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CONSTRAINTS ON LIGHT PARTICLES FROM KAON DECAYS

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ABSTRACT

We show that the decay $K^+ \rightarrow \pi^+ +$ unobserved neutrals(?) can give useful information about light neutral particles, including the number of neutrino types. Observation of the decay $K^+ \rightarrow \pi^+ + ?$ would give a lower bound on the number of neutrinos, which would be strengthened by improvement of the present lower bound on the top quark mass m_t . Non-observation of the decay $K^+ \rightarrow \pi^+ + ?$ gives an upper bound on the number of neutrinos whose associated leptons have masses outside a window whose width shrinks as m_t increases. Perturbative strong interactions are included in our calculations. We also estimate the rates for $K^+ \rightarrow \pi^+ +$ light neutral supersymmetric particles such as photinos, gravitinos and shiggses. We find that decays into photinos and gravitinos are suppressed relative to conventional neutrinos whereas decays into shiggses may be comparable with one conventional neutrino flavour. We stress the desirability of a new round of $K^+ \rightarrow \pi^+ + ?$ decay experiments.

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1. - INTRODUCTION

Many different types of light neutral particles are predicted by various theoretical ideas. In addition to the familiar neutrinos, ν , there are various other types of light neutral spin 1/2 particles expected in supersymmetric theories¹⁾ such as the photino $\tilde{\gamma}$, the goldstino $\tilde{G}$ and perhaps the neutral shiggs $\tilde{H}$ ²⁾. In supergravity theories the goldstino is eaten by the spin 3/2 gravitino to become the helicity $\pm 1/2$ states needed when it is massive³⁾. Various light spin 0 bosons have also been proposed, such as the axion⁴⁾ and its relatives the hypo- (or invisible) axion⁵⁾ and the Majoron⁶⁾.

There are many ways to seek information on these light particles, in astrophysics as well as in particle physics. The best particle physics limits on the numbers and properties of light neutral particles⁷⁾ come from the decays of heavier particles into unobserved neutrals (?). These include $K^+ \rightarrow \pi^+ + ?$, $J/\psi \rightarrow ?$, $T \rightarrow ?$ and in the future $Z^0 \rightarrow ?$. Astrophysical limits come both from conventional Big Bang cosmology and from comparisons between astronomical observations and theories of stellar evolution. For example, the most stringent constraint on the number N_ν of conventional neutrino types ($N_\nu \leq 3$ or ≥ 6600) comes from the success of Big Bang nucleosynthesis calculations⁷⁾, while the late stages of stellar evolution would look very different if N_ν were larger than about $10^{+2 \pm 1.8}$.

Not all these constraints are directly comparable, and they are in many cases complementary. This is because the various different processes are sensitive to different ranges of masses and coupling strengths. For example, the particle decay constraints are clearly sensitive to different ranges of "light" particle masses;

$$\left. \begin{array}{l} K^+ \rightarrow \pi^+ + ? : m_? \leq 350 \text{ MeV} \quad ; \quad J/\psi \rightarrow ? : m_? \leq 3.1 \text{ GeV} \\ \tau \rightarrow ? : m_? \leq 9.4 \text{ GeV} \quad ; \quad Z^0 \rightarrow ? : m_? \leq 90 \text{ GeV?} \end{array} \right\} \quad (1.1)$$

(These limits should be divided by two for light particles produced in pairs such as $? = \bar{\nu}\nu$.) Also, the sensitivities of decay experiments are inversely proportional to the coupling strengths of the particles ? concerned. Astrophysical constraints are often applicable to narrower ranges of masses $m_?$ and may depend on the ? coupling strengths in more complicated ways. For example, conventional Big Bang nucleosynthesis⁷⁾ severely constrains the number of particles with masses $m_? \leq 0(0.1) \text{ MeV}$. The present mass density of the Universe further requires that if $m_? \leq 0(0.1) \text{ MeV}$ then in fact $m_? \leq 0(100) \text{ eV}$ if the particle ? has weak

interactions of the usual strength⁹⁾. These constraints on numbers and masses are relaxed if the particles χ only have weaker interactions¹⁰⁾. This relaxation comes about because the χ particles decouple earlier and hence have lower number densities than conventional neutrinos. However, these number densities depend in a non-linear manner on the χ decoupling times and hence a fortiori on the χ coupling strengths. Constraints^{8),11)} from stellar evolution arise from objects with temperatures $T \lesssim 0(10^9)^{\circ}\text{K}$ and are hence only sensitive to $m_\chi \lesssim 0(100) \text{ keV}$. These constraints are again inversely proportional to the χ coupling strengths as long as they are weak enough to escape from the star without interacting. The very different temperature dependences of emission rates for spin 0 particles and for pairs of fermions mean that different types of star impose varying constraints on particles χ of different spins^{8),11)}.

Because of this diversification in the domains of mass and coupling strength constrained by different processes, it is desirable to understand all of them as best one can. Then one can make detailed comparisons which may pinpoint the characteristic "signature" of each different variety of particle χ . In the absence of direct observations of these elusive particles, our detective work may need to rely on these pieces of circumstantial evidence. In this paper we hone the forensic tool provided by $K^+ \rightarrow \pi^+ + \chi$ decays¹²⁾⁻¹⁴⁾.

The analyses of the other heavy particle decays J/ψ , T ^{15),16)} and $Z^0 \rightarrow \bar{\nu}\nu$ are theoretically uncontroversial and hence uninteresting¹⁶⁾. From the experimental bound¹⁷⁾ $\Gamma(J/\psi \rightarrow \chi)/\Gamma(J/\psi \rightarrow e^+e^-) \lesssim 1/10$ one can only deduce $N_\chi < 5 \times 10^5$, whereas if one could establish $\Gamma(T \rightarrow \chi)/\Gamma(T \rightarrow e^+e^-) < 1$ one could constrain $N_\chi < 5000$. The same bound on $\Gamma(J/\psi \rightarrow \chi)$ also provides restrictions on the coupling and hence the mass of the gravitino. By way of contrast, $K^+ \rightarrow \pi^+ + \chi$ decays¹²⁾⁻¹⁴⁾ are fraught with all the usual ambiguities of semi-leptonic weak interactions, including uncertainties in weak mixing angles, strong interaction corrections, hadronic matrix elements, etc. Therefore they are interesting. We do not consider the cases where χ is a spin 0 boson, since we regard the conventional axion⁴⁾ as moribund, partly because of limits¹⁸⁾ already obtained from $K^+ \rightarrow \pi^+ + \chi$ decays¹⁹⁾, and alternative spin 0 bosons^{5),6)} known to us cannot yield observable $K^+ \rightarrow \pi^+ + \chi$ decay rates. We concentrate on the cases where χ is a pair of fermions including $\bar{\nu}\nu$, $\bar{\gamma}\gamma$, $\bar{\gamma}\tilde{G}$, $\tilde{G}\tilde{G}$ and $\bar{H}H$.

We start our analysis in Section 2 by estimating as best we can the branching ratio for $K^+ \rightarrow \pi^+ +$ pairs of the known neutrinos $\bar{\nu}_e\nu_e$, $\bar{\nu}_\mu\nu_\mu$ and $\bar{\nu}_\tau\nu_\tau$. Using and checking the results of previous authors¹²⁾⁻¹⁴⁾, especially the detailed calculations of Inami and Lim¹⁴⁾ into which we incorporate perturbative QCD corrections²⁰⁾, we find that top quark loops dominate. Using the $K_L^0 \rightarrow \mu^+\mu^-$ decay rate and the

K_L, K_S mass difference Δm_K to constrain²¹⁾ the mass and mixing angles of the top quark, we find both upper and lower bounds on the $K^+ \rightarrow \pi^+ + \sum_{i=e,\mu,\tau} \bar{\nu}_i \nu_i$ decay rate:

$$B(K^+ \rightarrow \pi^+ + \sum_{i=e,\mu,\tau} \bar{\nu}_i \nu_i) = 3 \times (1.4 \text{ to } 11) \times 10^{-10} \quad (1.2)$$

(We recall that the branching ratios for $K_{S,L}^0 \rightarrow \pi^0 \bar{\nu} \nu$ should be¹²⁾ much smaller.) This range will be reduced if the top quark mass becomes known, or if the lower limit on its mass is significantly increased. We observe parenthetically that if one accepts the latest estimate by Donoghue et al.²²⁾ of the matrix element of the $|\Delta S| = 2$ operator relevant to Δm_K , then the recent upper bound on the top quark mass due to Buras²¹⁾ can only be satisfied if

$$m_t \approx 20 \text{ GeV!} \quad (1.3)$$

and one must lie at the top end of the range (1.2) of branching ratios. If the branching ratio for $K^+ \rightarrow \pi^+ + ?$ is found to lie outside the range (1.2) there must be new physics. The most conservative possibility to entertain is the existence of more neutrino species. If the corresponding heavier quarks do not contribute significantly to the $K^+ \rightarrow \pi^+ + \bar{\nu} \nu$ amplitude, then one replaces the factor 3 in Eq. (1.2) by N_ν if all the neutrinos' associated leptons are much lighter than the $W^\pm$ bosons. The $K^+ \rightarrow \pi^+ + \bar{\nu} \nu$ decay amplitude is smaller¹⁴⁾ if the associated lepton mass is non-negligible, turning the branching ratio (1.2) into an upper limit

$$B(K^+ \rightarrow \pi^+ + \sum_i \bar{\nu}_i \nu_i) \leq N_\nu \times 1.1 \times 10^{-9} \quad (1.4)$$

Thus observation of the decay $K^+ \rightarrow \pi^+ + ?$ above $O(10^{-9})$ would immediately establish a lower bound on the number of neutrinos (assuming that $\tilde{\gamma}$, $\tilde{G}$ or $\tilde{H}$, etc. do not confuse the issue, as shown in Section 3). For example, observation of $K^+ \rightarrow \pi^+ + ?$ close to the present upper limit of 1.4×10^{-7} would imply $N_\nu \geq O(100)$. Since the amplitude for $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ vanishes¹⁴⁾ for some value of m_ℓ , which depends on the value of m_t , it is not possible to establish an absolute lower limit on $B(K^+ \rightarrow \pi^+ + \bar{\nu}_\ell \nu_\ell)$. However, one can give such a lower limit for neutrinos whose associated charged leptons' masses do not lie in a band around 100 GeV. Hence one can give upper bounds on the number of neutrinos whose associated leptons lie outside this band. For example, one can conclude from the present upper limit $B(K^+ \rightarrow \pi^+ + ?) \leq 1.4 \times 10^{-7}$ that there are not more than 6000 neutrinos whose associated leptons have masses $\leq 30 \text{ GeV}$ or $\geq 30 \text{ TeV}$.

The above statements follow from neglecting loop amplitudes for Δm_K , $K_L^0 \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ involving heavier quarks. Including one heavier quark does not significantly weaken the upper limit (1.4), and hence the potential lower bound on N_ν , but can in principle invalidate any lower limit on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ decay rate, and hence any potential upper bound on N_ν . However, this would require unnatural cancellations in more than one one-loop amplitude, and we would consider it wilfully negativistic to increase the upper bound on N_ν by more than an order of magnitude. So much for neutrino counting.

In Section 3 we discuss $K^+ \rightarrow \pi^+ +$ pairs of supersymmetric particles. We argue that the decay $K^+ \rightarrow \pi^+ + \tilde{\gamma} \tilde{\gamma}$ is unlikely to give useful new lower limits on the squark masses²³⁾, and find that in the limit of large squark masses

$$B(K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}) \leq O\left(\frac{1}{10}\right) B(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e) \quad (1.5)$$

We also argue that because the goldstino probably has smaller couplings than the photino

$$B(K^+ \rightarrow \pi^+ \tilde{G} \tilde{G}) \leq \frac{1}{2} B(K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{G}) \leq B(K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}) \quad (1.6)$$

On the other hand, the branching ratio for $K^+ \rightarrow \pi^+ \tilde{H} \tilde{H}$ could be as large as that for a single neutrino flavour.

Looking on the negative side, these results mean that $K^+ \rightarrow \pi^+ + ?$ decays are not an excellent place to look for supersymmetric particles. Looking on the positive side, these results mean that our previous results on neutrino counting cannot be invalidated by the possible impact of supersymmetry.

Section 4 discusses prospects for future developments in this area. In particular, we stress that new experiments on $K^+ \rightarrow \pi^+ + ?$ decay are highly desirable as they can provide significant new information about light particles²⁴⁾.

2. - $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ DECAYS

The $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate has been studied previously¹²⁾⁻¹⁴⁾. Here we concentrate our efforts on obtaining useful conclusions about the number of light ($m_\nu \leq \leq 100$ MeV) neutrino flavours. Bounds on the rate-per-neutrino for the decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ derive from a careful unified study of $K_L \rightarrow \mu^+ \mu^-$ and the K_L, K_S mass difference (Δm_K). These bounds lead to conclusions regarding the number of neutrino flavours which become rather interesting either when $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is observed or the current experimental upper bound is considerably improved.

2.1 Lower Bound On The Number Of Neutrino Flavours

The most cogent and interesting bound would follow the detection of the decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$, and this would be a lower bound on the number of neutrino species needed to elevate the decay to an observable rate. This lower bound requires, of course, an upper bound on the rate-per-neutrino for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$, to which we address ourselves first.

The most thorough analysis of the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ decay amplitude to date is due to Inami and Lim¹⁴⁾. These authors address themselves to the higher order effects of quark and lepton masses, which are potentially non-negligible on the scale of m_W . Our own analysis confirms the results of these authors, to which we add the potentially important short-distance effects of QCD.

The decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is a second-order weak process which is dominated by diagrams involving heavy quark loops¹²⁾⁻¹⁴⁾. Such diagrams fall into two classes - Z exchange diagrams wherein quark loops produce an effective flavour-changing neutral current sdZ vertex (Fig. 1a), and the box diagrams of Fig. 1b whose contribution depends upon m_{χ} - the mass of the neutrino's charged weak partner¹³⁾. These diagrams generate an effective four-fermion interaction which we express, following the notation of Inami and Lim¹⁴⁾, as the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi \sin^2 \theta_W} \sum_{i=1}^N \tilde{D}_i (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_L \gamma_\mu \nu_L) \quad (2.1)$$

where i runs over neutrino flavours and the i dependence of $\tilde{D}_i$ reflects the fact that the coefficient function depends on the neutrinos' charged weak partner m_{χ_i} through Fig. 1b. In addition, the $\tilde{D}_i$ depend on the charge 2/3 quark masses and couplings:

$$\tilde{D}_i = \sum_{j=2}^M V_{js}^* V_{jd} \bar{D}(x_j, y_i) \quad (2.2)$$

where $x_j \equiv m_{q_j}^2 / m_W^2$ and $y \equiv m_{\chi_i}^2 / m_W^2$. The summation specifically excludes the $j = 1$ (up) quark whose mass is set equal to zero and whose contribution has been subtracted from each $\bar{D}_{ij}$ using the unitarity of the quark mixing matrix U . Direct evaluation of Figs. 1 yields¹⁴⁾

$$\begin{aligned} \bar{D}(x, y) = & -\frac{1}{8} \frac{yx}{y-x} \left(\frac{y-4}{y-1} \right)^2 \ln y \\ & + \frac{1}{8} \left[\frac{x}{y-x} \left(\frac{x-4}{x-1} \right)^2 + 1 + \frac{3}{(x-1)^2} \right] x \ln x + (\text{const}) \end{aligned} \quad (2.3)$$

$$+ \frac{x}{4} - \frac{3}{8} \left(1 + \frac{3}{y-1} \right) \frac{x}{x-1} \quad (2.3)$$

cont.

The branching ratio $B(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$ is obtained from $K^+ \rightarrow \pi^0 e^+ \nu$ by means of an isospin rotation¹⁴⁾:

$$\frac{B(K^+ \rightarrow \pi^+ \bar{\nu}_i \nu_i)}{B(K^+ \rightarrow \pi^0 e^+ \nu_e)} = \frac{2\alpha^2}{16\pi^2 \sin^4 \theta_w} \frac{|\tilde{D}_i|^2}{V_{us}^2} \quad (2.4)$$

and hence

$$B(K^+ \rightarrow \pi^+ \bar{\nu}_i \nu_i) = 0.61 \times 10^{-6} \frac{|\tilde{D}_i|^2}{V_{us}^2} \quad (2.5)$$

An important refinement we wish to add to this effective Lagrangian is the strong interaction radiative effects at short distances. These short-distance QCD renormalizations are numerically significant, relatively free of theoretical ambiguities, and constitute the most important class of radiative corrections to this process. The most direct approach to these corrections is outlined by Novikov et al.²⁰⁾ in the context of their analysis of $K_L \rightarrow \mu^+ \mu^-$. The only new complication which enters the present application is the occurrence of three distinct mass scales: m_{χ} , m_q and m_W . Furthermore, one must recognize that whichever hierarchy among m_{χ} , m_q and m_W one chooses to consider, the leading log corrections pertain only to the leading mass terms, and not to the subleading mass ratios contained within the full formula of Eq. (2.3). The correct procedure is to identify the leading mass terms in the context of a particular hierarchy (Table 1, columns a,c) and to replace these terms by their all-orders leading log expressions (columns b,d).

In fact, the leading log corrections are quite complementary to the sub-leading mass corrections contained in Eq. (2.3) (due to Inami and Lim¹⁴⁾). If, for instance, the mass ratios considered are truly large, the leading logs are substantial, while the subleading mass terms for which we have not derived radiative corrections are fortunately negligible. Conversely, when the mass scales are sufficiently degenerate that the subleading mass terms are genuinely important, the leading log corrections become inconsequential.

We conclude the discussion of radiative corrections with two last remarks. The first concerns our keeping terms in Table 1 which are strictly speaking not leading logarithms. Formally (i.e., if $\alpha_s/4\pi \log m_2^2/m_1^2$ were of order unity) these constant terms would be of the same order as α_s corrections to the leading logs which are neglected. In fact, however, for the energies and mass scales of interest, $\alpha_s/4\pi \log m_2^2/m_1^2$ tends to be considerably less than one and these sub-leading-but-zeroth-order-in-QCD terms are numerically important, and are retained for this reason. Finally it is encouraging to observe that our numerical results for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ are quite insensitive to the choice of QCD scale parameter Λ used to evaluate these corrections. This is due to the fact that the top quark (and not the charm quark) dominates in our analysis for the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate, and serves as the lower bound on the logarithmic domain of scaling. Ratios such as $\alpha(m_t^2)/\alpha(m_W^2)$ which appear in Table 1 do not care about the precise Q^2 at which α_s becomes $O(1)$. For numerical purposes we take Λ equal to 0.15 GeV.

We shall begin our discussion of the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude in the context of the six-quark model ($j = u, c, t$) and then consider the possible rôle of heavier quarks. The first remark to be made in connection with the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude is its obvious and close connection to the short-distance part of the $K_L \rightarrow \mu^+ \mu^-$ amplitude. Both receive the same contribution from Z exchange (Fig. 1a)^{12),14)} though the box diagrams differ, giving $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ a dependence on m_ℓ and a logarithmic dependence which cancels²⁰⁾ in the case of $K_L \rightarrow \mu^+ \mu^-$ if strong interactions are neglected (Table 1, columns a,c). These processes share a dependence on the same quark mixing angles $U_{qs}^* U_{qd}$ and both amplitudes grow roughly as $m_{q_{2/3}}^2$ (due to the GIM mechanism which is operative in both cases) and these similarities lead to a constraint for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ based on the $K_L \rightarrow \mu^+ \mu^-$ experimental branching ratio.

This constraint from $K_L \rightarrow \mu^+ \mu^-$ arises from the requirement that the short-distance contribution to the $K_L \rightarrow \mu^+ \mu^-$ branching ratio should not exceed the experimental branching ratio minus a calculable absorptive contribution from the decay chain $K_L \rightarrow \gamma \gamma \rightarrow \mu^+ \mu^-$, where $\gamma \gamma$ are on-shell physical photons²⁵⁾. The only uncertainty connected with this argument concerns the magnitude of the additional, long-distance contribution to the dispersive amplitude which occurs when the $\gamma \gamma$ are continued off-shell. This extra dispersive contribution certainly exists at some level and can interfere destructively with the short-distance piece, effectively permitting the latter to become larger²⁶⁾. In fact, Barger et al.²⁶⁾ have suggested that the analogous decay $\eta \rightarrow \mu^+ \mu^-$ seems to require a very large dispersive contribution from off-shell intermediate photons in order to explain the branching ratio for $\eta \rightarrow \mu^+ \mu^-$ reported earlier²⁷⁾. However, the experimental

situation regarding $\eta \rightarrow \mu^+ \mu^-$ is now significantly different²⁸⁾, and the measured branching ratio is now consistent with a vanishing dispersive contribution, although a small contribution is certainly allowed. The current status of this decay affords confidence in theoretical estimates²⁹⁾ which conclude that the long-distance dispersive contribution to $K_L \rightarrow \mu^+ \mu^-$ decay ought not to be too large.

Taking the experimental branching ratio for $K_L \rightarrow \mu^+ \mu^-$, subtracting the calculable absorptive part³⁰⁾ and adding errors in quadrature leads to a constraint on the short-distance contribution which can be written^{14),25)}

$$\left[\sum_{j=c,t} U_{js}^* U_{jd} \bar{C}(x_j) \right]^2 / U_{us}^2 \leq 0.8 \times 10^{-4} \quad (2.6)$$

where

$$\bar{C}(x) = \frac{3}{4} \left(\frac{x}{x-1} \right)^2 \ln x + \frac{x}{4} - \frac{3}{4} \frac{x}{x-1} \quad (2.7)$$

and the possible distinction between $U_{ts}^* U_{td}$ and its modulus may be ignored due to the strong reality constraint imposed by CP violation in $K^0-\bar{K}^0$ mixing. The short-distance QCD renormalization of $\bar{C}(x)$, which is given in Table 1, involves a straightforward generalization of the results of Ref. 20) to the case of six quarks. We concur with a statement by Buras²¹⁾ who finds such radiative corrections amount to a modest suppression in the $K_L \rightarrow \mu^+ \mu^-$ amplitude when $m_t \ll m_W$, plus we would like to point out the absence of leading logs when $m_t > m_W$.

The charm quark contribution to Eq. (2.6) is negligible - almost three orders of magnitude below the experimental bound. Its contribution to $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is also negligible for the present purpose of establishing an upper bound on the amplitude. In this respect the top quark is far more important, and its contribution is tightly constrained by Eq. (2.6).

The consequences of this constraint for the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate are shown in Fig. 2. In this graph we plot the branching ratio $B(K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell)$ [Eq. (2.5) together with radiative corrections] as a function of the neutrino's associated lepton mass m_ℓ for different values of m_t , where each curve is separately normalized according to the constraint upon $|U_{ts}^* U_{td}|$ which derives from Eq. (2.6). These curves are the quantitative upper bounds on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ branching ratio derived from $K_L \rightarrow \mu^+ \mu^-$ within the six-quark model. In addition to the general scale of branching ratios involved, one observes that the largest rates consistent

with $K_L \rightarrow \mu^+ \mu^-$ occur when m_ℓ and m_t are both light. Since m_t is known to be ≥ 20 GeV, this observation leads to an upper bound on the rate-per-neutrino of

$$B(K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell) \leq 1.1 \times 10^{-9} \quad (2.8)$$

One potentially non-trivial consequence would be that if the decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ were detected at a level near the current experimental bound¹⁹⁾ of 1.4×10^{-7} (for instance), the number of neutrinos needed to explain such a measurement would be at least 130, scaling proportionally to the branching ratio actually observed.

There are a number of issues which such a statement raises. The first issue concerns the possible rôle of heavier quark generations, and whether they could act to weaken this bound. In subsection 2.3 we shall consider the effect of heavier quarks on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate in detail, and show that their influence on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ upper bound is essentially negligible.

The second issue concerns possible new sources for the decay $K^+ \rightarrow (\pi^+ + \text{missing neutrals})$, where the neutrinos get replaced by other light neutrals such as photinos, goldstinos and shiggses. In the next section we address ourselves specifically to these decays and show that they can amount to at most the equivalent of a single neutrino flavour ($BR \lesssim 10^{-9}$), and thus would not help to explain an observation of $K^+ \rightarrow (\pi^+ + \dots)$ at a much higher level. The decay into $\pi + \text{axion}$ is another option, though we feel that if a light axion exists and can be seen at all, it would probably have been observed in this decay already¹⁸⁾.

2.2 Upper Bound On The Number Of Neutrino Flavours

We turn now to the more difficult search for a reliable lower bound on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate-per-neutrino, and for the upper limits on the number of neutrinos which such a bound would impose. This bound has the disadvantage of being less solidly founded phenomenologically, particularly with regard to heavy quark generations, but has the added interest of being valid now (as it relies only upon the non-observation of $K^+ \rightarrow \pi^+ \bar{\nu} \nu$) although the numbers will not be especially interesting until the experimental bound on $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is considerably improved.

To obtain a non-trivial lower bound on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate-per-neutrino one has to rely on the K_L, K_S mass difference to determine a lower bound on the top quark couplings $U_{ts}^* U_{td}$. In fact, such a lower bound has frequently been suggested^{31), 32), 21)}, and the motivation for this bound is becoming increasingly

well founded. The origin of the bound is simply that the charm quark contribution to the short-distance part of the K_L, K_S mass difference is insufficient by itself to explain the observed Δm_K . Leaving aside for the moment the question of the additional long-distance contribution to Δm_K from low mass intermediate states³³⁾, this statement concerning the charm quark contribution is probably true. Originally³²⁾, this assertion depended rather critically upon an MIT bag model evaluation of the matrix element $\langle \bar{K}^0 | [\bar{s} \gamma^\mu (1 - \gamma_5) d]^2 | K^0 \rangle$ which is, in fact, unreliable even with regard to sign³⁴⁾. However, since 1979, several advances have been made which significantly reinforce the conclusion. The short-distance QCD renormalization of $\mathcal{L}_{\text{eff}}^{\Delta S=2}$ by Gilman and Wise shows a substantial suppression which is quite insensitive to parameters³⁵⁾. In view of their result, it would appear that a top quark contribution to Δm_K is needed even for values of $\langle \bar{K}^0 | \dots | K^0 \rangle$ well in excess of the value obtained by vacuum insertion. In addition, there is a recent and convincing article by Donoghue et al.²²⁾ which extracts this matrix element from the known $\Delta I = 3/2$ decay rate using SU(3), giving 0.22 times the vacuum insertion result with small estimated errors. These developments reinforce the need for a significant top quark contribution to Δm_K , provided the uncertainty associated with the long-distance contribution is not too large.

The long-distance contribution to Δm_K is poorly known, and contributes the largest uncertainty to the bound we seek. However, several studies of Δm_K^{ld} have been undertaken which suggest that Δm_K^{ld} may be negative and/or small³⁶⁾. Either circumstance would support the need for a top quark contribution and hence a lower bound on the relevant couplings. We will proceed on the assumption that Δm_K^{ld} is small or negative, and consider the implications for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$.

The most conservative conclusions follow from taking $\langle \bar{K}^0 | \dots | K^0 \rangle$ as large as possible. For this purpose we choose the vacuum insertion result, although we will also make stronger statements based on smaller and perhaps more probable values. Our analysis of Δm_K combines both the effects of higher order quark masses¹⁴⁾ and QCD³⁵⁾ in the manner described in subsection 2.1. The results can be summarized as follows. The branching ratio for $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ as a function of m_ℓ and m_t has already been plotted in the context of an upper bound from $K_L \rightarrow \mu^+ \mu^-$ (Fig. 2). The lower bounds obtained in the present context amount to an over-all renormalization of each curve. The appropriate normalizations are found in Fig. 3 which plots the branching ratio $B(K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell)$ for $m_\ell \ll m_W$, equivalent to the y intercept of Fig. 2, as a function of m_t and contrasts this with the upper bound from $K_L \rightarrow \mu^+ \mu^-$. Several conclusions are worth noting:

1) The intersection in Fig. 3 at $m_t = 60$ GeV reflects the incompatibility of the upper bound for the couplings $U_{ts}^* U_{td}$ derived from $K_L \rightarrow \mu^+ \mu^-$ and the lower bound on the same coupling from Δm_K when $m_t > 60$ GeV. This is precisely the observation of Buras²¹⁾ who uses this same argument to place an upper bound on the top quark mass of $m_t \leq 33$ GeV. The difference in values simply reflects our use of a more conservative value for the matrix element $\langle \bar{K}^0 | \dots | K^0 \rangle$. For smaller values of this matrix element, such as the MIT bag model estimate used by Buras²¹⁾, the lower bound in Fig. 3 gets elevated, and the upper bound on the top quark mass gets reduced along with the uncertainty in the $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ rate [the difference between the lower bound from Δm_K and the upper bound which comes from $K_L \rightarrow \mu^+ \mu^-$ (see Fig. 3)]. We note that the value of $\langle \bar{K}^0 | \dots | K^0 \rangle$ obtained by Donoghue et al.²²⁾ requires $m_t \lesssim 20$ GeV, where the lower and upper bounds on $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ converge to yield a prediction.

2) For neutrinos whose associated leptons' masses are small compared to m_W (such as ν_e, ν_μ, ν_τ), Figure 3 shows that the branching ratio $B(K^+ \rightarrow \pi^+ \bar{\nu} \nu)$ is expected to be greater than $1.4 \cdot 10^{-10}$. This leads, in the context of the simplest six-quark, six-lepton model, to an expected branching ratio

$$B(K^+ \rightarrow \pi^+ \sum_{i=e,\mu,\tau} \bar{\nu}_i \nu_i) = 3 \times (1.4611) \times 10^{-10} \quad (2.9)$$

The uncertainty decreases as m_t becomes heavier (Fig. 3), or if smaller values of $\langle \bar{K}^0 | \dots | K^0 \rangle$ are used, in which case the lower bound is increased. Thus for $\langle \bar{K}^0 | \dots | K^0 \rangle = 0.22$ times vacuum insertion²²⁾ the lower bound coincides with the upper bound from $K_L \rightarrow \mu^+ \mu^-$ to yield the prediction

$$B(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = 3.3 \times 10^{-9} \quad (2.10a)$$

together with the prediction

$$m_t \lesssim 20 \text{ GeV} . \quad (2.10b)$$

3) The total number of neutrinos whose associated leptons' masses are light compared to m_W is obviously limited by

$$N_\nu (m_\ell^2 \ll m_W^2) < \frac{\text{experimental upper bound for } K^+ \rightarrow \pi^+ \bar{\nu} \nu}{1.4 \times 10^{-10} \text{ (or } 1.1 \times 10^{-9} \text{ for } \langle \bar{K}^0 | \dots | K^0 \rangle = 0.22 \text{ V.I.)}} \quad (2.11)$$

A more general limit on the number of neutrinos is difficult, since the rate for $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ vanishes for some value of m_ℓ which depends on m_t (fig. 2). One can nonetheless obtain bounds on the number of neutrinos whose associated leptons' masses lie outside a window in the neighbourhood of this intercept. These bounds are illustrated in Fig. 4 which shows the maximum number of neutrinos allowed whose m_ℓ lie outside a window of a given size. The larger numbers pertain to the present experimental bound of $B(K^+ \rightarrow \pi^+ \bar{\nu} \nu) < 1.4 \cdot 10^{-7}$ and are not very restrictive ($N_\nu \lesssim 6000$). They do however constitute the best constraints on N_ν coming directly from particle physics, and complement the cosmological bound from helium nucleosynthesis which requires $N_\nu \leq 3$ or $N_\nu \geq 6600$, helping to rule out the latter alternative [see also Ref. 8)]. Most importantly, these bounds can be improved by three orders of magnitude by improving the experimental bound on $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ 19). The smaller numbers in Fig. 4 apply to an experimental bound of $B(K^+ \rightarrow \pi^+ \bar{\nu} \nu) < 4.2 \cdot 10^{-10}$ (our most conservative theoretical lower limit) and are meant to illustrate the bounds on N_ν which can result from an improved experimental branching ratio.

A less conservative but nonetheless interesting bound on N_ν comes from taking the $\langle \bar{K}^0 | \dots | K^0 \rangle$ matrix element of Donoghue et al.²²⁾ rather than vacuum insertion. Here the presentation of the results is greatly simplified because the top quark mass is no longer a free parameter, but is constrained to be ≈ 20 GeV. Now the upper bound on N_ν for neutrinos whose associated leptons' masses equal (or lie outside) a given value is just the reciprocal of Fig. 2 for $m_t = 20$ GeV, drawn in Fig. 5. Here N_ν is normalized to the present experimental bound and suggests that $N_\nu \lesssim 1000$. The numbers improve proportionally with the experimental branching ratio.

2.3 Heavy Quarks

We now discuss the possible rôle of heavier quark generations and their possible effect upon the bounds presented in subsections 2.1 and 2.2. In general, one must consider the cases where the charge 2/3 quarks add constructively or destructively in the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude. Somewhat surprisingly, as far as the upper bound on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ branching ratio is concerned, constructive interference presents no danger. Constructive interference in $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ implies constructive interference in $K_L \rightarrow \mu^+ \mu^-_{sd}$, for both amplitudes $\bar{C}(x)$ and $\bar{D}(x, y \ll 1)$ are definite with regard to sign, so that constructive or destructive interference depends in both cases only on the relative sign of the charge 2/3 quark couplings $U_{js}^* U_{jd}$. However, because of the upper bound on $K_L \rightarrow \mu^+ \mu^-_{sd}$ [Eq. (2.6)], constructive interference from a heavy charge 2/3 quark reduces the allowed contribution to $K_L \rightarrow \mu^+ \mu^-_{sd}$ from the top quark (which is proportional

to $U_{ts}^* U_{td}$) and the top quark's contribution to $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ as well. Furthermore, Figs. 2 and 3 both show that the lighter, top quark is more efficient in producing $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ decay than a heavier quark, so that replacing some of the top quark's contribution to $K_L \rightarrow \mu^+ \mu^-$ with a heavy quark contribution only decreases the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate and strengthens our bound.

It is more interesting to consider the case of destructive interference from heavier quarks and its possible effect upon the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ upper bound. Destructive interference could permit an increased top quark contribution to $K_L \rightarrow \mu^+ \mu^-$, which could weaken the bound on the top quark couplings $U_{ts}^* U_{td}$ [Eq. (2.6)] and thus permit a larger contribution to $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ from the top quark. As discussed previously, this increased contribution to $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ from the top quark would only be partially compensated for by the destructive interference directly in $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ from the heavy quarks and the net result would be to weaken the upper bound on $K^+ \rightarrow \pi^+ \bar{\nu} \nu$.

In order to maximize the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate and establish a revised upper bound, one would like to make the top quark's contribution to $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ and to $K_L \rightarrow \mu^+ \mu^-$ as large as possible by choosing a large negative contribution from the heavy quark(s). However, this procedure soon does violence to the K_L, K_S mass difference, since one cannot make the top quark's (or any other quark's) contribution to Δm_K arbitrarily large. In this context the most conservative statements result from taking the matrix element $\langle \bar{K}^0 | \{ \bar{s} \gamma^\mu (1 - \gamma_5) d \}^2 | K^0 \rangle$ as small as possible, for which purpose we choose the result of Donoghue et al.²²⁾. However, we have previously shown that such a choice leads (in the six quark model) to a value for the top quark coupling $U_{ts}^* U_{td}$ which is as restrictive as the upper bound from $K_L \rightarrow \mu^+ \mu^-$ for $m_t = 20$ GeV (see Fig. 3). Thus, to increase the $U_{ts}^* U_{td}$ coupling above the six quark bound requires a double conspiracy - destructive interference from heavier quarks in both $K_L \rightarrow \mu^+ \mu^-$ and Δm_K simultaneously, and the analytic form of these very different constraints makes this difficult to arrange.

To make this argument quantitative we consider the case of four generations in detail. Here there are two unknown couplings $U_{ts}^* U_{td}$ and $U_{hs}^* U_{hd}$ (where 'h' stands for heavy) and the two constraints from $K_L \rightarrow \mu^+ \mu^-$ and Δm_K define the allowed domains in a two-dimensional $U_{ts}^* U_{td} - U_{hs}^* U_{hd}$ plane illustrated in Fig. 6 for $m_t = 20$ GeV and $m_h = 100$ GeV. The parallel lines define a bound within which the couplings $U_{ts}^* U_{td}$ and $U_{hs}^* U_{hd}$ must lie to satisfy the constraint from $K_L \rightarrow \mu^+ \mu^-$ [Eq. (2.6)]. The constraint from Δm_K [$\langle \bar{K}^0 | \dots | K^0 \rangle = 0.22$ times vacuum insertion] forms an ellipse within which the couplings must lie in order not to oversaturate the K_L, K_S mass difference. [It is obviously crucial here that a formula for Δm_K is used which is valid to all orders in the quark masses, such as is found in Ref. 14)]. The smaller ellipse [$\langle \bar{K}^0 | \dots | K^0 \rangle = 1.0$ times

vacuum insertion] defines a lower bound on the couplings which is not essential for the present argument. The shaded region represents the domain of couplings consistent with both the $K_L \rightarrow \mu^+ \mu^-$ rate and Δm_K . The dashed line shows values of the top and heavy quark couplings for which the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate vanishes - where there is complete destructive interference in the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude.

To maximize the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude and to establish an upper bound on the rate in the presence of four generations of quarks, one must move as far away from this line as possible. In the six quark model (where $U_{hs}^* U_{hd}$ is equal to zero) this was achieved by choosing the point labelled with a "1" in Fig. 6 where the $K_L \rightarrow \mu^+ \mu^-$ bound is completely saturated by the top quark. As noted, this solution is also (marginally) consistent with the extra constraint from Δm_K . In the eight quark model one may leave the $U_{ts}^* U_{td}$ axis by letting $U_{hs}^* U_{hd}$ be non-zero, but the perpendicular distance from the $K^+ \rightarrow \pi^+ \bar{\nu} \nu = 0$ line one can achieve in this way is an almost negligible improvement over the six quark model (see Fig. 6, point "2"). The explanation is straightforward. First, given the analytic form of Δm_K it is rather difficult to achieve substantial destructive interference from heavier quarks. Secondly, it is even more difficult to arrange destructive interference in Δm_K and $K_L \rightarrow \mu^+ \mu^-$ simultaneously. Finally, it is still more difficult to arrange destructive interference in both constraints subject to the further requirement that the $K_L \rightarrow \mu^+ \mu^-$ bound [Eq. (2.6)] be fully saturated, which is obviously necessary in order to maximize the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate, as can be seen immediately from Fig. 6 or by noting the strong similarity in the two amplitudes [Eqs. (2.3) and (2.7)]^{*)}. The $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ upper bound for eight quarks is shown in Fig. 7 as a function of m_h for $m_t = 20$ and 60 GeV, and contrasted with the six quark bound. As advertized, the six quark and eight quark bounds are numerically quite similar, and we conclude that adding a fourth generation does not significantly affect the six quark upper bounds on the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate that we have presented. Moreover, the general arguments we have given are not peculiar to eight quarks, but pertain to any number of generations. One can reasonably conclude that the potential effect of heavier quark generations upon the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ upper bound is limited.

The effect of heavy quark generations upon the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ lower bound is potentially more severe. In fact, it is clear from Fig. 6 that the six quark $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ lower bound goes away in the presence of four generations - the

*) We note in passing that our previous argument about the inutility of constructive interference between the t and h loops can be seen directly by looking at the first and third quadrants of Fig. 6.

$K^+ \rightarrow \pi^+ \bar{\nu} \nu = 0$ line crosses the shaded territory, indicating that complete destructive interference between the top and heavy quarks in the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude is possible. Hence one can only argue that a complete cancellation in the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ amplitude requires an unnatural act - fluke cancellations among loop diagrams with non-trivial mass and coupling constant dependence. We would consider an order-of-magnitude reduction in the $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ lower bound derived for six quarks as possible, though pessimistic. Anything more than an order of magnitude might be considered wilfully negativistic. Indeed, we note that if the heavy quark mixing angles are hierarchically smaller than the top quark mixing angles:

$$U_{hs}^* U_{hd} \lesssim O\left(\frac{m_c^2}{m_h^2}\right) \times U_{ts}^* U_{td} \quad (2.12)$$

then such a conspiratorial cancellation is essentially impossible. It is nonetheless intriguing that a $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ rate below our six quark bound would constitute evidence for the existence of more than three generations of quarks and leptons.

3. - DECAYS INTO SUPERSYMMETRIC PARTICLES

We saw in the previous section that the rate for the decay $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ is tightly constrained in the standard three-generation model, and that this constraint will be tightened as our knowledge of the top quark mass improves. A branching ratio for $K^+ \rightarrow \pi^+ + ?$ (unobserved neutrals) outside the predicted range would therefore be evidence for new physics. In the previous section we considered the conservative possibility that there are more neutrino flavours as well as leptons and quarks. In this section we examine the more interesting possibility that there might be other varieties of light neutral fermions, specifically those predicted by theories with broken supersymmetry¹⁾⁻³⁾. Since there are many manners and mass scales for breaking supersymmetry, there are considerable uncertainties in the possible types and masses of "light" neutral supersymmetric particles. Three generic possibilities can however be distinguished. One is the photino $\tilde{\gamma}$, the spin 1/2 partner of the photon which may be light ($\lesssim 0(1)$ keV?) and has³⁷⁾ couplings to particles X and their supersymmetric partners $\tilde{X}$ proportional to their conventional electric charges e_X :

$$\mathcal{L}_{X\tilde{X}\tilde{\gamma}} = \sqrt{2} e_X X \tilde{X}_L \lambda_L + (\text{herm. conj.}) \quad (3.1)$$

Then there is the goldstino $\tilde{G}$ which is massless in theories with spontaneously broken global supersymmetry. However, the $\tilde{G}$ acquires³⁾ a mass

$$m_{\tilde{G}} = O\left(\frac{d}{m_p}\right) \quad (3.2)$$

when it is eaten by the gravitino in supergravity theories which have local supersymmetry broken spontaneously at a $(\text{mass})^2$ scale d . We see from Eq. (3.2) that $m_{\tilde{G}}$ is less than $O(350)$ MeV, and hence accessible in K^+ decays, if $d < [O(10^9) \text{ GeV}]^2$. A potentially interesting range of d is the neighbourhood of the weak interaction scale $G_F^{-1} \approx (300 \text{ GeV})^2$, perhaps $d = [O(100 \text{ GeV to } 10 \text{ TeV})]^2$. We see from Eq. (3.2) that this range of d is comfortably accessible in K^+ decays. The couplings of the goldstino $\tilde{G}$ to particles X and their supersymmetric partners $\tilde{X}$ are analogous¹⁾ to those of the photino in Eq. (3.1) but with the substitution

$$\sqrt{2} e_x \rightarrow \frac{m_{\tilde{X}}^2 - m_X^2}{d} \quad (3.3a)$$

In realistic models one expects

$$\Delta m_{\tilde{X}}^2 \equiv m_{\tilde{X}}^2 - m_X^2 \leq O(e_x d) \quad (3.3b)$$

so that the goldstino couplings are unlikely to be larger than the photino couplings, and may be substantially smaller in models with a relatively large scale d of supersymmetry breaking³⁸⁾. Finally, we mention the shiggs $\tilde{H}$, a neutral spin 1/2 partner of the conjectured Higgs boson H . It may be the lightest supersymmetric particle in some spontaneously broken theories²⁾ with the mass spectrum dominated by radiative corrections. Its couplings to conventional quarks, leptons and their supersymmetric particles are expected to be small: $O(g(m_q \text{ or } m_l)/m_W)$, but it would have couplings of gauge strength to gauge bosons and their gaugino partners.

This completes our recital of possible light supersymmetric particles. Subsequent subsections deal with the branching ratios for $K^+ \rightarrow \pi^+(\tilde{\gamma} + \tilde{G})^2$ and for $K^+ \rightarrow \pi^+\tilde{H}\tilde{H}$.

3.1 Photinos, Goldstinos and Gravitinos

The one-loop supergraphs relevant to $K^+ \rightarrow \pi^+\tilde{\gamma}\tilde{\gamma}$ decay are shown in Fig. 8a,b,c. Each of these gives two independent component graphs with different pairs of supersymmetric particles circulating in the loops, as seen in Fig. 9. We use the following rules of thumb²³⁾ to estimate the orders of magnitudes of these graphs.

1) We assume that the squark masses are $\gg m_W$. We know already from experiment³⁹⁾ that all squark and slepton masses are $\geq O(15) \text{ GeV}$, and in the class^{2),38)} of spontaneously broken models that we prefer squark masses are larger than slepton masses by powers of (α_3/α_2) , and are generally expected to be $\geq O(m_W)$.

2) We assume the same Cabibbo-Kobayashi-Maskawa mixing angles for squarks as for quarks. This is a general feature⁴⁰⁾ of spontaneously broken supersymmetry models^{2),38)}, and is also strongly suggested by the gross features of the suppression of flavour-changing neutral interactions⁴⁰⁾.

3) We assume the same (mass)² splitting for squarks as for quarks, i.e.,

$$m_{\tilde{t}}^2 - m_{\tilde{c}}^2 \approx m_t^2 - m_c^2 ; m_{\tilde{c}}^2 - m_{\tilde{u}}^2 \approx m_c^2 - m_u^2, \text{etc.} \quad (3.4)$$

This assumption is also motivated by our favoured models^{2),38)} of spontaneous supersymmetry breakdown, as well as by the phenomenology of flavour-changing neutral interactions.

Using these assumptions it is evident that one can forget about the mixing angles $U_{ts}^* U_{td}$ common to both $K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}$ and $K^+ \rightarrow \pi^+ \tilde{\nu} \nu$ and compare the orders of magnitude of the supersymmetric diagrams of Fig. 9 directly with those of the conventional neutrino diagrams simply by counting powers of the large masses $m_{\tilde{Q}}$, m_W and $m_{\tilde{W}}$ that appear. The appropriate powers of $1/m_{\tilde{Q}}^2$ can be read directly from the figures, taking into account the expected super-GIM cancellations. The results are shown in Table II and may be compared with the conventional neutrino amplitude which is $O(G_F m_t^2/m_W^2)$. It is evident that the only diagram which has any possibility of becoming as large as the one-loop diagrams for neutrinos is that of Fig. 9 (c₁) and hence we study it in more detail. It has, in fact, two distinct contributions: one from a $\tilde{W}^-$ going downwards in Fig. 9 (c₁) and one from a $\tilde{W}^+$ going upwards. We evaluate these as

$$\begin{aligned} iA_{\lambda}^- &= U_{ts}^* U_{td} \left(\frac{g}{\sqrt{2}}\right)^2 e^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(\bar{d}_L \gamma^\mu (k+m_t) \gamma^\nu s_L) (\bar{\lambda}_L \gamma_\mu (k+m_{\tilde{W}}) \gamma_\nu \lambda_L)}{(k^2 - m_t^2) (k^2 - m_W^2)^2 (k^2 - m_{\tilde{W}}^2)} \\ &= U_{ts}^* U_{td} (-2e^2 g^2) m_t^2 \left[\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_t^2) (k^2 - m_W^2)^2 (k^2 - m_{\tilde{W}}^2)} \right] \\ &\quad \times (\bar{d}_L \gamma^\mu s_L) (\bar{\lambda}_L \gamma_\mu \lambda_L) \end{aligned} \quad (3.5^-)$$

and

$$\begin{aligned} iA_{\lambda}^+ &= U_{ts}^* U_{td} \left(\frac{g}{\sqrt{2}}\right)^2 e^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(\bar{d}_L \gamma^\mu (k+m_t) \gamma^\nu s_L) (\bar{\lambda}_L \gamma_\mu (k+m_{\tilde{W}}) \gamma_\nu \lambda_L)}{(k^2 - m_t^2) (k^2 - m_W^2)^2 (k^2 - m_{\tilde{W}}^2)} \\ &= U_{ts}^* U_{td} \left(\frac{e^2 g^2}{2}\right) m_t^2 \left[\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_t^2) (k^2 - m_W^2)^2 (k^2 - m_{\tilde{W}}^2)} \right] \\ &\quad \times (\bar{d}_L \gamma^\mu s_L) (\bar{\lambda}_L \gamma_\mu \lambda_L) \end{aligned} \quad (3.5^+)$$

respectively. When summed, these give a total amplitude

$$iA_\lambda = \frac{3e^2g^2}{32\pi^2m_W^2} U_{ts}^* U_{td} \frac{x_t}{y-x_t} \left\{ \frac{y}{(y-1)^2} \ln y - \frac{x_t}{(x_t-1)^2} \ln x_t - \frac{1}{y-1} + \frac{1}{x_t-1} \right\} \times (\bar{d}_L \gamma^\mu s_L) (\bar{\lambda}_L \gamma_\mu \lambda_L) \quad (3.6)$$

where $x_t \equiv m_t^2/m_W^2$ as in Section 2, and $y \equiv m_{\tilde{W}}^2/m_W^2$. In the limit of small x_t one obtains the following ratio of amplitudes

$$\frac{A(K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma})}{A(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e)} = (\sin^2 \theta_W) \frac{\left[\frac{2}{y-1} - \frac{2}{(y-1)^2} \ln y \right]}{\left[-\frac{1}{3} - \ln x_t \right]} \quad (3.7)$$

which is $\lesssim 0.1$ for $m_t = 20$ GeV and $y \geq 1$, and remains $\lesssim 0.5$ even when $m_{\tilde{W}}$ is decreased to its experimental lower bound of $m_{\tilde{W}} \gtrsim 15$ GeV. It is therefore unlikely that the rate for $K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}$ can exceed that for a single neutrino flavour given the mass hierarchy $m_{\tilde{q}} > m_W, m_{\tilde{W}}$ which we favour.

Moreover, one can relax our assumption regarding the squark masses without dramatically affecting this conclusion. If the stop mass $m_{\tilde{t}}$ is small compared to m_W , then it is Figs. 9a which dominate the $K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}$ amplitude, and these yield

$$\frac{\zeta_F}{\sqrt{2}} \propto \frac{(2\sqrt{2})^2}{\pi} F(m_t^2/m_{\tilde{t}}^2) \quad (3.8a)$$

which must be compared with the $K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e$ amplitude

$$\frac{\zeta_F}{\sqrt{2}} \propto \frac{1}{\pi} \left[\frac{1}{\sin^2 \theta_W} \left(-\frac{x}{2} - \frac{3}{2} x \ln x \right) \right] \quad (3.8b)$$

where the expression in square brackets is ≈ 1 for $m_t = 20$ GeV. One therefore expects that even for a light stop $m_{\tilde{t}}$ of order m_t the amplitude for $K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}$ should not be much larger than that for a single neutrino flavour.

Diagrams involving a photino and a goldstino, or just a goldstino pair, are precisely analogous to Figs. 8 and 9. According to Eqs. (3.3), we expect the amplitudes for $K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{G}$ and $K^+ \rightarrow \pi^+ \tilde{G} \tilde{G}$ to be no larger than those for

$K^+ \rightarrow \pi^+ \tilde{\gamma} \tilde{\gamma}$ and therefore we anticipate the hierarchy of decay rates (1.6). If d is much larger than $O(\sqrt{2}/G_F)$ as advocated by some authors³⁸⁾, then we would expect the goldstino rates to be much smaller than the purely photino rates.

3.3 Shiggses

In at least one broken supersymmetric scenario^{2),38)}, the lightest supersymmetric particle is a neutral shiggs $\tilde{H}$: a neutral spin 1/2 fermionic partner of a Higgs boson. In this class of models there are two light $I = 1/2$ chiral Higgs supermultiplets $H_{1,2}$ which give masses to the charge $+2/3$ and charge $-1/3$ quarks, respectively. One combination

$$\tilde{A}^0 \equiv \frac{v_1 \tilde{H}_1^0 - v_2 \tilde{H}_2^0}{v} \quad (3.9)$$

of neutral shiggses has Dirac mass terms mixing it with the $U(1)$ and neutral $SU(2)$ gauginos, while the other combination

$$\tilde{H} \equiv \frac{v_2 \tilde{H}_1^0 + v_1 \tilde{H}_2^0}{v} \quad (3.10)$$

remains massless in some approximation. In Eqs. (3.9) and (3.10) $v_{1,2}$ are the vacuum expectation values of the scalar components of the $H_{1,2}$ Higgs supermultiplets, and $v^2 = (2\sqrt{2} G_F)^{-1}$. In the limit of small quark masses the dominant diagrams are those of Fig. 10 which have obvious affinities with the $W^\pm$ box and Z^0 exchange diagrams of Fig. 1. The particles circulating in Fig. 10 a,b are charged shiggs fermions $\tilde{H}_L^\pm$. Their magnitudes relative to the corresponding neutrino diagrams are v_1^2/v^2 and v_2^2/v^2 , respectively. Since the $\tilde{H}_{1,2}^0$ have isospins $\mp 1/2$ respectively the coupling of $\tilde{H}^0$ to the Z^0 is in the ratio $(v_2^2 - v_1^2)/v^2$ relative to the conventional $\bar{\nu}\nu Z^0$ vertex. Therefore in an obvious notation one has

$$\begin{aligned} \frac{A(K^+ \rightarrow \pi^+ \tilde{H} \tilde{H})}{A(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e)} &= \frac{A_W \left(\frac{v_1^2 - \frac{1}{4} v_2^2}{v^2} \right) + A_Z \left(\frac{v_2^2 - v_1^2}{v^2} \right)}{A_W + A_Z} \\ &= \frac{(-7 - 5 \ln x_t) v_1^2 + (4 + 2 \ln x_t) v_2^2}{(-1 - 3 \ln x_t) v^2} \end{aligned} \quad (3.11)$$

if one takes small $H_L^\pm$ and ℓ_1 masses, and assumes $x_t \ll 1$.

We therefore see that the decay rate for $K^+ \rightarrow \pi^+ \tilde{H} \tilde{H}$ should be comparable to that for a single neutrino flavour. In particular, one might^{3),38)} hazard the guess that since $m_t \gg m_b$, and since $m_t \propto v_1$, $m_b \propto v_2$, perhaps also $v_1 \gg v_2$. In this case Eq. (3.11) indicates that for small $\tilde{H}_L^{\pm 2}$ and ℓ_i masses, and $m_t \approx 20$ GeV

$$\frac{B(K^+ \rightarrow \pi^+ \tilde{H} \tilde{H})}{B(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e)} \approx 0.88 \quad (3.12)$$

Once again we see that supersymmetry is unlikely to give rates for $K^+ \rightarrow \pi^+ + ?$ much larger than the bounds (2.9) and (2.10) obtained in the standard model. However, it is clear that a determination of $B(K^+ \rightarrow \pi^+ + ?)$ with the precision of an extra neutrino flavour, namely a few times 10^{-10} , could provide useful evidence regarding the existence of light neutral supersymmetric particles.

4. - CONCLUSIONS AND PROSPECTS

The results of this paper confirm the great interest of experimental searches⁴¹⁾ for the decays $K^+ \rightarrow \pi^+ +$ unobserved neutrals(?)²⁴⁾. We have seen that the standard three-generation Kobayashi-Maskawa model makes a relatively precise prediction for the branching ratio of $K^+ \rightarrow \pi^+ + \sum_i \bar{\nu}_i \nu_i$. If the top quark mass is measured, or if the lower bound on its mass is significantly increased, the possible range of branching ratios gets significantly restricted. A measurement of $K^+ \rightarrow \pi^+ + ?$ in this range can be regarded not only as a check on the number of neutrinos, but also a non-trivial check on the conventional Kobayashi-Maskawa phenomenology of flavour-changing neutral interactions. A branching ratio significantly above the expected range would be certain evidence for new physics. This would either be because there are more than three neutrino flavours, or else because there are other light neutral fermions such as the particles $\tilde{\gamma}$, $\tilde{G}$ or $\tilde{H}$ expected in various supersymmetric theories^{12),3),38)}. Of these possibilities, the decay $K^+ \rightarrow \pi^+ + \tilde{H} \tilde{H}$ seems most likely to bump up the branching ratio for $K^+ \rightarrow \pi^+ + ?$. If the branching ratio for $K^+ \rightarrow \pi^+ + ?$ is significantly below the expected range, this would also be very suggestive of new physics. It would signal a breakdown in conventional Kobayashi-Maskawa phenomenology. For the reasons described in the text we find it unlikely that the conventional c and t quark loop contributions to the $K^+ \rightarrow \pi^+ + \bar{\nu}_i \nu_i$ amplitude could exhibit significant cancellations. However, if there were four or more generations, heavy quark exchanges could perhaps cancel the conventional t quark loop. Hence, somewhat paradoxically, a branching ratio for $K^+ \rightarrow \pi^+ + ?$ significantly below our bound would be evidence for additional generations^{*)}.

^{*)} Though it would require seemingly unnatural cancellations to suppress the rate for $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ far below the three generation prediction.

Historically, rare kaon decays have provided us with a great deal of insight into the theory of the weak interactions. The decay $K^+ \rightarrow \pi^+ + ?$ is one of the last open frontiers among flavour-changing neutral interactions which has not yet been completely explored. We are sure that measuring a rate for this process will give us useful information. Perhaps it will be along the lines discussed in this paper, or perhaps it will be something different. We are convinced that in any case a continued experimental search⁴¹⁾ for the decays $K^+ \rightarrow \pi^+ +$ unobserved neutrals is highly desirable²⁴⁾.

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Table 1
QCD corrections to one-loop diagrams for $K_L^0 \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \bar{\nu} \nu$

$K \rightarrow \mu^+ \mu^- : \frac{\bar{c}}{x}$	sdZ_{bare}	sdZ_{QCD}	Box_{bare}	Box_{QCD}
$x < 1$	$\frac{1}{2} \log x + \frac{3}{2}$	$\frac{-2\pi}{\alpha(m_c^2)} [2(K_L^{-1/7} - K_L^{-2/7}) + \frac{1}{5}(K_L^{-1/7} - K_L^{4/7})]$ $- \frac{1}{2} K_L^{4/7} + K_L^{-2/7} + K_L^{-8/7}$	$-\frac{1}{2} \log x - \frac{1}{2}$	$\frac{-2\pi}{\alpha(m_c^2)} (K_L^{-1/7} - 1) - \frac{1}{2} K_L^{-8/7}$
$1 < x$	$\frac{1}{4}$	$\frac{1}{4}$	—	—
$K \rightarrow \pi \bar{\nu} \nu : \frac{\bar{D}}{x}$	sdZ_{bare}	sdZ_{QCD}	Box_{bare}	Box_{QCD}
$y < x < 1$	$\frac{1}{2} \log x + \frac{3}{2}$	$\frac{-2\pi}{\alpha(m_c^2)} [2(K_L^{-1/7} - K_L^{-2/7}) + \frac{1}{5}(K_L^{-1/7} - K_L^{4/7})]$ $- \frac{1}{2} K_L^{4/7} + K_L^{-2/7} + K_L^{-8/7}$	$-2 \log x - 2$	$\frac{-8\pi}{\alpha(m_c^2)} (K_L^{-1/7} - 1) - 2 K_L^{-8/7}$
$x < y < 1$	$\frac{1}{2} \log x + \frac{3}{2}$	—	$-2 \log y - 2$	$\frac{-8\pi}{\alpha(m_c^2)} (K_L^{-1/7} - K_2^{-1/7}) - 2 K_L^{-8/7}$
$x < 1 < y$	$\frac{1}{2} \log x + \frac{3}{2}$	—	$-\frac{1}{8} \log y - \frac{7}{8}$	$\frac{-\pi}{2\alpha(m_c^2)} (K_L^{-1/7} - K_2^{-1/7}) - \frac{7}{8} K_L^{-8/7}$
$y < 1 < x$	$\frac{1}{4}$	$\frac{1}{4}$	—	—
$1 < y < x$	$\frac{1}{4}$	$\frac{1}{4}$	—	—
$1 < x < y$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8} \log \frac{x}{y}$	$\frac{-\pi}{2\alpha(m_c^2)} (1 - K_2^{-1/7})$

In the above table $K_1 \equiv \frac{\alpha(m_c^2)}{\alpha(M_W^2)}$, $K_2 \equiv \frac{\alpha(m_b^2)}{\alpha(M_b^2)}$

Table 2
Orders of magnitude of supersymmetric loops

Fig. 8	(a_1)	(a_2)	(b_1)	(b_2)	(c_1)	(c_2)
Magnitude	$G_F \left(\frac{m_t}{m_Q} \right)^2$	$G_F \left(\frac{m_L}{m_Q} \right)^2 \left(\frac{m_W}{m_Q} \right)^2$	$G_F \left(\frac{m_L}{m_Q} \right)^2$	$G_F \left(\frac{m_t}{m_Q} \right)^2$	$\min(G_{F_{W/2}}^2, G_{F_{W/2}}^2 \frac{m_t^2}{m_W^2})$	$\min(G_{F_{W/2}}^2 \frac{m_t^2}{m_Q^2}, G_{F_{W/2}}^2 \frac{m_t^2}{m_Q^2} \frac{m_W^2}{m_Q^2})$

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It was this proposal which stimulated the present study.

FIGURE CAPTIONS

- Fig. 1 : One loop graphs for $\bar{s}d \rightarrow \bar{\nu}\nu$. (a) Z exchange; the effective $Z\bar{s}d$ vertex appears as a circle. (b) Box diagrams including unphysical charged Higgs exchange.
- Fig. 2 : Upper bounds for the branching ratio $B(K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell)$ imposed by the $K_L \rightarrow \mu^+ \mu^-$ rate, drawn as a function of the neutrino's associated lepton mass, for $m_t = 20, 50$ and 100 GeV.
- Fig. 3 : Upper and lower bounds on $B(K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell)$ from $K_L \rightarrow \mu^+ \mu^-$ and ΔM_K for $m_\ell \ll m_W$. The lower bounds are drawn for $\langle \bar{K}^0 | \{\bar{s}\gamma^\mu(1-\gamma_5)d\}^2 | K^0 \rangle$ equal to 1.0, 0.42 and 0.22 times the vacuum insertion result.
- Fig. 4 : Neutrino bounds from $K^+ \rightarrow \pi^+ \bar{\nu}\nu$. Curves indicate the maximum number of neutrinos whose associated leptons' masses lie outside a window in the neighbourhood of 100 GeV, whose size shrinks as the top quark becomes larger. Large numbers on curves pertain to the current experimental bound¹⁹⁾ of $B(K^+ \rightarrow \pi^+ \bar{\nu}\nu) < 1.4 \cdot 10^{-7}$. The small numbers apply to a hypothetical experimental bound of $B(K^+ \rightarrow \pi^+ \bar{\nu}\nu) < 4.2 \cdot 10^{-10}$.
- Fig. 5 : Neutrino bounds from $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ based on $\langle \bar{K}^0 | \{\bar{s}\gamma^\mu(1-\gamma_5)d\}^2 | K^0 \rangle = 0.22$ times the vacuum insertion value and $m_t = 20$ GeV. The ordinate shows the maximum number of neutrinos permitted whose associated leptons' masses lie outside a window defined by the two abscissae.
- Fig. 6 : Constraints on the top quark and heavy quark couplings $U_{ts}^* U_{td}$, $U_{hs}^* U_{hd}$ for $m_t = 20$ GeV and $m_h = 100$ GeV. The regions outside the parallel lines are excluded by the $K_L \rightarrow \mu^+ \mu^-$ branching ratio. The regions outside the large ellipse and within the small ellipse are excluded by the K_L, K_S mass difference. The shaded domain is consistent with both constraints. The dashed line represents values of the couplings where the $K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e$ rate vanishes. The $K^+ \rightarrow \pi^+ \bar{\nu}\nu$ rate is maximized in the six quark model by choosing the point labelled with a "1", and in the eight quark model by choosing the point marked "2".
- Fig. 7 : Upper bounds for $B(K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e)$ in the eight quark model consistent with the known $K_L \rightarrow \mu^+ \mu^-$ rate and the K_L, K_S mass difference.

Fig. 8 : Supergraphs contributing to the $\bar{s}d\tilde{\gamma}\tilde{\gamma}$ vertex. Here the straight lines represent chiral supermultiplets, while the wavy lines represent gauge supermultiplets.

Fig. 9 : Component diagrams corresponding to the supergraphs of Fig. 8. Here the straight lines represent fermions, the wavy lines gauge bosons and the dashed lines spin-zero particles.

Fig. 10 : Component diagrams contributing to the $\bar{s}d\tilde{H}\tilde{H}$ vertex. We neglect loops involving other supersymmetric particles.

(a)

(b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

(a)

(b)

(c)

Fig. 10