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INFLATION CAN SOLVE THE ROTATION PROBLEM

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ABSTRACT

If the Universe can rotate, why does it rotate so slowly? We formulate this "rotation problem" by introducing a dimensionless measure of the angular velocity in the early Universe which must have been less than 5×10^{-28} in order to be compatible with present-day upper limits on the rotation. Hitherto the slow rate of rotation has only been explained by invoking Mach's principle, which decrees that it vanish. Here we point out that an inflationary epoch in the very early Universe, proposed as a solution to other cosmological problems, could also solve the "rotation problem", and thus make Mach redundant. However, in contrast to Mach's principle, inflation could allow observable vorticity in the present-day Universe.

The cosmologist's job is to tell us what the Universe was like in the past, why it is what it is today, and what it will become. The roll-call of cosmological problems to be investigated has often included the homogeneity, isotropy, flatness and age of the present-day Universe ¹⁾. Why is the Universe now so homogeneous and isotropic on scales which have only recently entered our horizon and were previously held incommunicado by conventional Big Bang cosmology? Why is our Universe nearly flat, having an energy density within one or two orders of magnitude of the critical closure density, despite having lived to the ripe old age of $0(10^{61})$ Planck times? Why is the present-day vacuum energy density so small?

In connection with the observed isotropy, one can be more specific and ask why are the shear ²⁾ and rotational ^{3),4)} anisotropies so small today. In general one can add terms to the Einstein field equations which take into account both shear and rotation. The Hubble expansion parameter of an isotropic Universe can be expressed as ⁵⁾

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{R^2} + \frac{\Lambda}{3} + \dots \quad (1)$$

where R is the Robertson-Walker scale factor, ρ is the mass-energy density, k is the curvature constant ($k=0, \pm 1$ corresponding to a flat, closed or open Universe) and Λ is the cosmological constant. We can add to Eq. (1) possible terms involving $\omega(\sigma)$, the magnitudes of the rotation (shear) tensors. All of the cosmological problems referred to above can be translated into the single question: why (when they are put into dimensionless forms) are the quantities k , Λ , ω and σ so small today?

We can infer from observation ⁶⁾ that today the expansion of the Universe is governed by the matter density (i.e., we are not curvature dominated or in a de Sitter space today). If we take as a conservative upper limit on the mass energy density $\Omega = \rho/\rho_c < 2$, where $\rho_c \approx 8.1 \times 10^{-47} \text{ GeV}^4 h_0^2$ and $h_0 = (H_0/100) < 1$ where H_0 is the present value of the Hubble parameter in $\text{km sec}^{-1} \text{Mpc}^{-1}$, then the matter density term in Eq. (1) is bounded today by

$$\frac{8\pi G}{3} \rho < 9 \times 10^{-84} \text{ GeV}^2 \quad (2)$$

We must then require that all other terms in Eq. (1) are also less than the bound (2). For example, the bound on the cosmological constant (in dimensionless units) becomes

$$\Lambda \lesssim 2 \times 10^{-121} \quad (3)$$

If Λ is the result of a series of cancellations due to the microphysics at very early times, it represents perhaps the highest degree of fine tuning found in physics today. Of course there might be some reason why $\Lambda=0$ exactly. As there is still no satisfactory explanation to this problem, we will not address it any further.

Let us next consider the curvature term in Eq. (1). This term is what brings about the frequently discussed flatness and age problems ¹⁾. Very simply, because it falls off as $1/R^2$ while in the early radiation-dominated era $\rho \sim 1/R^4$, under "natural" initial conditions $k/R^2 = O(m_{\text{Planck}}^2)$ at the Planck time, the Universe would have become curvature dominated very quickly. For $k=+1$ this would mean that the Universe would have collapsed long before it had a chance to make a baryon asymmetry, etc.. Once again we must demand that today this term be smaller than the matter density. Since $R \sim T^{-1}$, we can write the dimensionless quantity

$$\hat{k} \equiv \frac{k}{R^2 T^2} \lesssim \frac{9 \times 10^{-84}}{T_0^2} \lesssim 2 \times 10^{-58} \quad (4)$$

where we have used $T_0 = 2.7 \text{ }^\circ\text{K} = 2.3 \times 10^{-13} \text{ GeV}$. This number $\hat{k}$ remains constant during the adiabatic expansion of a Friedmann Universe, and hence must have been that small initially. But one might expect that all dimensionless quantities at the Planck time typically took a value $O(1)$.

Possible extra terms in Eq. (1) describing the rotation (vorticity) and shear are normally neglected. The shear term, however, falls off quite rapidly ³⁾ $\sigma^2 \sim 1/R^6$ and put in the form of a dimensionless constant $\hat{\sigma}$ the bound (2) becomes

$$\hat{\sigma} \equiv \frac{\sigma}{T^3} \lesssim 6 \times 10^{34} \quad (5)$$

which comfortably accommodates the "natural" initial value of $O(1)$. A potentially stronger constraint ²⁾ on σ comes about when one considers the effect of shear on the expansion rate at the time of Big Bang nucleosynthesis ⁷⁾. In much the same way that the number of neutrino flavours are limited (so as not to overproduce ${}^4\text{He}$) the magnitude of the shear tensor can also be limited. One finds ²⁾ that this corresponds to a limit today

$$\frac{\sigma^2}{3} < 8 \times 10^{-106} \text{ GeV}^2 \quad (6)$$

to be compared with the $9 \times 10^{-84} \text{ GeV}^2$ in Eq. (2). In constant dimensionless units, however, this only yields the limit

$$\hat{\sigma} \leq 6 \times 10^{23} \quad (7)$$

Hence because $\sigma \propto 1/R^3$ a value of unity for $\hat{\sigma}$ at the Planck epoch would have reduced to such an extent that shear anisotropy would not interfere at all with Big Bang nucleosynthesis or our matter dominated era today. Hence there is no "shear problem".

We now consider a possible rotation term in Eq. (2). Its behaviour as a function of T (or R) is slightly more complicated than that of shear because the rotation also depends on the equation of state ⁵⁾. For a matter dominated Universe ($p=0$, isotropic pressure) $\omega \sim 1/R^2$ while for a radiation dominated Universe ($p=\rho/3$) $\omega \sim 1/R$. There are therefore two dimensionless constants we must calculate. The first is ω/T^2 which is constant from the time when the energy density of non-relativistic matter began to dominate over the radiation (approximately the time t_d of photon decoupling) up to today. The second is ω/T which is constant from the Planck era to decoupling in a Friedmann Universe. The bound (2) for rotation becomes

$$\frac{\omega}{T^2} \leq 850 \quad (t_d \leq t \leq t_0) \quad (8.a)$$

$$\hat{\omega} \equiv \frac{\omega}{T} \leq 6 \times 10^{-25} \quad (t_p \leq t \leq t_d) \quad (8.b)$$

where $t_p \approx \frac{1}{2} \times 10^{-43}$ s is the Planck time, $t_d \approx 10^{12}$ s is the time of decoupling and $t_0 \approx 3 \times 10^{17}$ s is the present age of the Universe.

If one considers ³⁾ the effect of rotation on the microwave background radiation, one obtains the limit $\omega < 5 \times 10^{-21} \text{ s}^{-1}$. For larger vorticities, we would have an unacceptably large apparent peculiar velocity. Using this limit one obtains

$$\frac{\omega}{T^2} \leq 0.75 \quad (t_d \leq t \leq t_0) \quad (9.a)$$

$$\hat{\omega} \leq 5 \times 10^{-28} \quad (t_p \leq t \leq t_d) \quad (9.b)$$

The question we pose here as the "rotation problem" is : why was the value of $\hat{\omega} \equiv \omega/T$ as small as $O(10^{-28})$? Again we would expect a value of the order of unity for $\hat{\omega}$ at the Planck epoch.

Before one panics at the sight of yet another cosmological problem, it is worth while asking whether or not the effects of viscosity could damp an initially large vorticity. In the presence of a shear viscosity, the vorticity will be damped ³⁾ by

$$\exp \left[-16\pi \int \eta dt \right] \quad (10)$$

where η is the coefficient of shear viscosity ⁸⁾

$$\eta = \frac{4\pi^2}{450} N(T) T^4 \tau \quad (11)$$

Here $N(T)$ is the number of interacting degrees of freedom and τ is the typical mean free time. There were perhaps three different epochs during which one might have expected a possibility for damping. The last was during the period of decoupling. In this case, $N(T)=2$, and $\tau = (n_Y \sigma_T)^{-1}$ where $n_Y \approx 0.24 T^3$ and the Thompson cross-section is just $\sigma_T \approx 1.7 \times 10^{-25} \text{ GeV}^{-2}$ so that $\eta = 4.27 \times 10^{-4} T$. Conservatively, we might say that the viscosity occurred at $T = 10^5 \text{ }^\circ\text{K}$ for a period of time lasting 10^5 years. In this case, the damping factor (10) is only $\exp[-2 \times 10^{-12}]$ which is negligible. One might also expect viscosity due to neutrinos ⁹⁾ to be important in the lepton era. We use the weak interaction cross-section $\sigma_{\text{weak}} = G_F^2 T^2$ so that $\eta \approx 10^{10}/T$. Using the relation $t \approx 10^{18}/T^2 \text{ GeV}^{-1}$ and integrating up to the time of neutrino decoupling, we find that the factor (10) is at most $O(1)$ [in agreement with Hawking's analysis ³⁾]. Finally, one might expect a contribution to the viscosity damping ¹⁰⁾ some time during the Grand Unified era. In this case it has been estimated that $\eta = 650 T^3$ and integrating over a period ¹¹⁾ $\Delta T \approx 10^{15} \text{ GeV}$ we find once again that the damping factor (10) is only about $O(1/10)$. Hence we conclude that an initial vorticity would not be damped by shear viscosity and we are left with the "rotation problem" introduced above.

One could choose to ignore this problem, as was done until recently for many other cosmological problems. Or one might simply invoke Mach's principle ¹²⁾ which would guarantee that $\hat{\omega} = 0$. However, it is well known by now that many cosmological problems can be resolved by microphysics ^{1), 13)}, especially if there was a super-cooled phase transition during which the Universe passed through a de Sitter or exponentially expanding period at an early stage of its evolution. During such a stage, the dimensionless quantities introduced in Eqs. (4)-(9) are no longer constant. Instead, they fall off exponentially in ways related to their dependences on R , and can be reduced from unity to observationally acceptable values. If the scale factor R increased from R_i to R_f during this phase of exponential expansion, then

$$\frac{\hat{R}_f}{\hat{R}_i} = \left(\frac{R_i}{R_f}\right)^2, \quad \frac{\hat{\sigma}_f}{\hat{\sigma}_i} = \left(\frac{R_i}{R_f}\right)^3, \quad \frac{\hat{\omega}_f}{\hat{\omega}_i} = \left(\frac{R_i}{R_f}\right) \quad (12)$$

However, in order for this inflationary scenario to work, it is necessary that the Universe did not become curvature, shear or rotation dominated before the inflationary epoch got underway. Therefore one needs

$$\hat{K}_i < O\left(\frac{T_{DS}}{T_P}\right)^2, \quad \hat{\omega}_i < O\left(\frac{T_{DS}}{T_P}\right) \quad (13)$$

where T_{DS} is the temperature at which the vacuum energy driving the de Sitter expansion starts to dominate the mass-energy density ρ in the expansion equation (1). If $T_{DS} = O(10^{-3} \text{ to } 10^{-4})T_P$ as in many inflationary scenarios based on grand unified theories, then equation (13) tells us that we still need some fine-tuning of $\hat{K}$ and $\hat{\omega}$ to small values¹⁴⁾, the reason being, that for values of $\hat{K}$ and $\hat{\omega}$ larger than (13), the Universe might not have had the chance to inflate. This is especially true for $k=+1$ where the Universe would recollapse before inflation. Because of the sign of the possible ω term in (1), rotation could also produce the same effect. However, these modest versions of the "problems" can also be avoided if inflation took place at the Planck epoch¹⁵⁾: $T_{DS} \approx T_P$. If we ignore for the moment the possibility (13) that $\hat{K}_i$ and $\hat{\omega}_i$ were $\ll 1$, then the present-day bound (4) and the scaling law (12) tell us that

$$\left(\frac{R_f}{R_i}\right) \gtrsim O(10^{29}) \quad (14)$$

would be sufficient inflation to solve the flatness and age problems related to curvature. In this case, Eq. (12) also tells us that

$$\hat{\omega}_f = O\left(\frac{R_i}{R_f}\right) \lesssim O(10^{-29}) \quad (15)$$

which is consistent with the bound (9.b) that we deduced earlier, thus solving the "rotation problem".

Many inflationary scenarios actually give much more inflation than the minimum amount (14). Thus they predict that the curvature term in the expansion equation (1) is negligible today, implying that the Universe should now have a density very close to the critical value $\Omega = \rho/\rho_c = 1$. The value of Ω is the subject of much observational controversy⁶⁾, and it seems distinctly possible that $\Omega < 1$, and may be $O(1/10)$. This does not kill all inflationary scenarios, but suggests that the inflation factor (R_f/R_i) may actually be very

close to the lower bound (14). In this case the value of $\hat{\omega}_F$ might be very close to the upper bound (15), and hence the present-day rotation ω might not be much smaller than the bound (9.a) deduced from the microwave background radiation. In this connection we recall the recent claim ⁴⁾ to have observed evidence for rotation close to the microwave background limit (9.a).

We have seen in this note that the cosmological inflation introduced ¹⁾ for other reasons may also solve the "rotation problem". In this sense it makes Mach redundant, since we no longer need to invoke his principle ¹²⁾ to understand why distant objects rotate so slowly relative to a bucket of water whose surface is flat, or a ballet-dancer whose hands fall limply at his sides ⁸⁾. Moreover, inflation offers the enticing possibility that Mach's principle might be slightly false. There is even the faint suggestion that distant objects may rotate at an observable rate relative to a truly flat water-bucket or a truly limp ballet-dancer. If this turns out to be the case, it could be construed as evidence for the validity of some kind of inflationary cosmology.

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