Ref.TH.3431-CERN

GLUON CONDENSATION PARAMETERS - INDICATORS FOR PROBING

THE VACUUM STRUCTURE OF QCD

M. Müller-Preussker *)
CERN - Geneva

A B S T R A C T

Predictions of the modified instanton gas model involving hard-core repulsion, as well as dipole-like interactions, are discussed with respect to gluon condensation parameters. The model provides a compensation mechanism which might explain the small value of the topological susceptibility found in recent lattice Monte Carlo simulations.

*) On leave of absence from Joint Institute for Nuclear Research, Dubna 141980, USSR.

1. - INTRODUCTION

Monte Carlo simulations carried out during recent years have shown with surprising consistency that QCD, regularized on a lattice of moderate size and spacing, reasonably describes a lot of non-perturbative phenomena. For example, the extraction of the value of the gluon condensate, and recently of the tri-gluon condensation parameter from SU(2) ¹⁾⁻³⁾ and partly from SU(3) ⁴⁾ MC data, has led to numbers close to those found from QCD charmonium sum rules ^{5),6)}.

However, in the case of the topological susceptibility playing a fundamental rôle in the solution of the $U_A(1)$ problem ⁷⁾,

$$\chi_t = - \left. \frac{d^2 P(\theta)}{d\theta^2} \right|_{\theta=0} = \int d^4x \langle Q(x) Q(0) \rangle, \quad (1)$$

$$Q(x) = \frac{g^2}{32\pi^2} G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x)$$

(P and θ denote the pressure and the phase of the vacuum state, resp.) the SU(2) lattice result ⁸⁾ failed by a factor of ~ 100 . Preliminary data of an analogous calculation in the SU(3) case seem to give a similar result ⁹⁾. The main criticism of these lattice calculations might be that the employed lattice definitions for the topological charge density $Q(x)$ have no topological significance. This is obviously related to the existence of a perturbative tail to be subtracted in order to isolate the non-perturbative signal. Nevertheless, independently of the search for more convenient lattice definitions, it is reasonable to look for predictions from the continuum point of view, too. For example, Lüscher has conjectured a lower bound for χ_t ¹⁰⁾, unfortunately of the same order of magnitude as the derived lattice result.

In this paper we want to show that the instanton gas model reported in Ref. 11) predicts SU(3) gluon condensation parameters consistent with available phenomenological and lattice results. On the other hand, it provides a compensation mechanism for χ_t due to dipole-like interactions, which at least yields an upper bound slightly smaller than the phenomenologically estimated number.

In order to make this paper more or less self-consistent, we decided to present the main assumptions of the model in section 2. Section 3 is devoted to the discussion of the results.

2. - THE INSTANTON GAS MODEL

We assume the vacuum state of SU(N) Yang-Mills theories to be described by the grand partition function for a dilute instanton gas ¹²⁾ put into a large volume V

$$Z[V] = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{j=1}^n \left(\sum_{\epsilon_j=\pm} \int \frac{d\rho_j}{\rho_j} \int_V d^4 z_j d_0(\rho_j) e^{i\epsilon_j \theta} \int [dR_j] e^{-\sum_i U(i,j)} \right) \quad (2)$$

where the standard notations for the collective coordinates and for the single instanton amplitude $d_0(\rho)$ ¹³⁾ are understood. We require the dominant contributions to $U(i,j)$ to be dipole-like interactions U_{dip} between instantons and anti-instantons at large distances and repulsive forces at small distances - represented for simplicity as a hard core $U_{\text{h.c.}}$. We have

$$U_{\text{dip}}(i,j) = \frac{1}{2} \int d^4 x F_{i\mu}^{+a}(x-z_i) \square_x F_{j\mu}^{-a}(x-z_j) \\ \sim -8\pi^2 D_{\mu\nu}^{+(i)a} D_{\mu\nu}^{-(j)a} \left(\delta_{\nu\nu'} - 4 \frac{\Delta_\nu \Delta_{\nu'}}{\Delta^2} \right) \Delta^{-4} \quad (3)$$

with A^+ and D^+ the one-instanton solution written in the singular gauge and its "dipole moment", respectively ($\Delta_\nu = (z_i - z_j)_\nu$). For $U_{\text{h.c.}}$ the following ansatz is chosen.

$$U_{\text{h.c.}}(i,j) = \begin{cases} 0 \\ \infty \end{cases} \quad \text{for} \quad \begin{matrix} \Delta^4 > a' \rho_i^2 \rho_j^2 \\ \Delta^4 < a' \rho_i^2 \rho_j^2 \end{matrix} \quad (4)$$

a' is a dimensionless parameter related to the fraction of space-time filled up by instantons. (The condition $|U_{\text{dip}}| \leq 2 \cdot 8\pi^2/g^2$ yields a lower bound $a' \geq 6$.)

In principle, a repulsive force between instantons can be extracted directly from the classical action. It rises logarithmically as Δ tends to zero. One expects its careful consideration to fix the hard core parameter a' as an effective one ¹⁴⁾.

Here we shall determine it by comparison with lattice results without referring to phenomenological numbers.

In Ref. 11) we have shown how the interplay of both types of interactions can be dealt with. One linearizes U_{dip} by an averaging procedure, the functional integration of which has to be carried out after solving the hard core problem. The latter is done in a self-consistent way. One assumes a ρ -distribution concentrated around an average $\bar{\rho}$. Then one can argue for a one-instanton distribution of the form

$$d^{\pm}(\varrho) = d_0(\varrho) e^{\pm i\theta} \exp \left(-P_{\text{h.c.}}(\theta) \cdot \frac{\pi^2}{2} a' \bar{\varrho}^2(\theta) \cdot \varrho^2 \right) \quad (5)$$

where large scales are exponentially suppressed by the hard core pressure $P_{\text{h.c.}}$. The latter is directly related to the average scale size by

$$\frac{\pi^2}{2} a' (\bar{\varrho}^2)^2 = \left(\frac{b_N}{2} - 2 \right) P_{\text{h.c.}}^{-1}, \quad b_N = \frac{11 N_c}{3} \quad (6)$$

By variation of the hard core partition function with respect to a constant chemical potential, one is able to derive a simple differential equation for $P_{\text{h.c.}}$, the solution of which can be represented as ¹¹⁾

$$a^4 P_{\text{h.c.}} \sim \left(\exp - \frac{8\pi^2}{g_0^2} \right)^{\frac{4}{b_N}} \cdot (\cos \theta)^{\frac{4}{b_N}} \quad (7)$$

[We are adopting here the "lattice renormalization scheme" for the definition of the coupling constant. It is characterized by the Λ ratios ¹⁵⁾ $\Lambda_{\text{PV}}/\Lambda_{\text{L}} = 21.55$ (31.31) for SU(2) (SU(3)). This seems to be convenient because we are going to compare finally instanton with lattice results. Furthermore, this scheme yields small couplings such that the unknown two-loop contribution to $d_0(\rho)$ might be really small. In this sense, a can be understood as a lattice regularization parameter.] Relation (7) agrees with what one would expect from renormalization group arguments. The variation with respect to $1/g_0^2$ provides the gluon condensate in accordance with the trace anomaly of the energy-momentum tensor (if dipole interactions are neglected).

The θ dependence in (7) can be questioned. In Ref. 16), we also speculated in favour of a $\cos 4\theta/b_N$ dependence. For this one had to assume a factorization of the partition function Z into self-dual and anti-self-dual contributions without mutual interactions, which seems to be a rather unnatural requirement.

Using relation (6) we rewrite the distribution (5) as

$$d^{\pm}(\varrho) = d_0(\varrho) e^{\pm i\theta} e^{-a_N \frac{\varrho^2}{\bar{\varrho}^2} (\cos \theta)^{2/b_N}}$$

$$a_N = \frac{b_N}{2} - 2, \quad \bar{\varrho}^2 \equiv \bar{\varrho}^2(\theta=0) \quad (8)$$

Finally, one has to carry out the functional integration previously mentioned. This can be done within the approximation that others apart from Gaussian fluctuations of the averaging field will be neglected. Furthermore, we disregard contributions due to two-particle correlations in the partition function. Then we end up with a total pressure

$$P(\theta) = P_{h.c.}(\theta) - \frac{3}{16\pi^2} (N_c^2 - 1) (\bar{\varrho}^2)^{-2} \int_0^\infty dx x^3 \log(1 - (\pi^2 \chi(x, \theta))^2) \quad (9)$$

with the "susceptibility"

$$\chi(x, \theta) = \frac{1}{N_c^2 - 1} \frac{8\pi^2}{g^2(\bar{\varrho})} \int_0^\infty \frac{d\varrho}{\varrho} d_0(\varrho) \varrho^4 \exp\left(-a_N \frac{\varrho^2}{\bar{\varrho}^2} (\cos \theta)^{2/b_N}\right) F^2\left(x \frac{\varrho}{\bar{\varrho}}\right) \quad (10)$$

where

$$F(y) = \frac{4}{y^2} \left(1 - \frac{y^2}{2} K_2(y)\right)$$

is due to the Fourier transform of the instanton solution in the singular gauge. The logarithm in Eq. (9) reminds the reader of the determinant found in the functional averaging procedure. It corresponds to an expansion in rings of interacting pairs of instantons and anti-instantons. Therefore, the phase factor $e^{\pm i\theta}$ has been cancelled out everywhere.

It is easy to see that the expressions (9) and (10) lead to an effective one-instanton distribution ($\theta = 0$)

$$D_{\text{eff}}(\varrho) \simeq d^{\pm}(\varrho, \theta) \left\{ 1 + \frac{3}{16} \frac{\varrho^4}{(\bar{\varrho}^2)^2} \frac{8\pi^2}{g^2(\bar{\varrho})} \pi^2 \chi(\varrho, \theta) \int_0^\infty dx x^3 \frac{F^4(x)}{1 - F^4(x) (\pi^2 \chi(\varrho, \theta)^2)} \right\} \quad (11)$$

Equations (9) and (11) characterize our model given in the "one-instanton approximation".

Finally, one has to determine the diluteness parameter a' . It is related to $\bar{\rho}^2$ via the space-time packing fraction

$$f_0(\bar{\rho}) = 2 \int_0^\infty \frac{d\rho}{\rho} d_0(\rho) \exp(-q_N \frac{\rho^2}{\bar{\rho}^2}) \cdot \frac{\pi^2}{2} \rho^4 = (2 - \frac{4}{b_N}) a'^{-1} \quad (12)$$

In a recent paper ¹⁷⁾ we investigated the static force between quarks and antiquarks, both from the instanton gas and the lattice point of view. We found that the instanton gas describes the lattice Monte Carlo data well up to intermediate distances [~ 0.4 fm for SU(3)]. Contributions due to dipole-like interactions according to Eq. (11) turned out to be important, whereas the packing fraction was small enough ($f_0 \lesssim 4\%$). Since we discussed only qualitative features of the static force, we replaced, in Ref. 17), the exponential scale-size cut-off by the usual one, for simplicity. Here we would like to discuss the predictions of the model more quantitatively. As far as the ratios between expectation values of the operators of higher dimension g^{nG^n} strongly depend upon the cut-off convention applied, we are enforced to take expression (8) seriously. Apart from this we proceed analogously to Ref. 17) and assume that the expectation value of a Wilson loop $W(T, R)$ with the Euclidean time extension $T \gg R$ can be computed by factorizing the single instanton contributions along the Euclidean time axis. The parameters a' or $\bar{\rho}$ are then adjusted such that the total static force

$$F(R) = \frac{N_c^2 - 1}{8\pi N_c} \cdot \frac{g^2(R)}{R^2} + F_{inst}(R) \quad (13)$$

[with $g^2(R)$ given according to Ref. 18)] follows the lattice data up to some distance R_{max} .

Figures 1 and 2 show the result in the cases of SU(2) and SU(3), resp. In the SU(2) case, for comparison, we used here the data recently discussed in Ref. 19). The points indicated in Fig. 1 in fact indicate the error of the fit carried out in Ref. 19) and reflect the statistical errors only to a certain extent. For the SU(3) gauge group we use our own data for the Creutz' ratios $\chi(4,2)$, $\chi(5,2)$ found on a lattice with size 8^4 with the help of Pietarinen's heat bath method ²⁰⁾. Compared with Ref. 17), we have improved the statistics for the points with $RA_L > 4 \cdot 10^{-3}$ (averages over ~ 60 sweeps).

The corresponding instanton scales sizes are determined to be

$$\bar{\rho} \Lambda_L = \begin{cases} (5.25 \pm .10) \cdot 10^{-3} & \text{for } SU(2) \\ (5.30 \pm .15) \cdot 10^{-3} & \text{for } SU(3) \end{cases} \quad (14)$$

At least for $SU(3)$, $\bar{\rho}$ is of the same magnitude as $R_{\max}$. Since we measured the static force on the lattice at distances $R \sim \sqrt{2}a$, we conclude that instantons cover areas of several lattice units. $\bar{\rho}$ turns out to be fixed tightly and approximately independent of N_c . This is in accordance with the usually assumed large N_c limit. The corresponding hard core parameters are

$$a' \simeq \begin{cases} 117 \pm 15 & \text{for } SU(2) \\ 43 \pm 13 & \text{for } SU(3) \end{cases} \quad (15)$$

(Numerical deviations from the numbers quoted in Ref. 17) are due to the different ρ cut-offs applied.)

In Ref. 16) the large N_c limit has been discussed for $P_{h.c.}$ and for the corresponding gluon condensate. Assuming $a' = \text{const.}$ we found $P_{h.c.} \sim O(N_c)$ in contrast to $O(N_c^2)$, which is usually expected. The a' values according to Eq. (15) could indicate $a' \sim O(N_c^{-1})$. Thus, one cannot exclude the fact that the instanton gas model obeys the correct N_c limit. Unfortunately, this problem cannot be solved until it is understood how instanton interactions can be dealt with in this limit.

3. - RESULTS

With the parameters a' and $\bar{\rho}$ fixed by Eqs. (14) and (15), respectively, our model predicts the gluon condensation parameters without relying on any phenomenological information. The one-instanton approximation yields

$$\langle g^n G^n \rangle = 2 \int_0^\infty \frac{d\rho}{\rho} D_{\text{eff}}(\rho) \mathcal{O}_n(\rho) \quad (16)$$

where

$$\mathcal{O}_n(\rho) = \int d^4x \int [dR] g^n (G_{\text{inst}}(\rho, x, R))^n \quad (17)$$

(G_{inst} denotes the instanton field strength with a given group orientation R .) For the gauge invariant operators $g^2 G_{\mu\nu}^a G_{\mu\nu}^a$, $g^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c$, $(g^2 f^{abc} G_{\mu\alpha}^b G_{\mu\beta}^c)^2$ and $(g^2 f^{abc} G_{\mu\nu}^b G_{\alpha\beta}^c)^2$ needed in the determination of the gluonium spectrum via QCD sum rules the reader finds easily $\mathcal{O}_n(\rho) = 32\pi^2$, $-76.8\pi^2\rho^{-2}$, $292.6\pi^2\rho^{-4}$ and $585.1\pi^2\rho^{-4}$, respectively.

The numerical results of the ρ integration (16) are collected in the Table. For comparison, we have also shown the available lattice and phenomenological values. It is convenient to use Λ_L as a universal unit. Then the phenomenological values are affected by the large error in relation to the string tension $\sqrt{\sigma} \approx 420$ MeV

$$\Lambda_L = \begin{cases} (.013 \pm .002) \sqrt{6}^{21)} & \text{for } SU(2) \\ (.006 \pm .001) \sqrt{6}^{22)} & \text{for } SU(3) \end{cases}$$

The "phenomenological" values shown for $SU(2)$ are translated from the real $SU(3)$ case by assuming $(1/N_c) \langle g^{n,n} \rangle$ and χ_t to be independent of N_c .

First we remark that the $SU(3)$ predictions for the gluon condensate and for the tri-gluon condensation parameter agree very well with the lattice, as well as with phenomenological results ($\langle g^3 G^3 \rangle$ turns out to be negative in the Euclidean world!). For the operators of the type $g^4 G^4$, expectation values are predicted roughly satisfying the hypothesis of vacuum dominance, which is tantamount to the factorization $\langle g^4 G^4 \rangle \sim (\langle g^2 G^2 \rangle)^2$. In this respect, different models based on the dilute gas picture yield results, which strongly deviate from each other.

The naive dilute gas model does not include dipole-like interactions and uses a very large scale size cut-off ($\rho_c \sim (170 \text{ MeV})^{-1}$)²³⁾ in order to saturate the gluon condensate to its phenomenological value. It then predicts essentially smaller values for $\langle g^3 G^3 \rangle$ and $\langle g^4 G^4 \rangle$ than our model.

The recently proposed "instanton liquid model"²⁴⁾ takes the effect of dipole-like interactions into account in a rough manner and shrinks the one-instanton distribution to a δ function at some reasonably small $\bar{\rho}$ adjusted as well by means of the gluon condensate. It provides very large numbers for the former quantities (for $\langle g^4 G^4 \rangle$ by a factor of ~ 10 larger than our value).

It is obvious that lattice calculations analogous to those performed in Ref. 3), with appropriately defined operators on the lattice, would give better insight into the vacuum structure. One should be able to discriminate between different models.

In the $SU(2)$ case, our model yields a reasonable gluon condensate, too. However, the absolute value of the tri-gluon condensation parameter becomes too large whereas $\langle (g^2 G^2)^2 \rangle$ even diverges. This divergence should be related to a short-distance singularity of the local operator product and manifests itself as

an ultra-violet divergence of the scale size integration. We remind the reader that the ρ distribution is of the form, $\rho^{bN-4} \exp(-a_N \rho^2 / \bar{\rho}^2)$ becoming a narrow one for N_c large enough. The observed singularity is surely "felt" in the result for $\langle g^3 G^3 \rangle$, too, which explains the overestimated value. Similarly, we should expect our $SU(3)$ $\langle g^4 G^4 \rangle$ value to be overestimated.

We conclude this discussion with the remark that $N_c=3$ might be an optimal case for the validity of the instanton gas picture. For $N_c=2$ we run into trouble with ultra-violet singularities for operators of higher dimensions. On the other hand, the gas turned out to be rather dilute. For large N_c , the ρ distribution avoids the appearance of such singularities, but the gas becomes possibly too dense.

Finally let us turn to the discussion of the topological susceptibility. Differentiating twice the expression (9) with respect to θ we find the dipole interaction contribution to χ_t to be negative in sign and numerically of the same order of magnitude as

$$-\frac{d^2}{d\theta^2} P_{h.c.}(\theta) \Big|_{\theta=0} = \frac{4}{b_N} P_{h.c.}(0) \sim \theta(10^6) \quad (18)$$

(compare with the Table). This cancellation points in the same direction as the small value found in Ref. 8). Of course, our instanton model with the given crude approximations cannot be used for any quantitative estimates for the magnitude of χ_t . Nevertheless, Eq. (18) can be interpreted at least as an upper bound which, in the case of $SU(3)$, is somewhat smaller than the phenomenologically derived value.

Anyway, the solution of the $U_A(1)$ problem requires further study. So far we can draw the conclusion that instantons, although not providing confining forces, are presumably strongly related to chiral symmetry breaking (for a thorough discussion see also Ref. 24)), because they seem to be those field excitations responsible for saturating gluon condensation parameters.

ACKNOWLEDGEMENTS

The author would like to express his deep gratitude to the staff of the Theory Division of CERN for the kind hospitality. He gratefully acknowledges discussions with B. Berg, R. Bertlmann, G. Bhanot, N.V. Krasnikov, A. Martin, K.J.M. Moriarty, R. Petronzio, I. Stamatescu and G. Veneziano.

REFERENCES

- 1) A. Di Giacomo and G.C. Rossi - Phys.Letters 100B (1981) 481;
A. Di Giacomo and G. Paffuti - Phys.Letters 108B (1982) 327.
- 2) R. Kirschner et al - Leipzig preprint KMU-HEP 82-06 (1982).
- 3) A. Di Giacomo, K. Fabricius and G. Paffuti - Pisa preprint IFUP-TH 13/82 (1982).
- 4) E.-M. Ilgenfritz and M. Müller-Preussker - Dubna preprint E2-82-598 (1982).
- 5) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Nucl.Phys. B147 (1979)
385,447.
- 6) S.N. Nikolaev and A.V. Radyushkin - Phys.Letters 110B (1982) 476.
- 7) E. Witten - Nucl.Phys. B156 (1979) 269;
G. Veneziano - Nucl.Phys. B159 (1979) 213;
P. Di Vecchia - Phys.Letters 85B (1979) 213;
see also Refs. cited in 8).
- 8) P. Di Vecchia et al - Nucl.Phys. B192 (1981) 392; Phys.Letters 108B
(1982) 323.
- 9) N. Makhdaliani and M. Müller-Preussker, work in progress.
- 10) M. Lüscher - Nucl.Phys. B205 [FS5] (1982) 483.
- 11) E.-M. Ilgenfritz and M. Müller-Preussker - Nucl.Phys. B184 (1981) 443.
- 12) C. Callan, R. Dashen and D. Gross - Phys.Rev. D17 (1978) 2717.
- 13) G. 't Hooft - Phys.Rev. D14 (1976) 3432;
C. Bernard - Phys.Rev. D19 (1979) 3013.
- 14) M. Dyakonov, private communication.
- 15) R. Dashen and D. Gross - Phys.Rev. D23 (1981) 2340;
A. Hasenfratz and P. Hasenfratz - Phys.Letters 93B (1980) 165.
- 16) E.-M. Ilgenfritz and M. Müller-Preussker - Phys.Letters 99B (1981) 128.
- 17) E.-M. Ilgenfritz and M. Müller-Preussker - Dubna preprint E2-82-473 (1982).
- 18) A. Billoire - Phys.Letters 104B (1981) 472.
- 19) J. Stack - Urbana preprint I11-(TH)-82-21 (1982).
- 20) E. Pietarinen - Nucl.Phys. B190 [FS3] (1981) 349.
- 21) M. Creutz - Phys.Rev. D21 (1980) 2308; Phys.Rev.Letters 45 (1980) 313;
B. Berg and J. Stehr - Z.Phys. C - Particles and Fields 9 (1981) 333.
- 22) M. Creutz and K.J.M. Moriarty - Brookhaven preprint (1982).
- 23) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov - Phys.Letters 76B (1978) 471.
- 24) E.V. Shuryak - Nucl. Phys. B203 (1982) 93,116; CERN preprint TH.3351 (1982).

expectation values	SU(2)			SU(3)		
	instanton gas	lattice	"phenomenology"	instanton gas	lattice	phenomenology
$\langle \frac{g^2}{4\pi^2} G_{\mu\nu}^a G_{\mu\nu}^a \rangle > \Lambda_L^{-4}$	$(5.7 \pm 0.8) 10^7$	$(1.4 \pm 0.1) 10^7$ $(0.70 \pm 0.28) 10^7$	$(1.47 \pm 0.93) 10^7$	$(3.8 \pm 1.3) 10^8$	$(2.27 \pm 0.14) 10^8$	$(4.9 \pm 3.3) 10^8$
$-\langle g^3 f^{abc} G_{\mu\nu}^a G_{\mu\nu}^b G_{\mu\nu}^c \rangle > \Lambda_L^{-6}$	$(2.29 \pm 0.13) 10^{14}$	$(1.45 \pm 0.13) 10^{12}$	$(6.6 \pm 6.1) 10^{12}$	$(1.1 \pm 0.4) 10^{15}$	?	$(1.0 \pm 1.0) 10^{15}$
$\langle (g^2 f^{abc} G_{\mu\alpha}^a G_{\mu\beta}^b G_{\mu\gamma}^c)^2 \rangle > \Lambda_L^{-8}$	divergent	?	?	$(2.0 \pm 0.4) 10^{20}$	?	?
$\langle (g^2 f^{abc} G_{\mu\nu}^a G_{\mu\nu}^b G_{\mu\nu}^c)^2 \rangle > \Lambda_L^{-8}$	divergent	?	?	$(4.0 \pm 0.8) 10^{20}$	?	?
$\int d^4x \langle Q(x) Q(0) \rangle > \Lambda_L^{-4}$	$(2.07 \pm 0.10) 10^6$ $-O(10^6)$	$(1.0 \pm 0.6) 10^4$	$(1.2 \pm 0.7) 10^6$	$(7.6 \pm 1.3) 10^6$ $-O(10^7)$	$O(10^5)$	$(2.6 \pm 1.7) 10^7$

- Table -

Fig. 1

Fig. 2

FIGURE CAPTIONS

- Fig. 1 : The static $Q\bar{Q}$ force for $SU(2)$ pure Yang-Mills theory,
curve A : perturbation theory only,
curve B : includes instanton contributions according to Eq. (13);
points taken from the fit of Ref. 19).
- Fig. 2 : The same for $SU(3)$;
data points with statistical errors from own MC simulation (cf. Ref. 17)).