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SUPERSYMMETRIC GRAND UNIFICATION

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ABSTRACT

We construct $SU(5)$ grand unified theories in which the hierarchy problem is solved by supersymmetry broken spontaneously at a very large mass scale. Primordial supersymmetry breaking at the tree level in a gauge singlet $O'Raifeartaigh$ sector is fed to gauge non-singlet particles by radiative corrections. Light particle masses are related to the primordial supersymmetry breaking scale by products of chiral and gauge couplings. Squarks and sleptons are naturally heavier than their quark and lepton partners, while gauginos may or may not be heavier than conventional gauge particles. Radiative corrections can relate the scales of $SU(5)$ and supersymmetry breaking and naturally give negative $(\text{mass})^2$ to Higgs fields if the top quark is heavier than $O(25)$ GeV. We work out in some detail the phenomenology of the low-energy effective theory, discussing in particular the masses and couplings of light fermions and Higgs bosons, and mentioning some of their experimental signatures. We also discuss cosmological aspects of our models, emphasizing in particular that finite temperature effects select an intermediate energy $SU(3) \times SU(2) \times U(1)$ invariant phase.

1. - INTRODUCTION

It has recently been suggested that $N = 1$ global supersymmetry^{1),2)} may help to solve^{3),4)} the gauge hierarchy problem: how come m_W/m_{Planck} or $m_W/m_X \ll 1$? The advantage of a supersymmetric field theory is that the masses and couplings of the scalars in the theory have no intrinsic renormalization⁵⁾. Hence, scalars which are light at the tree level do not get large masses from radiative corrections. Some steps have been taken towards building supersymmetric grand unified theories (GUTs). In the first models⁴⁾ it was assumed that supersymmetry is unbroken down to energies $O(m_W)$ in order to keep the Weinberg-Salam Higgs doublets light. After these doublets acquire a vacuum expectation value (v.e.v.) $SU(2) \times U(1)_Y$ is broken down to the $U(1)$ of electromagnetism.

Various ways of breaking supersymmetry are known, such as an explicit soft breaking of supersymmetry, and spontaneous supersymmetry breaking arising from terms in the effective potential due to $U(1)$ gauge⁶⁾ or Yukawa⁷⁾ interactions. Soft supersymmetry breaking is inelegant and subject to phenomenological constraints⁸⁾ forcing it into a mould which is naturally fitted by models with spontaneous supersymmetry breaking. If this is done using the Fayet-Iliopoulos $U(1)$ mechanism⁶⁾ one may hope to get a realistic spectrum at the tree level. However, the $U(1)_Y$ of the Weinberg-Salam model is not a suitable starting point as it would generate⁹⁾ negative $(\text{mass})^2$ for some spin-zero supersymmetric partners of quarks and leptons (squarks and sleptons), leading to unacceptable symmetry breaking patterns. Possible solutions to this problem include the introduction of a new $\tilde{U}(1)$ gauge symmetry⁹⁾.

An alternative scenario was suggested in Ref. 10) in which $SU(2) \times U(1)$ symmetry breaking is induced radiatively once supersymmetry is broken. The gauge interactions give positive radiative $(\text{mass})^2$ to the squarks and sleptons which automatically yield acceptable mass patterns with squarks and sleptons and perhaps gauginos heavier than their well-known supersymmetric partners. Scalar Higgs doublets get additional negative $(\text{mass})^2$ from Yukawa couplings which permit $SU(2) \times U(1)_Y$ breaking. Other similar scenarios have also been proposed recently^{11),12)}, and it seems most attractive to trigger the supersymmetry breaking through a Fayet-O'RaiFeartaigh mechanism⁷⁾ in the sector of chiral supermultiplet self-interactions.

In early schemes¹⁰⁻¹²⁾ within this philosophy, although the scale m_W of the weak interactions is generated radiatively, one still has two vastly different mass scales m_S (supersymmetry breaking) and m_X (grand unification)

put into the theory by hand. In a previous letter¹³⁾ we have argued that the supersymmetry breaking scale m_S may be as large as m_X (or m_{Planck}) if supersymmetry breaking first occurs in a gauge-singlet sector in such a way that it is transmitted to the usual world of quarks and leptons through multi-loop diagrams involving many chiral couplings. Other scenarios with large supersymmetry breaking scales have been proposed by other authors¹⁴⁾⁻¹⁷⁾.

In this paper we discuss in more detail the scenario for large supersymmetry breaking that we introduced in Ref. 13). The paper is organized as follows. In Section 2 we display the supermultiplet contents of two sample GUT models, based on the group $SU(5)$, as well as the superpotential which breaks supersymmetry through an O'Raifeartaigh mechanism⁷⁾. In Section 3 we discuss in detail the pattern of supersymmetry and gauge symmetry breaking. An important rôle is played in the early Universe by finite temperature effects which select the correct symmetry breaking pattern $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ at the grand unification mass. We also discuss how the gauginos and spin-zero partners of quarks and leptons gain radiative masses of order 10^2 to 10^3 GeV. In Section 4 the effective low-energy $SU(3) \times SU(2) \times U(1)$ theory¹⁸⁾ is studied, including the expected low-energy fermion and boson spectra, as well as some phenomenological signatures. Section 5 mentions some prospects for the future.

2. - STRUCTURE OF THE MODELS

2.1 The Supermultiplet Structure

In order to build an $N = 1$ globally supersymmetric version of the standard $SU(5)$ model, it is necessary to assign each of the $SU(5)$ multiplets to an $N = 1$ supermultiplet, either chiral or vector²⁾. A vector supermultiplet consists of a (massless) two-component vector field together with a two-component spinor. A chiral supermultiplet consists of a two-component spinor together with two real scalar fields (or one complex scalar field). These supermultiplets will be assigned internal symmetry transformation properties so that both fermion and boson components have the same internal symmetry.

The adjoint representation of gauge bosons of $SU(5)$ must be assigned to a vector supermultiplet V , their fermion partners also forming an adjoint representation, called the "gauginos". Quarks and leptons in $SU(5)$ are assigned to three generations transforming as $\bar{5}$ and 10 . As the only vectors allowed are gauge bosons, we must choose the superpartners of these fermions to be scalars, the so-called "squarks and sleptons". Thus each generation of quarks

and leptons is assigned to two chiral supermultiplets ψ_a^α and $\chi_{a\alpha\beta}$ transforming as $\bar{5}$ and 10 under $SU(5)$, respectively. Here a is the generation index and α and β are $SU(5)$ indices. Our chiral supermultiplets have left-handed fermions.

In order to complete the model we must choose the representation of Higgs scalars necessary to generate the required pattern of symmetry breaking and assign them to chiral supermultiplets. To break $SU(2) \times U(1)$ and give quarks and leptons masses it is necessary⁴⁾ in a supersymmetric theory to introduce a 5 and a $\bar{5}$ of scalars. [In pre-supersymmetric $SU(5)$ only one 5 was necessary¹⁹⁾.] The colour triplet and antitriplet Higgs fields mediate proton decay and must have a very large mass ($\geq O(10^{10} \text{ GeV})$ ²⁰⁾). The doublets however must be relatively light ($O(10^2 \text{ GeV})$) in order to generate $SU(2) \times U(1)$ breaking. One of the main difficulties in constructing a viable model is to generate this split multiplet pattern in a natural way. The fermion partners must also be split (supersymmetry breaking is only of order 10^3 GeV in this sector) and consequently they cannot be the quarks or leptons. For this reason we must introduce two new chiral supermultiplets H_1 and H_2 transforming as 5 and $\bar{5}$ under $SU(5)$.

To break $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ we need an adjoint chiral supermultiplet Σ together with an $SU(5)$ singlet chiral superfield Z .

Finally, we include chiral supermultiplets to break supersymmetry via the Fayet-O'Raifeartaigh⁷⁾ mechanism. There is considerable freedom in this choice, but the minimal set necessary requires chiral singlet superfields A, B, C together with a 5 and a $\bar{5}$ chiral superfield ϕ_1 and ϕ_2 , respectively. The latter are included to couple the supersymmetry breaking into the gauge sector.

This completes the particle spectrum necessary to build a viable $SU(5)$ model. The multiplets used are summarized in Table 1. For a slight variant of the model with different high temperature behaviour we will also need two additional singlet chiral supermultiplets X and Y .

2.2 The Superpotential

To specify the model completely it is necessary to write down the Yukawa and scalar couplings. These are related by supersymmetry and it is most convenient to describe them via the superpotential P . P is a gauge invariant function of dimension ≤ 3 constructed from the chiral superfields of the model (but not their complex conjugates). Then the Yukawa and scalar couplings are given²⁾ in terms of P by

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i,j} \frac{\partial^2 P}{\partial \phi_i \partial \phi_j} \psi_i \psi_j \quad (2.1)$$

$$\mathcal{L}_{\text{scalar}} = \left| \sum_i \frac{\partial P}{\partial \phi_i} \right|^2 = \sum_i F_i^* F_i \quad (2.2)$$

Here ψ, ϕ refer to the (LH) fermion and scalar components of the chiral supermultiplets, respectively and the sums over i, j run over all the chiral supermultiplets. The F_i are the auxiliary fields.

For the minimal set of supermultiplets of Table 1 P is taken to be

$$P = P_{\text{OR}} + P_{\text{GD}} + P_{\text{SM}} + P_{\text{SZ}}$$

where

$$P_{\text{OR}} = \lambda_1 A B M + \lambda_2 (A^2 - M^2) C \quad (2.3)$$

$$P_{\text{GD}} = \lambda_3 \phi_1^a \phi_{2a} A + \lambda_4 A M^2 \quad (2.4)$$

$$P_{\text{SM}} = m_{ij}^{(d)} \psi_i^\alpha \chi_{j\alpha\beta} H_2^\beta + m_{ij}^{(u)} e^{\alpha\beta\gamma\delta\epsilon} \chi_{i\alpha\beta} \chi_{j\gamma\delta} H_{1\epsilon} \quad (2.5)$$

$$P_{\text{SZ}} = H_1 (\gamma_1 Z + \gamma_{24} \Sigma) H_2 \quad (2.6)$$

Supersymmetry is broken at the tree level through P_{OR} . P_{GD} couples this supersymmetry breaking to the gauge non-singlet fields ϕ_1, ϕ_2 and includes a soft term breaking residual R invariance and allowing the gauginos to acquire radiative masses. P_{SZ} generates quark and lepton masses once H_1 and H_2 develop vacuum expectation values (v.e.v.'s)*). We will discuss the details of these various terms in subsequent sections.

The scale of supersymmetry breaking is given by the mass parameter M in Eq. (2.3) which we may hope¹³⁾ to choose to be up to $O(m_{\text{Planck}})$ [modulo problems²¹⁾ with local supersymmetry]. We also choose the other dimensionful

*) Because the superpotential cannot use conjugate fields, we see from Eq. (2.5) the necessity⁴⁾ to have two Higgs fields transforming as a 5 and a $\bar{5}$. In $SU(5)$ without¹⁹⁾ the constraints of supersymmetry a single 5 suffices.

parameter in Eq. (2.4) to be M . All other mass scales will be derived from this primordial scale in terms of the couplings of the theory. In particular, we will find that $SU(5)$ breaking occurs at a scale $m_X \approx M$ and $SU(2) \times U(1)$ breaking will naturally occur at a scale $m_W \ll M$. In this way we solve the hierarchy problem for it is not necessary to include two vastly different mass scales in the original Lagrangian.

We turn now to a discussion of the symmetries of the superpotential. In choosing P , particularly in the gauge singlet sector P_{OR} , there is considerable freedom. It is necessary, if gluinos are to receive mass in any order, for there to be no residual R symmetry²²⁾. It is also necessary for the scalar potential to be positive definite so that supersymmetry be broken. These requirements do not specify P uniquely. In non-supersymmetric theories one must include all terms in the Lagrangian which are not excluded by a symmetry, global or local. However, in a supersymmetric theory, if a term is absent in the original superpotential, it will not be generated in any order of perturbation theory⁵⁾. Thus, although the initial absence of a term may not be natural in the usual sense because there may be no obvious symmetry forbidding it, it is natural in the weaker sense of not appearing through radiative corrections. Once the "final" theory applicable at $O(M_{\text{Planck}})$ is known, the reason for the initial absence of the relevant term may be revealed, and the (broken) supersymmetry then ensures that the term does not get generated at an unacceptable level.

For this reason it is not clear that one needs to restrict P to be natural in the strong sense. However, in choosing P as in Eqs. (2.3)-(2.6), all terms of dimension 3 allowed by the (R) symmetries of such terms are included. The R symmetries of P may be expressed in the form

$$P(\phi_i) = e^{i\beta} P(e^{i\theta_i} \phi_i) \quad (2.7)$$

For P given in Eqs. (2.3)-(2.6) the θ_i for the various fields are given in Table 2. We see that all dimension 4 gauge-invariant terms consistent with these symmetries are included in P .

Since a gaugino mass term transforms like $e^{2i\beta}$ it is necessary to introduce further terms to break this symmetry if gauginos are to acquire radiative masses²²⁾. We choose to break these symmetries by including soft terms in P as in Eqs. (2.3) and (2.4). Such terms would be expected if the full theory had an R invariance which was spontaneously violated at high energy. Since the breaking is soft, no divergent terms will appear which violate these symmetries and all such effects are calculable.

Including these terms we find $\alpha = \beta = 0$ and so there is no residual R invariance forbidding gluino masses.

The superpotential of Eqs. (2.3)-(2.6) represents the simplest interactions for a viable $SU(5)$ model (Model I). However, for reasons to be discussed in subsection 3.3 we will consider a more complicated model (Model II) in which the $SU(5)$ breaking is introduced at the tree level. For this, in addition to the terms of Eqs. (2.3)-(2.6), we have the terms

$$P_5 = \frac{\beta_3}{3} \text{Tr}(\Sigma^3) + \frac{\beta_2}{2} X \text{Tr}(\Sigma^2) + \beta_1 X M^2 \quad (2.8)$$

$$P_{5T} = \beta_4 X Y^2 + \beta_5 M Y^2 \quad (2.9)$$

P_5 breaks $SU(5)$ at the tree level. P_{5T} governs the high temperature behaviour of the theory and forces the symmetry breaking to choose the $SU(3) \times SU(2) \times U(1)$ minimum. The R invariances of this model are given in Table 3. The most general dimension 3, 4 terms consistent with these invariances are not included in P so this model is only weakly natural. This could easily be rectified by including an extra adjoint chiral supermultiplet but, for simplicity, we choose not to do so here.

3. - SYMMETRY BREAKING AND MASS GENERATION

3.1 The Pattern of Spontaneous Symmetry Breaking

In order to discuss the predictions of the models introduced in Section 2, it is necessary to consider in detail the breaking of the underlying symmetries. This happens in several stages.

At a scale $M \lesssim O(M_{\text{Planck}})$, P_{OR} induces spontaneous breakdown of the $N = 1$ global supersymmetry, but only in the gauge singlet sector involving the A supermultiplet. At this stage in Model I the symmetry of the other fields is $SU(5) \otimes [N = 1 \text{ global supersymmetry}]$. For Model II the terms of Eq. (2.8) give a potential with multiple degenerate minima corresponding to the symmetries $SU(4) \times U(1)$, $SU(3) \times SU(2) \times U(1)$ and others²³⁾. As we will discuss in subsection 3.3, what is relevant²⁴⁾ in deciding which of these minima is chosen in our Universe is the temperature-dependent potential and the terms of Eq. (2.9) will select the $SU(3) \times SU(2) \times U(1)$ one. At this stage there is still an unbroken $N = 1$ global supersymmetry in the gauge non-singlet sector.

Subsequently, at a much lower scale, radiative corrections involving the interactions generated by P_{GD} break supersymmetry and lift the fermion/boson degeneracy in the gauge non-singlet sector¹⁰⁻¹²⁾ giving an $SU(5)$ symmetric theory in Model I and an $SU(3) \times SU(2) \times U(1)$ symmetric theory in Model II. Finally, further radiative corrections involving both the gauge interactions and the Yukawa interactions of P_{5M} will break the $SU(5)$ in Model I to $SU(3) \times SU(2) \times U(1)$ at a scale $m_X \approx M$ and also break $SU(3) \times SU(2) \times U(1)$ to $SU(3) \times U(1)_{EM}$ at a much lower scale $m_W \ll M$ in both Models I and II. Here m_W is related to the fundamental scale M of the model by high powers of the coupling constants and is expected to be hierarchically small relative to it¹³⁾. In Model I, m_X does not involve such high powers of coupling relative to M and so is naturally of the scale M . Thus in both models there is no need to fine tune parameters to get the hierarchy of mass scales and they "solve" the hierarchy problem.

We now turn to a detailed discussion of these various stages of symmetry breaking. For this we need the full scalar potential at tree level. Using Eqs. (2.2)-(2.6) we find for Model I the following auxiliary fields

$$F_A = \lambda_1 B M + 2\lambda_2 A C + \lambda_3 \phi_1 \phi_2 + \lambda_4 M^2 \quad (3.1)$$

$$F_B = \lambda_1 A M \quad (3.2)$$

$$F_C = \lambda_2 (A^2 - M^2) \quad (3.3)$$

$$F_{\phi_1^a} = \lambda_3 \phi_{2a} A \quad (3.4)$$

$$F_{\phi_{2a}} = \lambda_3 \phi_{1a} A \quad (3.5)$$

$$F_{\chi_i^\alpha} = m_{ij}^{(d)} \chi_{j\alpha\beta} H_2^\beta \quad (3.6)$$

$$F_{\chi_{j\gamma\delta}} = m_{ij}^{(d)} \chi_i^\gamma H_2^\delta + 2m_{ij}^{(u)} \epsilon^{\alpha\beta\gamma\delta} \chi_{j\alpha\beta} H_{1\epsilon} \quad (3.7)$$

$$F_{H_{1\alpha}} = (\gamma_1 \Sigma_\beta^\alpha + \gamma_{24} \Sigma_\beta^\alpha) H_2^\beta + m_{ij}^{(u)} \epsilon^{\alpha\beta\gamma\delta} \chi_{i\beta\gamma} \chi_{j\delta\epsilon} \quad (3.8)$$

$$F_{H_2^\alpha} = (\gamma_1 \Sigma_\alpha^\beta + \gamma_{24} \Sigma_\alpha^\beta) H_{1\beta} + m_{ij}^{(d)} \chi_i^\beta \chi_{j\beta\alpha} \quad (3.9)$$

$$F_Z = \gamma_1 H_{1\alpha} H_2^\alpha \quad (3.10)$$

$$F_{\Sigma_\alpha^\beta} = \gamma_{24} H_{1\beta} H_2^\alpha \quad (3.11)$$

For the more complicated Model II the auxiliary fields are given by Eqs. (3.1)-(3.10) plus

$$F_{\Sigma^B_\alpha} = \beta_3 (\Sigma^2)_\beta^\alpha + \beta_2 X \Sigma^\alpha_\beta + \gamma_{2\alpha} H_{1\beta} H_2^\alpha \quad (3.12)$$

$$F_X = \frac{\beta_2}{2} \text{Tr}(\Sigma^2) + \beta_4 Y^2 + \beta_5 M^2 \quad (3.13)$$

$$F_Y = 2Y (\beta_4 X + \beta_5 M) \quad (3.14)$$

The scalar potential is given in terms of these auxiliary fields by Eq. (2.2). In addition there are contributions to the scalar potential coming from the D terms of Eq. (2.1)

$$\begin{aligned} V^{(D)} = & \frac{g^2}{4} \text{Tr} \left[\left\{ [\Sigma, \Sigma^\dagger] + (H_1 H_1^\dagger - \frac{1}{5} H_1^\dagger H_1) \right. \right. \\ & - (H_2^\dagger H_2 - \frac{1}{5} H_2 H_2^\dagger) + (\phi_1 \phi_1^\dagger - \frac{1}{5} \phi_1^\dagger \phi_1) \\ & \left. \left. - (\phi_2^\dagger \phi_2 - \frac{1}{5} \phi_2 \phi_2^\dagger) \right\}^2 \right] \\ & + \text{terms involving matter fields} \end{aligned} \quad (3.15)$$

Here g is the $SU(5)$ gauge coupling. With this scalar potential we can now discuss the details of the symmetry breaking.

3.2 Supersymmetry Breaking At The Tree Level

Supersymmetry is broken because $F_B^* F_B + F_C^* F_C$ cannot be zero for any choice of the v.e.v. $\langle A \rangle$ of A . V is positive definite and supersymmetry is broken by the Fayet-O'Raifeartaigh mechanism⁷⁾.

Since Eqs. (3.1)-(3.5) are the only ones to mention A , B , C , ϕ_1 and ϕ_2 , we may minimize them independently of the other terms. We find

$$\langle \text{Re} A \rangle^2 = \left\{ \frac{2|\lambda_2|^2 - |\lambda_1|^2}{2|\lambda_2|^2} \right\} M^2 \quad (3.16)$$

$$\langle \text{Im} A \rangle = 0 \quad (3.17)$$

$$\lambda_1 \langle B \rangle M + 2\lambda_2 \langle A \rangle \langle C \rangle + \lambda_4 M^2 = 0 \quad (3.18)$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0 \quad (3.19)$$

At the tree level the combination of fields $\{-2\lambda_2 \langle A \rangle B + \lambda_1 M C\}$ has an undetermined v.e.v.. One-loop radiative corrections give this combination a positive mass²⁵⁾ and thus it has zero v.e.v. Thus

$$2\lambda_2 \langle A \rangle \langle B \rangle = \lambda_2 M \langle C \rangle = \frac{-2\lambda_1 \lambda_2 \lambda_4 M^3 \langle A \rangle}{(4\lambda_2^2 \langle A \rangle^2 + \lambda_1^2 M^2)} \quad (3.20)$$

In the approximation $\lambda_1, \lambda_2 \gg \lambda_3, \lambda_4$ the fermions have mass² 0 and $\lambda_1^2 M^2 + 4\lambda_2^2 \langle A \rangle^2$. These correspond to the goldstino, the massless Goldstone fermion which is generated when global supersymmetry is broken, and to a massive Dirac fermion. These states are given respectively by

$$\frac{1}{[4\lambda_2^2 \langle A \rangle^2 + \lambda_1^2 M^2]^{1/2}} (2\lambda_2 \langle A \rangle B - \lambda_1 M C) \quad (3.21)$$

and

$$\frac{1}{[4\lambda_2^2 \langle A \rangle^2 + \lambda_1^2 M^2]^{1/2}} (\lambda_1 M B + 2\lambda_2 \langle A \rangle C)_L + \bar{A}_R \quad (3.22)$$

If (as we expect) the global supersymmetry we are considering is a relic of a local supersymmetry at a fundamental level, the goldstino will be "eaten" by the gravitino, the spin 3/2 partner of the graviton. (Local supersymmetry includes Einstein's gravity.) The goldstino acquires a mass²⁶⁾

$$m_{\text{gravitino}} \simeq \frac{M^2}{m_{\text{Planck}}} \quad (3.23)$$

In the scalar sector there are two real scalar states ($\equiv$ one complex state) of mass² $(\lambda_1^2 M^2 + 4\lambda_2^2 \langle A \rangle^2)$ given by

$$\frac{1}{[4\lambda_2^2 \langle A \rangle^2 + \lambda_1^2 M^2]^{1/2}} [\lambda_1 M B + 2\lambda_2 \langle A \rangle C] \quad (3.24)$$

There are two massless scalars (which acquire mass in higher order) given by

$$\frac{1}{[4\lambda_2^2 \langle A \rangle^2 + \lambda_2^2 M^2]^{\frac{1}{2}}} \left[2\lambda_2 \langle A \rangle B - \lambda_2 B M \right] \quad (3.25)$$

Finally, there are two states, $\text{Re}(A)$ and $\text{Im}(A)$ with masses

$$\text{and } \left. \begin{aligned} \lambda_2^2 M^2 + 6\lambda_2^2 \langle \text{Re} A \rangle^2 - 2\lambda_2^2 M^2 \\ \lambda_2^2 M^2 + 2\lambda_2^2 \langle \text{Re} A \rangle^2 + 2\lambda_2^2 M^2 \end{aligned} \right\} \quad (3.26)$$

respectively. We see from these masses that supersymmetry is broken at the tree level for the A chiral supermultiplet, the scalar fields being split on either side of their fermion partners and the masses satisfying the sum rule

$$2 \sum_i m_{F_i}^2 = \sum_i m_{B_i}^2 \quad (3.27)$$

where $m_{F(B)}^2$ refer to fermion and boson mass², respectively.

3.3 SU(5) Breaking And The Rôle Of Finite Temperature Effects

In the non-supersymmetric $SU(5)$ model¹⁹⁾ the symmetry is broken to $SU(3) \times SU(2) \times U(1)$ at a scale $m_v \approx 0(10^{15} \text{ GeV})$ when the adjoint of scalars develops a v.e.v.. To see if this happens in the supersymmetric models introduced above, we note that, at tree level, the supersymmetry is unbroken in the gauge non-singlet sector and therefore the minimum of the potential occurs when each of the terms $F_i^* F_i$ is zero.

It is convenient to discuss Model II first. From Eq. (3.10) we see that $F_{\Sigma}^* F_{\Sigma}$ may be zero for multiple choices of the v.e.v. of Σ . In particular,

$$\langle \Sigma \rangle = 0 \quad (3.28)$$

$$\langle \Sigma \rangle = \frac{\beta_2}{3\beta_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \langle X \rangle \quad (3.29)$$

$$\langle \Sigma \rangle = \frac{\beta_2}{\beta_3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \langle X \rangle \quad (3.30)$$

will leave $F_{\Sigma}^* F_{\Sigma}$ zero. These v.e.v.'s leave $SU(5)$ unbroken, break $SU(5)$ to $SU(4) \times U(1)$ or to $SU(3) \times SU(2) \times U(1)$, respectively. It has been pointed

out²³⁾ that there are further possibilities which may be obtained from the above by an $SL(5,6)$ transformation which breaks the gauge symmetry further. We will consider these other possibilities later.

It is a characteristic of many supersymmetric models, following from the positive definiteness of the potential, that there should be multiple degenerate minima before supersymmetry breaking effects are included. Which of the minima will be selected? One might hope that radiative corrections will split the degeneracy and the absolute minimum will be the one required. However, due to the non-renormalization of terms in the superpotential in the supersymmetry limit, this degeneracy will only be lifted by supersymmetry breaking effects. In the gauge sector supersymmetry breaking must be very small on the scale of M_{Planck} , typically of order 1 TeV, otherwise the hierarchy problem will reappear. Thus any splitting of the degenerate minima will be of order 1 TeV, whereas the barrier between the minima is of typical height $\lesssim O(m_X)$. This means that even though the true minimum may be the desired $SU(3) \times SU(2) \times U(1)$ one, if the inverse is initially in one of the other minima the probability for tunnelling into the true one, proportional to $\exp[-O(m_X/1 \text{ TeV})^4]$, will be quite negligible^{24),27)}. In this case, it is the temperature-dependent terms in the effective potential that determine the minima chosen²⁴⁾. These terms, irrelevant at present temperatures, were crucial in the evolution of the Universe after the Big Bang and split the degeneracy of the zero temperature potential. To a good approximation the temperature-dependent terms are given by²⁸⁾

$$V_1(T) = -\frac{\pi^2}{90} (N_B + \frac{7}{8} N_F) + \sum_{i,j} \left| \frac{\partial^2 P}{\partial \phi_i \partial \phi_j} \right|^2 \frac{T^2}{8} + \frac{T^2}{4} g^2 \sum_i C(\phi_i) |\phi_i|^2 \quad (3.31)$$

where $N_{B,F}$ are the numbers of light (mass $\ll T$) degrees of freedom, the sum i, j runs over these light states and the $C(\phi_i)$ are the quadratic Casimirs of the multiplets.

At very high temperature, the T^4 terms dominate and the $SU(5)$ invariant solution is preferred. At lower temperatures the T^2 terms become important. Consider the terms involving X and Y coming from P_5 and P_{5T} in Eq. (2.8) and (2.9)

$$V_a T^2 = \frac{1}{8} \left\{ 8 |\beta_4|^2 |Y|^2 + 4 |\beta_4 X + \beta_5 M|^2 + 24 |\beta_2|^2 |X|^2 \right\} T^2 \quad (3.32)$$

This clearly favours $\langle Y \rangle = 0$ and $\langle X \rangle \neq 0$. In addition there are terms involving the Σ field

$$V_\Sigma T^2 = \frac{1}{8} \left\{ (4|\beta_3|^2 + 2|\kappa_{24}|^2 + 2|\beta_2|^2 + 10g^2) \text{Tr}(\Sigma^2) \right\} T^2 \quad (3.33)$$

Let us consider the potential in the neighbourhood of one of the minima of Eqs. (3.28)-(3.30). There the finite temperature potential has the form

$$V_M(T) = 4|\beta_4 X + \beta_5 M|^2 |Y|^2 + \left| \frac{\beta_2}{2} \text{Tr} \Sigma^2 + \beta_1 M^2 + \beta_4 Y^2 \right|^2 + (V_a + V_b) T^2 + O(NT^4) + \left(\text{terms vanishing at the minimum} \right) \quad (3.34)$$

For temperatures $T_C^2 \lesssim 1/N(V_a + V_b)$, the T^2 terms will be the most important in splitting the zero temperature degeneracy. If β_1 and β_5 are small relative to the other terms, the favoured minimum is $\langle \Sigma \rangle = \langle X \rangle = \langle Y \rangle = 0$ and $SU(5)$ remains unbroken. If β_1, β_4 and β_5 are large, the minimum of Eq. (3.34) is at

$$\langle Y^2 \rangle \approx 0 \quad (3.35)$$

$$\langle \beta_4 X + M \beta_5 \rangle \approx 0 \quad (3.36)$$

$$\langle \text{Tr}(\Sigma^2) \rangle = -\frac{2\beta_1}{\beta_2} M^2 \quad (3.37)$$

If

$$\frac{2\beta_1}{\beta_2} \gtrsim \frac{30\beta_2^2}{\beta_3^2} \frac{\beta_5^2}{\beta_4^2}$$

this minimum lies closer to the $SU(3) \times SU(2) \times U(1)$ minimum of Eq. (3.30) and the temperature-dependent terms will drive the vacuum towards this minimum^{*)}. As the temperature falls, this minimum will be the only one significantly occupied. The phase transition occurs at $T \approx T_C$.

*) The above analysis does not use the full form²⁹⁾ of $V_M(T)$ valid for T in the neighbourhood of mass thresholds. We have verified that our scenario works if a more complete form of $V_M(T)$ is used. In particular, the T_C^4 terms cannot trap the Universe in a false minimum.

For the case of Model I the symmetry breaking pattern is different for the T^2 temperature-dependent effects always favour the unbroken case. They are, from Eq. (2.6)

$$V_c T^2 = \left\{ (\delta, \tau)^2 + |\delta_2|^2 \text{Tr}(\mathcal{Y}^2) \right\} T^2 \quad (3.38)$$

In this case, as the temperature falls below m_X , the $SU(5)$ symmetric theory persists. However, at a temperature $\approx 10^{10}$ GeV the $SU(5)$ coupling becomes large. It has been argued³⁰⁾ that since one expects, in analogy with QCD, that $SU(5)$ be a confining theory at this stage, the T^4 terms will disfavour the $SU(5)$ minimum as there will be no massless (composite) states below $\approx 10^{10}$ GeV. In this case the theory favoured will be $SU(4) \times U(1)$ having the most light degrees of freedom after $SU(5)$. It too, however, confines at a scale $\approx 10^9$ GeV and the next favoured group is $SU(3) \times SU(2) \times U(1)$. Thus, in this scheme, $SU(3) \times SU(2) \times U(1)$ is selected because it is the largest subgroup of $SU(5)$ which does not confine at a scale $\geq m_W$. As originally proposed³⁰⁾, this scenario failed^{24), 27)} because it was applied to the case discussed above with large potential barriers between the minima, but since in this model, in the absence of v.e.v.'s for H_1 and H_2 , the potential involving Σ is flat at the tree level, there is no tunnelling problem³¹⁾. We expect temperature-dependent effects to choose the $SU(3) \times SU(2) \times U(1)$ minimum at a temperature $\approx 10^9$ GeV, much lower than that expected in Model II. This will give significant differences in the cosmology of these models.

3.4 Radiative Masses And Supersymmetry Breaking In The Gauge Sector

As discussed in the previous two sections, at the tree level the supersymmetry is broken in the A supermultiplet and $SU(5)$ is broken to $SU(3) \times SU(2) \times U(1)$ in the gauge sector but supersymmetry exact. Radiative corrections now communicate the supersymmetry breaking to the gauge sector and lift the degeneracy between the fermion and boson partners of Table I.

The pattern of the resultant spectrum is easy to see. After supersymmetry breaking, we expect any particle not protected by a symmetry to acquire mass of order $\beta^n M$ where β denotes couplings in the theory and M is the universal scale. For the vector supermultiplet we expect the $SU(3) \times SU(2) \times U(1)$ gauge bosons will remain massless due to the residual unbroken gauge invariance of the theory. Their fermion partners, in the absence of an R invariance, are not protected by this gauge invariance after supersymmetry is broken and as a result

we expect them to acquire a radiative mass. For the chiral supermultiplets the unbroken $SU(2)$ symmetry prevents quarks and leptons from acquiring mass, but their scalar partners can and do get radiative masses¹⁰⁾⁻¹³⁾.

Thus the general pattern is encouraging; all new supersymmetric states will naturally acquire radiative masses which may be large (~ 1 TeV) and would account for them not having been seen.

We turn now to a quantitative evaluation of those masses in the models proposed. We will find that the scalar quarks and leptons acquire large positive masses but the Higgs scalars get negative mass squared triggering $SU(2) \times U(1)$ breaking¹⁰⁾. In this way the scale of the splitting of the weak and electromagnetic interactions is naturally related to the fundamental supersymmetry by a high power of the couplings, giving the possibility of explaining why the ratio m_W/M_{Planck} is small. In Model I we also find that the v.e.v. of Σ is driven by radiative corrections to a scale of $O(M)$ and is thus naturally comparable to the supersymmetry breaking scale. In Model II this is put in by hand.

Before turning to the actual computation, let us first discuss the allowed radiative corrections in a supersymmetric theory. The non-renormalization theorems⁵⁾ forbid F type terms of the type coming from the superpotential P . Only D type terms can arise, i.e.,

$$\int d^4\theta [\text{local operator in } \Theta]$$

where θ and θ^* are the anticommuting variables (F type terms are of the form $\int d^2\theta P$). If, however, an auxiliary field develops a v.e.v., as for example F_B and F_C in Eqs. (3.2) and (3.3) do [cf. Eqs. (3.16) and (3.17)] then supersymmetry is broken and D type terms can generate effective F type terms, e.g.,

$$\int d^4\theta (PB^+) = F_B^+ \int d^2\theta P \rightarrow \langle F_B^+ \rangle \int d^2\theta P \quad (3.39)$$

In calculating such terms, we must consider graphs which couple to F_B or F_C and this occurs through the massive propagator of the supermultiplet A . (The mass splitting is proportional to $\langle F_B \rangle$ or $\langle F_C \rangle$.)

Consider, for example, the contributions to a mass matrix mixing ϕ_1 and ϕ_2 . At the tree level there is no such term. The R invariances of Table 2 forbid such a term so it may only arise as a result of the dimension four terms

in the Lagrangian and must be finite. If we consider the case $\lambda_3, \lambda_4 \ll \lambda_1, \lambda_2$ there are no one-loop fermion graphs contributing in $O(\lambda_3)$, but there are one-loop scalar graphs as in Fig. 1a. However, from Eqs. (2.2) and (3.1), we see that it couples $\phi_1 \phi_2$ to F_A^* which does not develop a v.e.v. in this order. The first contributing graph is that of Fig. 1b which gives, using Eqs. (3.16) and (3.20),

$$m_{\phi_1 \phi_2}^2 = \frac{1}{12\pi^2} \frac{\lambda_1^2 \lambda_3 \lambda_2^2 \lambda_4 (2\lambda_2^2 - \lambda_1^2)}{(4\lambda_2^2 - \lambda_1^2)^2} M^2 \quad (3.40)$$

$$= O\left\{\left(\frac{\lambda_4 \lambda_3 \lambda_2^2}{12\pi^2}\right) M^2\right\}$$

where λ denotes λ_1 or λ_2 .

The scalars ϕ_1 and ϕ_2 and their fermion partners get masses $\lambda_3 \langle A \rangle$ at the tree level. The degeneracy will be broken at the one-loop level, but since they are superheavy we will not bother to compute it here. Gluinos acquire mass through their coupling to ϕ_1 and ϕ_2 as in Fig. 2. Again, as the R invariance forbidding gluino mass is softly broken, the gluino mass is finite and calculable. Since $\phi_1 \phi_2$ mixing occurs only in one-loop order, the gluino mass^{*)} occurs at two-loop order and is given by

$$m_{\tilde{g}} = \frac{\alpha_3}{12\pi} \frac{m_{\phi_1 \phi_2}^2}{m_{\phi_1}^2} \lambda_3 \langle A \rangle \quad (3.41)$$

$$\approx \frac{\alpha_3}{12\pi} \frac{\lambda_2^2 \lambda_4 M}{12\pi^2}$$

Similarly, the winos and bino acquire a radiative mass related to the gluino mass by ratios of their couplings.

$$m_{\tilde{g}} : m_{\tilde{w}} : m_{\tilde{b}} = \alpha_3 : \alpha_2 : \frac{5}{3} \alpha_1 \quad (3.42)$$

^{*)} We note in passing that the gluino mass is probably uncomfortably small in the supersymmetric GUT of Dine and Fischler¹⁵⁾ (M. Dine, private communication).

Here the couplings are those for SU(3), SU(2) and U(1) and should be evaluated at a scale of order m_ϕ^2 .

Scalar quarks and leptons similarly get mass from their couplings to F_B and F_C . In three-loop order they have a contribution from the graphs of Fig. 3. The diagrams of Figs. 3a to 3j correspond to the supergraph in Fig. 3k. Together they yield

$$m_{\tilde{q}}^2 \approx \left(\frac{\alpha_3}{4\pi} \right)^2 \frac{\lambda_3^2 \lambda_2^2}{16\pi^2} M^2 \quad (3.43)$$

In addition to these diagrams there is the class of diagrams represented by the supergraph of Fig. 3l: we hope the reader will forgive us for not calculating the rest of the alphabet. In any case, the squarks also get a contribution from graphs of Fig. 4. Since the gluino is now massive the fermion contribution of Fig. 5 is suppressed leaving a finite positive remainder

$$m_{\tilde{q}}^2 = \frac{4\alpha_3}{3\pi} \ln \left(\frac{m_{\phi_1}^2}{m_{\tilde{g}}^2} \right) m_{\tilde{g}}^2 \quad (3.44)$$

From Eq. (3.41) we see that $m_{\tilde{q}}^2$ is, from Eq. (3.43), of order $\alpha_s^3 \lambda^6$ and might naively be expected to be much smaller than the contribution, Eq. (3.42), of the graphs of Fig. 3. However, the dominant contribution to Eq. (3.43) comes from virtual momenta associated with the large scale M ($k^2 \approx M^2$), as is easily seen since there are no $\log(m_{\tilde{g}}^2)$ terms in Eq. (3.43), demonstrating insensitivity to the low mass scale. Accordingly, all couplings in Eq. (3.43) should be evaluated at the large scale M of supersymmetry breaking.

In Eq. (3.44) on the other hand, the appearance of the logarithmic term $\log(m_{\phi_1}^2/m_{\tilde{g}}^2)$ signals that this term does depend significantly on momentum scales $k^2 \approx m_{\tilde{g}}^2$. One might suppose that care in establishing the dominant momentum scale is irrelevant considering the difference in order of perturbation between Eq. (3.43) and Eq. (3.44), but this is not so, for in addition to determining the scale at which the couplings should be evaluated, there also arise power suppressions of light masses over heavy masses. In the very simple version described above we have [cf. Eqs. (3.24) and (3.26)] masses $\lambda^2 M^2 \equiv M_0^2$, where $\lambda = \lambda_1, \lambda_2$ for the O'Raifeartaigh scalars, and masses $\lambda_3^2 M^2 = M_0^2 \lambda_3^2 / \lambda^2$ for the $\phi_1 \phi_2$ scalars connecting the O'Raifeartaigh sector to the gauge sector. It is easy to construct variations in which $\phi_1 \phi_2$ are lighter. For example, if we replace Eq. (2.4) by

$$(m_{\tilde{q}}^2, m_{\tilde{H}_{1,2}}^2) : (m_{\tilde{L}}^2, m_{H_{1,2,i}}^2) \approx \left(\frac{4}{3} \alpha_3^2\right) \alpha_3 : \left(\frac{3}{4} \alpha_2^2\right) \alpha_2 \quad (3.49)$$

where $H_{1,2}$ are the colour triplet components of $H_{1,2}$ and $H_{1,2,i=4,5}$ denote the colour singlet, weak doublet, components of the $H_{1,2}$ SU(5) multiplets.

There are still further contributions to these masses which we expect to be important, particularly for the colour singlet fields, coming from the graphs of Fig. 6. Although these are insensitive to loop momenta of order $m_{\tilde{g}}$ they are sensitive to momenta of order m_{ϕ} and will not be power suppressed relative to the terms of Fig. 4. They give additional contributions

$$m_{\tilde{q}}^2 = \frac{\alpha_3^2}{6\pi} \frac{(m_{\phi_1\phi_2}^2)^2}{\lambda_3^2 M^2} + O(\alpha_1, \alpha_2) \quad (3.50)$$

$$m_{\tilde{L},\tilde{H}}^2 = \frac{1}{8\pi} \left(\frac{3}{4} \alpha_2^2 + \frac{5}{6} \alpha_1^2 \right) \frac{m_{\phi_1\phi_2}^2}{\lambda_3^2 M^2}, \quad m_{\tilde{L}_R}^2 = \frac{5\alpha_1^2}{12\pi} \frac{m_{\phi_1\phi_2}^2}{\lambda_3^2 M^2}$$

where $m_{\phi_1\phi_2}^2$ is given by Eq. (3.40). Finally, Higgs scalars get contributions, in addition to those of Eqs. (3.46) and (3.50), coming from their Yukawa couplings to quarks as in Fig. 7. In leading log order this gives a net contribution of

$$m_{H_{1,2}}^2 = -\frac{3}{8\pi^2} |h_{t,b}|^2 \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{q}}^2}\right) \times \frac{4\alpha_3^2}{3\pi} \frac{\lambda_1^4 \lambda_4^2}{(144\pi^3)^2} M^2 \quad (3.51)$$

$$+ (eq. (3.43)) + (eq. (3.44)) + (eq. (3.50))$$

with all the obvious substitutions of coupling constants and Clebsch-Gordan coefficients made in the quoted equations. Here, $h_{t,b}$ are the Yukawa couplings for the top and bottom quarks coming from the diagonalization of $M^{(u)}$ and $M^{(d)}$ in Eq. (2.5). Note that the contribution of Fig. 7 is negative^{*)}, as may be simply seen because the cancellation of the graphs of Fig. 7 is spoilt by the

^{*)} There is a similar negative radiative term for the right-handed τ lepton but it is very much suppressed because it involves the exchange of a very heavy Higgs colour-triplet superfield. We thank Mike Chase for a comment on this point.

$$P_{GD'} = \lambda'_3 \phi_1 \phi_2 E + \lambda'_4 E^2 A \quad (3.45)$$

with E a new singlet chiral superfield, in which case ϕ_1 and ϕ_2 acquire (masses)² of order $(\lambda_3^2 \lambda^2 / (4\pi^2) M^2)$. With $m_\phi^2 \ll M_0^2$ there is a suppression of Eq. (3.43) relative to Eq. (3.44) by a factor (m_ϕ^2 / M_0^2) . The origin of this is again the fact that the graphs of Fig. 4 are dominated by the scale $k^2 \simeq m_\phi^2$ so that the result for $m_{\tilde{g}}$ in Eq. (3.41) is inversely proportional to m_ϕ^2 , leading to the factor $(\lambda_3^2 / \lambda^2)$ when comparing with the result of evaluating the graphs of Fig. 3 which are sensitive only to the scale M_0^2 . Putting all this together we find, from Fig. 5, for Eq. (3.44), the form

$$m_{\tilde{g}}^2 = M^2 \frac{4\alpha_3^3 [m_{\tilde{g}}^2] \lambda^4 [m_\phi^2] \lambda^2 [m_\phi^2]}{3\pi (144\pi^3)^2} \left[\frac{\alpha_3 [m_{\tilde{g}}^2]}{\alpha_3 [m_\phi^2]} \right]^{18/7} \ln(m_\phi^2 / m_{\tilde{g}}^2) \quad (3.46)$$

where we have indicated in square brackets the expected scale at which the couplings should be evaluated and the additional factor

$$\propto \left[\frac{\alpha_3 [m_{\tilde{g}}^2]}{\alpha_3 [m_\phi^2]} \right]^{18/7}$$

includes gluino mass renormalization due to higher order gluon contributions. From Eq. (3.42) and Eq. (3.44) we estimate the relative contribution r to $m_{\tilde{g}}^2$ from Figs. 3 and 4 respectively as

$$r = \frac{(144)^2 \times 3 \times \pi^3}{16^2 \times 4} \left[\frac{\alpha_3 (M_0^2)}{\alpha_3 [m_{\tilde{g}}^2]} \right]^2 \left[\frac{\alpha_3 [m_\phi^2]}{\alpha_3 [m_{\tilde{g}}^2]} \right]^{18/7} \left[\frac{m_\phi^2}{M_0^2} \right] \frac{1}{\alpha_3 [m_{\tilde{g}}^2] \lambda^2 [m_\phi^2] \ln(m_\phi^2 / m_{\tilde{g}}^2)} \quad (3.47)$$

with $M_0 \approx 10^{17}$ GeV this gives

$$r \approx \left[\frac{m_\phi^2}{M_0^2} \right] \frac{1}{\lambda^2 [m_\phi^2]} \quad (3.48)$$

Clearly for a range of parameters giving $m_\phi^2 \ll M_0^2$ r will be small and the graphs of Fig. 4 will dominate (we will return to a discussion of what these parameters must be in a realistic theory at the end of this section). Similar graphs to those of Figs. 4 and 5 give masses to the sleptons and Higgs giving, up to logarithmic corrections and $U(1)$ contributions

fact that $\tilde{q}$ acquires a supersymmetry breaking mass [cf. Eqs. (3.44) and (3.48)] reducing the contribution of Fig. 7b and leaving a net negative contribution. If this negative piece is bigger than the wino and zeno contributions [Eq. (3.47) and (3.48)] the mass^2 of the Higgs will be negative inducing $\text{SU}(2) \times \text{U}(1)$ breaking. It is only because $\alpha_{1,2}(m_{\phi,\tilde{g}}) < \alpha_3(m_{\phi,\tilde{g}})$ that this can happen; for the colour triplet components of H_1 and H_2 the gluino contribution of course dominates, giving them a positive mass. Thus we have a very simple explanation why $\text{SU}(3)$ remains unbroken while $\text{SU}(2) \times \text{U}(1)$ does not. In Section 4 we consider in detail the resultant $\text{SU}(2) \times \text{U}(1)$ breaking. The v.e.v.'s generated for H_{15} and H_{25} will give masses to the $W^\pm$ and Z^0 bosons as well as additional contributions to their fermion partners. The terms of Eq. (2.5) will also give masses to the quarks and leptons.

3.5 Stability Of The Gauge Hierarchy

We now turn to the necessity of arranging in a natural¹⁰⁾ way that $m_W \ll m_X$, and ensuring that this hierarchy is not disturbed by radiative corrections. First we note¹⁰⁾ that in both of the models introduced in subsection 2.2 minimization of the tree level potential coming from $F_{H_1}^\dagger F_{H_1}$ and $F_{H_2}^\dagger F_{H_2}$ in Eqs. (3.8) and (3.9) leads to the condition

$$\gamma_1 \langle Z \rangle + \gamma_{24} \langle \Sigma_5^S \rangle = 0 \quad (3.52)$$

At the tree level these are the only terms to involve $\langle Z \rangle$ so Eq. (3.42) will be obeyed by adjusting $\langle Z \rangle$ whatever the value of $\langle \Sigma_5^S \rangle = \langle \Sigma_4^4 \rangle$. If $\langle \Sigma_5^S \rangle$ is large, as is required to break $\text{SU}(5)$, Eq. (3.52) is just the necessary condition that the doublet components of H_1 and H_2 should remain light so as to be available to break $\text{SU}(2) \times \text{U}(1)$. On the other hand, the triplet components acquire masses

$$m_{\underline{H}}^2 = (\gamma_1 \langle Z \rangle + \gamma_{24} \langle \Sigma_5^S \rangle)^2 \quad (3.53)$$

Combining Eqs. (3.52) and (3.53) gives

$$m_{\underline{H}}^2 = \left(\gamma_{24} \frac{5}{3} \langle \Sigma_5^S \rangle \right)^2 \quad (3.54)$$

Thus the triplet components receive a large mass as do their fermionic partners. This is crucial if proton decay mediated by these states is not to proceed too fast.

As we discussed in subsection 3.3, in Model II $SU(5)$ is automatically broken at the tree level and Σ develops a v.e.v. as in Eqs. (3.30) and (3.37). For the more svelte Model I the tree level potential does not determine the v.e.v. of Σ and it will only be determined by radiative corrections. In this case the dominant contribution to the neutral Σ and Z fields comes from the graphs of Fig. 8. In evaluating these we should recall that the graphs in Fig. 7 may cause the supersymmetry breaking mass splittings $\delta m_{H_{1,2}}^2$ of the H_1 and H_2 supermultiplets to be $SU(5)$ non-invariant. From the doublet components of H_1 and H_2 we can get a term in the effective potential of the form

$$-\frac{1}{4\pi^2} (\gamma_1 Z + \gamma_{24} \Sigma_5^5)^2 \ln \left(\frac{M_0^2}{(\gamma_1 Z + \gamma_{24} \Sigma_5^5)^2} \right) \{ \delta m_{H_{2,1}}^2 + \delta m_{H_{2,2}}^2 \} \quad (3.55a)$$

where we expect, on the basis of subsection 3.4, that $\delta m_{H_{2,1,2}}^2 < 0$ so that (3.55a) favours $\gamma_1 \langle Z \rangle + \gamma_{24} \langle \Sigma_5^5 \rangle = 0$ as in Eq. (3.52). On the other hand, the corresponding term

$$-\frac{3}{8\pi^2} (\gamma_1 Z + \gamma_{24} \Sigma_1^1)^2 \ln \left(\frac{M_0^2}{(\gamma_1 Z + \gamma_{24} \Sigma_1^1)^2} \right) \{ \delta m_{H_{3,1}}^2 + \delta m_{H_{3,2}}^2 \} \quad (3.55b)$$

coming from the triplet components of H_1 and H_2 is expected to have $\delta m_{H_{3,1,2}}^2 > 0$ (because the gauge couplings of the coloured triplets can dominate the Yukawa coupling to the top quark). Therefore, Eq. (3.55b) favours a v.e.v. $O(M_0)$ for the combination $\gamma_1 \langle Z \rangle + \gamma_{24} \langle \Sigma_1^1 \rangle$. We do not necessarily have to worry whether the breaking of $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ is energetically preferred over that to $SU(4) \times U(1)$, since we can rely on finite temperature effects to select the desired minimum, and tunnelling to $SU(4) \times U(1)$ would be suppressed. We recall that in Model II the $SU(5)$ breaking scale has already been fixed to the $O(M_0)$ through the extra superpotential terms (2.8) and (2.9).

One must worry lest the condition (3.52) be violated in higher orders of perturbation theory. This might be unacceptable since $\langle Z \rangle$ and $\langle \Sigma_5^5 \rangle$ are both very large and even a small mismatch in the cancellation (3.52) would give the $H_{1,2}$ doublets too large a mass. Higher orders surely do change the form of Eq. (3.52). The field Z only occurs through its couplings to H_1 and H_2 . Hence it will appear in higher orders of the effective potential only ^{*)} in

^{*)} We only consider minima in the neighbourhood of the minimum (3.30) preserving $SU(3) \times SU(2) \times U(1)$.

combinations of the form $(\gamma'_1 Z + \gamma'_{24} \Sigma_5^5)^2$ and $(\gamma'_1 Z + \gamma'_{24} \Sigma_1)^2$ where γ'_1 and γ'_{24} are the couplings of H_1 and H_2 to Z and Σ respectively as evaluated through the order in which we are working. Since both $\langle Z \rangle$ and $\langle \Sigma_5^5 \rangle$ are free parameters in Model I, it now seems that we have to minimize a function of two variables with two unknowns. The combination $(\gamma'_1 Z + \gamma'_{24} \Sigma_5^5)^2$ occurs via Eqs. (3.8) and (3.9) - now also with γ'_1 and γ'_{24} computed through the order to which we are working - plus small higher order corrections of the form of Eqs. (3.55). Hence the minimum appears to be of the form

$$\gamma'_1 \langle Z \rangle + \gamma'_{24} \langle \Sigma_5^5 \rangle = 0 \quad (3.52')$$

to replace Eq. (3.52), which is still of the right form to keep the masses of the doublet members of H_1 and H_2 zero through the order to which we are working. However, this picture is incomplete because Σ may also appear in radiative corrections arising from its gauge couplings. These lead to an effective potential which is a function also of the variable $(\Sigma_1^{1^2} + \Sigma_5^{5^2})$ as well^{*)} as of $(\gamma'_1 Z + \gamma'_{24} \Sigma_5^5)^2$ and of $(\gamma'_1 Z + \gamma'_{24} \Sigma_1)^2$. In this case it will not, in general, be favourable for Eqs. (3.52') to be identically true.

The level at which Eq. (3.52) is broken depends on the magnitude of these additional terms compared to those involving $(\gamma'_1 \langle Z \rangle + \gamma'_{24} \langle \Sigma_5^5 \rangle)$. The terms involving the latter contribute to the effective potential in order $m_W^2 (\gamma'_1 \langle Z \rangle + \gamma'_{24} \langle \Sigma_5^5 \rangle)^2$. The terms involving the combinations $(\gamma'_1 \langle Z \rangle + \gamma'_{24} \langle \Sigma_1^1 \rangle)^2$ and $(\Sigma_1^{1^2} + \Sigma_5^{5^2})$ do not contribute at this order for they couple only to heavy particles (mass $\approx m_X$) and thus the $\delta m_{H_{1,2}}^2$ in Eqs. (3.55) will not be as big as those found for the light scalar quarks Eq. (3.43), both because the couplings must all be evaluated at $O(m_X)$ and, principally, because the denominator of Eq. (3.44) will involve the heavy scale m_X^2 and not m_ϕ^2 . As a result, the dominant graphs contributing will be those of Fig. 3. They lead to a contribution of order:

$$\frac{3}{8\pi^2} (\gamma'_1 \langle Z \rangle + \gamma'_{24} \langle \Sigma_1^1 \rangle)^2 r m_W^2 \left(\frac{M_0}{m_X} \right)^4 \quad (3.56)$$

*) The term $\gamma'_{24} \text{Tr}(\Sigma^2)$ does not occur because the original Lagrangian is invariant under a transformation $\gamma_{24} \rightarrow e^{iX} \gamma_{24}$, $H_1 \rightarrow e^{-iX} H_1$. Similarly, one can prove that no variables other than those listed appear in any order of perturbation theory.

where r in the simple model of Eq. (2.4) is as in Eq. (3.47). A similar result is obtained for the $(\Sigma_1^{1^2} + \Sigma_5^{5^2})$ terms. For $(M_0/m_X)^4 3r/8\pi^2 m_X^2 \lesssim m_W^2$ it will be energetically favourable to have $(\gamma_1' \langle Z \rangle + \gamma_{24}' \langle \Sigma_5^5 \rangle) \lesssim O(m_W^2)$ as is required for a realistic model. This condition gives

$$m_\phi^2 \lesssim O\left(M_0^2 \left(\frac{m_X}{M_0}\right)^4 \left(\frac{m_W}{m_X}\right)^2\right) \quad (3.57)$$

where we have set $8\pi^2\lambda^2/3 = 1$. For the slightly more devious model of Eq. (2.4) the condition is not so stringent but still qualitatively true. It is evident from Eq. (3.57) that if the scale of supersymmetry breaking is such that

$$M_0 < m_X \left(\frac{m_W}{m_X}\right)^{1/2} \Leftrightarrow M_0^2 < m_X m_W \quad (3.58)$$

then there will be no strong constraint on m_ϕ ($m_\phi < M_0$ only) and the suppression of proton decay due to the colour triplet components of H_1 and H_2 being heavy follows naturally. This corresponds to the Geometric Hierarchy suggested by various authors^{16),17)}. On the other hand, if we wish to have $M_0 \approx m_X \approx M_{\text{Planck}}$, Eq. (3.57) shows that this is possible, while still maintaining the massiveness of the triplet components of H_1 and H_2 , provided

$$m_\phi^2 \lesssim m_W^2 \left(\frac{m_X}{M_0}\right)^2 \quad (3.59)$$

i.e., the gauge non-singlet fields should not be hierarchically different in mass from the W boson mass in this case. However, as our simple example in Eq. (2.4') showed, it is very easy to arrange that ϕ should get its mass only in high order in perturbation theory, in which case this constraint would be naturally satisfied.

Depending as it does on the details of many higher orders of radiative corrections, the stability of the gauge hierarchy clearly deserves a more detailed and formal analysis than we have given here. We have seen that the stability requires either $M_0 \ll m_X$ (the geometrical hierarchy) or else $m_\phi = O(m_W)$. It is possible that still more conditions may be needed before the stability is guaranteed. For this reason it is worth emphasizing that most of the discussion in the rest of this paper does not hinge upon details of the conditions necessary for stabilizing the gauge hierarchy. Indeed, the pattern of mass generation by radiative corrections described earlier in this section, and the resulting phenomenology of the next section, could go through in models

analogous to those of Ref. 4) where the doublet H_1 and H_2 masses were "fine-tuned" to zero at the tree level. However, we would regard such models only as stop-gap solutions on the way to a stable solution of the gauge hierarchy problem along the lines discussed in this section.

4. - SYMMETRY BREAKING AT THE WEAK INTERACTION SCALE

4.1 The Effective Higgs Potential

We have seen in the previous section how¹³⁾ supersymmetry and the $SU(5)$ gauge symmetry can be broken at large mass scales, leaving us with a residual $SU(3) \times SU(2) \times U(1)$ invariant low-energy theory. We will now concentrate on Model II which is simpler to analyze. In this section we see how the $SU(3) \times SU(2) \times U(1)$ symmetry naturally breaks down to $SU(3) \times U(1)$ with the natural components of the Higgs multiplets H_1 and H_2 acquiring vacuum expectation values, while the squark and slepton fields do not. We start by recalling the essential features of the effective Higgs potential obtained after the radiative corrections discussed in the previous section. We restrict our analysis to the $SU(3)$ singlet and $SU(2)$ doublet components of the $H_1 H_2$ and ψ superfields, which are seen from subsection 3.4 to be the only spin-zero fields which offer any prospect of breaking $SU(3) \times SU(2) \times U(1)$.

When $SU(5)$ is broken down to $SU(3) \times SU(2) \times U(1)$ the D terms (3.15) become, after decoupling of the heavy degrees of freedom,

$$\begin{aligned} V^{(D)} = & \left(\frac{g_2^2 + g_1^2}{8} \right) \left[(|H_1|^2)^2 + (|H_2|^2)^2 + \sum_a (|\psi_a|^2)^2 \right] \\ & + \left(\frac{g_2^2 - g_1^2}{4} \right) \left[|H_1|^2 |H_2|^2 + \sum_a |H_1|^2 |\psi_a|^2 - \sum_a |H_2|^2 |\psi_a|^2 \right. \\ & \quad \left. - \sum_a \sum_b |\psi_a|^2 |\psi_b|^2 \right] \\ & + \left(-\frac{g_2^2}{2} \right) \left[|H_1 \cdot H_2|^2 + \sum_a |H_1 \cdot \psi_a|^2 - \sum_a |H_2 \cdot \psi_a|^2 - \sum_{a,b} |\psi_a \cdot \psi_b|^2 \right] \end{aligned} \quad (4.1)$$

where the ψ_a are the different generations of slepton fields coming from the 5 matter representations of $SU(5)$. Turning now to the F terms in the low-energy effective potential, only the $SU(5)$ singlet Z remains light (mass $\ll m_X$)*). It couples to the H_1 and H_2 superfields through a truncated version of P_{5Z} (2.6), namely the superpotential term

*) There is no term in the low-energy effective potential coming from the F term of the Σ supermultiplet, as was assumed in Ref. 15), because it is superheavy.

$$P_{22} = \gamma_1 H_1 \bar{Z} H_2 \quad (4.2)$$

We also have a superpotential term from P_{5M} (2.5) which couples together the ψ_a and H_2 multiplets. From these two pieces in the superpotential we deduce

$$V^{(F)} = \gamma_1^2 |H_1 H_2|^2 + \sum_a h_a^2 |\bar{H}_2 \psi_a|^2 \quad (4.3)$$

where the h_a are the Yukawa couplings for the different generations of leptons ℓ_a . To within logarithmic corrections

$$h_a = \text{diag}(\bar{m}_{ij}^{(d)}) \quad (4.4)$$

and is probably quite small.

Finally, in the low-energy effective potential we have the masses generated by the radiative corrections¹⁰⁾⁻¹³⁾ described in subsection 3.4:

$$V^{(R)} \equiv m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + \sum_a m_a^2 |\psi_a|^2 \quad (4.5)$$

If the diagrams of Fig. 5 dominate the Higgs, squark and slepton masses, then in Eq. (4.5) we have

$$m_{H_{1,2}}^2 \approx \mu^2 \left(1 - \frac{C_q}{R^2} h_{t,b}^2 \right) \quad (4.6)$$

with

$$\mu^2 \equiv \left(\frac{3\alpha_2^3}{4\pi} + \frac{25\alpha_1^3}{36\pi} \right) \frac{\lambda_2^4 \lambda_4^2}{(144\pi^3)^2} \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{q}}^2}\right) M^2 \quad (4.7)$$

[cf. Eqs. (3.44) and (3.46)], and

$$C_q \equiv \frac{3}{8\pi^2} \ln\left(\frac{m_{\phi_1}^2}{m_{\tilde{q}}^2}\right) \quad (4.8)$$

[cf. Eq. (3.51)], and finally

$$R^2 = \left(\left(\frac{3}{4}\right) \alpha_2^3 + \frac{75}{144} \alpha_1^3 \right) / \alpha_3^3 \quad (4.9)$$

In cannibalizing Eqs. (4.6)-(4.9) from Section 3 we have neglected the scale dependences of the coupling constants exhibited in Eq. (3.46), as well as the power of $\alpha_3[m_g^2]/\alpha_3[m_\phi^2]$. This is for reasons of simplicity and also because, as discussed in subsection 3.3, the mass scale m_ϕ may not be very different from m_g . Real life probably has more complications, with the diagrams of Fig. 6 making the dominant contributions to the slepton and Higgs masses [cf. Eqs. (3.49) and (3.50)]. We obtain from Eq. (3.50) the revised estimates

$$\mu^2 = 18 \left(\frac{3}{4} \alpha_2^2 + \frac{5}{6} \alpha_1^2 \right) \frac{\lambda_1^4 \lambda_4^2}{(144 \pi^3)^2} M^2 \quad (4.7')$$

$$R^2 = \frac{18 \left(\frac{3}{4} \alpha_2^2 + \frac{5}{6} \alpha_1^2 \right)}{\ln(m_{\phi_1}^2/m_{\tilde{g}}^2) 4\alpha_3^2/3\pi} \quad (4.9')$$

These modifications may not be very significant phenomenologically, since we treat μ^2 as a free parameter and the ball-park estimate of R^2 is not greatly altered. Therefore we will use Eqs. (4.6)-(4.9) for numerical estimates in the rest of this section. In Eq. (4.5) we also have

$$m_a^2 \approx \mu^2 \quad (4.10)$$

so that while the Higgses (especially H_1) have the possibility that $m_H^2 < 0$, the slepton masses are definitely positive.

We now turn to the pattern of symmetry breaking engendered by the low-energy potential

$$V^{(L)} \equiv V^{(D)} + V^{(F)} + V^{(R)} \quad (4.11)$$

We first examine the possibility that the sleptons might acquire a vacuum expectation value. Since the D terms $V^{(D)}$ (4.1) are symmetric under rotations in the $(N_a + 1)$ dimensional space of the (H_2, ψ_a) , we must study how the F terms (4.4) and the radiative corrections (4.5) break this symmetry. We assume that the h^2 in Eq. (4.4) are negligible compared with γ_1^2 . The effective (mass)² generated by (4.4) and (4.5) for the H_2 is

$$M_{H_2}^2 = \gamma_1^2 v_1^2 \cos^2 \theta_{12} + m_{H_2}^2 \quad (4.12)$$

where $v_{1,2} \equiv |\langle 0|H_{1,2}|0\rangle|$ (which we have not yet demonstrated to be non-zero) and θ_{12} is the relative orientation of $\langle 0|H_1|0\rangle$ and $\langle 0|H_2|0\rangle$. The sleptons will not acquire a vacuum expectation value if

$$\mu^2 > m_{H_2}^2 \quad (4.13)$$

Equations (4.6) and (4.10) guarantee that this condition is obeyed when $v_1^2 \cos^2 \theta_{12} = 0$. Assuming this condition is satisfied we must then ask whether charge is conserved by $\langle 0|H_1|0\rangle$ and $\langle 0|H_2|0\rangle$, i.e., whether these v.e.v.'s are parallel (or antiparallel) such that $\cos^2 \theta_{12} = 1$. Setting $\langle 0|\psi_a|0\rangle = 0$ on the basis of Eq. (4.13) we see that the only term in $V^{(2)}$ which depends on θ_{12} is the first term on the third line of Eq. (4.1), which clearly prefers $\langle 0|H_1|0\rangle$ and $\langle 0|H_2|0\rangle$ to be aligned so that charge is conserved. This result is therefore guaranteed if our initial assumption (4.13) is satisfied, namely if

$$\mu^2 > \delta^2 v_1^2 + m_{H_2}^2 \quad (4.14a)$$

or

$$\frac{C_a}{R^2} h_t^2 \mu^2 > \delta^2 v_1^2 \quad (4.14b)$$

To facilitate further analysis we now drop the redundant ψ_a terms in $V^{(L)}$ and restrict ourselves to the part relevant for determining $v_{1,2}$:

$$V_0^{(L)} = \left(\frac{g_2^2 + g_1^2}{8} \right) (v_1^2 - v_2^2)^2 + m_{H_1}^2 v_1^2 + m_{H_2}^2 v_2^2 + \delta^2 v_1^2 v_2^2 \quad (4.15)$$

We can rewrite $V_0^{(L)}$ in terms of the combinations

$$v^2 \equiv v_1^2 + v_2^2, \quad \delta^2 \equiv v_1^2 - v_2^2 \quad (4.16)$$

as follows:

$$\begin{aligned} V_0^{(L)} = & \left(\frac{g_2^2 + g_1^2}{8} - \frac{\delta^2}{4} \right) \delta^4 + \left(\frac{m_{H_1}^2 - m_{H_2}^2}{2} \right) \delta^2 \\ & + \frac{\delta_1^2}{4} v^4 + \left(\frac{m_{H_1}^2 + m_{H_2}^2}{2} \right) v^2 \end{aligned} \quad (4.17)$$

In order to ensure that $v^2 > 0$ we must impose

$$m_{H_1}^2 + m_{H_2}^2 < 0 \quad (4.18a)$$

or

$$h_t^2 + h_b^2 > \frac{2R^2}{c_q} \quad (4.18b)$$

in the notations of Eqs. (4.8) and (4.9). If $h_t > h_b$, then $m_{H_1}^2 < m_{H_2}^2$ and hence $\delta^2 = v_1^2 - v_2^2 > 0$, resulting in $m_t > m_b$ as we desire. To ensure that $v^2 > \delta^2$ we must ensure that

$$g_2^2 + g_1^2 > 2\delta_1^2 \quad (4.19)$$

and impose

$$\delta_1 m_{H_1}^2 > \left(\frac{g_2^2 + g_1^2}{4} \right) (m_{H_1}^2 + m_{H_2}^2) \quad (4.20a)$$

or

$$\delta_1^2 \left(\frac{R^2}{c_q} - h_t^2 \right) > \left(\frac{g_2^2 + g_1^2}{4} \right) \left(\frac{2R^2}{c_q} - h_t^2 - h_b^2 \right) \quad (4.20b)$$

in the notations of Eqs. (4.8) and (4.9). We thus obtain our desired pattern of $SU(2) \times U(1)$ symmetry breaking if the (not very stringent) conditions (4.14), (4.18)-(4.20) are obeyed.

4.2 Estimate Of The Top Quark Mass

Obtaining the desired pattern of $SU(2) \times U(1)$ symmetry breaking requires that the negative contributions of Fig. 7 to the Higgs (mass)² (4.6) overwhelm the positive contributions of Figs. 5 and 6. This scenario will only work if the quark masses are sufficiently large, and in particular if m_t is large enough. To see how large m_t must be^{13),18)}, we first denote

$$h_{t,b}^2 \equiv \frac{R^2}{c_q} (1 + \epsilon_{t,b}) \quad (4.21)$$

inspired by the bound (4.21). Using the following expressions

$$v^2 = \frac{\epsilon_t + \epsilon_b}{\delta_1^2}, \quad \delta^2 = \frac{\epsilon_t - \epsilon_b}{(g_2^2 + g_1^2)/2 - \delta_1^2} \quad (4.22)$$

for v^2 and δ^2 obtained from Eqs. (4.17) and (4.21) it is then easy to show that

$$\begin{aligned}
 m_t^2 + m_b^2 &= h_t^2 v_1^2 + h_b^2 v_2^2 \\
 &= \frac{R^2}{c_q} \left\{ (1 + \epsilon_t) \left[\frac{\epsilon_t + \epsilon_b}{2\alpha_1^2} + \frac{\epsilon_t - \epsilon_b}{g_2^2 + g_1^2 - 2\alpha_1^2} \right] \right. \\
 &\quad \left. + (1 + \epsilon_b) \left[\frac{\epsilon_t + \epsilon_b}{2\alpha_1^2} - \frac{\epsilon_t - \epsilon_b}{g_2^2 + g_1^2 - 2\alpha_1^2} \right] \right\}
 \end{aligned} \tag{4.23}$$

which can easily be re-expressed as

$$m_t^2 + m_b^2 = \frac{R^2 v^2}{c_q} \left\{ 1 + \left(\frac{\epsilon_t + \epsilon_b}{2} \right) \left[1 + \frac{(\epsilon_t - \epsilon_b)^2 2\alpha_1^2}{(\epsilon_t + \epsilon_b)^2 (g_2^2 + g_1^2 - 2\alpha_1^2)} \right] \right\} \tag{4.24}$$

The second term in the curly parentheses $\{ \}$ of Eq. (4.24) is positive because of the conditions (4.18) and (4.19). We therefore conclude that

$$m_t^2 + m_b^2 > \frac{R^2 v^2}{c_q} \tag{4.25}$$

which yields, when we insert the plausible values $\alpha_2 = 1/30$, $\alpha_3 = 1/7$ and $\ln(m_{\phi_1}^2/m_{\tilde{q}}^2) = 30$

$$m_t > \alpha(25) \text{ GeV} \tag{4.26}$$

One can re-express $m_t^2 + m_b^2$ in terms of other observables such as the SU(2) gaugino mass $m_{\tilde{W}}$, $v_2 = |\langle 0 | H_2 | 0 \rangle|$ and $\alpha_Y \equiv \gamma_1^2/4\pi$. Using Eqs. (4.7) and (4.9) we get

$$\begin{aligned}
 m_t^2 + m_b^2 &= \frac{\pi}{\alpha_2} \frac{3(\alpha_2^3)}{4(\alpha_3)} \frac{m_{\tilde{W}}^2}{\ln(m_{\phi_1}^2/m_{\tilde{q}}^2)} \left\{ 1 + \frac{4\pi}{3\alpha_2^2 \ln(m_{\phi_1}^2/m_{\tilde{q}}^2)} \left(\frac{m_{\tilde{W}}^2}{m_{\tilde{W}}^2} \right) \right. \\
 &\quad \left. \times \left[2\alpha_Y + (1 - 2v_2^2/v^2)^2 (\alpha_2 + \alpha_1 - 2\alpha_3) \right] \right\}
 \end{aligned} \tag{4.27}$$

and the numerical result of using the alternatives (4.7') and (4.9') is comparable. In the plausible limit $2\alpha_Y \ll \alpha_2 + \alpha_1$ the curly parentheses $\{ \}$ become approximately

$$\{ \} \approx 1 + \frac{4\pi}{3} \left(\frac{m_{\tilde{W}}^2}{m_{\tilde{W}}^2} \right) \tag{4.28}$$

which is not likely to be much larger than unity. The lower bound (4.26) may therefore be almost saturated.

4.3 Fermion Mass Spectrum

The low-energy sector of our GUT contains many spin 1/2 particles whose spectrum¹⁸⁾ we now investigate: see Table 4 for a summary of the results. We parametrize the masses of the unmixed SU(3), SU(2) and U(1) gauginos as

$$m_{\tilde{g}} = \alpha_3 \mu_0, \quad m_{\tilde{W}} = \alpha_2 \mu_0, \quad m_{\tilde{B}} = \frac{5}{3} \alpha_1 \mu_0 \quad (4.29)$$

where, from Eq. (3.39) we have

$$\mu_0 = \frac{\lambda^2 \lambda_4 M}{144 \pi^3} \quad (4.30)$$

We then obtain the following mass matrix for the left-handed spin 1/2 gauginos $\tilde{W}_L^+$ and shiggses $\tilde{H}_{2L}^-$ and $\tilde{H}_{1L}^+$:

$$\begin{pmatrix} \tilde{W}^+, \tilde{H}_1^+ \end{pmatrix}_L \begin{pmatrix} \alpha_2 \mu_0 & g_2 v_2 \\ g_2 v_1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_2^- \end{pmatrix}_L \quad (4.31)$$

In a previous paper¹⁸⁾ we have analyzed this mass matrix under the assumption that μ_0 is large, which need not be the case if the primordial R breaking term in the superpotential (2.4) is small. Therefore we analyze here for completeness a more general situation allowing for the possibility that μ_0 is small. The eigenstates of the matrix (4.31) have masses

$$m_{\pm} = \frac{\alpha_2 \mu_0 \pm \sqrt{(\alpha_2 \mu_0)^2 + 4g_2^2 v_1 v_2}}{2} \quad (4.32)$$

which become

$$\approx \left(m_{\tilde{W}} = \alpha_2 \mu_0, \quad - \frac{g_2^2 v_1 v_2}{m_{\tilde{W}}} \right) \quad (4.33a)$$

in the limit of large $m_{\tilde{W}} = \alpha_2 \mu_0$, and

$$(g_2 v_1, g_2 v_2) \quad (4.33b)$$

in the limit of small $m_{\tilde{W}}$. The eigenstates are

$$\begin{cases} (\tilde{W}^+ + \frac{g_2 v_2}{\alpha_2 \mu_0} \tilde{H}_1^+ + \dots)_L \\ (\tilde{W}^- + \frac{g_2 v_1}{\alpha_2 \mu_0} \tilde{H}_2^+ + \dots)_L \end{cases} \quad \begin{cases} (\tilde{H}_1^+ - \frac{g_2 v_2}{\alpha_2 \mu_0} W^+ + \dots)_L \\ (\tilde{H}_2^- - \frac{g_2 v_1}{\alpha_2 \mu_0} W^- + \dots)_L \end{cases} \quad (4.34a)$$

in the limit of large $m_{\tilde{W}}$, and Dirac eigenstates

$$\tilde{W}_R^- + \tilde{H}_{2L}^-, \quad \tilde{W}_R^+ + \tilde{H}_{1L}^+ \quad (4.34b)$$

in the limit of small $m_{\tilde{W}}$. As pointed out in Ref. 18), the charged fermion masses (4.32) must both be larger than 0(15) GeV since no new charged fermion has been seen at PETRA³²⁾. This is not a significant constraint in the case (4.33b) of small $m_{\tilde{W}}$, but imposes the limit

$$\frac{g_2^2 v_1 v_2}{m_{\tilde{W}}} = \frac{4\pi v_1 v_2}{\mu_0} \gtrsim 15 \text{ GeV} \quad (4.35)$$

in the limit of large $m_{\tilde{W}}$. As pointed out in Ref. 18), the mass eigenstates (4.34) have distinctive neutral weak couplings which will make them easy to distinguish from a conventional sequential heavy lepton.

The neutral shiggs and gaugino mass matrix is somewhat more complicated:

$$(\tilde{W}^3, \tilde{B}^0, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{Z}) \begin{pmatrix} \alpha_2 \mu_0 & 0 & \frac{1}{2\sqrt{2}} g_2 v_1 & -\frac{1}{2\sqrt{2}} g_2 v_2 & 0 \\ 0 & \frac{5}{3} \alpha_1 \mu_0 & \frac{1}{2\sqrt{2}} g_2 v_1 & -\frac{1}{2\sqrt{2}} g_2 v_2 & 0 \\ \frac{1}{2\sqrt{2}} g_2 v_1 & \frac{1}{2\sqrt{2}} g_2 v_1 & 0 & \frac{\epsilon}{2} & \gamma v_2 \\ -\frac{1}{2\sqrt{2}} g_2 v_2 & -\frac{1}{2\sqrt{2}} g_2 v_2 & \frac{\epsilon}{2} & 0 & \gamma v_1 \\ 0 & 0 & \gamma v_2 & \gamma v_1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{W}^3 \\ \tilde{B}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \\ \tilde{Z} \end{pmatrix} \quad (4.36)$$

where we have included the neutral fermion from the Z superfield (4.2) in the mass matrix. The off-diagonal $\tilde{H}_1^0 \tilde{Z}$ terms in (4.36) come directly from Eq. (4.2), while we expect an $\tilde{H}_1^0 \tilde{H}_2^0$ mixing term ϵ (see Fig. 9 for contributing graphs) and also a $\tilde{Z}\tilde{Z}$ diagonal mass term η to appear in a complete treatment of the light singlet sector sketched in Section 3, emanating from radiative corrections to the superpotential term P_{5Z} of Eq. (2.6). It is convenient to introduce the linear combinations

$$\tilde{A}^0 = \frac{v_1 \tilde{H}_1^0 - v_2 \tilde{H}_2^0}{v} \quad (4.37a)$$

$$\tilde{S}^0 = \frac{v_2 \tilde{H}_1^0 + v_1 \tilde{H}_2^0}{v} \quad (4.37b)$$

of neutral shiggses, in terms of which the matrix (4.36) becomes

$$(\tilde{W}^3, \tilde{B}^0, \tilde{A}^0, \tilde{S}^0, \tilde{Z}) \begin{pmatrix} \alpha_2 \mu_0 & 0 & \frac{g_2 v}{2\sqrt{2}} & 0 & 0 \\ 0 & \frac{5}{3} \alpha_1 \mu_0 & \frac{g_1 v}{2\sqrt{2}} & 0 & 0 \\ \frac{g_2 v}{2\sqrt{2}} & \frac{g_1 v}{2\sqrt{2}} & -\frac{v_1 v_2}{v^2} \epsilon & \frac{(v_1^2 - v_2^2)}{2v^2} \epsilon & 0 \\ 0 & 0 & \frac{v_1^2 - v_2^2}{2v^2} \epsilon & \frac{v_1 v_2}{v^2} \epsilon & \frac{\gamma v}{2} \\ 0 & 0 & 0 & \frac{\gamma v}{2} & \eta \end{pmatrix} \begin{pmatrix} \tilde{W}^3 \\ \tilde{B}^0 \\ \tilde{A}^0 \\ \tilde{S}^0 \\ \tilde{Z} \end{pmatrix} \quad (4.38)$$

In the limit $\epsilon \rightarrow 0$ the $(\tilde{W}^3, \tilde{B}^0, \tilde{A}^0)$ and $(\tilde{S}^0, \tilde{Z})$ submatrices decouple, and we see that the $\tilde{S}^0$ combination of neutral shiggses would be massless without the intervention of the light singlet multiplet Z (4.2). In the limit of large M_0 but small ϵ and η the eigenstates of the matrix (4.38) become

$$\begin{aligned} \tilde{W}^3 + \frac{g_2 v}{2\sqrt{2} \alpha_2 \mu_0} \tilde{A}^0 &: m \approx \alpha_2 \mu_0 \\ \tilde{B}^0 + \frac{3g_1 v}{10\sqrt{2} \alpha_1 \mu_0} \tilde{A}^0 &: m \approx \frac{5}{3} \alpha_1 \mu_0 \\ \tilde{A}^{0'} \equiv \tilde{A}^0 - \frac{g_2 v}{2\sqrt{2} \alpha_2 \mu_0} \tilde{W}^3 - \frac{3g_1 v}{10\sqrt{2} \alpha_1 \mu_0} \tilde{B}^0 &: m \approx \frac{g_2^2 v^2}{5 \alpha_2 \mu_0} \\ \tilde{S}_\pm^0 \equiv \frac{1}{\sqrt{2}} (\tilde{S}^0 \pm \tilde{Z}) &: m \approx \frac{\gamma v}{2} \pm \frac{1}{2} \left(\eta + \frac{v_1 v_2}{v^2} \epsilon \right) \end{aligned} \quad (4.39a)$$

In the limit of small M_0 , ϵ and η the eigenstates of the matrix (4.38) become

$$\begin{aligned} \tilde{\gamma} &\equiv \frac{g_1 \tilde{W}^3 - g_2 \tilde{B}^0}{\sqrt{g_1^2 + g_2^2}} : m = 0 \\ \tilde{Z}_\pm^0 &\equiv \frac{g_2 \tilde{W}^3 + g_1 \tilde{B}^0 \pm \sqrt{g_1^2 + g_2^2} \tilde{A}^0}{\sqrt{2(g_1^2 + g_2^2)}} : m \approx \sqrt{g_1^2 + g_2^2} \frac{v}{2\sqrt{2}} \\ \tilde{S}_\pm^0 &= \frac{1}{\sqrt{2}} (\tilde{S}^0 \pm \tilde{Z}) : m \approx \frac{\gamma v}{2} \pm \frac{1}{2} \left(\eta + \frac{v_1 v_2}{v^2} \epsilon \right) \end{aligned} \quad (4.39b)$$

There are no important particle physics constraints on the neutral fermions (4.39), but there are some potentially significant cosmological constraints.

The lightest supersymmetric particle is stable, and hence should not be charged, since there are no charged stable relics from the Big Bang with masses $\leq 0(1) \text{ TeV}$ ³³⁾. Any stable neutral weakly interacting particle which decouples around the same time as the neutrinos, should either weigh less than $0(100) \text{ eV}$ ³⁴⁾, or else weigh more than 1 to 10 GeV ³⁵⁾. Also, Big Bang nucleosynthesis does not allow more than one extra two-component neutral fermion weighing less than $0(100) \text{ eV}$ ³⁶⁾. In the case of large μ_0 we therefore deduce

$$\frac{g_2 v_1 v_2}{m_{\tilde{W}}} \gtrsim \min \left(\frac{g_2^2 v^2}{5 m_{\tilde{W}}}, \frac{\delta_1 v}{2} - \frac{1}{2} \left(3 + \frac{v_1 v_2}{v^2} e \right) \right) \gtrsim 0(1 \text{ to } 10) \text{ GeV} \quad (4.40a)$$

whereas in the case of small μ_0 we deduce that

$$g_2 \sqrt{v_1 v_2} \gtrsim m_{\tilde{g}} \begin{cases} \lesssim 0(100 \text{ eV}) \\ \text{or} \\ \gtrsim 0(1 \text{ to } 10) \text{ GeV} \end{cases} \quad (4.40b)$$

While restrictive, neither of the sets of conditions (4.40) is difficult to satisfy, and we infer that either of the scenarios of large μ_0 or of small μ_0 is feasible.

To see this point in somewhat more detail, we explore somewhat further the constraints (4.40a) applicable in the limit of large μ_0 . The vacuum expectation value v_2 can be calculated in terms of the parameters γ_1 (4.2) and μ^2 (4.6) entering into the low-energy effective potential $V^{(L)}$ (4.11). It is easy to check that

$$v_2^2 = \frac{\left(\frac{g_2^2 + g_1^2}{2} v^2 - 2\delta_1^2 v^2 - 2\mu^2 \right) \pm \sqrt{\left(\frac{g_2^2 + g_1^2}{2} v^2 - 2\delta_1^2 v^2 - 2\mu^2 \right)^2 + 16 \left(\frac{g_2^2 + g_1^2}{2} - \delta_1^2 \right) \left(\mu^2 m_b^2 \frac{c_g}{R^2} \right)}}{2(g_2^2 + g_1^2 - 2\delta_1^2)} \quad (4.41)$$

which becomes

$$v_2^2 \approx \frac{m_b^2 \frac{c_g}{R^2}}{\left(1 - \frac{(g_2^2 + g_1^2 - 4\delta_1^2) v^2}{4\mu^2} \right)} \quad (4.42)$$

in the limit of small m_b^2 . In the limit of large μ^2 the expression (4.42) becomes

$$v_2^2 \approx m_b^2 \frac{c_q}{R^2} \quad (4.43)$$

and putting in the approximate values

$$c_q \approx \frac{3}{4}, \quad R^2 \approx \frac{1}{50} \quad (4.44)$$

derived from the expressions (4.8) and (4.9) we finally obtain a limiting value of

$$v_2 \rightarrow 0(30) \text{ GeV} \quad (4.45)$$

when μ gets large. It is worth recalling that in the preferred scenario of Section 3 where the R breaking is not small

$$\mu^2 \approx \frac{3}{8\pi} \alpha_2 \ln\left(\frac{m_b^2}{m_{\tilde{W}}^2}\right) m_{\tilde{W}}^2 \approx 0\left(\frac{1}{10}\right) m_{\tilde{W}}^2 \quad (4.46)$$

Therefore the limit of large μ_0 in this case drives μ (4.46) up and v_2 (4.42) down, and hence the mass (4.35) of the lightest charged supersymmetric fermion also decreases as μ_0 increases. Demanding the phenomenological condition (4.35) we finish with an upper limit on μ_0 and a lower limit on v_2 :

$$m_{\tilde{W}} \leq 0(3) m_W, \quad v_2 \geq 0(40) \text{ GeV} \quad (4.47)$$

It is apparent that "large μ_0 " does not mean very large, although the approximations (4.33a) to the eigenvalues are still accurate to better than 10% for $m_{\tilde{W}}$ saturating the bound (4.47).

The final phenomenological discussion of this section concerns the two light neutral shiggses of Eq. (4.39a). We do not know the magnitude of the two small parameters ϵ and η which might be $\ll m_W$. In this case the $\tilde{S}_{\pm}^0$ would be almost degenerate in mass and very likely the lightest supersymmetric particles. The heavier one $\tilde{S}_-^0$ will then decay into the lighter one $\tilde{S}_+^0$ via Z^0 exchange as seen in Fig. 10a with a lifetime

$$\frac{\tau(\tilde{S}_+^0)}{\tau(c)} = 0 \left[\left(\frac{\eta + \frac{v_1 v_2}{v^2} \epsilon}{m_c} \right)^5 \right] \quad (4.48)$$

As observed in Ref. 18) the $\tilde{S}_+^0$ lifetime could easily be much longer than the generic charm lifetime $\tau(c) = 0(10^{-13})$ seconds. If $\tau(\tilde{S}_+^0) > 0(10^{-10})$ sec. one can expect the $\tilde{S}_+^0$ to decay outside the apparatus. In this case it would be very difficult to distinguish the $\tilde{S}_\pm^0$ in any experiment. For example, in Z^0 decay they would be pair-produced with the same rate as a single $\tilde{S}^0$ state in the absence of mixing with the $\tilde{Z}$. On the other hand, if the unknown parameters ϵ and η are large, it could be that only the $\tilde{S}_-^0$ could be pair-produced in Z^0 decay with a decay rate

$$\Gamma(Z^0 \rightarrow \tilde{S}_-^0 \tilde{S}_-^0) = \frac{1}{2} \Gamma(Z^0 \rightarrow \bar{\nu}_a \nu_a) \quad (4.49)$$

It is therefore desirable that neutrino counting experiments at the Z^0 be sufficiently precise to spot half an extra neutrino.

4.4 Higgs Boson Mass Spectrum

We now turn to the spectrum of charged and neutral Higgs bosons expected from the potential $V^{(L)}$ as exhibited in Table 5. The charged Higgs boson mass matrix is [after using the conditions (4.22)]:

$$\frac{g_2^2}{2} (H_1^+, H_2^+) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix} \quad (4.50)$$

One combination of these states has zero mass so as to be eaten to give the $W^\pm$ masses, while the other combination is:

$$H^\pm = \frac{v_2 H_1^\pm - v_1 H_2^\pm}{v} : m_{H^\pm}^2 = \frac{g_2^2}{2} v^2 \quad (4.51)$$

We deduce from Eq. (4.51) a feature which is generic in models with two Higgs multiplets, one giving masses to the charge 2/3 quarks and one to the charge -1/3 quarks, namely that

$$\Gamma(H^+ \rightarrow q_{2/3} q'_{-1/3}) \propto |U_{qq'}|^2 \left(\frac{v_1^2 m_{2/3}^2 + v_2^2 m_{-1/3}^2}{v^2} \right) \quad (4.52)$$

where $U_{qq'}$ is the appropriate element of the Kobayashi-Maskawa mixing matrix.

To the extent that $v_2 < v_1$ as we expect, the $H^\pm$ couplings will depend more on the charge $-1/3$ quark involved than on the charge $+2/3$ quark. For example:

$$\frac{\Gamma(H^+ \rightarrow c\bar{b})}{\Gamma(H^+ \rightarrow c\bar{s})} \approx \frac{|V_{cb}|^2 m_b^2}{|V_{cs}|^2 m_s^2} > 1? \quad (4.53)$$

whereas one might have expected a more equal ratio if $v_2/v_1 \approx 0(1)$.

Now we turn to the mixing matrix for the scalar fields H_1^0, H_2^0 :

$$(H_1^0, H_2^0) \begin{pmatrix} \frac{g_2^2 + g_1^2}{2} v_1^2 & (2\gamma^2 - \frac{g_2^2 + g_1^2}{2}) v_1 v_2 \\ (2\gamma^2 - \frac{g_2^2 + g_1^2}{2}) v_1 v_2 & \frac{g_2^2 + g_1^2}{2} v_2^2 \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix} \quad (4.54)$$

We neglect mixing with the scalar component of the Z multiplet which can be arbitrarily small. The eigenstates of (4.54) are, for γ_1 small,

$$H^0 \equiv \frac{v_1 H_2^0 + v_2 H_1^0}{v} : m_{H^0}^2 \approx 4\gamma_1^2 v^2 \quad (4.55a)$$

$$H^{0'} \equiv \frac{v_1 H_1^0 - v_2 H_2^0}{v} : m_{H^{0'}}^2 \approx \frac{g_2^2 + g_1^2}{2} v^2 \quad (4.55b)$$

We notice another possibly generic feature of analogous Higgs systems, namely that the (probably) lighter state H^0 has a larger coupling to charge $-1/3$ quarks than to charge $2/3$ quarks because of the factors of v_1 and v_2 in (4.55a). This means for example that the toponium $\rightarrow H^0 + \gamma$ decay rate³⁷⁾ is suppressed by comparison with that expected in a single Higgs model:

$$\frac{\Gamma((\bar{t}t) \rightarrow H^0 + \gamma)}{\Gamma((\bar{t}t) \rightarrow \gamma^* \rightarrow e^+ e^-)} = \frac{G_F m_t^2}{\sqrt{2} \pi \alpha} \left[1 - \frac{m_{H^0}^2}{m_{\bar{t}t}^2} \right] \left(\frac{v_2^2}{v^2} \right) \quad (4.56a)$$

In view of the possible limiting case (4.47) there could be a suppression of H^0 production by more than an order of magnitude. On the other hand, m_{H^0} (4.55a) might be so small that the H^0 could be produced in $T(\bar{b}b)$ decay, in which case one would expect almost the canonical decay rate³⁷⁾

$$\frac{\Gamma(\Upsilon \rightarrow H^0 + \gamma)}{\Gamma(\Upsilon \rightarrow \delta^* \rightarrow e^+ e^-)} = \frac{G_F m_b^2}{\sqrt{2} \pi \alpha} \left[1 - \frac{m_{H^0}^2}{m_\Upsilon^2} \right] \left(\frac{v_1^2}{v^2} \right) \quad (4.56b)$$

We see from Eq. (4.55b) that the heavier scalar Higgs H^0 is expected to have a mass similar to the Z^0 . It therefore cannot be produced in toponium decay unless $m_t > 50$ GeV. On the other hand, the H^0 is produced much more strongly than the H^0 [by a factor $O(v_1^2/v^2)$] in association with the Z^0 via the reaction³⁸⁾ $e^+ e^- \rightarrow Z^{0*} \rightarrow Z^0 + H^0$.

As for the pseudoscalar components η_i of the H_1 and H_2 multiplets, one combination $(\eta_2 v_2 + \eta_1 v_1)/v$ is massless and eaten by the Z^0 , while the axion-like³⁹⁾ orthogonal combination $a \equiv (\eta_1 v_2 - \eta_2 v_1)/v$ is uneaten. The terms we have so far written down in the low-energy potential $V^{(L)}$ (4.11) do not give a mass to a , but one can be generated by diagrams like that in Fig. 11 which gives an $H_1 H_2$ coupling in $V^{(R)}$ of order $\gamma_{24}^2/h_t^2 \mu^2$. This can give a mass to a which makes it sufficiently heavy ($m_a > 350$ MeV) to have avoided detection to date while not disturbing substantially previous parts of our analysis. We will not discuss the a in greater detail, except to observe that it also couples preferentially to charge $-1/3$ quarks and is hence difficult to detect in toponium or charmonium decays.

Finally we turn to the neutral spin-zero components of the Z supermultiplet. These can have masses up to $O(m_W)$. They couple very weakly to ordinary matter because they do not couple to quarks and leptons at the tree level [see the superpotential term (4.2)]. They can acquire matter couplings through one-loop diagrams as in Fig. 12, giving $\bar{q}q$ couplings of the order of magnitude

$$g_{\bar{q}q} = \left(\frac{\alpha}{\pi} \right) \gamma_1 \left(\frac{m_q}{m_W} \right)^3 \left(\frac{v_2}{v_1} \right)^{1/2} \lesssim O(10^{-12}) \gamma_1 \quad (4.57)$$

which makes them unobservable. Since they are gauge singlets they also do not have significant couplings to the massive gauge bosons and hence we can forget about their low-energy phenomenology. This and other aspects of our phenomenological analysis are summarized in Tables 4 and 5.

5. - PROSPECTS

In this paper we have given examples of supersymmetric GUTs in which the hierarchy problem is solved using supersymmetry broken at a very high energy scale. We have demonstrated that the pattern of masses generated by radiative corrections triggered by primordial spontaneous supersymmetry breaking in a gauge singlet Fayet-O'Raifeartaigh sector⁷⁾ is consistent with low-energy phenomenology. We have shown that not only is supersymmetry able to "protect" the light Higgs boson masses, but also that one can make them naturally massless at the tree level without resorting to a fine-tuning of parameters. The numerical value of the hierarchy $m_W/m_X = O(10^{-14})$ is explained as a product of Yukawa and gauge coupling constants. It is possible to construct models which are consistent with the basic cosmological constraints that the Universe cools down into an intermediate energy $SU(3) \times SU(2) \times U(1)$ invariant state, that there not be too many light neutral particles at the epoch of Big Bang nucleosynthesis, and that the lightest stable supersymmetric particle be neutral and not weigh between about 100 eV and 1 to 10 GeV. Our class of models makes testable predictions for forthcoming generations of e^+e^- and other experiments.

The complexity of the radiative corrections in this class of models makes them difficult to analyze completely. Thus there are many details, particularly of the $SU(3) \times SU(2) \times U(1)$ singlet sector of Model I, that we have not worked out. There are considerable ambiguities in the structure of these models, particularly at the grand unification scale. Therefore it may not make much sense to analyze these models before one has some more phenomenological indications whether one is on the right track, and if so how to proceed. It is difficult to believe that the O'Raifeartaigh sector of the superpotential is truly fundamental, and one might like to find alternative sources of supersymmetry breaking in a gauge singlet sector. We have not investigated all possible cosmological issues, and in particular we have not addressed the possibility of having an inflationary epoch⁴⁰⁾ in the very early Universe in an attempt to solve the large-scale isotropy, homogeneity and flatness problems. We have not addressed the question of compatibility with local supersymmetry. This arises first at the level of breaking local $N = 1$ supersymmetry so that the goldstino gets eaten by the gravitino to acquire²⁶⁾ a mass $O(m_S^2/m_P)$ where m_S is the primordial supersymmetry breaking scale. It has been argued²¹⁾ that if radiative corrections are neglected the gravitino can be no heavier than any of the spin-zero bosons of $O(100)$ GeV which means that m_S can be no longer than $O(10^{10})$ GeV. This problem must be confronted if one eventually wishes to push m_S close to the grand unification scale or the Planck mass. An even more open problem is that of constructing a truly superunified⁴¹⁾ theory in which gravity and particle interactions are not simply glued on to one another.

Despite the problems ahead, we are excited by the vistas opened up by supersymmetric grand unification. We hope that Nature will soon vouchsafe to us some iota of evidence that supersymmetry plays any rôle in the Grand Design.

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Rôle	Notation	SU(5) Representation Content
Matter	$\psi_a^\alpha, \chi_a{}_{\alpha\beta}$	$N_G \times (\underline{5} + \underline{10})$
Higgs	$H_{1\alpha}, H_2^\alpha$ Σ_α^β Z	$(\underline{5} + \underline{\bar{5}})$ $\underline{24}$ $\underline{1}$
O'Raifeartaigh Sector	A, B, C	$3 \times (\underline{1})$
Coupling to Gauge Sector	$\phi_{1\alpha}, \phi_2^\beta$	$(\underline{5} + \underline{\bar{5}})$
High Temperature Behaviour	X, Y	$2 \times (\underline{1})$

Table 1 : Chiral supermultiplets used in model-building

Field	θ_i
A	α
C	$-2\alpha + \beta$
ϕ_1	γ
ϕ_2	$-\gamma + \beta - \alpha$
Σ	$\beta/3 + \delta$
Z	$\beta/3 + \delta$
H_1	$\rho + \beta/3 - \delta$
H_2	$-\rho + \beta/3$
χ	$\beta/3 - \rho/2 + \delta/2$
ψ	$\beta/3 + 3\rho/2 - \delta/2$

Table 2 : R Invariances of the superpotential in
Eqs. (2.3)-(2.6)

Y	$\beta/3$
Z	$\beta/3$

Table 3 : R Invariances of the additional fields
in the superpotential Eqs. (2.8) and
(2.9). The others are as in Table 2
with $\delta = 0$.

Fermions	Mass	Characteristics
$\tilde{W}^\pm + \dots$	$m_{\tilde{W}} + \dots$	Mainly $I = 1$, vector-like neutral weak couplings
$\tilde{H}^\pm + \dots$	$\frac{g_2^2 v_1 v_2}{m_{\tilde{W}}}$	Mainly $I = \frac{1}{2}$, vector-like neutral weak couplings decaying into $\tilde{H}^0 + (\ell\nu \text{ or } \bar{q}q)$
$\tilde{W}^3 + \dots$	$m_{\tilde{W}} + \dots$	Mainly $I = 1$ wino
$\tilde{B}^0 + \dots$	$\frac{5}{3}(\frac{\alpha_1}{\alpha_2})m_{\tilde{W}} + \dots$	Mainly $I = 0$ bino
$\tilde{A}^0 \equiv \frac{v_1 \tilde{H}_1^0 - v_2 \tilde{H}_2^0}{v} + \dots$	$\frac{g_2^2 v^2}{5m_{\tilde{W}}} + \dots$	Mainly $I = \frac{1}{2}$ shiggs decaying into $\tilde{H}^\pm + (\ell\nu \text{ or } \bar{q}q)$
$\tilde{S}_\pm^0 \equiv \frac{1}{\sqrt{2}}(\tilde{S}^0 \pm \tilde{Z}^0)$	$\frac{\lambda v}{2} \pm \frac{1}{2}(\eta + \frac{v_1 v_2}{v^2} \epsilon) + \dots$	Mixture of $I = \frac{1}{2}$ and $I = 0$, lighter one stable, heavier one quasistable de- caying into lighter one $+(\ell^+ \ell^- \text{ or } \bar{q}q)$, end of all decay chains

Table 4 : Light Fermion Spectroscopy

Bosons	(Mass) ²	Characteristics
$H^0 \equiv \frac{v_1 H_1^0 + v_2 H_2^0}{v} + \dots$	$4\lambda^2 v^2 + \dots$	$I = \frac{1}{2}$, mainly coupled to charge $-\frac{1}{3}$ quarks, small coupling to Z^0
$H^{0'} \equiv \frac{v_1 H_1^0 - v_2 H_2^0}{v}$	$\frac{g_2^2 + g_1^2}{2} v^2 + \dots$	$I = \frac{1}{2}$, mainly coupled to charge $+\frac{2}{3}$ quarks, large coupling to Z^0
$a \equiv \frac{\eta_1 v_2 - \eta_2 v_1}{v}$	few GeV?	Pseudoscalar coupling, larger for charge $-\frac{1}{3}$ quarks
$H^\pm \equiv \frac{v_1 H_2^\pm - v_2 H_1^\pm}{v}$	$\frac{g_2^2}{2} v^2 + \dots$	Conventional charged Higgs boson
Z	$O(\lambda^2 v^2)?$	$I = 0$ scalar and pseudoscalar, very weakly coupled to light quarks

Table 5 : Light Boson Spectroscopy

FIGURE CAPTIONS

- Figure 1 : One-loop scalar graphs which a) do not, and b) do contribute to $m_{\phi_1\phi_2}^2$ as in Eq. (3.40).
- Figure 2 : Gluinos (and other gauginos) acquire masses from loops involving the ϕ_1 and ϕ_2 supermultiplets:
a) is a component graph and b) the relevant supergraph.
- Figure 3 : Three-loop contributions to the squark and slepton masses. The component graphs a) to j) correspond to the supergraph k). The supergraph l) gives another set of diagrams which are obtainable from the authors on microfiche.
- Figure 4 : More graphs contributing to the squark and slepton masses via the gauge boson/gaugino mass splittings.
- Figure 5 : An explicit contribution to squark and slepton masses obtained by substituting Fig. 2 into the fermion diagram in Fig. 4.
- Figure 6 : Still another contribution evaluated in Eq. (3.50).
- Figure 7 : Additional contribution to Higgs masses coming from Yukawa couplings of matter fields. They are net negative because the fermions in graph a) are lighter than the bosons in graph b).
- Figure 8 : Radiative corrections to the effective potential for the Z and Σ fields. The relative signs of a) fermion and b) boson diagrams may be different for triplet and doublet $H_{1,2}$ loops.
- Figure 9 : a) A component graph, and
b) a supergraph contributing to $\tilde{H}_1^0 - \tilde{H}_2^0$.
- Figure 10 : a) Diagram leading to $\tilde{S}_+^0 \rightarrow \tilde{S}_-^0 + \ell^+\ell^-(\bar{q}q)$ decay at the tree level and
b) a possible experimental signature for $e^+e^- \rightarrow \tilde{S}_+^0 \tilde{S}_+^0$ and decay.
- Figure 11 : Supergraph contributing to $H_1 - H_2$ mixing.
- Figure 12 : Lowest order one-loop contribution to $Z\bar{q}q$ couplings.

FIG. 3

FIG. 1

FIG. 2

FIG. 7

FIG. 8

FIG. 9

FIG.10

FIG.11

FIG.12

FIG.4

FIG.5

FIG.6