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THE VALUE OF ϵ'/ϵ IN MODELS BASED ON
 $SU(2)_L \times SU(2)_R \times U(1)$

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ABSTRACT

We emphasize that the ϵ'/ϵ ratio can play a crucial rôle in distinguishing between various electroweak models. It is shown that $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories provide a natural scenario for obtaining either vanishing or very small values for the ϵ'/ϵ ratio. They also have the attractive feature of establishing a link between the strength of CP and P breaking.

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1. - INTRODUCTION

The experimentally observed CP violation can be easily incorporated into the standard electroweak gauge theory of Glashow, Salam and Weinberg. This can be done by hand by considering complex Yukawa couplings in the Lagrangian which in turn lead to a CP violating Kobayashi-Maskawa (KM) phase¹⁾ when at least three generations of quarks are considered. Alternatively, one may incorporate CP breaking by introducing two²⁾ or more³⁾ Higgs doublets. In this case, one may start with a CP invariant Lagrangian and have CP spontaneously broken by the vacuum expectation values of the Higgs fields. A common feature of models based on $SU(2)_L \times U(1)$ is that since the left-right symmetry is broken from the onset, no relation exists between P and CP violation. Such a relation was suggested⁴⁾ some time ago in the framework of an $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory with four quarks.

It is clearly important to be able to distinguish experimentally between these theoretical schemes. The ratio ϵ'/ϵ can play a crucial rôle in establishing this distinction. Indeed, it has recently been emphasized⁵⁾ that in the framework of $SU(2) \times U(1)$ the three scalar doublets model with natural flavour conservation and spontaneous CP violation⁶⁾ predicts values of ϵ'/ϵ (i.e., $\epsilon'/\epsilon \approx 0.05$) at the limit of inconsistency with the present experimental data⁷⁾ ($|\epsilon'/\epsilon| = 0.003 \pm 0.015$). The popular KM model also predicts⁸⁾ a non-vanishing value for ϵ'/ϵ ($0.002 \leq \epsilon'/\epsilon \leq 0.02$), which should be tested by the next generation of experiments. Smaller values of ϵ'/ϵ (i.e., $\epsilon'/\epsilon \leq 10^{-3}$) can be obtained in models which are not natural in the technical sense, involving, in general, flavour changing neutral currents mediated either by scalar bosons or by gauge particles pertaining to some horizontal gauge interaction. However, as has recently been emphasized⁹⁾, in the simplest realization of these models in the context of $SU(2) \times U(1)$, if one does not allow for any tuning of the parameters, the neutral scalar boson masses are required to be in the TeV region and for these values of scalar boson masses one expects the scalar self-interactions to become strong at high energies. In the framework of $SU(2)_L \times SU(2)_R \times U(1)$ models, it was shown some time ago⁴⁾ that ϵ'/ϵ would automatically vanish in the case of four quarks if one neglects W_L - W_R mixing and scalar boson contributions. In general, this is no longer true when six quark flavours are considered.

In this paper we show that $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories provide a natural scenario for having a small value for the ϵ'/ϵ ratio ($\leq 10^{-4}$). This result applies to a large number of theories based on $SU(2)_L \times SU(2)_R \times U(1)$ and with an arbitrary number of quark flavours, provided the left-handed KM matrix (K_L) can be made orthogonal. This can be realized either by "fitting" or in a technically natural way. Having K_L orthogonal will automatically guarantee that

the interesting connection between P and CP violation continues to be valid even when more than four quark flavours are considered. Furthermore, we will show that if in addition we impose manifest left-right symmetry, then a vanishing value of ϵ'/ϵ is obtained. The paper is organized as follows. Section 2 establishes our notation and provides some useful relations between the left- and right-handed mixing matrices. In Section 3 we calculate the ϵ'/ϵ ratio in the framework of left-right symmetric theories with spontaneous CP violation. Section 4 is devoted to the question of naturalness. In Section 5 we evaluate ϵ'/ϵ in $SU(2)_L \times SU(2)_R \times U(1)$ theories where manifest left-right symmetry is not imposed. Some further comments and our conclusions are contained in Section 6. Finally, in the two Appendices we give examples of how these ideas can be implemented in a technically natural way.

2. - THE QUARK MIXING MATRICES

For completeness and in order to establish our notation, we next make a brief analysis of the quark mixing matrices in $SU(2)_L \times SU(2)_R \times U(1)$ models, for an arbitrary number of quark flavours. Consider the Yukawa interaction:

$$\mathcal{L}_Y = \sum_{i,j,\alpha} g_{ij}^\alpha \bar{\psi}_{iL}^\alpha \psi_{jR}^\alpha \phi_\alpha + h.c. \quad (2.1)$$

where $\psi_{iL,R}^\alpha$ denote the left-handed and right-handed fermion doublets and ϕ_α are scalar bosons transforming as $(1/2, 1/2)$ under $SU(2)_L \times SU(2)_R \times U(1)$. After the scalars ϕ_α have acquired vacuum expectation values:

$$\langle \phi^\alpha \rangle = \begin{pmatrix} v'_\alpha e^{i\eta_\alpha} & 0 \\ 0 & v_\alpha e^{i\zeta_\alpha} \end{pmatrix} \quad (2.2)$$

the mass matrices for the up and down quarks are respectively given by:

$$(M^u)_{ij} = \sum_\alpha g_{ij}^\alpha v'_\alpha e^{i\eta_\alpha}, \quad (M^d)_{ij} = \sum_\alpha g_{ij}^\alpha v_\alpha e^{i\zeta_\alpha} \quad (2.3)$$

In general, these matrices are diagonalized by bi-unitary transformations:

$$U_L^{u,d} M^{u,d} U_R^{u,d\dagger} = D^{u,d} \quad (2.4)$$

The physical states are then given by

$$\psi_{L(R)}^{u,d} = U_{L(R)}^{u,d} \psi_{L(R)}^{u,d} \quad (2.5)$$

Then, the left-handed (right-handed) mixing matrices are defined as

$$K_{L(R)} = U_{L(R)}^u U_{L(R)}^{d\dagger} \quad (2.6)$$

In general, an $n \times n$ unitary matrix is described by $1/2 n(n-1)$ angles and $1/2 n(n+1)$ phases. By choice, a maximal number of phases are usually removed from the left-handed mixing matrix K_L . This is done through a redefinition of the left-handed physical fields, which allows us to eliminate $(2n-1)$ phases. Clearly this implies corresponding transformations on the right-handed physical quarks in order to keep the diagonal mass matrix real.

The number of CP violating parameters in $K_{L(R)}$ is then:

$$\begin{aligned} \frac{1}{2} (n-1) (n-2) & \text{ for } K_L \\ \frac{1}{2} n (n+1) & \text{ for } K_R \end{aligned} \quad (2.7)$$

We must further remark that in the limit where there is no $W_L - W_R$ mixing, the over-all phase of K_R is unobservable, bringing us then to a total of $n(n-1)$ CP violating phases. At this point it is worth noting that obtaining a naturally vanishing $W_L - W_R$ mixing at the tree level continues to be an open question¹⁰⁾. It is clear that for $n = 2$, K_L can always be made real but, as has been emphasized some time ago⁴⁾, one may still have CP violation through W_R gauge interactions. For $n > 2$, both K_L and K_R will, in general, contain CP violating phases and an important contribution to CP breaking will obviously come from W_L interactions, as is the case in the KM model.

So far, we have analyzed the general case, without introducing any constraints in the $SU(2)_L \times SU(2)_R \times U(1)$ model. If we require the theory to have a manifest left-right symmetry and spontaneous CP violation, the mixing matrices K_L and K_R are related. This will be shown in the following theorem.

Theorem: In a gauge theory based on $SU(2)_L \times SU(2)_R \times U(1)$, with manifest left-right symmetry and spontaneous CP violation, the left- and right-handed mixing matrices are related by ^{*)}:

$$K_R = J^u K_L^* J^{d\dagger} \quad (2.8)$$

where $J^{u,d}$ are, in general, unrelated unitary diagonal matrices.

Proof: In theories with manifest left-right symmetry and spontaneous CP violation, we have symmetric mass matrices, which implies

$$M M^\dagger = (M^\dagger M)^* \quad (2.9)$$

From Eq. (2.4) we then obtain

$$(U_R^* U_L^\dagger) D^2 = D^2 U_R^* U_L^\dagger \quad (2.10)$$

As quark masses are non-degenerated, we immediately conclude that the unitary matrix $U_R^* U_L^\dagger$ is diagonal, namely

$$\begin{aligned} U_R^u &= J^u U_L^{u*} \\ U_R^d &= J^d U_L^{d*} \end{aligned} \quad (2.11)$$

From (2.6) and (2.11) one finally obtains the result of (2.8). Note that the requirement of spontaneous CP violation plays here a crucial rôle. If one had introduced complex Yukawa couplings, but real vacuum expectation values for the scalar fields, then the requirement of a real K_L would immediately imply a real K_R and therefore no CP violation through gauge interactions.

3. - AN ALGEBRAIC MECHANISM

We will now describe a mechanism for obtaining $\epsilon'/\epsilon = 0$ in a class of models with left-right symmetry and spontaneous CP violation. A clear prerequisite for this is to exclude CP violation via the usual KM mechanism. Therefore we require that K_L can be made real. We will discuss in Section 4 and in Appendices A and B how this can be achieved in a technically natural way. We thus set

^{*)} A similar property was established some time ago¹¹⁾, although the previous derivation is only valid on a special basis where the left-right symmetry is not explicit.

$$K_L = O_L \quad (3.1)$$

which, in view of Eq. (2.8), implies:

$$K_R = J^u O_L J^{d\dagger} \quad (3.2)$$

where

$$\begin{aligned} J^u &= \delta_{ij} \exp(i\chi_i) \\ J^d &= \delta_{ij} \exp(i\varphi_i) \end{aligned} \quad (3.3)$$

Next we examine the implications of (3.1) and (3.2) for the value of $|\epsilon'|$ which is usually¹²⁾ defined as:

$$|\epsilon'| = \frac{1}{\sqrt{2}} \frac{\text{Im} \langle 2\pi, I=2 | H_{\text{eff}} | K^0 \rangle}{\langle 2\pi, I=0 | H_{\text{eff}} | K^0 \rangle} \quad (3.4)$$

where a phase convention intervening in the definition of the K^0 state was chosen in such a way that the $I = 0$ amplitude is real. In other words, ϵ' will vanish if the $I = 2$ and $I = 0$ amplitudes bear the same phase. We neglect for the moment the small corrections to ϵ'/ϵ which will originate from Higgs boson exchanges and W_L - W_R mixing (see, however, Section 5 for further comments). Considering, therefore, only the contributions arising from gauge interactions, we give some representative contributions to H_{eff} . The graph of Fig. 1a contributes both to $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes, while the graph 1b only contributes to the $\Delta I = 1/2$ transition. At this point, we would like to emphasize that the reasoning we will use to obtain $\epsilon' = 0$ can be generalized to any diagram involving only one weak boson propagator and any number of gluons. Furthermore, our argument will in no way be contingent on the dominance of the penguin diagram. We first note that once the summation over intermediate quarks is performed, the momentum running in the heavy gauge boson propagator can safely be neglected. No imaginary part arises from the Feynman diagrams themselves and therefore potential phases only originate from the mixing matrices. For the two graphs of Fig. 1 we obtain:

$$G^{1.a} = \sum_K O_{K2} O_{K1} \left[g_{LK}^{1.a} + g_{RK}^{1.a} e^{i(\varphi_2 - \varphi_1)} \right] \quad (3.5)$$

$$G^{1.b} = \sum_K O_{K2} O_{K1} \left[g_{LK}^{1.b} + g_{RK}^{1.b} e^{i(\varphi_2 - \varphi_1)} \right]$$

where $g_k^{(i)}$ is the contribution to the effective $\Delta S = 1$ Hamiltonian coming from graph (i), with the $Q = 2/3$ quark K (all mixing angles excluded), while $G^{(i)}$ is the total contribution of graph (i). The essential point is that, as a result of (3.2) in an L-R symmetric model, the contributions due to W_L and W_R exchange can be obtained from each other through the substitution

$$\begin{aligned} M_{W_L} &\longleftrightarrow M_{W_R} \\ \gamma_5 &\longleftrightarrow -\gamma_5 \end{aligned} \quad (3.6)$$

and by the inclusion of a phase entirely determined by the external legs. Since we are interested in the $K \rightarrow 2\pi$ transition, namely in a pure s wave parity violating process (which implies the intervention of only one γ_5), and taking into account the fact that any number of gluon exchanges will affect identically graphs with W_L or W_R , it follows that:

$$\langle 2\pi | g_K^{(i)} | K \rangle = - \frac{M_{W_L}^2}{M_{W_R}^2} \langle 2\pi | g_{LK}^{(i)} | K \rangle \quad (3.7)$$

If we denote by $A^{(i)}$ the contribution of graph (i) to the $K \rightarrow 2\pi$ transition amplitude, we can write:

$$A^{(i)} = A_L^{(i)} \left[1 - \frac{M_{W_L}^2}{M_{W_R}^2} e^{i(\varphi_2 - \varphi_1)} \right] \quad (3.8)$$

irrespective of the amplitude ($I = 0$ or $I = 2$) they are contributing to. It is thus clear that the relative phase between the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes vanishes in the present case, implying $\epsilon' = 0$.

At this point, we would like to dissipate one reasonable fear. Could it be that all CP violating effects would effectively vanish in the above scheme? In order to answer this question, we now concentrate our attention on the $K^0 - \bar{K}^0$ system. The dominant contributions to the $\Delta S = 2$ transition is given in Fig. 2. If one denotes by $a_{LL}^{k\bar{l}}$, $a_{LR}^{k\bar{l}}$, $a_{RR}^{k\bar{l}}$ the absolute value of the amplitudes corresponding respectively to Figs. 2a, 2b and 2c, one obtains:

$$A(K^0 \bar{K}^0) = \sum_{kl} \left[a_{LL}^{kl} + a_{LR}^{kl} e^{i(\varphi_2 - \varphi_1)} + a_{RR}^{kl} e^{2i(\varphi_2 - \varphi_1)} \right] \quad (3.9)$$

This result again reflects the fact that only external legs give rise to phases. On dimensional grounds one obtains

$$\begin{aligned} a_{RR}^{kl} &= \left(\frac{M_{WL}^2}{M_{WR}^2} \right)^2 a_{LL}^{kl} \\ a_{LR}^{kl} &= -2 r^{kl} \frac{M_{WL}^2}{M_{WR}^2} a_{LL}^{kl} \end{aligned} \quad (3.10)$$

where r^{kl} are enhancement factors¹³⁾ which, somewhat surprisingly, turn out to be very large^{*)}. The u quark contribution is known to be negligible, while the c quark contribution turns out to be

$$2 r^{cc} \gtrsim 400$$

This implies a non-zero CP violation strength defined by

$$|\varepsilon| \approx \frac{1}{2\sqrt{2}} \frac{\text{Im } A(K^0 \bar{K}^0)}{\text{Re } A(K^0 \bar{K}^0)} \quad (3.11)$$

in the basis where $\langle 2\pi; I = 0 | H_{\text{eff}}^{\Delta S=1} | K^0 \rangle$ is made real^{**)}.

*) The authors of Ref. 13) have observed that the box diagram involving one left and one right gauge boson exchange (Fig. 2b) is strongly enhanced with respect to its left-left analogue, due to the conjunction of different factors, namely the counting of graphs, PCAC equations and, more important, asymptotically logarithmic behaviour. They then use the requirement that this left-right contribution be at most of the same order as the pure left-handed exchange contribution to provide a lower limit on the right-handed gauge boson mass $M_{WR} \gtrsim 20 M_{WL}$. This is of course contingent on some assumptions: saturation of $\Delta M(K^0 \bar{K}^0)$ by the c exchange diagram [see, in this respect, Ref. 14)], absence of unlikely cancellation with t or even scalar exchange diagrams. This lower bound on M_{WR} , combined with the evolution equations for the coupling constants in the simplest $SO(10)$ unification model¹⁵⁾, would lead to a stronger limit $M_{WR} \gtrsim 10^9$ GeV. This would clearly obliterate the rôle of the right-handed sector in the weak interaction processes considered here. We note, however, that there are no convincing arguments in favour of a minimal embedding in $SO(10)$. In particular, generalization to supersymmetric models or to models involving new interactions at some intermediary scale would invalidate that extrapolation. It is thus our feeling that any phenomenological analysis of the possible existence and implications of right-handed currents has to be considered independently of this last limit.

**) If we had $r^{cc} = 1$ (no enhancement) we should find

$$A(K^0 \bar{K}^0) \approx \sum_{kl} a_{LL}^{kl} \left[1 - \frac{M_{WL}^2}{M_{WR}^2} e^{i(\varphi_2 - \varphi_1)} \right]^2$$

such that a suitable redefinition of the $K^0, \bar{K}^0$ phases would make $\langle 2\pi; I = 0, 2 | H_{\text{eff}} | K^0 \rangle$ and $A(K^0 \bar{K}^0)$ simultaneously real.

4. - THE QUESTION OF NATURALNESS

We have seen that for three or more generations of quarks, both K_L and K_R will, in general, contain CP violating phases. Since in this work we are particularly interested in theories where K_L is orthogonal for at least three quark generations, the question arises of how to achieve this in a technically natural way. At this point it should be emphasized that the counting of phases that leads to Eq. (2.7) presupposes an arbitrary Yukawa interaction. Indeed, this arbitrariness just reflects our ignorance about the mechanism of fermion mass generation. In a more sophisticated theory, the Yukawa interactions may have some extra symmetries, either global or discrete. Our point is that some of these symmetries may naturally lead to an orthogonal K_L even when more than two generations of quarks are considered. A sufficient condition in order to obtain an orthogonal K_L is to have

$$U_L^{u,d} = O_L^{u,d} \quad (4.1)$$

Although (4.1) is only a sufficient condition, we find it reasonable to assume it, since otherwise one would have to consider some conspiracy between phases coming from U_L^u and U_L^d (see, however, Appendix A). From Eqs. (2.4) and (2.11) we find

$$O^{u,d} M^{u,d} O^{u,d T} = D^{u,d} J^{u,d} \quad (4.2)$$

Since $M^{u,d}$ are symmetric matrices, this implies

$$[\text{Re } M^{u,d}, \text{Im } M^{u,d}] = 0 \quad (4.3)$$

If we work on the "nearly physical" basis where the mass matrices are diagonal but still complex, we will have $K_L = K_R$ and orthogonal, with phases now appearing in the quark masses:

$$m_j^u = |m_j^u| e^{i\varphi_j} \quad (4.4)$$

Since G^{1a} is independent of quark masses, while G^{1b} is only a function of $|m_k|$, this naturally ensures that $\epsilon' = 0$. However, one still obtains a non-vanishing value for ϵ in the $K^0 - \bar{K}^0$ system since the diagram of Fig. 2b depends on m_k due to its left-right structure. So, although $M^{u,d+} M^{u,d}$ and $M^{u,d} M^{u,d+}$ are real matrices, we can still have CP violation in $SU(2)_L \times SU(2)_R \times U(1)$ models. Clearly, this situation is impossible to achieve in the $SU(2)_L \times U(1)$ framework. A simple solution^{*)} to (4.3) is, of course, given by

$$M^u = D^u J^u, \quad M^d \text{ real} \quad (4.5)$$

A detailed model will be given in Appendix A.

5. - ϵ'/ϵ SUPPRESSION DUE TO $\Delta S = 2$ ENHANCEMENT

We will now consider another class of theories also based on $SU(2)_L \times SU(2)_R \times U(1)$, but where the constraints of manifest left-right symmetry and spontaneous CP violation will not be imposed. However, we will keep the requirement that CP violation only arises from W_R interactions (i.e., $K_L = 0$). It will be seen that in this class of theories, the ratio ϵ'/ϵ will no longer vanish, but a very small value is obtained under quite reasonable assumptions. Our argument will be based on the fact already stressed in the previous section, that there is an enhancement factor in the graph of Fig. 2b, while no such enhancement occurs in $\Delta S = 1$ processes.

Before actually estimating the value of ϵ'/ϵ , we would like to make a few comments about this class of theories, concerning in particular our requirement of having a real K_L matrix. As we have stressed before, one has essentially two options in achieving an orthogonal left-handed mixing matrix. One way is to require that the model be technically natural and to introduce some symmetries in the Lagrangian which would guarantee the reality of K_L , at least in the tree approximation. It turns out that since here we do not require the theory to be manifestly left-right symmetric, it is somewhat easier to implement technical naturality in this class of theories. Indeed, we show in Appendix B that in the case of n generations a simple Z_n symmetry is sufficient for our purposes. In technically natural theories one has usually to introduce various scalar multiplets. Some of these scalars may, in principle, have rather light masses and give important contributions to CP violation. An analysis of these scalar effects has to be done in a case by case analysis and is made difficult by the complexity of the scalar potential.

^{*)} It is worth noticing that for this solution, the $D^0 - \bar{D}^0$ system has no CP violation.

An alternative way to have an orthogonal K_L is obviously by simply "fitting" Yukawa couplings which are, in general, arbitrary. This procedure is in fact similar to what is usually done in the standard model in order to obtain a correct fermion mass spectrum. The above scenario can be achieved by requiring, for example, that one of the mixing angles in K_L vanish (or at least be very small), which is not in disagreement with present experimental data. In this case, only one $\phi(1/2, 1/2)$ multiplet is needed, since here we do not require either spontaneous CP violation nor left-right symmetry. After symmetry breaking, this scalar multiplet splits into two $SU(2) \times U(1)$ doublets, one being superheavy, with a mass of the order of M_{WR} . In this case, the constraints required to guarantee a small W_L - W_R mixing are also sufficient to ensure the absence of neutral scalar flavour changing interactions¹⁶⁾.

We now turn to the actual computation of ϵ'/ϵ . In order to keep our arguments as general as possible, at this stage we will not consider any specific model. Although the right-handed mixing matrix is an arbitrary unitary matrix, an order of magnitude estimate of ϵ'/ϵ can be made. Going back to Eq. (3.10), we note that the strong enhancement in a_{LR} arises when internal c or t quarks are exchanged. If we denote by δ a typical phase associated with the right-handed mixing matrix in that sector, we obtain:

$$|E| \approx \sin \delta \frac{a_{LR}}{a_{LL}} \quad (5.1)$$

$$\approx \sin \delta \frac{M_{WL}^2}{M_{WR}^2} 2r$$

where r is a typical enhancement factor introduced in Eq. (3.10). If we assume $\delta = O(1)$ (which we believe to be a reasonable assumption), Eq. (5.1) enables us to obtain a constraint on M_{WR}^2 , namely:

$$M_{WR}^2 \approx 10^5 M_{WL}^2 \quad (5.2)$$

In order to get an order of magnitude estimate for ϵ'/ϵ , we observe that one expects:

$$\frac{\text{Im} \langle 2\pi, I=2 | H_{\text{eff}} | K^0 \rangle}{\text{Re} \langle 2\pi, I=2 | H_{\text{eff}} | K^0 \rangle} \approx \frac{\text{Im} \langle 2\pi, I=0 | H_{\text{eff}} | K^0 \rangle}{\text{Re} \langle 2\pi, I=0 | H_{\text{eff}} | K^0 \rangle} \approx \sin \sigma \frac{M_{WL}^2}{M_{WR}^2} \quad (5.3)$$

where σ is a typical phase occurring in U_R . Since, in general, we do not expect any cancellation between the phases of the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes, one obtains:

$$\varepsilon' = \frac{1}{\sqrt{2}} \frac{\text{Im} \langle 2\pi, I=2 | H_{\text{eff}} | K^0 \rangle}{\langle 2\pi, I=0 | H_{\text{eff}} | K^0 \rangle} \approx \frac{1}{20} \sin \sigma \frac{M_{WL}^2}{M_{WR}^2} \quad (5.4)$$

where a redefinition of the K^0 state phase has been done in such a way that the $\Delta I = 1/2$ amplitude is made real. The numerical factor in Eq. (5.4) stands for the enhancement of the $\Delta I = 1/2$ amplitude with respect to the $\Delta I = 3/2$ amplitude. From (5.1) and (5.4) we finally obtain

$$\frac{\varepsilon'}{\varepsilon} \approx \frac{\sin \sigma}{20 \sin \delta (2r)} \quad (5.5)$$

If we now assume that σ and δ are of the same order of magnitude and take $r \sim r^{\text{cc}} \sim 200$, we get:

$$\frac{\varepsilon'}{\varepsilon} \approx 10^{-4} \quad (5.6)$$

Since the essential of the remarks of this section rely on dimensional arguments, one might be led to think that in this case it is possible to dispose of the requirement of negligible left-right mixing. However, this is not true since, in the presence of CP violating phases, left-right mixing does contribute to P violating processes like $K \rightarrow 2\pi$ in Fig. 1a even in the case of left-right symmetry. It has been shown¹⁷⁾ that such a contribution is usually strongly enhanced for reasons related to PCAC and would thus upset the above conclusions.

6. - COMMENTS AND CONCLUSIONS

We would like to add here a few comments on the issue of the value of the neutron electric dipole moment (d_n^e). In models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$, a non-vanishing value for d_n^e can arise due to:

- i) a non-zero strong CP parameter θ_{strong} or, equivalently, a complex value for $\det M^u \cdot \det M^d$;
- ii) radiative corrections involving a W_L - W_R mixing (Fig. 3).

The first issue can be disposed of essentially in two ways. Starting from a Lagrangian which exhibits explicit left-right symmetry, one is naturally led to require a vanishing θ_{strong} parameter¹⁸⁾ since it appears as the coefficient of the parity violating term $F_{\mu\nu} \tilde{F}^{\mu\nu}$. However, the mass matrix which results from spontaneous symmetry breakdown still, in general, introduces phases. One must therefore require $\det M^u \cdot \det M^d$ to be real on the basis where the explicit left-right symmetry is formulated.

Alternatively, one could rely on a Peccei-Quinn-like symmetry¹⁹⁾ to rotate away the first contribution. The corresponding axion can be decoupled from light hadronic matter in one of the usual ways. The cosmological problem recently raised²⁰⁾ can be solved by generalizing the Kim model²¹⁾ to $SU(2)_L \times SU(2)_R \times U(1)$. Once this problem is fixed, one stays with the other source of d_n^e , namely the W_L - W_R mixing in gauge boson exchanges, which has been neglected in the present work. This issue has been studied²²⁾ in a framework similar to that of Section 3 where the authors consider only a four-quark model. Considering that the leading contribution to ϵ'/ϵ comes from W_L - W_R mixing, they related the neutron electric dipole moment to the value of ϵ' in the following way:

$$d_n^e \approx 10^{-21} \cdot 3 \epsilon'$$

The present experimental limit

$$d_n^e < 10^{-25}$$

would then require

$$\epsilon'/\epsilon < 10^{-2}$$

In conclusion, we have shown in this paper that in electroweak interaction models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$, the gauge contribution to ϵ'/ϵ is automatically suppressed by a factor of about 10^4 provided that:

- i) W_L - W_R mixing is negligible;
- ii) the left-handed mixing matrix K_L can be made real.

In addition, if manifest left-right symmetry is imposed, leading to an equality of the real part of K_L and K_R (Section 2), an algebraic mechanism cancels the contribution to ϵ' due to the exchange of gauge bosons.

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APPENDIX A - A NATURAL MODEL WITH MANIFEST LEFT-RIGHT SYMMETRY

The previous approach, based on the mixing matrices $K_{L,R}$, leads only to a sufficient condition (4.3) to implement our basic scenario, namely

$$\begin{aligned} M^{u,d} &= M^{u,d\tau} \\ [Re M^{u,d}, Im M^{u,d}] &= 0 \Rightarrow K_L = O_L \end{aligned} \quad (A.1)$$

Starting now with more general symmetric mass matrices, $M^{u,d}$, it is, however, possible to derive a weaker constraint. Let us indeed consider a manifest left-right symmetric theory with

$$M^{u,d} = I_L^{u,d\dagger} M_{real} I_R^{u,d} \quad (A.2)$$

where $I_{L,R}^{u,d}$ are unitary diagonal matrices. These mass matrices thus contain at most $2n-1/2 n(n-1)$ independent phases. Since the most general symmetric matrix has $n(n+1)/2$ independent phases, we already conclude that additional constraints are needed for $n \geq 3$. However, the present approach certainly includes the previous one (A.1) because we now obtain

$$[Re M^{u,d}, Im M^{u,d}] = Im M^{u,d} M^{u,d\dagger} \quad (A.3)$$

with

$$\begin{aligned} U_L^{u,d} &= O^{u,d} I_L^{u,d} \\ U_R^{u,d} &= O^{u,d} I_R^{u,d} \end{aligned} \quad (A.4)$$

So (A.1) may be generalized in the following way

$$\begin{aligned} M^{u,d} &= M^{u,d\tau} \\ M^{u,d} &= I_L^{u,d\dagger} M_{real} I_R^{u,d} \Rightarrow K_L = O^u I_L^u I_L^{d\dagger} O^{d\tau} \end{aligned} \quad (A.5)$$

We may find new natural solutions in addition to those already quoted, by considering the useful property

$$\begin{aligned} [O^{u,d}, I] &= 0 \\ I_{ii} \neq I_{jj} &\Rightarrow O^{u,d} = \mathbb{1} \end{aligned} \quad (A.6)$$

Let us consider, for example, the case

$$[O^u, I_L^u I_L^{d\dagger}] = 0 \quad (A.7)$$

with six quark flavours ($n=3$). From (A.5) it is clear that if (A.7) is satisfied then K_L can be made real by a simple phase transformation. This can be achieved by the following mass matrices:

$$M^u = \begin{pmatrix} d\langle\phi_1^u\rangle & 0 & 0 \\ 0 & a\langle\phi_2^u\rangle & c\langle\phi_2^u\rangle \\ 0 & c\langle\phi_2^u\rangle & b\langle\phi_2^u\rangle \end{pmatrix} \quad (A.8)$$

$$M^d = \begin{pmatrix} 0 & e'\langle\phi_1^d\rangle & f'\langle\phi_1^d\rangle \\ e'\langle\phi_1^d\rangle & a'\langle\phi_2^d\rangle & c'\langle\phi_2^d\rangle \\ f'\langle\phi_1^d\rangle & c'\langle\phi_2^d\rangle & b'\langle\phi_2^d\rangle \end{pmatrix} \quad (A.9)$$

These are naturally implemented by imposing, for example, discrete transformations only on

$$\begin{aligned} \psi_{1L} &\rightarrow e^{-i\frac{2\pi}{3}} \psi_{1L} \\ \psi_{1R} &\rightarrow e^{i\frac{2\pi}{3}} \psi_{1R} \\ \phi_1 &\rightarrow e^{i\frac{2\pi}{3}} \phi_1 \end{aligned} \quad (A.10)$$

identifying $\phi_1^u = \tilde{\phi}_1^d \equiv \phi_1$ with $v'_{1,2} \ll v_{1,2}$ in (2.2). In this particular application, at least one new scalar multiplet, uncoupled to fermions, is needed in order to explicitly break the accidental $U(1)$ symmetry of the potential:

$$\phi_1 \rightarrow e^{i\alpha(x)} \phi_1 \quad (A.11)$$

This will simultaneously ensure complex vacuum expectation values for ϕ_1 and ϕ_2 .

APPENDIX B - A NATURAL MODEL WITHOUT MANIFEST LEFT-RIGHT SYMMETRY

In this Appendix we will give an example of how to implement the ideas of Section 5 in a technically natural way. We will indeed construct a class of models where the left-handed mixing matrix is orthogonal for an arbitrary number of quarks without imposing the constraint of manifest left-right symmetry.

Consider the following Yukawa interaction:

$$\mathcal{L}_Y = \sum_{i,j} g_{ij} \bar{\psi}_{iL} \psi_{jR} \phi_j + h.c. \quad (B.1)$$

After spontaneous symmetry breaking, the mass matrices for the up and down quarks can be diagonalized in the following way:

$$\begin{aligned} O_L^u M^u I_R^{u\dagger} O_R^{uT} &= D^u \\ O_L^d M^d I_R^{d\dagger} O_R^{dT} &= D^d \end{aligned} \quad (B.2)$$

$I_R^{u,d}$ being again diagonal unitary matrices. The left-handed mixing matrix is $K_L = O_L^u O_L^{dT}$ and therefore orthogonal. There is no KM mechanism in the left-handed sector, while there will be CP violating effects due to W_R exchange processes, as explained in Section 5.

We now discuss the question of naturalness. The Yukawa interaction of Eq. (B.1) has an $U(1) \times U(1) \times U(1)$ global symmetry under which

$$\phi_j \rightarrow e^{i\chi_j} \phi_j ; \psi_{jR} \rightarrow e^{-i\chi_j} \psi_{jR} \quad (B.3)$$

If one imposes this symmetry in the whole Lagrangian, there would be Goldstone bosons when spontaneous breaking occurs. This can easily be avoided by requiring that the Lagrangian be only invariant under some discrete subgroup of the global symmetry (B.3). In the case of n generations, a possible choice is Z_n under which

$$\phi_j \rightarrow e^{i \frac{2\pi}{n} j} \phi_j ; \psi_{jR} \rightarrow e^{-i \frac{2\pi}{n} j} \psi_{jR} \quad (B.4)$$

It is clear that $\mathcal{L}_Y$ given by (B.1) is still the most general Yukawa interaction consistent with (B.4). Moreover, for only three generations of quarks, the scalar potential will contain terms like $\text{Tr}(\phi_3^\dagger \phi_2) \text{Tr}(\phi_3^\dagger \phi_1)$ and permutations which will, on the one hand give mass to the would-be Goldstone bosons and, on the other, generate the CP violating phases.

REFERENCES

- 1) M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
- 2) T.D. Lee, Phys. Rev. D8 (1973) 1226 and Phys. Rep. 96 (1974) 143;
P. Sikivie, Phys. Lett. 65B (1976) 141.
- 3) S. Weinberg, Phys. Rev. Lett. 31 (1976) 657.
- 4) R.N. Mohapatra and J.C. Pati, Phys. Rev. D11 (1975) 566.
- 5) A.I. Sanda, Phys. Rev. D23 (1981) 2647;
N.G. Deshpande, Phys. Rev. D23 (1981) 2653;
J.F. Donoghue, J.S. Hagelin and B.R. Holstein, Phys. Rev. D25 (1982) 195;
D. Chang, Carnegie Mellon Report No C00-3066-172 (1981).
See, however:
Y. Dupont and T.N. Pham, CNRS report No A498-0482 (1982).
- 6) G.C. Branco, Phys. Rev. D22 (1980) 2901.
- 7) Particle Data Group, Rev. Mod. Phys. 52 (1980) 51.
- 8) F. Gilman and M.B. Wise, Phys. Lett. 83B (1979) 83;
B. Guberina and R.D. Peccei, Nucl. Phys. B163 (1980) 289.
- 9) G.C. Branco and A.I. Sanda, Rockefeller Univ. Rep. No 82/B/23, VPI-HEP-82/2
(1982).
- 10) G.C. Branco and G. Senjanović, Phys. Rev. D18 (1978) 1621;
G.C. Branco and J.-M. Gérard, in preparation.
- 11) R.N. Mohapatra, F.E. Paige and D.P. Sidhu, Phys. Rev. D17 (1978).
- 12) See, for instance:
K. Kleinknecht, Ann. Rev. Nucl. Sci. 26 (1976) 1.
- 13) G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48 (1982) 843.
- 14) J. Ellis and J. Hagelin, CERN preprint
- 15) T.G. Rizzo, Phys. Rev. D24 (1981) 704 and Phys. Rev. Lett. 46 (1981) 1315.
- 16) G. Senjanović and P. Senjanović, Phys. Rev. D21 (1980) 3253.
- 17) I.I. Bigi and J.-M. Frère, Phys. Lett. 110B (1982) 255.
- 18) M.A.B. Bég and H.S. Tsao, Phys. Rev. Lett. 41 (1978) 278;
R.N. Mohapatra and G. Senjanović, Phys. Lett. 79B (1978) 283.
- 19) R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16
(1977) 1791.
- 20) P. Sikivie, Phys. Rev. Lett. 48 (1982) 1156.
- 21) J.F. Kim, Phys. Rev. Lett. 43 (1979) 103;
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166 (1980) 493.
- 22) G. Beall and A. Soni, Phys. Rev. Lett. 47 (1981) 552.

FIGURE CAPTIONS

Figure 1 : Diagrams which contribute to $K \rightarrow 2\pi$.

Figure 2 : Diagrams which contribute to an effective $\Delta S = 2$ interaction.

Figure 3 : Diagram which contributes to the neutron electric dipole moment.

Fig. 1

1a

1b

Fig. 2

2a

2b

2c

Fig. 3