad (4.12)$$

where t_N will denote the age of a given neutron star.

In order to derive limits from neutron stars, we must first be sure that GUMs do indeed get trapped while passing through. While the energy transfer from the GUM to light particles may be $\beta \times 0(1)$ GeV per collision, this may have either sign depending on the direction of the three-momentum transfer q . Averaged over many collisions the energy loss by a monopole per collision is

$$\Delta E \approx -\beta^2 \text{ GeV} \quad (4.13)$$

Thus the total energy loss while passing through the star is

$$R_N \Delta E / \lambda = 8 \times 10^{16} \beta_N \sigma_0 n^{2/3} M^{1/3} \text{ GeV} \quad (4.14)$$

which should be compared with the initial energy $\frac{1}{2} m \beta^2$. We distinguish between the two β 's because the velocity of the GUM entering the star β_N will be largely independent of its asymptotic value β . In general $\beta_N \approx 1/3$, so that the condition for stopping a GUM becomes (for $n = 1, M = 1$)

$$\beta^2 < \beta_t^2 \equiv 5.3 \times 10^{16} \sigma_0 / m \quad (4.15)$$

All of the limits which follow therefore will require $\beta < \beta_t$. For $\beta > \beta_t$, only the limits from Subsection 4.1 and limits from present-day proton-decay experiments are valid. The constraint $\beta < \beta_t$ (4.15) is rather non-trivial. For example, if $m = 10^{17}$ GeV as one might expect in a supersymmetric GUT³⁵⁾, then if $\sigma_0 = 0(10^{-5})$, GUMs with $\beta > 0(4 \times 10^{-3})$ can pass through a neutron star. If $m = 10^{20}$ GeV then all GUMs with $\beta > 0(10^{-4})$ can pass through a neutron star.

To begin with, we will present two limits containing what we consider negligible uncertainties. For simplicity we take $M = n = 1$.

The first merely comes from the observation that neutron stars exist. At least we will assume the Crab pulsar to be a neutron star with age $t_N \approx 1000$ yrs. Thus we may require that the total number of interactions in the star

$$I = \int_0^{t_N} N_m \Gamma dt = \Gamma_c \Gamma t_N^2 / 2 \quad (4.16)$$

is less than the total number of nucleons in the star. From Eqs. (4.8)-(4.12) and (4.16), we find that in order not to totally disintegrate a neutron star

$$(4 + \beta^{-2}) F \sigma_0 \leq 64 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.17)$$

so that even for $\beta = 10^{-5}$

$$F \sigma_0 < 6.4 \times 10^{-9} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.18)$$

which is well above the Cabrera limit²⁰⁾ with $\sigma_0 = 1$, and hence not particularly interesting.

A somewhat better limit comes when one considers that there exist upper limits³⁷⁾ on the temperatures of neutron stars of the order of $T_{\text{max}} \approx 2.3 \times 10^6$ °K (for the Crab pulsar). In general, GUMs captured by neutron stars will tend to heat them up. The energy generation rate within the neutron star is

$$Q = m_B \Gamma N_m = 1.8 \times 10^{42} (4 + \beta^{-2}) F \sigma_0 \text{ erg s}^{-1} \quad (4.19)$$

As most of the energy produced is not in the form of neutrinos, it does not escape easily but rather becomes thermalized with a temperature given by

$$T^4 = Q / (4\pi R_N^2 a) \text{ (}^\circ\text{K)}^4 \quad (4.20)$$

where a is the Stephan-Boltzmann constant. Thus insisting that $T < 2.3 \times 10^6$ °K from (4.19) and (4.20) we find that

$$F \sigma_0 (4 + \beta^{-2}) < 6.6 \times 10^{-8} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.21)$$

which for $\beta = 10^{-3}$ gives a reasonably strong constraint $F \sigma_0 < 6.6 \times 10^{-14} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. This is not inconsistent with the Cabrera flux for the plausible range (3.9) of σ_0 discussed in Section 3.

It has recently¹⁴⁾ been pointed out that an argument such as the one above could be strengthened by considering the effect of GUMs on a population of very old neutron stars with $t_N \sim 10^{10}$ yrs. The argument here runs as follows. Neutron stars which would have formed at the very early stages of the galaxy should still be present. If during this time, the neutron star continually gathers GUMs, as would be expected if it were bathed in a continuous galactic flux, corresponding to $\beta > 0(10^{-4})$ at the Earth's surface, it could heat up to such an extent as to overwhelm the present X-ray background³⁸⁾. The limit on the total energy emission per neutron star (for $E > 0.2$ keV) is 6×10^{49} ergs. From this Kolb et al.¹⁴⁾ have argued that by requiring

$$Q t_N < 6 \times 10^{49} \text{ erg} \quad (4.22)$$

one has the limit on the flux^{*)}

$$F \sigma_0 (4 + \beta^{-2}) \leq 10^{-17} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.23)$$

This limit is slightly relaxed, however, when one realizes that the energy output must be thermalized before it escapes. In fact, at the flux limit (4.23) the temperature of the neutron star is only $T = 4.6 \times 10^5 \text{ }^\circ\text{K} \approx 0.04 \text{ keV}$ and hence will not disturb the limits³⁸⁾ on the X-ray background. As a more conservative limit we will require $T < 2.3 \times 10^6 \text{ }^\circ\text{K} = 0.2 \text{ keV}$ using similar equations as (4.19)-(4.21) but with $t_N = 10^{10}$ yrs. In this case we find

$$F \sigma_0 (4 + \beta^{-2}) < 6.6 \times 10^{-15} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.24)$$

We emphasize that this limit applies to velocities $\beta > 0(10^{-4})$ at the Earth's surface.

Let us now consider what happens when $\beta \leq 10^{-4}$. It has been argued recently¹³⁾ that the Cabrera flux could be explained if the GUMs were due to a local source such as the sun. If we suppose that a cloud of GUMs were trapped in the solar system, they would eventually be caught by the sun or expelled because of the magnetic field $B \approx 3 \times 10^{-5}$ Gauss due to the solar wind. The capture time scale is given by^{39),13)}

$$t_c = \beta^2 T^{-1} \omega^{-2} \text{ sec} \quad (4.25)$$

^{*)} This is slightly stronger than the limit quoted by Kolb et al.¹⁴⁾ but has been adapted for the parameters considered here.

where $T \approx 10^6$ sec is the period over which the magnetic field fluctuates and $\omega \approx gB/m \text{ sec}^{-1}$. For $m = 10^{16}$ GeV and $\beta = 10^{-4}$, $t_c \approx 10^8$ yrs. The number of GUMs within the Sun must then be

$$N_m = \frac{t_s}{t_c} V \left(\frac{4\pi F}{\beta} \right) \quad (4.26)$$

where $t_s \approx 5 \times 10^9$ yrs is the age of the sun and V is an appropriate volume which we take to have a radius of 1 A.U. (this is the minimum radius needed to explain Cabrera's flux²⁰⁾). Thus $N_m \approx 3 \times 10^{56} F/(m^2 \beta^3)$.

If we suppose, however, that such a picture can be extended to other solar systems as well, we see that such an explanation is incompatible with our previous arguments concerning neutron stars. The reasoning is simple: if all the neutron stars contain at least $3 \times 10^{56} F/(m^2 \beta^3)$ GUMs in their interior when they are born by the collapse of a conventional star like the sun, they will be subject to the same kind of heating due to baryon number violating interactions. We consider this a minimum for N_m since in general neutron stars were created from stars more massive than the Sun and would hence capture more GUMs. We also neglect any GUMs captured after the main sequence lifetime of the star^{*)}. From (4.19) we have

$$Q \approx 10^{75} F \sigma_0 / m^2 \beta^3 \text{ erg s}^{-1} \quad (4.27)$$

corresponding to a temperature

$$T \approx 2.2 \times 10^6 (F \sigma_0)^{1/4} / m^{1/2} \beta^{3/4} \text{ } ^\circ\text{K} \quad (4.28)$$

which for $m = 10^{16}$ GeV and $\beta = 10^{-4}$ at Cabrera's flux²⁰⁾ gives $T \approx 10^9 \sigma_0^{1/4} \text{ } ^\circ\text{K} \approx 10 \text{ keV}$ thus easily radiating X and γ rays and overwhelming the observed limits.

We can turn this argument around, however, to get a bound on the GUM flux for the case $\beta < 10^{-4}$ when GUMs should be bound to solar systems. In this case the number of GUMs trapped by stars (through their magnetic fields) will be given by (4.26). If we demand that the energy generated by these GUMs (4.27) does not exceed the X-ray limits then we have the following limit on the flux of slow moving GUMs.

^{*)} Capture might continue after the collapse to a neutron star, but we do not know how to estimate it. Is there a solar wind from the surface? Are there Jupiters orbiting the neutron star to perturb GUM orbits? Is it a member of a binary system where there would be large perturbations?

$$F\sigma_0/m^2\beta^3 < 1.3 \times 10^{-40} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.29)$$

which for $m = 10^{16}$ GeV, $\beta = 10^{-4}$ yields

$$F\sigma_0 < 1.3 \times 10^{-20} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.30)$$

which is comparable with our previous limits.

Figure 3 shows a compilation of our various astrophysical limits on $F\sigma_0$ as a function of β for different choices of GUM mass, compared with the sensitivity of a baryon detector able to see a nucleon lifetime of 10^{33} yrs. We see in Fig. 3a that GUMs of mass 10^{16} GeV are best bounded by neutron stars for $\beta < 10^{-2}$, and above this by the galactic magnetic field and by the mass density of the Universe. Figure 3b shows a similar picture for GUMs of mass 10^{17} GeV. Figure 3c shows that GUM-catalyzed baryon decay can be detectable for $\beta < 10^{-4}$, and this possibility also exists in the range $10^{-4} < \beta < 10^{-3}$ for heavier GUMs (cf. Fig. 3d)*). Thus while astrophysical considerations make life difficult for baryon decay experiments, they do not make it impossible. Note also that the conservative limits on $F\sigma_0$ coming from known neutron stars such as the Crab are much less stringent.

4.3 Monopoles and the Sun

Having considered extensively the effects of GUMs in neutron stars, it is worth while here to consider briefly their possible effects in the Sun. There are two ways in which GUMs can affect the Sun. First, they can heat up the Sun and enhance its luminosity or perhaps produce an observable amount of neutrinos. GUMs will produce these effects by either stopping in the Sun and collecting there much as they might do in neutron stars, or by simply passing through the Sun and interacting occasionally. Let us first see under what conditions a GUM will stop in the Sun. Through its baryon violating interactions the total energy loss for a GUM will be

$$R_0 \Delta E/\lambda = 2.3 \times 10^7 \beta_s \sigma_0 \text{ GeV} \quad (4.31)$$

where $\beta_s \approx 2 \times 10^{-3}$ and a mean solar density of $\rho = 1.41 \text{ g cm}^{-3}$ was used. This is to be compared with $\frac{1}{2}m\beta^2$ so that the condition to stop is

*) The best constraint on slow, heavy GUMs $F\sigma_0 < 2.3 \times 10^{-12} \beta \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ comes from present-day baryon decay experiments.

$$\beta^2 < 9.2 \times 10^4 \sigma_0/m < 9.2 \times 10^{-12} \sigma_0 \quad (4.32)$$

which for $\sigma_0 < 10^{-3}$ gives $\beta \leq 10^{-7}$. We conclude that $\Delta B \neq 0$ interactions could not stop a GUM in the Sun. It is, however, possible that monopoles may lose energy by conventional electromagnetic processes in which case the energy loss could be as large as⁴⁰⁾

$$\frac{dE}{dx} \approx 100 \beta \text{ GeV/cm}$$

and monopoles would be stopped for

$$\beta^2 < 2.8 \times 10^{10}/m < 2.8 \times 10^{-6} \quad (4.33)$$

From the limits on the flux, we can compute the maximum luminosity produced by GUMs in the Sun for $\beta \leq 10^{-3}$. The luminosity is given by (4.19)

$$Q = 5 \times 10^{46} F \sigma_0 \text{ erg s}^{-1} \quad (4.34)$$

For $m = 10^{16}$ GeV, $\beta = 10^{-3}$ we see from Fig. 3a that $F \sigma_0 < 6.6 \times 10^{-21} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ so that $Q < 3.3 \times 10^{26} \text{ erg/s}$ which is only $\leq 10^{-7} L_\odot$. The flux of neutrinos which one would expect (assuming one ν per "decay" emanating from the secondary decays of primary decay products: $\pi \rightarrow \mu\nu$, $K \rightarrow \mu\nu$, πev , ...) at this level is

$$F_\nu = 10^{21} F \sigma_0 < 6 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.35)$$

which may just possibly be a detectable amount. For larger GUM masses and lower velocities the limits are eased somewhat so that for $m = 10^{19}$ GeV, $\beta \leq 5 \times 10^{-5}$ we get $F \sigma_0 < 10^{-16} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ and $Q \leq 5 \times 10^{30} \text{ erg/s} \approx 10^{-3} L_\odot$, still consistent with the solar model. In this case the neutrino flux may be as high as $F_\nu \approx 10^5 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ which is probably detectable. We note that most of the neutrinos are expected to be muonic.

For larger β , GUMs will not be trapped by the Sun, but baryon violating interactions may occur as they pass through. The rate of interactions in this case is

$$\Gamma_I = 4\pi^2 R_\odot^3 F/\lambda = 4.4 \times 10^{30} \sigma_0 F/\beta \text{ s}^{-1} \quad (4.36)$$

and the energy generation rate is

$$Q = m_B \Gamma_I = 6.5 \times 10^{27} \sigma_0 F / \beta \text{ erg s}^{-1} \quad (4.37)$$

which yields the very unrestrictive limit

$$F \sigma_0 < 6 \times 10^5 \beta \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

for $Q < L_\odot$. The neutrino flux for passing monopoles is just

$$F_\nu = O(100) \frac{F \sigma_0}{\beta} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (4.38)$$

and hence is unobservable.

We find therefore (see also Fig. 3) that if GUMs are stopped by the Sun and are heavier than 10^{16} GeV, the flux limits for $\beta < 10^{-4}$ soften to the extent that baryon number violating interactions in the Sun might produce a detectable ν flux. This comes about because at large masses and slow (asymptotic) velocities $\beta \sim 5 \times 10^{-5}$ GUMs may be caught by the Sun. As we consider neutron stars to have only a limited source of GUMs for $\beta < 10^{-4}$, the direct neutron star limit does not apply. In fact, in this case the proton "decay" limit is stronger than (4.29). Thus we are independent of σ_0 for the neutrino flux limit which then becomes $F_\nu < 10^5 \text{ cm}^2 \text{ s}^{-1} \text{ sr}^{-1}$. In all other cases, due to the strong limits imposed by neutron stars, GUM-catalyzed baryon decay will not be observed by solar processes, unless some unknown mechanism can be invented for stopping in the Sun those GUMs with velocities $\beta > O(10^{-3})$.

5. - CONCLUSIONS AND SPECULATIONS

In this paper we have investigated theoretical, experimental and astrophysical aspects of the large $\Delta B \neq 0$ cross-sections expected^{17)-19),21)} to be associated with GUMs. We saw in Section 2 that B violating condensates may form around monopoles even in the absence of a "grand unified" core. GUTs enable one to pick out which of these condensates actually appear. In Section 3 we estimated the magnitude of the B violating GUM-nucleon cross-sections and discussed the hierarchy of possible catalyzed baryon "decay" modes. We also emphasized the possibility of observing a chain of baryon "decays" occurring in a large baryon decay detector. As seen in Fig. 2 a successful search for such chains may require reducing the dead time of the detector²⁴⁾ after recording a baryon decay candidate. In Section 4 we examined astrophysical constraints on the flux and $\Delta B \neq 0$ cross-sections of GUMs and found that while neutron stars impose severe

constraints¹⁴⁾ they do not exclude the observation of baryon "decays" catalyzed by GUMs, as seen in Fig. 3. It may also be possible to detect a flux of $0(100)\text{MeV}$ neutrinos coming from the Sun¹⁷⁾ if GUMs get stuck there, as may happen if they are sufficiently light [$m \sim 10^{16} \text{ GeV}$, $\beta < 0(10^{-4})$]? In view of all the uncertainties, we do not think that anyone should allow these astrophysical consideration to dissuade her(him) from an experimental search for evidence of the large $\Delta B \neq 0$ cross-sections expected for GUMs.

We finish with a short discussion of the speculative possibility²²⁾ of exploiting baryon "decays" catalyzed by GUMs. Every GUM has the capability of triggering an indefinite number of baryon "decays", releasing 1 GeV of energy each time. If GUMs exist, could this capacity for energy release be harnessed? The first problem is to capture the GUMs: stopping a GUM of mass m and velocity β requires an integrated magnetic field

$$\int B dl = 2.4 \times 10^4 \beta^2 m \quad (5.1)$$

In the most favourable case $m = 10^{16} \text{ GeV}$ and $\beta = 10^{-5}$ this requirement is about about 24 Tesla-kilometre. Clearly even if one could achieve such an integrated field, there are formidable obstacles in the way of aligning the field so as to cancel very precisely the momentum vector of an incoming GUM. Suppose a GUM is stopped: what rate of energy generation can it provide? Since the $\Delta B \neq 0$ cross-section is proportional to $1/\beta$, the rate of energy production Q by a GUM eating away material of density ρ is independent of β :

$$Q = m_B \Gamma \quad \text{per GUM} \quad (5.2)$$

where the interaction rate

$$\Gamma = 7 \times 10^6 \rho \sigma_0 \text{ s}^{-1} \quad (5.3)$$

with σ_0 as defined in Eq. (3.7) and ρ expressed in gm/cm^3 . In units familiar to electric power companies,

$$Q = 1.0 \times \rho \sigma_0 \text{ milliWatts / GUM} \quad (5.4)$$

meaning that it requires at least 10^{11} GUMs to replace a nuclear power station. Therefore, GUMs are unlikely to solve the "energy crisis" in the near future.

GUMs are an inescapable feature²⁾ of GUTs⁴⁾⁻⁶⁾. It is therefore very important to look for them in all possible ways. The discovery^{17),18)} that they can catalyze $\Delta B \neq 0$ reactions with large cross-sections provides a new handle on GUMs. Astrophysics provides severe constraints on the flux and $\Delta B \neq 0$ cross-sections of GUMs, and the very early Universe may have furnished us with orders of magnitude fewer GUMs. Nevertheless we believe that one should push ahead with experimental searches for the $\Delta B \neq 0$ interactions of GUMs.

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FIGURE CAPTIONS

Fig. 1 : Sketch of the condensate strengths around a monopole. The monopole looks like an Abelian Dirac monopole outside the core radius $O(1/m_X)$. Condensates involving fermions of mass m_2 are exponentially damped at radii $>1/m_2$, cf. the $F_1 F_2$ condensate exhibited. The F_1 condensate is damped by a power of $m_2 r$ at radii $<1/m_2$, and is exponentially damped at $r > 1/m_1$. The chromomagnetic field is presumably screened at some radius between 1 GeV^{-1} and 10 GeV^{-1} .

Fig. 2 : Indication of the capabilities of baryon decay experiments to search for B "decays" catalyzed by GUMs. The probable (?) range of β and the possible (?) range of the $\Delta B \neq 0$ cross-section strength σ_0 (3.7) are indicated. Experiments can observe a catalyzed "decay" in the region marked 1E if GUMs have the Cabrera²⁰⁾ flux. In the region 2E two events can occur within the same apparatus. The solid lines refer to the NUSEX experiment, the dashed lines to the IMB experiment.

Fig. 3 : Plots of the astrophysically allowed regions for the quantity F (measured in $\text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$) for different GUM velocities and masses (a) 10^{16} GeV , (b) 10^{17} GeV , (c) 10^{18} GeV and (d) 10^{19} GeV . Catalyzed B "decays" and $O(100) \text{ MeV}$ neutrinos from the Sun (if GUMs stop inside it) are detectable above the indicated lines corresponding to an apparent "lifetime" of 10^{33} years and $F_\nu = 1 \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. The astrophysical bounds come from neutron stars (NS), the galactic magnetic field (GB), the density of the Universe (DU) and the missing mass in the galaxy (MM). Also shown on Fig. 3d is the upper limit on $F\sigma_0$ coming from present-day baryon decay experiments (BD).

Fig. 1

Fig. 2

(a)

Fig. 3a

Fig. 3b

Fig. 3c

(d)

Fig. 3d