Ref.TH.3396-CERN

LEPTON VIOLATING $\beta^-\beta^-$, $\beta^+\beta^+$ DECAYS, (e^-,e^+) CONVERSION
AND DOUBLE ELECTRON CAPTURE IN GAUGE THEORIES

J.D. Vergados
CERN -- Geneva
and
Physics Department, University of Ioannina,
Ioannina, Greece

ABSTRACT

The lepton violating processes $\beta^-\beta^-$, $\beta^+\beta^+$, (e^-,e^+) and double electron capture have been investigated in the context of modern gauge theories. Mechanisms involving light or heavy intermediate Majorana neutrinos, with or without right-handed currents, as well as Higgs particles, have been studied. The lepton violating emission of light bosons, recently proposed by Georgi, Glashow and Nussinov, has also been analyzed. From the analysis of the $^{48}\text{Ca} - ^{48}\text{Ti}$ data the following limits emerge: $|\langle m_\nu \rangle| < 80 \text{ eV}$, $m_N > (2-20) \times 10^3 \text{ GeV}$, $m_{WR} > 400 \text{ GeV}$ and $g_{\nu_e \bar{\nu}_e X^0} < 5 \times 10^{-3}$. The above limits are then used to predict the lifetimes for $\beta^+\beta^+$, (e^-,e^+) and double electron capture in the $A = 58, 92$ and 96 systems employing realistic nuclear models.

Ref.TH.3396-CERN
26 August 1982

1. - INTRODUCTION

Recent developments in gauge theories have rekindled interest in the question of whether lepton number (charge) is conserved¹⁾⁻⁴⁾, and revived attempts to observe neutrinoless double β decay (ov). Most of the theoretical and experimental efforts have hitherto focused on double electron emission

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- \quad (\text{ov}) \quad (1)$$

which violates lepton number by two units. This process must be observed against the background originating from the two neutrino (2ν) decay

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \tilde{\nu}_e + \tilde{\nu}_e \quad (2\nu) \quad (2)$$

which is allowed by lepton conservation. Experimentally, (ov) can easily be distinguished from (2ν), given sufficiently fast rates, by measuring the energies of the two simultaneously emitted electrons^{1),2)}. In a world with maximal lepton violation ($\eta=1$), the ov process is expected to be favoured over the 2ν one by large factors ($\sim 10^6$). In the actual world, however, with $\eta \ll 1$ the possibility of observing the ov mode depends on the size of lepton violation. Neither process ov nor 2ν has yet been observed in laboratory experiments⁵⁾, but the predicted rates for the 2ν process are within present experimental limits. Double $\beta\beta$ decay has, however, definitely been "seen" in geochemical measurements^{6),7)}, which, unfortunately, cannot distinguish between (ov) and (2ν) modes. We shall see that even from such total lifetime measurements one can, in principle, extract the lepton violating parameter η . We thus know from such experiments that η must be small ($\eta \sim 10^{-5}$). In practice, however, the extraction of precise lepton violating parameters from the total lifetimes appears difficult. This can only come from laboratory measurements of ov lifetimes.

In recent years double $\beta\beta$ decay has become fashionable and experimentalists with high energy backgrounds are beginning to be involved. These people are expected to apply new techniques involving large target masses and focus their attention on reactions which emit positrons rather than electrons. Such processes are:

Double Positron Emission:

$$(A, Z) \rightarrow (A, Z-2) + e^+ + e^+ \quad (\text{ov}) \quad (3)$$

Electron Positron Conversion

$$e_b^- + (A, Z) \rightarrow (A, Z+2) + e^+ \quad (4)$$

and

Double Electron Capture

$$e_b^- + e_b^- + (A, Z) \rightarrow (A, Z-2)^* \xrightarrow{\gamma} \gamma \quad (5)$$

(The electron capture processes are always energetically allowed when double positron decay is.) These processes must be distinguished from

$$(A, Z) \rightarrow (A, Z+2) + e^+ + e^+ + \nu_e + \bar{\nu}_e \quad (6)$$

$$e_b^- + (A, Z) \rightarrow (A, Z-2) + e^+ + \bar{\nu}_e + \nu_e \quad (7)$$

$$e_b^- + e_b^- + (A, Z) \rightarrow (A, Z-2) + \nu_e + \bar{\nu}_e \quad (8)$$

which are allowed by lepton conservation and are always present. Again, reactions (3)-(5) have a different experimental signature than (6)-(9), since in the latter some energy is lost to the neutrinos. In reaction (3) one needs to measure the sum of the energies of the two positrons and in reaction (4) the energy of the positron as well as of the hard X-ray following the de-excitation of the atom. In reaction (5) one should measure the nuclear γ -ray as well as the two hard X-rays following the de-excitation of the final system. In all cases, population of excited final nuclear states is preferred since then one has the extra signature of the emitted γ -ray. In the case of reaction (5) we will show that one should look for a γ -ray with energy characteristic of the lepton violating process which does not coincide with any of the γ -rays expected from the de-excitation of the final nucleus populated in a traditional fashion (see Section 7). In the absence of right-handed currents this should be a γ -ray leading to the first excited 2^+ state. Measurement of the subsequent de-excitation of this 2^+ state will only serve to show that this level has in fact been populated. Processes like (3)-(5) have, of course, been qualitatively discussed a long time ago⁸⁾. We feel, however, that somewhat more quantitative predictions are necessary, in particular those that are motivated by recent developments in gauge theories^{3),4)}. It is interesting that capture reactions have also become fashionable as new ways of extracting the conventional neutrino masses⁹⁾.

The present paper will be organized as follows. The basic and common features of all the above neutrinoless processes will be examined in Section 2. Expressions for the $\beta^-\beta^-$ and $\beta^+\beta^+$ decay rates will be given in Section 3. An analysis of the data for the $^{48}\text{Ca} - ^{48}\text{Ti}$ decay will be given in Section 4 and lepton violating parameters will be extracted from such data. In Section 5 these parameters will be used in making predictions for the $\beta^+\beta^+$ decay of ^{58}Ni and ^{96}Ru . The basic features of the electron-positron conversion process will be discussed in Section 6. Results of calculations for the $^{58}_{28}\text{Ni}$, $^{92}_{42}\text{Mo}$ and $^{96}_{46}\text{Ru}$ will also be given. Double electron capture will be discussed in Section 7 along with numerical results for such capture on ^{58}Ni , $^{92}_{42}\text{Mo}$ and $^{96}_{46}\text{Ru}$. Some of the inessential details regarding the nuclear models will be given in the Appendix.

2. - LEPTON VIOLATING MECHANISMS

The lepton violating processes mentioned in the previous section will be discussed as second order weak interactions mediated by intermediate neutrinos or Higgs particles. For them to occur one needs at least one of two possibilities:

- i) Majorana neutrinos, i.e., neutrinos which are self-conjugate particles, i.e.,

$$\nu_m = \nu_m^c \quad ; \quad \nu_m^c = C (\bar{\nu}_m)^T \quad (9)$$

where C is the charge conjugation operator. In addition, at least one of the following:

- a) These neutrinos have non-zero mass;
 - b) There exist right-handed currents in weak interactions;
 - γ) There exist very light physical lepton violating Higgs particles which couple to these neutrinos.
- ii) There exist lepton violating double charged Higgs which can decay into two charged leptons and which can appreciably couple to quarks.

2.1 The Neutrino Mass Contribution

The mechanism for such double β decay is given in Fig. 1. The charged current Lagrangian takes the form

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (J^\lambda W_\lambda + J_\lambda W^\lambda) + \text{H.C.} \quad (10a)$$

where J_λ is the usual left-handed hadronic current, assumed to be of the V-A structure, W^λ is the charged vector boson and j^λ is the lepton violating leptonic current which is assumed to be of the type

$$j^\lambda = \bar{\Psi}_{\nu L} \gamma^\lambda \Psi_{eL} \quad , \quad \Psi_L = \frac{1}{2}(1-\gamma_5)\Psi \quad (10b)$$

where $\Psi_\nu = \nu_{eL} + N_{eL}$ is the neutrino field associated with the electron which can be written in terms of mass eigenstates as follows

$$\nu_e = \sum_j U_{ej}^{(1)} \nu_j \quad (\text{"light" component}) \quad (10c)$$

$$N_e = \beta \sum_j U_{ej}^{(2)} N_j \quad (\text{"heavy" component}) \quad (10d)$$

The separation into light and heavy components (compared say to MeV/c^2) is necessary because the transition operator is different in the two cases. β can be thought of as an effective mixing angle coupling the light and the heavy sector and is assumed to be $\ll 1$. It was separated out so that

$$\sum_j |U_{ej}^{(1)}|^2 \approx 1 \quad \sum_j |U_{ej}^{(2)}|^2 \approx 1$$

This mechanism to leading order contributes only to $0^+ \rightarrow 0^+$ transitions with an amplitude which takes the form

$$\mathcal{M} = \left(\frac{G_F}{\sqrt{2}} \cos \theta_c \right)^2 \left(-\frac{1}{2\pi} \right) \frac{m_e f_A^2}{R_0 A^{1/3}} \ell_0 \sum_\alpha \eta_\alpha \langle f | \Omega_\alpha | i \rangle \quad (11a)$$

[For notational convenience the Fermi function was omitted. (See Section 3.)] where θ_c is the Cabibbo angle, $R_0 = r_0 A^{1/3}$ is the nuclear radius, $\langle f | \Omega_\alpha | i \rangle$, $i = \nu, N$ (light, heavy), are the nuclear matrix elements, η_α = lepton violating parameters and

$$\ell_0 = \frac{1}{\sqrt{2}} (1 - p_{12}) [\bar{u}(p_1)(1+\gamma_5)u^c(p_2) - \bar{u}(p_2)(1+\gamma_5)u^c(p_1)] \quad (11b)$$

The separation of the matrix element into η_α and Ω_α can be accomplished only in the limit when the neutrino is either very heavy or very light. The lepton violating parameters are then defined as follows

$$\eta_\nu = \frac{\langle m_\nu \rangle}{m_e} \quad \langle m_\nu \rangle = \sum_j (U_{ej}^{(1)})^2 m_j \quad (12)$$

$$\eta_N = \beta^2 \frac{m_A}{m_N} \quad ; \quad \frac{1}{m_N} = \sum_j (U_{ej}^{(2)})^2 \frac{1}{m_j} \quad (13)$$

The quantity $m_A = 0.84 \text{ GeV}$ will be discussed below. The nuclear matrix elements involve the operator

$$S_\alpha = \sum_{i \neq j} \tau_{+(i)} \tau_{+(j)} \left[\frac{f_V^2}{f_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] \frac{R_0}{r_{ij}} F_\alpha(r_{ij}), \alpha = \nu, N \quad (14)$$

The nuclear radius $R_0 = r_0 A^{1/3}$ ($r_0 = 1.1 \text{ fm}$) was introduced for convenience as well as for historical reasons. The above derivation is valid under quite general and very reasonable assumptions in the case of light intermediate neutrinos¹⁰⁾. The heavy Majorana neutrino case is a little tricky. If one treats the nucleons as pointlike particles, one finds that $F_\alpha(r_{ij}) \propto \exp\{-mr_{ij}\}$, where m is the mass of the lightest particle in the loop of Fig. 1b. Since all particles are heavy, $F_\alpha(r_{ij})$ is essentially a δ function. In the presence of the nuclear hard core, one then gets a negligible contribution. Furthermore, the results depend on poorly-understood physics. There are, however, two ways out of this problem.

- a) Include a nucleon form factor. One should drop the unnecessary and unreasonable approximation of treating the nucleons as pointlike particles^{11),12)}. The two interacting quarks can, in fact, come close so that the matrix element is not damped off. Even though this can be taken care of in a bag-model type calculation, it is sufficient for our purposes to incorporate such effects by including the nucleon form factor in the Fourier integrals. The form factor of interest to us can be adequately described by a dipole shape¹³⁾

$$F(q^2) = \frac{1}{(1 - q^2/m_A^2)^2} \quad , \quad M_A = 0.85 \text{ GeV} \quad (15)$$

Then, assuming that $m_j \gg m_A$, one can drop the momentum in the neutrino propagator. Then one gets the mass dependence given by Eq. (13) where $F_N(r_{ij})$ is independent of the neutrino mass and takes the form

$$F_N(r_{ij}) = F_1(x_A) \quad , \quad x_A = M_A r_{ij} \quad (16a)$$

$$F_1(x) = \frac{m_A}{m_e} \frac{1}{48} x (x^2 + 3x + 3) e^{-x} \quad (16b)$$

(we have decided to absorb m_A/m_e in $F_1(x)$ rather than in η_N so that the nuclear matrix elements have reasonable magnitude).

- β) Include other particles in the nuclear soup. Such particles should have isospin $> 1/2$ so that they can undergo a change of charge by two units. Since the isobars cannot contribute to $0^+ \rightarrow 0^+$ double β decay processes, the best candidates are pions in flight between the two interacting nucleons. The relevant basic reaction is¹⁴⁾

$$\pi^- \rightarrow \pi^+ + e^- + e^- \quad (17a)$$

$$\pi^+ \rightarrow \pi^- + e^+ + e^+ \quad (17b)$$

and the corresponding Feynmann diagram is given in Fig. 2. This process can again be described in terms of the lepton violating parameter η_N given above, but the nuclear matrix element is now that of the operator

$$Q_\pi = \alpha_\pi \sum_{i,j} \tau_+(i) \tau_+(j) \frac{R_0}{r_{ij}} \left[\vec{\sigma}_i \cdot \vec{\sigma}_j F_2(x_\pi) + \sqrt{\frac{24\pi}{5}} \left[\vec{r}_i^2 \cdot \vec{r}_j^2 + (\vec{\sigma}_i \otimes \vec{\sigma}_j)^2 \right] F_3(x_\pi) \right] \quad (17c)$$

where $\alpha_\pi = 0.007 - 0.06$ gives the probability¹⁴⁾ of finding such a particle inside the nucleus and

$$F_2(x) = \frac{m_A}{m_e} (x-2) \bar{e}^x, \quad F_3(x) = \frac{m_A}{m_e} (x+1) \bar{e}^x, \quad x_\pi = m_\pi r_{ij} \quad (17d)$$

We note that the smallness of α_π is compensated for by the shorter range of the operator (m_π instead of m_A). So we expect the two treatments to give equal contributions.

Before concluding this subsection we should mention that the neutrino masses entering in double β decay are "average" quantities [see Eqs. (12) and (13)]. Therefore limits on such parameters extracted from $\beta\beta$ decay may differ from those extracted from other experiments (triton decay or neutrino oscillations). To see this we repeat an argument given in the case of the neutrino flavours. In such a situation one can show^{15),16)} that $|\langle m_\nu \rangle|$ takes the form

$$|\langle m_\nu \rangle| = \sqrt{m_1^2 + m_2^2 + 2m_1 m_2 \cos 2\varphi}$$

Thus $|\langle m_\nu \rangle| = |m_1 + m_2|$ and $|m_1 - m_2|$ for $\varphi = 0$ and $\varphi = \pi/2$, respectively. In the absence of CP violation the effect of the two neutrinos tend to cancel¹⁵⁾

(some of the quantities U_{ej} are pure imaginary). Also in the case of Dirac neutrinos ($\varphi = 0$, $m_1 = -m_2$) we get $\langle m_\nu \rangle = 0$ consistent with what we have said at the beginning of this section.

2.2 Contribution From Light Higgs Bosons

Models have been proposed whereby B-L symmetry is spontaneously broken by a small expectation value of a Higgs triplet⁴⁾. In such theories light or massless neutral Higgs particles survive the Higgs mechanism as actual particles. Such particles are the majoron or its light counterpart. They will subsequently be denoted by χ^0 . These particles couple to the neutrinos and for our purposes have the same effect as a mass insertion term (see Fig. 3). They can thus induce lepton violation which does not explicitly depend on the mass of the Majorana neutrinos. Thus reactions (1)-(8) could have this extra χ^0 particle in the final state. Unfortunately such particles interact very weakly with anything else and thus, like neutrinos, they will go undetected. Thus a lepton violating reaction involving such particles, e.g.,

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \chi^0 \quad (18)$$

will experimentally be confused with the (2ν) reaction of Eq. (2). In other words the width $\Gamma_{2\nu}^{\text{exp}}$ assigned to the (2ν) process will be exaggerated⁴⁾. Unfortunately however, in those cases where total lifetimes are available, the $\Gamma_{2\nu}^{\text{th}}$ calculated theoretically already exceeds¹⁷⁾ the total $\Gamma_{\text{tot}}^{\text{exp}}$ and one need not, in such cases, invoke χ^0 . We shall see, however, that we may be able to extract the coupling g of this particle to neutrinos from the available data in ^{48}Ca . The nuclear matrix element is the same as in the case of 0ν (light neutrino) if one ignores the momentum carried away by this particle in the relevant Fourier integrals. The kinematical dependence on the energy is, of course, different¹²⁾ since one more particle is produced (see later sections).

2.3 Decay of Doubly Charged Higgs Particles

For a number of reasons, for example to get an understanding of the neutrino mass as we saw in the previous section, one has to devise gauge models or extend the existing ones so that in addition to the well-known Higgs doublet, an isospin triplet is also included. This triplet contains a doubly charged member which can decay into two leptons and thus give a signature similar to that of β decay without the need for intermediate Majorana neutrinos. Whether such a mechanism

contributes significantly to $\beta\beta$ decay depends, of course, on the parameters of such models and, in particular, on the masses of such particles and their couplings to quarks and leptons. Two such models have recently been proposed^{18),19)}. Even though it seems that the $SU_L(2) \times U(1)$ versions of such models give negligible^{20),21)} contributions, they may still contribute significantly in left-right symmetric theories. Therefore we will consider them in our discussion. Our purpose will primarily be to extract limits on the gauge parameters involved here from the $\beta\beta$ decay data of ^{48}Ca and to see what this implies for the effect of such mechanisms on other systems.

We will begin with a mechanism which was recently shown by Piccioto and Sahir¹⁹⁾ to be the most dominant among a group of Higgs mediated $\beta\beta$ decay modes (see Fig. 4). It is based on an $SU_L(2) \times SU_R(2) \times U(1)$ model analogous to that of Ref. 18). Their treatment is along the lines of Ref. 20) except that they have at their command left- and right-handed doublets and triplets. The coupling to the quarks is now achieved via the right vector bosons and thus the amplitude associated with such a diagram is going to be proportional to κ^2 , $\kappa = (m_{W_L}/m_{W_R})^2$ where m_{W_L} = the mass of the ordinary charged vector boson (~ 80 GeV). As far as the nucleons are concerned, this amplitude differs from that of the intermediate heavy Majorana neutrino discussed earlier only by a scale factor. It can therefore be cast into the form (11a) if one defines the lepton violating parameter

$$\eta_H = \frac{1}{\sqrt{2} G_F m_{W_L}^2} \frac{\lambda_0 M_0 m_\Delta}{m_\Delta^2} \kappa^2 \quad (19a)$$

where m_Δ is the mass of the doubly charged Higgs particles Δ_R^{--} and $\lambda_0 M_0$ is the cubic mass coupling between the right-handed vector bosons and the Δ_R^{--} . With "reasonable" choice of the above parameters $m_\Delta \approx 100$ GeV, $\lambda_0 M_0 \sim 200$ GeV, $\kappa = 0.01$ one finds that

$$\eta_H \sim 10^{-5} \quad (19b)$$

In the other model (see Fig. 5) the coupling of the doubly charged Higgs to the quarks is achieved via its coupling to ordinary Higgs¹⁸⁾. The latter couple to the quarks via primarily a pseudoscalar coupling. The leptonic matrix element is the same as that of Eq. (11a) but the effective two nucleon transition operator is different. After some straightforward algebra one finds that this operator takes the form:

$$\Omega_\Delta = \sum_{i,j} \tau_+(i) \tau_+(j) \frac{R_0}{r_{ij}} \left\{ \alpha_s F_1(x_A) + \alpha_p \left[\vec{\sigma}_i \cdot \vec{\sigma}_j F_4(x_A, x_\pi) + \sqrt{\frac{24\pi}{5}} \gamma^2(\hat{r}_{ij}) \cdot (\vec{\sigma}_i \times \vec{\sigma}_j) F_5(x_A, x_\pi) \right] \right\} \quad (20a)$$

where $F_1(x)$ has been defined previously [Eq. (16b)] and

$$F_4(x_A, x_\eta) = \frac{m_A}{m_e} \left\{ \frac{1}{2} [(x_\eta - 2) \bar{e}^{x_\eta} + (x_A - 2) \bar{e}^{x_A}] - 2 \left[\left(\frac{m_\pi}{m_A} \right)^2 \bar{e}^{x_\eta} \bar{e}^{x_A} \right] \right\} \quad (20b)$$

$$F_5(x_A, x_\pi) = \frac{m_A}{m_e} \left\{ \frac{1}{2} [(x_\eta + 1) \bar{e}^{x_\eta} + (x_A + 1) \bar{e}^{x_A}] + 2 \left[\left(\frac{m_\pi}{m_A} \right)^2 Z(x_\eta) - Z(x_A) \right] \right\} \quad (20c)$$

with

$$Z(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \bar{e}^x.$$

The quantities α_s and α_p are related to the scalar and pseudoscalar form factors at zero momentum transfer as follows¹⁸⁾

$$\alpha_s = \frac{1}{f_A^2} [F_s^3(0)]^2, \quad \alpha_p = \frac{1}{12 f_A^2} [F_p^3(0)]^2 \left(\frac{m_\pi}{m_A} \right)^4 \quad (20d)$$

These form factors are not accurately known. Here we will adopt the values obtained by Adler et al.²²⁾, i.e.,

$$\alpha_s = 0.25, \quad \alpha_p = 0.40$$

Then the amplitude takes the form¹⁸⁾ of Eq. (11a) with

$$\eta_\Delta = \frac{\lambda V_R m_A h_q^2 h_\ell}{m_H^4 m_\Delta^2 G_F^2} \quad (21a)$$

The meaning of the various parameters is apparent from Fig. 5. With the "reasonable" choice¹⁸⁾

$$\lambda V_R = 10^3 \text{ GeV}, \quad m_\Delta \approx m_H \approx 100 \text{ GeV}, \quad h_q^2 = 10^{-2}, \quad h_\ell = 1$$

we get

$$\eta_\Delta \approx 10^{-9} \quad (21b)$$

This mechanism appears to be less important than the previous one but we note that the nuclear matrix element will be larger due to the fact that Ω_Δ has a somewhat longer range.

2.4 The Right-Handed Interaction

In the earlier treatments of $\beta\beta$ decay the mass insertion mechanism described at the beginning of this section was not emphasized. Most of the emphasis was put on the possible right-handed current admixture to the predominantly left-handed current. Since this aspect has been extensively studied previously^{1),10),16),23)}, we are not going to give many details here. The charged current part of the weak interaction [Eq. (10a)] becomes

$$\mathcal{L}_W^{cc} = \frac{g}{\sqrt{2}} (j_L^\mu W_\mu^L + j_R^\mu W_\mu^R + J_\mu^L W_\mu^L + J_\mu^R W_\mu^R) + \text{H.C.} \quad (22)$$

where j_R^μ and J_μ^R refer to the leptonic hadronic right-handed current and $W_\mu^L(W_\mu^R)$ is the field for the left- (right-) handed boson. The boson fields can be expressed in terms of the mass eigenstates $W_\mu^{(1)}$ and $W_\mu^{(2)}$ as follows

$$W_\mu^L = c W_\mu^{(1)} - s W_\mu^{(2)}, \quad W_\mu^R = s W_\mu^{(1)} + c W_\mu^{(2)} \quad (23)$$

where $c = \cos \theta$, $s = \sin \theta$ (θ = mixing angle). Retaining terms up to linear in $\epsilon = s/c$ and $\kappa = m_L^2/m_R^2$ (see Fig. 6) we see that for the L-R combination in the leptonic sector one can have both L-R and L-L contributions in the hadronic sector. Except for the appearance of a term proportional to ϵ , our results are similar to those of Ref. 16).

Since, instead of the mass, we now pick the momentum of the intermediate neutrino, only the "light" components of the left-handed and right-handed neutrinos contribute significantly and the lepton violating parameter takes the form¹⁶⁾

$$\eta_{RL} = c^4 \sqrt{\kappa^2 + \epsilon^2} \sum_j U_{ej}^{(1)} V_{ej}^{(1)} \quad (24)$$

The leptonic current now has a different Lorentz structure and the effective hadronic operator^{1),14),16)} is proportional to the relative separation of the interacting hadrons ($\sim \hat{r}_{ij}$). For positive parity transitions one picks out the dipole term of the plane wave solution of the outgoing electrons¹⁾. In principle we could now have nuclear transitions $0^+ \rightarrow J^\pi$, where $J^\pi = 0^-, 1^-, 2^-, 0^+, 1^+, 2^+$ and 3^+ . Only a few of them, however, are energetically allowed. In the present paper we will restrict ourselves to the case of the $0^+ \rightarrow 0^+$ transition for which the amplitude takes the form

$$m = \left(\frac{G_F \cos \theta_c}{\sqrt{2}} \right)^2 \frac{\eta_{RL}}{2\pi} \frac{m_e f_A^2}{r_0 A^{1/3}} \left[2 \ell_S \langle f | \Omega_S | i \rangle + 2 \ell_A \langle f | \Omega_A | i \rangle \right] \quad (25)$$

where

$$\ell_S = \frac{1}{\sqrt{2}} \left[\bar{u}(p_1) \frac{\vec{\gamma} \cdot \vec{q}}{q m_e} u^c(p_2) + \bar{u}(p_2) \frac{\vec{\gamma} \cdot \vec{q}}{q m_e} u^c(p_1) \right] \quad (26a)$$

$$\ell_A = \frac{1}{\sqrt{2}} \left[\bar{u}(p_1) \frac{\vec{\gamma} \cdot \vec{Q}}{q m_e} \gamma_5 u^c(p_2) - \bar{u}(p_2) \frac{\vec{\gamma} \cdot \vec{Q}}{q m_e} \gamma_5 u^c(p_1) \right] \quad (26b)$$

with

$$\vec{q} = \frac{1}{2} (\vec{p}_1 - \vec{p}_2), \quad \vec{Q} = (\vec{p}_1 + \vec{p}_2) \quad (27)$$

and

$$\Omega_S = \frac{\varepsilon - \kappa}{\sqrt{\varepsilon^2 + \kappa^2}} \Omega_1 + \frac{\kappa}{\sqrt{\varepsilon^2 + \kappa^2}} \Omega_2, \quad \Omega_A = \frac{\varepsilon}{\sqrt{\varepsilon^2 + \kappa^2}} \Omega_3 \quad (28)$$

$$\Omega_1 = \sum_{i \neq j} \tau_+(i) \tau_+(j) \frac{R_0}{r_{ij}} \left[3 \frac{f_V^2}{f_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j + 2 \sqrt{\frac{24\pi}{5}} \vec{\gamma}(\hat{r}_{ij}) \cdot (\vec{\sigma}_i \otimes \vec{\sigma}_j) \right] \quad (29a)$$

$$\Omega_2 = 6 \frac{f_V^2}{f_A^2} \sum_{i \neq j} \tau_+(i) \tau_+(j) \frac{R_0}{r_{ij}} \quad (29b)$$

$$\Omega_3 = 3 \frac{f_V}{f_A} \sum_{i \neq j} \tau_+(i) \tau_+(j) \frac{R_0 R_{ij}}{r_{ij}^2} (\vec{\sigma}_i - \vec{\sigma}_j) \cdot (i \hat{r}_{ij} \times \hat{R}_{ij}) \quad (29c)$$

with

$$\vec{r}_{ij} = (\vec{r}_i - \vec{r}_j) \quad \vec{R}_{ij} = \frac{1}{2} (\vec{r}_i + \vec{r}_j) \quad (30)$$

Again we have decided to scale the nuclear matrix elements with the nuclear radius and multiply them by three so that Ω_1 is reduced to the Gamow-Teller operator in the limit in which the radial dependence is ignored (a compensating factor of 1/3 together with the 1/3 coming from the above-mentioned dipole expansion has been absorbed in the lepton current). We note that the space antisymmetric operator

Ω_A is absent when $\epsilon \rightarrow 0$. Also in that limit, except for the trivial modification just mentioned, our operator Ω_S coincides with X_4 of Ref. 16).

In the actual computation we are going to use three ranges of κ and ϵ .

- i) $\kappa \ll \epsilon$ which will be referred to as case i);
- ii) $\kappa \gg \epsilon$ which will be referred to as case ii);
- iii) $\kappa = 10\epsilon$ which will be referred to as case iii).

3. - TRANSITION RATES FOR e^-e^- AND e^+e^+ DECAYS

Given the structure of the amplitude of Eqs. (11a) and (25), it is not difficult to perform the lepton spin sums and integrate over the leptonic four momenta to obtain an expression for the width associated with the reactions (1) or (3). This can be cast into the form

$$\Gamma_{0\nu} = \Gamma_0(0\nu) \frac{f_{0\nu}(\epsilon_0, Z \pm 2)}{A^{2/3}} |ME|_{0\nu}^2 \quad (31)$$

with

$$\Gamma_0(2\nu) = \frac{m_e}{(2\pi)^5} (G_F \omega_s \theta_c m_e^2)^4 \left(\frac{1}{m_e \eta}\right)^2 \quad (32a)$$

$$f_{0\nu}(\epsilon_0, Z \pm 2) = \int_0^{\epsilon_0} \chi(Z \pm 2, \xi) \chi(Z \pm 2, \epsilon_0 - \xi) (\epsilon + 1)^2 (\epsilon_0 - \epsilon + 1)^2 d\epsilon \quad (32b)$$

$$\chi(Z, \xi) = F(Z, \xi), \quad \xi = \frac{E}{P} = \frac{\epsilon + 1}{\sqrt{\epsilon(\epsilon + 2)}} \quad (32c)$$

ϵ = lepton kinetic energy (in units of $m_e c^2$) and

$$\epsilon_0 = \frac{1}{m_e} \Delta m, (\beta^- \beta^-) ; \quad \epsilon_0 = \frac{\Delta m}{m_e} - 4, (\beta^+ \beta^+) \quad (33)$$

$F(Z, \xi)$ is the well-known Fermi function. It is convenient to give an expression for the lifetime

$$T_{1/2}(0\nu) = K_{0\nu} / |ME|_{0\nu}^2 \quad (34)$$

$$K_{0\nu} = \frac{\ln 2}{\Gamma_0(2\nu)} \frac{A^{2/3}}{f_{0\nu}(\epsilon_0, Z \pm 2)}, \quad \frac{\ln 2}{\Gamma_0(2\nu)} = 1.1 \times 10^{16} \text{ yr}$$

In this paper when we talk about lifetime we mean the half-life.

In the above expression we have explicitly written out the energy dependence of $f_{0\nu}(\epsilon_0, Z \pm 2)$ which characterizes the left-handed interaction. The additional energy dependence of the right-handed contribution, as well as all the interesting physics, is implicit in the expression $|ME|_{0\nu}^2$.

Before proceeding further we mention that in essentially all previous $\beta^-\beta^-$ decay the following approximation was employed

$$\chi(Z, \epsilon) \approx \frac{2\pi\alpha Z}{1 - \exp(-2\pi\alpha Z)} \quad (\text{R.P.}) \quad (35a)$$

Then the $f_0(\epsilon, Z)$ becomes

$$f_{0\nu}(\epsilon_0, Z) = \left(\frac{2\pi\alpha Z}{1 - \exp(-2\pi\alpha Z)} \right)^2 \frac{\epsilon_0}{30} (\epsilon_0^4 + 10\epsilon_0^3 + 40\epsilon_0^2 + 60\epsilon_0 + 30) \quad (35b)$$

It has recently¹⁷⁾ been shown, however, that this approximation substantially underestimates the 2ν width. We find the same thing for the 0ν decays as well (see Table 1). In any case such an approximation is unacceptable for positron decay and electron-positron conversion. We will therefore use the tabulated values of the Fermi function²⁴⁾.

The quantity $|ME|_{0\nu}^2$ takes the form

$$\begin{aligned} |ME|_{0\nu}^2 = & |\eta_{RL}|^2 [I_S |\langle f | \Omega_S | i \rangle|^2 + I_A |\langle f | \Omega_A | i \rangle|^2] \\ & - I_{OS} \text{Re } \eta_{RL} \eta_{LL}^* \langle f | \Omega_S + 2\Omega_A | i \rangle \langle f | \Omega_L | i \rangle^* \\ & + |\eta_{LL}|^2 |\langle f | \Omega_L | i \rangle|^2 \end{aligned} \quad (36)$$

with

$$\eta_{LL} = \sqrt{|\eta_V|^2 + |\eta_W + \eta_H|^2 + |\eta_A|^2} \quad (37a)$$

$$\langle f|\Omega_L|i\rangle = \frac{\eta_V}{|\eta_{LL}|} \langle f|\Omega_V|i\rangle + \frac{\eta_N+\eta_H}{|\eta_{LL}|} \langle f|\Omega_N+\Omega_H|i\rangle + \frac{\eta_D}{|\eta_{LL}|} \langle f|\Omega_D|i\rangle \quad (37b)$$

(for the definition of the various quantities see the previous section). The quantities I_S , I_A and I_{OS} are functions of ϵ_0 and $Z\pm 2$ and for some cases of interest to us they are given in Table 1. These quantities become negligible when ϵ_0 is small, which is another way of saying that the right-handed interaction will be suppressed when the available energy is small.

For orientation purposes we give an expression for the lifetime of the 2ν process

$$T_{1/2}(2\nu) = \frac{K_{2\nu}}{f_{2\nu}(\epsilon_0, Z\pm 2)} \frac{1}{|ME|_{2\nu}^2} \quad (38)$$

$$K_{2\nu} = \frac{8\pi^7}{(G_F \cos\theta_c m_e)^4} \frac{\mu_0^2}{m_e} \frac{1}{f_A^4} = 3.3 \times 10^9 \text{ yr } \mu_0^2 \quad (39)$$

with

$$(ME)_{2\nu} = \sum_n \langle f|\vec{Y}|n\rangle \cdot \langle n|\vec{Y}|i\rangle \frac{\mu_0}{\mu_n} \quad (40)$$

where $|n\rangle$ are the intermediate $(A, Z\pm 1)$ states, μ_n is their excitation (in units of $m_e c^2$) and $\vec{Y}$ is the Gamow-Teller operator. More precisely,

$$\mu_n \approx \frac{1}{m_e} (E_{xn} + m_n - M_i) + \frac{\epsilon_0}{2} \quad (41a)$$

$$\vec{Y} = \sum_i \tau_+(i) \vec{\sigma}_i \quad (41b)$$

μ_0 is a suitable average excitation which can be calculated in the spirit of Ref. 23). Estimates of μ_0 for the nuclear systems of interest to us are presented in Table 1. In the spirit of such approximations $(ME)_{2\nu} = \langle f|\vec{Y} \cdot \vec{Y}|i\rangle$. The kinematical function $f_{2\nu}$ is given by

$$f_{2\nu} = \int_0^{\epsilon_0} (\epsilon_1+1)^2 \chi(Z\pm 2, \epsilon_1) d\epsilon_1 \int_0^{\epsilon_0-\epsilon_1} (\epsilon_2+1)^2 \chi(Z\pm 2, \epsilon_2) \frac{1}{30} (\epsilon_0-\epsilon_1-\epsilon_2)^5 d\epsilon_2 \quad (41c)$$

Under the R-P approximation [Eq. (35a)] it becomes¹⁶⁾

$$f_{2\nu} = \left(\frac{2\pi\alpha Z}{1 - \exp(-2\pi\alpha Z)} \right)^2 \frac{2^4}{11!} \epsilon_0^7 (\epsilon_0^4 + 22\epsilon_0^3 + 220\epsilon_0^2 + 990\epsilon_0 + 1980) \quad (41d)$$

Finally we will give an expression for the lifetime associated with the neutral Higgs χ^0 emission (see subsection 2.2). It takes the form:

$$T_{1/2}(0\nu, \chi^0) = K\chi^0 / |< f | g \Omega_\nu | i >|^2 \quad (42)$$

The quantity K_{χ^0} can be easily computed (see Table 1). In fact, $\Gamma_0(0\nu, \chi^0)$ is obtained by multiplying $\Gamma_0(2\nu)$ by $\frac{1}{2} 1/(2\pi)^2$ while the energy-dependent function $f_{0\nu, \chi^0}(\epsilon_0, Z+2)$ is given by Eq. (41d) after replacing $\frac{1}{30}(\epsilon_0 - \epsilon_1 - \epsilon_2)^5$, with $\epsilon_0 - \epsilon_1 - \epsilon_2$, i.e., the energy of χ^0 .

4. - DOUBLE β DECAY OF ^{48}Ca

The decay $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ has been considered as one of the most favourable from the experimental point of view, even though ^{48}Ca is very expensive, because of the large available energy ($\epsilon_0 \approx 8.4$). In fact, the experimental limits

$$T_{1/2}^{\text{exp}}(0\nu) > 2 \times 10^{21} \text{ yr} \quad (43)$$

$$T_{1/2}^{\text{exp}}(2\nu) > 3.6 \times 10^{19} \text{ yr} \quad (44)$$

for the 0ν and 2ν modes were obtained a long time ago²⁵⁾⁻²⁷⁾.

It is clear, of course, that the $^{48}\text{Ca} - ^{48}\text{Ti}$ decay is favoured when compared to systems with lower energy, e.g., $^{128}\text{Te} - ^{128}\text{Xe}$ (see Table 1). However, such advantages are not overwhelming when we compare it with other systems and they may easily be offset by other effects, e.g., the nuclear matrix elements or the distortion of the Fermi function (the latter favours large Z and A). In any case, the importance of large available energy was exaggerated in the old days when people were thinking in terms of right-handed current admixtures which, as we have seen, are more favoured by large energy.

Large available energy may have some disadvantages. The first is a theoretical one. If the contribution of the right-handed currents is smaller than, but comparable to, the mass insertion contribution, a large available energy will make the extraction of the neutrino mass $\langle m_\nu \rangle$ more difficult. The second is an experimental one. Large energy will tend to favour much more strongly the 2ν process,

thus making the reaction background more formidable. However, this second general argument may not apply in the special case of ^{48}Ca since, as we shall see, the 2ν nuclear matrix element is much smaller than the corresponding one for 0ν decay. All these may be irrelevant, however, if one takes the point of view that it will in itself be exciting to see in the laboratory double β decay neutrinoless or not, mediated by right-handed currents or not.

Let us begin by estimating the 2ν lifetime for ^{48}Ca . Using the small nuclear matrix element obtained previously²³⁾, i.e., $|\text{ME}|^2 = 1.7 \times 10^{-2}$, we obtain

$$T_{1/2}^{+h}(2\nu) = 2.7 \times 10^{20} \text{ yr}$$

which is an order of magnitude longer than the experimental limit of Eq. (43).

The main purpose of this paper is to use the $^{48}\text{Ca} - ^{48}\text{Ti}$ system as a theoretical laboratory. It is suitable for this purpose since it has the simplest possible nuclear structure and one can easily calculate the matrix elements of fairly complicated operators like those of Eqs. (14) and (16b), (17c) and (19). Thus one may be able to draw general conclusions about the matrix elements of such operators because they mainly arise from $J = 0$ pairs and they are not themselves subject to cancellations between the $J = 0$ and $J = 2$ pairs like those occurring in the case of operators Ω_1 , Ω_2 , Ω_V and especially the Gamow-Teller operator $\vec{Y} \cdot \vec{Y}$ defined above. Thus such matrix elements, given proper treatment of short-range correlations, can be reliably estimated. Then, from the experimental data²⁵⁾⁻²⁷⁾ via Eqs. (43) and (44) one may be able to extract the lepton violating parameters η_V , $\eta_N + \eta_H$, η_A , η_{RL} . Finally, we will use these parameters in making predictions about the other processes mentioned in the Introduction.

The nuclear model we are going to use has been described elsewhere^{11),12),24)}. We shall, however employ a somewhat more realistic two-body short-range correlation function $c(|\vec{r}_i - \vec{r}_j|)$ defined by

$$\psi_{\text{cor}}(\vec{r}_i, \vec{r}_j) = [1 - c(|\vec{r}_i - \vec{r}_j|)] \psi_{\text{unc}}(\vec{r}_i, \vec{r}_j) \quad (45a)$$

To be specific, we will employ in all our calculations the function²⁸⁾

$$c(r) = e^{-ar^2} (1 - br^2), \quad a = 1.1 \text{ fm}^{-2}, \quad b = .68 \text{ fm}^{-2} \quad (45b)$$

The nuclear matrix elements obtained are given in Table 2. They are somewhat smaller than the values $\langle f|\Omega|i\rangle = 1.0$ and $\langle |\Omega_1|i\rangle = 4.0$ obtained without such correlations or the values $\langle f|\Omega_N+\Omega_\pi|i\rangle = 72 + 94 = 166$ and $\langle f|\Omega_\Delta|i\rangle = 530$ obtained with the simple step function correlation previously employed^{11),12),14)}. Suppose now that there are no right-handed currents ($\eta_{RL}=0$). Then, from Eqs. (36), (37a) and (38) and with the experimental data and our data in Tables 1 and 2, we get

$$|.62 \eta_\nu + 122 (\eta_H + \eta_N) + 262 \eta_\Delta|^2 \leq 76 \times 10^{-10}$$

Assuming further that only one lepton violating parameter is non-zero, we obtain

$$|\eta_\nu| \leq 1.4 \times 10^{-4} \quad \text{or} \quad |\langle m_\nu \rangle| < 71 \text{ eV}$$

$$|\eta_H + \eta_N| < 1.0 \times 10^{-6}, \quad |\eta_\Delta| < 3 \times 10^{-7}$$

If, in addition, one assumes that $\eta_H = 0$ (no Higgs contribution) one gets the bound

$$\beta^2 \frac{m_A}{m_N} < 10^{-6}$$

If the mixing angle is not unreasonably small, e.g., $\beta^2 \sim 10^{-3} - 10^{-2}$, one gets a limit on the heavy Majorana neutrino mass [in the sense of Eq. (13)] as follows

$$m_N \approx (2-20) \times 10^3 \text{ GeV}$$

No useful constraint can be extracted if the mixing angle is much smaller.

Conversely, if one assumes $\eta_N = 0$ (no heavy Majorana neutrino contribution) one gets $\eta_H < 10^{-6}$. If one further takes $m_\Delta \sim 100 \text{ GeV}$ and $\lambda_0 M_0 = 200 \text{ GeV}$, one finds that $\kappa < 0.04$ which is smaller than the estimate of 0.12 of Ref. 19) and the value of 0.1 of Mohapatra and Senjanovich²⁹⁾.

The inclusion of the right-handed currents makes life harder and to simplify matters we will retain only the two parameters $\eta_{LL} = \eta_\nu$ and η_{RL} . The following special cases will be considered.

i) $\kappa \ll \epsilon$ (see subsection 2.4) then

$$\langle f|\Omega_S|i\rangle = \langle f|\Omega_1|i\rangle = 3.3, \quad \langle f|\Omega_\Delta|i\rangle = -6.3$$

Using the functions I_S , I_A and I_{OS} of Table 1 we get the constraint

$$|\eta_{LL}|^2 + 4.8 \operatorname{Re}(\eta_{LL}^* \eta_{RL}) + 272 |\eta_{RL}|^2 \leq 1.9 \times 10^8$$

This is a constraint of four complex numbers. Making the simplifying assumption that they are real, we get

$$|\eta_{LL}| < 1.4 \times 10^4, \quad |\eta_{RL}| < 0.85 \times 10^5, \quad |\langle m_\nu \rangle| < 71 \text{ eV}$$

ii) $\kappa \gg \epsilon$. Now

$$\langle f | \Omega_5 | i \rangle = \langle f | \Omega_2 - \Omega_1 | i \rangle = -2.7, \quad \langle f | \Omega_A | i \rangle = 0$$

$$|\eta_{LL}|^2 + 1.3 \operatorname{Re}(\eta_{LL}^* \eta_{RL}) + 2.2 |\eta_{RL}|^2 < 1.9 \times 10^8$$

i.e.,

$$|\eta_{LL}| < 1.5 \times 10^4, \quad |\eta_{RL}| < 1.0 \times 10^4, \quad |\langle m_\nu \rangle| < 78 \text{ eV}$$

iii) $\kappa \approx 10\epsilon$ [see Ref. 29)]. Then

$$\langle f | \Omega_5 | i \rangle = -2.4, \quad \langle f | \Omega_A | i \rangle = -0.67$$

$$|\eta_{LL}|^2 + 1.8 \operatorname{Re}(\eta_{LL}^* \eta_{RL}) + 4.5 |\eta_{RL}|^2 < 1.9 \times 10^8$$

i.e.,

$$|\eta_{LL}| < 1.5 \times 10^4, \quad |\eta_{RL}| < 0.72 \times 10^4, \quad |\langle m_\nu \rangle| < 77 \text{ eV}$$

We thus see that in our system, which is perhaps the most favourable case for the manifestation of right-handed currents, the bound on $|\eta_{LL}|$ does not depend sensitively on the right-handed currents even though its actual value does. The allowed range of η_{LL} ($\eta_{LL} > 0$) and η_{RL} for case ii) is the area inside the curve of Fig. 7. Thus, in order to extract the neutrino mass from the data, one should know independently the value of η_{RL} (e.g., from $0^+ \rightarrow 2^+$ transitions) or employ a nuclear system with small value of ϵ_0 (see next section).

The above limit on the neutrino mass $\langle m_\nu \rangle < 78 \text{ eV}$ is not as good as the limit of $|\langle m_\nu \rangle| < 15 \text{ eV}$ obtained from the $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ and $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ data. It is expected to improve a little bit if one improves the nuclear wave functions by including the entire 1f-2p shell. It is clear, however, that the lifetime bound [Eq. (42)] should be improved.

We will briefly comment on the value $|\langle m_\nu \rangle| = 10 \text{ eV}$ extracted¹⁷⁾ from the ratio $R = (1.59 \pm 0.05) \times 10^3$ of the total lifetime of the $A = 128$ divided by that of the $A = 130$ system⁷⁾. This ratio can be cast into the form

$$R = \frac{\Lambda_{2\nu}(130) + \Lambda_{0\nu}(130) X^2}{\Lambda_{2\nu}(128) + \Lambda_{0\nu}(128) X^2}, \quad X = \eta_\nu \frac{\langle f | \Omega_\nu | i \rangle}{\langle f | \vec{\gamma} \cdot \vec{\gamma} | i \rangle} \quad (46a)$$

assuming that the two nuclear matrix elements involved in the two systems are equal²³⁾. We have $\Lambda_{2\nu} = K_{2\nu}^{-1}$ and $\Lambda_{0\nu} = K_{0\nu}^{-1}$ (see Table 1). In order to extract the value of η_ν from the above relation, R should be in the range $R_{\min} = 30$ (maximum lepton violation, $x \rightarrow \infty$) and $R_{\max} = 5200$ ($x=0$). Indeed, the above measurement is in this range and it was possible to extract¹⁷⁾ the above value of $|\langle m_\nu \rangle|$. However, the situation has recently changed since Kirsten³⁰⁾ finds $R = (1.0^{+0.2}_{-0.5}) \times 10^4$ which is outside the above range or perhaps consistent with $\eta_\nu = 0$.

Furthermore, one cannot reliably extract the lepton violating parameter η from a total lifetime measurement of a single system using a relation of the type

$$\Gamma_{\text{tot}}^{\text{exp}} = \Lambda_{0\nu} |\text{ME}|_{0\nu}^2 + \Lambda_{2\nu} |\text{ME}|_{2\nu}^2 \quad (46b)$$

since the quantity $\Gamma_{\text{tot}}^{\text{exp}} - \Lambda_{2\nu} |\text{ME}|_{2\nu}^2$ is very small. In fact, in the interesting cases¹⁷⁾ it becomes negative. In order to utilize (46b) one needs better accuracy than currently available. This is why we believe that laboratory experiments measuring $\Gamma_{0\nu}^{\text{exp}}$ are more promising in this regard.

Before concluding this section we remark that the problems associated with the extraction of η from Eq. (46a) cannot be eliminated by considering χ^0 . In fact, the ratio R , assuming χ^0 dominance, becomes $R = 25$. It cannot help in extracting η_ν from (46b) either since, as we mentioned, $\Lambda_{2\nu} |\text{ME}|_{2\nu}^2$ already exceeds $\Gamma_{\text{tot}}^{\text{exp}}$. In the case of ^{48}Ca , however, this quantity is positive. We can thus extract the value of the coupling g (see subsection 2.2) from a relation of the type

$$\frac{1}{T_{1/2}(0\nu, \chi^0)} = \frac{1}{T_{1/2}^{\text{exp}}(2\nu)} - \frac{1}{T_{1/2}^{1h}(2\nu)} \quad (47)$$

Using Eqs. (42) and (47) and using the value of K_{χ^0} given in Table 1, we get, from Eq. (47)

$$|g| < 5 \times 10^{-3}$$

which is in good agreement with the value of $|g| \sim 10^{-3}$ of Ref. 4) extracted with entirely different arguments.

5. - DOUBLE POSITRON DECAY

Everything we have said about $\beta^-\beta^-$ decay applies equally well to the case of two positron decay ($\beta^+\beta^+$). The available energy is given by Eq. (33) and is usually smaller than that of $\beta^-\beta^-$ decay. Also the function $\chi(Z, \epsilon)$ for non-relativistic energies is not now independent of ϵ , but behaves like

$$\chi(Z, \epsilon) = 2\pi\alpha Z \exp\left(-2\pi\alpha Z \frac{\epsilon+1}{\sqrt{\epsilon(\epsilon+2)}}\right) \approx 2\pi\alpha Z \exp\left(-\frac{2\pi\alpha Z}{\sqrt{\epsilon}}\right) \quad (49)$$

Thus it is necessary to use the exact Fermi function. In the present work we will apply the bounds obtained in the previous section to the study of the ${}^{58}_{28}\text{Ni} \rightarrow {}^{58}_{26}\text{Fe}$ and ${}^{96}_{44}\text{Ru} \rightarrow {}^{96}_{42}\text{Mo}$ systems. A brief description of the relevant nuclear wave function is given in the Appendix.

a) ${}^{58}_{28}\text{Ni} \rightarrow {}^{58}_{26}\text{Fe} + e^+ + e^+$ transition. The available energy is very tiny, $\epsilon_0 = 0.16$, so that the reaction barely makes it. The experimentalists, however, believe they can overcome the kinematical disadvantages (see Table 1) by being able to use large amounts of ${}^{58}\text{Ni}$ with sufficient purity as part of their detection device³¹⁾. Fortunately, as we shall see, with the small energy ϵ_0 there comes the bonus of a very small reaction background. The relevant half-life is given by

$$T_{1/2}(0\nu) = 1.4 \times 10^{20} \text{ yr} / |ME|_{0\nu}^2 \quad (50)$$

Using the nuclear matrix elements given in Table 1 and the right-handed current case i) (see subsection 2.4) we get

$$\begin{aligned} |ME|_{0\nu}^2 &= .008 |\eta_{RL}|^2 + .133 \text{Re}(\eta_{RL}\eta_{LL}^*) < f|\Omega_{LL}| + |\eta_{LL}| < f|\Omega_{LL}|^2 \\ &< f|\Omega_{LL}| = 2.2 \frac{\eta_V}{\eta_{LL}} + 110 \frac{\eta_N + \eta_H}{|\eta_{LL}|} + 210 \frac{\eta_D}{|\eta_{LL}|} \end{aligned} \quad (51)$$

As expected, the right-handed interaction is negligible since ϵ_0 is so small.

Using now the limits of η_ν , $\eta_N + \eta_H$ and η_Δ obtained in the previous section, we get the following lifetimes for the no-neutrino decay $T_{1/2}(\text{ov}) > 1.4 \times 10^{27}$, 1.1×10^{28} and 3.8×10^{28} yr for each of the three mechanisms discussed above. If, on the other hand, we assume that all three of them operate simultaneously, we obtain $T_{1/2}(\text{ov}) > 6.0 \times 10^{26}$ yr. This lifetime becomes $T_{1/2}(\text{ov}) > 3 \times 10^{28}$ yr if it is properly scaled so that it corresponds to the value of $|\langle m_\nu \rangle| = 10$ eV obtained from the analysis of the Te isotopes (see previous section).

β) ${}^{96}_{44}\text{Ru} \rightarrow {}^{96}_{42}\text{Mo} + e^+ + e^+$. Here the available energy is $\epsilon_0 = 1.3$. This is bigger than the previous one but still a bit smaller than favourable energies in $\beta^-\beta^-$ decay. The ov lifetime is

$$T_{1/2}(\text{ov}) = \frac{3.1 \times 10^{17} \text{ yr}}{|ME|_{\text{ov}}^2} \quad (52)$$

Using the parameters extracted from the $\beta\beta$ decay data of ${}^{98}\text{Ca}$ we again find that the right-handed contribution is less than 8%. The limits in the lifetimes associated with each mechanism are $T_{1/2}(\text{ov}) > 1.0 \times 10^{26}$, 5.6×10^{25} and 1.5×10^{26} yr. If all mechanisms are assumed to contribute, we get $T_{1/2}(\text{ov}) > 1.0 \times 10^{25}$ yr.

The nuclear matrix elements associated with the two neutrino decays are $|ME|_{2\nu}^2 \approx 1.8$ and 5.0×10^{-2} , respectively for the $A = 58$ and the $A = 96$ systems which lead to the following lifetimes

$$T_{1/2}(2\nu) = 1.6 \times 10^{36} \text{ yr} \quad (A=58), \quad T_{1/2}(2\nu) = 1.6 \times 10^{28} \text{ yr} \quad (A=96) \quad (53)$$

Clearly, in particular for the $A = 58$ system, this is an insignificant background. This is a great advantage since, due to the small energy available, one may not be able to distinguish experimentally the ov from the 2ν events.

6. - ELECTRON POSITRON CONVERSION

This process, whereby a bound (mainly 1s) electron is converted into a positron, is always energetically allowed whenever $\beta^+\beta^+$ decay occurs⁸⁾. In fact,

$$\epsilon = \frac{1}{m_e} (\Delta m - b - E_x) - 2 \quad (54)$$

where b is the binding energy of the electron (a few tens of keV), E_x is the excitation energy of the final nuclear state and Δm the (atomic) mass difference.

Transitions to excited states are preferred because the de-excitation γ -ray which follows provides an extra signal against background, the other two being the positron energy and the emitted X-ray.

The formalism developed in the previous section can be carried over with the following minor modifications.

i) Set $P_1 = 0$ $P_2 = P_e$ and ignore the $1/\sqrt{2}(1-P_{12})$ operator in Eqs. (11b), (26a) and (26b).

ii) Replace $F(Z-2, \epsilon)$ by $C_{\text{dist.}} \langle \phi_{1s} \rangle$ where ϕ_{1s} is the hydrogenic wave function and $C_{\text{dist.}}$ a distortion factor which takes into account all possible deviations of the actual wave function from ϕ_{1s} . These factors can be obtained from existing tables²⁵⁾ but they do not significantly change the result, since for the cases considered here we find values between 1 and 2.

iii) The energy of the positron is now completely specified and no integration is required.

Thus one easily finds that the lifetime for this process takes the form

$$\tau_{1/2}(e^- \rightarrow e^+, 0\nu) = \tau_0 \frac{A^{2/3}}{C_{\text{dist}}(\alpha Z)^3} \frac{1}{\chi(Z-2, \epsilon)} \frac{1}{(\epsilon+1)^2} \frac{1}{|ME|_{0\nu}^2} \quad (55a)$$

where

$$\tau_0 = \frac{\ln 2}{\Gamma_0} = 8.2 \times 10^{14} \text{ yr}, \quad \Gamma_0 = \frac{m_e}{(2\pi)^4} (G_F \cos \theta_c m_e^2)^4 \frac{f_A^4}{(m_e r_0)^2} \quad (55b)$$

The quantity $|ME|_{0\nu}^2$ takes the same form as before except that now

$$I_S = \frac{1}{182} \frac{\epsilon(\epsilon+2)}{\epsilon+1}, \quad I_A = \frac{\epsilon^2(\epsilon+2)}{81(\epsilon+1)}, \quad I_{0S} = \frac{\epsilon(\epsilon+2)}{9} \quad (56)$$

Similarly, the corresponding lifetime for χ^0 emission takes the form

$$\tau_{1/2}(e^- \rightarrow e^+, 0\nu, \chi^0) = \tilde{\tau}_0 \frac{A^{2/3}}{C_{\text{dist}}(\alpha Z)^3} \frac{1}{\tilde{f}_{\chi^0}(\epsilon)} \frac{1}{|\langle f | g \Omega_\nu | i \rangle|^2} \quad (57a)$$

with

$$\tilde{f}_{\chi^0}(\epsilon) = \int_0^\epsilon (\epsilon'+1)^2 d\epsilon' (\epsilon-\epsilon') \chi(Z-2, \epsilon') d\epsilon' \quad (57b)$$

$$\tilde{\tau}_0 = 2(2\pi)^2 \tau_0 = 6.5 \times 10^{16} \text{ yr} \quad (57c)$$

In the approximation of Eq. (35a) which, however, may not be appropriate when ϵ is fairly small, we get

$$\tilde{f}_{\chi^0}(\epsilon) \approx \left(\frac{2\pi\alpha(Z-2)}{\exp(2\pi(Z-2)-1)} \right) \frac{\epsilon^2}{12} (\epsilon^2 + 4\epsilon + 6) \quad (57d)$$

We are now going to apply the above formula to some simple systems.

$\alpha)$ $e_b^- + {}^{58}_{28}\text{Ni} \rightarrow {}^{58}_{26}\text{Fe} + e^+$. Only transitions to the ground state ($\epsilon = 2.15$) and the excited 2^+ state at 0.81 MeV ($\epsilon = 0.56$) are energetically allowed. The excited state can be populated only via the right-handed interaction and in line with the spirit of this paper it will not be considered. From the Tables, we find $C_{\text{dist.}} = 1.26$ and $\chi(26, 2.15) = 0.62$. The contribution of the right-handed current is negligible ($< 1\%$). Using the lepton violating parameters extracted from the ${}^{98}\text{Ca}$ and the nuclear matrix elements discussed in the previous section, we get

$$T_{1/2}(0\nu) > 2.6 \times 10^{24} \text{ yr}, 2.1 \times 10^{25} \text{ yr}, 6.9 \times 10^{25} \text{ yr}, 1.1 \times 10^{24} \text{ yr}$$

for each of the mechanisms (separately) and all of them combined respectively.

The only experimental limit available is that of Fiorini's group³²⁾ $T_{1/2} > 4 \times 10^{19} \text{ yr}$ which is not so good and applies only for transitions to the 2^+ state. The corresponding lifetime for χ^0 emission is $T_{\chi^0} = 1.9 \times 10^{23} \text{ yr}$, i.e., an order of magnitude smaller.

$\beta)$ $e_b^- + {}^{42}_{22}\text{Mo} \rightarrow {}^{42}_{20}\text{Zr} + e^+$. Only the ground state transition is allowed ($\epsilon = 1.2$). We find²⁵⁾ $C_{\text{dist.}} = 1.73$ and $\chi(40, 1.2) = 0.31$ $b = 0.036 \text{ MeV}$. From the matrix elements of Table 1 we find

$$\langle f | \Omega_{LL} | i \rangle = .33 \frac{\eta_V}{|\eta_{LL}|} + 53 \frac{\eta_{N+H}}{|\eta_{LL}|} + 100 \frac{\eta_D}{|\eta_{LL}|} \quad (58)$$

and, for case i),

$$|ME|_{0\nu}^2 = .29 |\eta_{RL}|^2 + 1.0 \text{Re}(\eta_{RL}\eta_{LL}^*) \langle f | \Omega_{LL} | i \rangle + |\eta_{LL}| \langle f | \Omega_{LL} | i \rangle^2 \quad (59)$$

The coefficients become 0.006 and 0.8 (case ii) and 0.008 and 1.6 (case iii). Since the available energy is small, the contribution of the right-handed currents is less than 7%. The lifetimes associated with each of the three mechanisms are

$T_{1/2}(\text{ov}) > 2.0 \times 10^{26}$ yr, 1.5×10^{26} yr and 4.6×10^{26} yr, respectively. The lifetime due to all the three mechanisms combined is $> 2.5 \times 10^{25}$ yr. If we consider in addition the right-handed currents, we get $T_{1/2}(\text{ov}) > 2.2 \times 10^{25}$ yr. The corresponding lifetime for χ^0 emission is 2.4×10^{25} yr.

$\gamma)$ $e_b^- + {}^{26}_{44}\text{Ru} \rightarrow {}^{96}_{42}\text{Mo} + e^+$. If we limit ourselves to 0^+ transitions we have two possibilities:

$\gamma 1)$: Ground state transition. Here $\epsilon = 2.8$, $\chi(42, 2.8) = 0.5$ and $^{25)} C_{\text{dist.}} = 1.82$. Using the nuclear matrix elements of Table 2 we get, for case i):

$$|ME|_{\text{ov}}^2 = 4.6 |\eta_{RL}|^2 + 2.9 \text{Re}(\eta_{RL} \eta_{LL}^*) \langle f | \Omega_{LL} | i \rangle + |\eta_{LL} \langle f | \Omega_{LL} | i \rangle|^2$$

For case ii) the coefficients are 0.014 and 0.15 while for case iii) they become 0.05 and 0.43, respectively. Also,

$$\langle f | \Omega_{LL} | i \rangle = 37 \frac{\eta_V}{|\eta_{LL}|} + 67 \frac{\eta_N + \eta_H}{|\eta_{LL}|} + 140 \frac{\eta_D}{|\eta_{LL}|}$$

The contribution of the right-handed currents is 14% [case i)], 9% [case ii)] and 17% [case iii)]. The lifetime associated with each mechanism is

$$T_{1/2}(\text{ov}) > 1.1 \times 10^{25} \text{ yr}, 1.2 \times 10^{25} \text{ yr and } 1.8 \times 10^{25} \text{ yr}$$

while all three mechanisms yield $T_{1/2}(\text{ov}) \geq 1.0 \times 10^{24}$ yr. The corresponding χ^0 emission lifetime is 1.5×10^{24} yr.

$\gamma 2)$: Transition to the 0^+ state at 1.15 MeV. Here $\epsilon = 1.0$ and $\chi(42, 1.0) = 0.3$. The right-handed currents make a negligible contribution. The predicted lifetimes for each mechanism are $T_{1/2}(\text{ov}) > 1.0 \times 10^{26}$ yr, 6.8×10^{26} yr and 3.2×10^{27} yr, respectively, while the total of all mechanisms is $> 4.0 \times 10^{25}$ yr. The lifetime is now an order of magnitude longer than the corresponding one for the ground state transition. We have, however, the additional signature of the 0.370 MeV γ -ray which follows the de-excitation of the populated 1.15 MeV state to the first 2^+ state with a lifetime of 0.06 sec. This will be of help in discriminating against the background. The corresponding lifetime for χ^0 emission is comparable to the previous one, i.e., 2.3×10^{25} yr. It may not be easy to discriminate between the two, but both of them are lepton violating.

We will conclude this section by briefly examining the two-neutrino process [Eq. (7)]. The decay rate takes the form

$$\Gamma_{2\nu}(e^-, e^+) = \frac{2}{(2\pi)^6} (G_F \cos \theta_c m_e^2)^4 m_e f_A^V(\alpha Z)^3 C_{\text{dist}}^3 (\alpha Z)^3 \mu_0^2 f_{2\nu}(e^-, e^+) |ME|_{2\nu}^2$$

where

$$f_{2\nu}(e^-, e^+) = \frac{1}{30} \int_0^\epsilon (\epsilon' + 1)^2 (\epsilon - \epsilon')^5 \chi(Z-2, \epsilon') d\epsilon'$$

and everything else has been explained before. In the spirit of the approximation (57d) above, we get

$$f_{2\nu}(e^-, e^+) = \frac{2\eta\alpha(Z-2)}{\exp(2\eta\alpha(Z-2))-1} \frac{1}{5040} \epsilon^6 (\epsilon^2 + 8\epsilon + 28)$$

The associated lifetime can be written as

$$T_{1/2}(e^-, e^+; 2\nu) = 0.88 \times 10^{26} \text{ yr} \frac{1}{C_{\text{eff}}(\alpha Z)^3} \frac{\mu_0^2}{(ME)_{2\nu}^2} \frac{1}{f_{2\nu}(e^-, e^+)}$$

Using the relevant Gamow-Teller matrix elements of Table 1, we obtain 1.9×10^{24} yr, 1.3×10^{27} yr and 2.6×10^{24} yr for the ground state transitions of the $A = 58, 92$ and 96 , respectively. These are comparable to the $\nu\nu$ rates predicted above and one may have to worry about the reaction background. Other kinds of background can be eliminated only by measuring the X-rays following the de-excitation of the atom. The predicted lifetime for the 2ν transition to the above-mentioned excited state of ^{96}Mo is $T_{1/2} \approx 4.8 \times 10^{30}$ yr. So, in addition to the other advantages, there is essentially no reaction background for this transition.

7. - DOUBLE ELECTRON CAPTURE

It has been suggested⁴⁾ that this process may shed some light on the precise mechanism of lepton violation. Since the binding energies of the electrons are small compared to typical mass differences, double electron capture will always be possible whenever single capture is. More precisely,

$$\epsilon = \frac{1}{m_e} (\Delta m - 2b)$$

This process involves two stages. In the first stage two neutral atomic species - the initial one with Z protons and Z electrons and the final one with $Z-2$ protons and $Z-2$ electrons (two $1s$ electron holes), get mixed via the lepton violating weak interaction. In the second stage, the atom de-excites emitting two hard X-rays and the nucleus de-excites emitting one γ -ray. Measured from the ground state of the final nucleus, the initial state (i) has an energy $E_i = \Delta m - 2b$ and the admixed nuclear states $|n\rangle$ have energies E_n . After the weak lepton violating interaction is turned on, one gets admixtures in the initial state which are of the form

$$|\psi_i'\rangle = \sum_n \frac{\Gamma_{ni}}{E_n - E_i} |n\rangle \quad (60)$$

where Γ_{ni} is the weak interaction width. Thus the amplitude for electromagnetic decay to a given final nuclear state $|f\rangle$ [process (5)] will be of the form

$$\mathcal{M}(i \rightarrow f) = \sum_n \frac{\Gamma_{ni}}{E_n - E_i} \langle f | H_\gamma | n \rangle \quad (61)$$

where H_γ is the electromagnetic interaction. The γ -ray energy will be $E_\gamma^* = E_i - E_f$ and will not coincide with any of the γ rays of the final system ($E \neq E_n, E_{\gamma n} = E_n - E_f$) (see Fig. 8 for a clarification).

The excited atom will de-excite in ways which are known from electron capture and will not be discussed here [see also Ref. 9)]. We are primarily interested in evaluating the quantity Γ_{ni} . In the presence of left-handed weak currents only 0^+ states can get admixed if one considers capture from s orbits. (The probability of capturing from the $2s$ orbit is down by a factor of 10. Higher orbits contribute even less.) There are such 0^+ states in the neighbourhood of E_i and they will get admixed. Capture from one $1p$ orbit and one $1s$ orbit can admix states $|n\rangle$ with negative parity. However, in spite of their large radiative widths, such states are far away in energy and this, combined with the atomic physics suppression, will make their contribution small. The right-handed interaction by itself can couple to negative parity states. In principle, even though the giant dipole resonance is high in excitation energy (~ 20 MeV), it could contribute due to its large radiative width. This possibility however, is not very interesting because one will not observe very narrow lines. In any event, unlike double β decay, the right-handed interaction cannot couple to positive parity states (except for capture from $1p$ orbits).

The width Γ_{ni} for the mixing with a given 0^+ state of the final nucleus is given by

$$\Gamma_{ni} = \gamma_0 C_{dist} \frac{(\alpha Z)^3}{A^{1/3}} \eta_{LL} \langle n | \Omega_L | i \rangle \quad (62a)$$

where

$$\gamma_0 = \frac{\sqrt{2}}{(2\pi)^2} (G_F \omega \delta_c m_e^2)^2 m_e \frac{f_A^2}{m_e r_0} \approx 6.0 \times 10^{-22} \text{ MeV} \quad (62b)$$

Then, given the nuclear wave functions, one can evaluate the amplitude of Eq. (60). We choose, however, to utilize as much as possible the available experimental data. Thus, neglecting all possible interference between the various admixtures $|n\rangle$ we can express our answer in terms of the radiative widths of the levels n decaying to f . Thus we obtain

$$T_{1/2}(i e^- e^- \rightarrow f \gamma) = 2.3 \times 10^{22} \text{ yr} \frac{A^{2/3}}{[C_{dist} (\alpha Z)^3]^2} \frac{1}{|ME|_{ov}^2} \quad (63)$$

where

$$|ME|_{ov}^2 = \sum_n \left(\frac{E_n^*}{E_{\gamma n}} \right)^5 \frac{1}{\xi(n \rightarrow f)} \frac{1}{(\epsilon_n - \epsilon_i)^2} |\eta_{LL} \langle n | \Omega_L | i \rangle|^2 \quad (64)$$

where $E_i = (E_i/m_e) = \epsilon$, $\epsilon_n = E_n/m_e$, $\xi(n \rightarrow f)$ is the radiative half-life (in psec) for the EM decay $|n\rangle \rightarrow |f\rangle$ [note that $\xi(n \rightarrow f)/\xi(f \rightarrow n) = 2J_f + 1/2J_n + 1$]. The extra energy dependence is characteristic of quadrupole radiation.

Before proceeding to our numerical estimates, we will write down the rates for the two competing mechanisms, mainly 2ν and χ^0 decay. Since these are genuine reactions and not state mixings à la $\bar{K}^0 - K^0$, they are expected to dominate ⁴⁾. We obtain

$$\Gamma_{2\nu}(e^-e^-i \rightarrow f) = \frac{4}{(2\pi)^5} (G_F \cos \theta_c m_e^2)^2 f_A^4 m_e (C_{dist}(\alpha Z)^3)^2 \frac{\epsilon^5}{30} \frac{|ME|_{2\nu}^2}{\mu_0^2} \quad (65a)$$

or

$$T_{1/2}(e^-e^-i \rightarrow f, 2\nu) = 3.3 \times 10^{22} \text{ yr} \frac{1}{\epsilon^5} \frac{1}{[C_{dist}(\alpha Z)^3]^2} \frac{\mu_0^2}{|ME|_{2\nu}^2} \quad (65b)$$

similarly

$$\Gamma_{\chi^0}(e^-e^-i \rightarrow f) = \frac{1}{(2\pi)^5} (G_F \cos \theta_c m_e^2)^4 m_e f_A^4 \frac{[C_{dist}(\alpha Z)^3]^2}{(m_e r_0)^2 A^{2/3}} \epsilon |\langle f | g \Omega_V | i \rangle|^2 \quad (66)$$

i.e.,

$$R_1 = \frac{\Gamma_{\chi^0}}{\Gamma_{2\nu}} \approx 10^6 \frac{1}{\epsilon^4} \frac{\mu_0^2}{|ME|_{2\nu}^2} \frac{|\langle f | g \Omega_V | i \rangle|^2}{A^{2/3}} \quad (67)$$

Thus, for $g \sim 10^{-3}$, χ^0 emission is in general going to compete with 2ν emission except when the energy is small when it may dominate.

We are now going to present numerical estimates for the simple $A = 58, 92$ and 96 systems.

a) $A = 58$. We are going to consider the admixture of only two 0^+ states. The ground state ($\epsilon_0 = 0$, $\xi_0 = 1/5 \times 9 \approx 2$, $E_{\gamma_0} = 0.81$ MeV) and the first excited ³³⁾ 0^+ state at 2.26 MeV ($\epsilon_1 = 4.4$, $E_{\gamma_1} = 1.44$ MeV). The radiative width of this state is not known experimentally. We will take it to be $\xi_1 = 2$ even though it may be smaller due to its larger decay energy. The total available energy is

$\epsilon = 4.2$ but the γ decay of interest to us is that to the 2^+ state with $E_\gamma^* = 1.315$ MeV. Of course this final state can further decay to the ground state with a half-life of 9 psec. The relevant nuclear lepton violating matrix element to the $|n = 0\rangle$ (gs) state was computed in the previous section. The one to the excited 0^+ state can be computed similarly using the data of Table 2. We thus get

$$T_{1/2}(\nu\nu) > 3.5 \times 10^{34}, 2.8 \times 10^{35}, 1.4 \times 10^{36}, 1.6 \times 10^{34} \text{ yrs}$$

for each of the three mechanisms (separately) and all of them combined respectively. Similarly, we obtain

$$T_{1/2}(2\nu) = 1.5 \times 10^{25} \text{ yr}, \quad T_{1/2}(\chi^0) > 1.0 \times 10^{25} \text{ yr}$$

β) $A = 92$. Here $E_\gamma^* = 0.69$ for decay to the 2^+ state at 0.935 MeV and $\epsilon = 3.2$. We allow admixtures of the ground state ($\epsilon_0 = 0$, $E_{\gamma 0} = 0.94$ MeV, $\xi_0 = \frac{1}{5} \times 5 = 1$) and the first excited 0^+ state at 1.38 MeV ($\epsilon_1 = 2.7$, $E_{\gamma 1} = 0.45$ MeV, $\xi_1 = 170$ [see Ref. 33]). Proceeding analogously as before, we obtain

$$T_{1/2}(2\nu) > 4.8 \times 10^{34}, 1.7 \times 10^{35}, 7.9 \times 10^{36}, 1.6 \times 10^{34} \text{ yr}$$

$$T_{1/2}(\nu\nu) = 8.5 \times 10^{25}, \quad T_{1/2}(\chi^0) > 5.2 \times 10^{25} \text{ yr}$$

γ) $A = 96$. Here $\epsilon = 5.24$ and $E_\gamma^* = 1.90$ MeV for decay to the 2^+ state at 0.778 MeV. We consider admixtures of three 0^+ states: the ground state ($\epsilon_0 = 0$, $E_{\gamma 0} = 0.78$ MeV, $\xi_0 = \frac{1}{5} \times 1.7 = 0.34$), the 0^+ excited state³³⁾ at 1.15 MeV ($\epsilon_1 = 2.3$, $E_{\gamma 1} = 0.37$ MeV, $\xi_1 = 60$) and the 0^+ state at 2.75 MeV ($\epsilon_2 = 5.38$, $E_{\gamma 2} = 1.97$ MeV). Its lifetime is not known. We will assume it to be comparable to that of the 0_1^+ state scaled by its larger decay energy, i.e., $\xi_2 = 60(0.37/1.97)^5 \sim 0.01$. Since, in addition, this level is (by nuclear standards) degenerate with the initial state, it dominates the transition. We thus obtain

$$T_{1/2}(\nu\nu) > 5.5 \times 10^{30}, 2.3 \times 10^{31}, 1.0 \times 10^{32}, 1.9 \times 10^{30} \text{ yrs}$$

(in the above order). The other two lifetimes are

$$T_{1/2}(2\nu) = 7.7 \times 10^{27} \text{ yr}, \quad T_{1/2}(\chi^0) > 5.0 \times 10^{24} \text{ yr}$$

(for ground state transitions). For transitions to the excited 0_1^+ state 1.15 MeV, we find

$$T_{1/2}(2\nu) = 2.7 \times 10^{27} \text{ yr}, \quad T_{1/2}(\chi^0) > 7.4 \times 10^{26} \text{ yr}$$

[in agreement with our expectations, see Eq. (66), due to the small energy $\epsilon = 2.3$].

We believe that, due to the almost degeneracy of the O_2^+ with the initial state, the $A = 96$ system is one of the most favourable for double e capture. It is, however, evident that the neutrinoless double e capture, without χ^0 emission, is beyond the experimental feasibility. Even lepton violation with χ^0 emission proceeds with lifetimes much longer than the presently attained limits of ordinary $\beta\beta$ decay. So it does not win in competition with the 2ν decay which is experimentally indistinguishable from it (except when the energy is small). Thus, double electron capture does not, at this point, promise to develop into a very useful probe.

8. - CONCLUSIONS

In the present paper we have discussed various double β decays with different experimental signatures like $\beta^-\beta^-$ and $\beta^+\beta^+$ decay, (e^-, e^+) conversion and double electron capture. Our discussion focussed on some attractive features of present-day gauge theories. We thus considered various lepton violating mechanisms including light and heavy Majorana neutrino, Higgs mediated processes, possible contributions of right-handed currents and models with unconventional neutrino masses (majoron emission). We then used the $^{48}\text{Ca} - ^{48}\text{Ti}$ as a theoretical laboratory to extract constraints on the various lepton violating parameters from the data. The following limits emerged from our calculation

$$|\langle m_\nu \rangle| < 80 \text{ eV} \quad (\text{light neutrino mass})$$

$$|M_{RL}| < 10^{-4} \quad (\text{admixture of R-H currents})$$

$$g_{\nu e \tilde{\nu} e \chi^0} \leq 5 \times 10^{-3} \quad (\text{Majoron-neutrino coupling})$$

$$m_N > (2-20) \times 10^3 \text{ GeV} \quad (\text{heavy neutrino mass})$$

$$\left(\frac{m_{WL}}{m_{WR}}\right)^2 < 0.04 \quad \text{or} \quad m_{WR} > 400 \text{ GeV} \quad (\text{mass of right vector bosons})$$

the last two predictions do not sensitively depend on nuclear physics, but they depend on assumptions about the other gauge parameters. We have seen that total lifetime measurements, like the available geochemical rates, may not be good ways of extracting lepton violating parameters. The reason is that the measured rates are close to (and sometimes¹⁷⁾, unfortunately, less than) those calculated for the lepton allowed 2ν process. We have also seen that complications will arise even if the 0ν rate is measured. For example, one may not be able to extract the neutrino mass without information about the right-handed currents. This, however,

can be done if one looks for $0^+ \rightarrow J \neq 0$ transitions which occur only if there is a right-handed interaction, or by looking at systems with small available energies, for example ^{58}Ni , where the right-handed interaction will be suppressed. This, of course, implicitly assumes that the disadvantages of the long lifetimes may be overcome by employing new experimental techniques which allow the use of large and pure targets. Our lifetime predictions, based on the parameters extracted from the $^{48}\text{Ca} - ^{48}\text{Ti}$ decays, are summarized in Table 3 for the reader's convenience.

It appears that the traditional $\beta^-\beta^-$ decay experiments have reached their ultimate limits, i.e., $T_{1/2} \sim 10^{22}$ yr which constitutes the longest limit attained experimentally^{32),34)}. One should therefore look at the other processes discussed in the present work. Double electron capture appears to be hopelessly slow without providing the compensation of an additional desirable signature. Positron emission may prove to provide great advantages. The double positron decay of ^{58}Ni with a tiny available energy may proceed with lifetimes of the order of 10^{28} yr. If such a lifetime does not deter the experimental efforts, the experimentalists will get the bonus of a negligible 2ν reaction background ($\sim 10^{36}$ yr). Furthermore we have seen that in this case one need not worry about the contribution of the right-handed interaction if the neutrino mass is not much smaller than 10 eV. Also, at the same time, single positrons originating from electron capture should be looked for. The fact that the lifetimes associated with such processes are about 10^3 longer than typical $\beta^-\beta^-$ lifetimes and may be compensated for by the signature of the positron. Transitions to excited states will provide the extra signature of the emitted de-excitation γ -ray to discriminate against background. The lifetime for populating the 0_1^+ state of ^{96}Mo at 1.15 MeV is predicted to be 10^{25} yr, i.e., only ten times slower than the corresponding ground state transition, while the 2ν reaction background will proceed with lifetimes of 10^{24} yr and 10^{27} to the ground and excited states, respectively.

Last but not least we strongly suggest that such an experimental set up may be used to look also for high-energy positrons (~ 1 GeV) which are expected from double proton decay $(A, Z) \rightarrow (A-2, Z-2) + e^+ + e^+$ which, unlike $\beta\beta$ decay, is a B-L conserving process. Thus it may reasonably be expected to proceed with a lifetime³⁵⁾ of 10^{34} yr.

In all the positron emitted decays discussed in this work, the contribution of the right-handed currents appears to be small. It is unlikely that this will merely reflect the bias of the $^{48}\text{Ca} - ^{48}\text{Ti}$ system which was used in extracting the lepton violating parameters. Finally, we conclude that it will be very difficult to distinguish among themselves the other mechanisms discussed in this

work (light neutrino, heavy neutrino, Higgs contribution, χ^0 production). But such questions may be premature at this point since the observation of lepton violation, regardless of the mechanism, will make big news.

$\beta^-\beta^-$ decay has by now a history of 50 years [see the Introduction of Zdesenko's article in Ref. 2)]. It is not easy to assess the theoretical and experimental progress made on this complicated subject in the above period.

Given the expectations from double β decay experiments in this post-gauge-theory era, we hope that the experimental and theoretical efforts will intensify. Decays involving positrons may contribute in this direction.

ACKNOWLEDGEMENTS

The author is indebted to E. Fiorini for very useful discussions, to Dr. L. Skouras for providing the nuclear wave functions for the $A = 96$ system, and to Dr. Dr. A. de Rújula for a careful reading of the manuscript and useful comments.

APPENDIX

In this Appendix we will present a brief discussion of the nuclear models entering in the nuclear matrix elements of the various systems discussed in the main text.

i) A = 48 System: the nuclear model employed has been described at length elsewhere^{11),22)}. The wave functions used were

$${}^{48}\text{Ca}(\text{gs}) = |f_{7/2}^8(n) J=0^+\rangle$$

$${}^{48}\text{Ti}(\text{gs}) = .845|1\rangle - .523|2\rangle + .109|3\rangle - .001|4\rangle$$

with

$$|K\rangle = |f_{7/2}^6(n) J_K; f_{7/2}^2(p) J_K; J=0^+\rangle, \quad J_K = 0, 2, 4, 6$$

The smallness in the matrix elements of the operators $\Omega_V, \Omega_1, \Omega_2$ and $\tilde{Y}$ mentioned in the text results from the fact that the $J_L = 0$ and 2 components subtract from each other. No such large cancellation occurs for the operators Ω_N, Ω_π and Ω_Δ , since the $J = 0$ component of the wave function dominates.

ii) A = 58 System: ${}^{58}\text{Ni}$ was assumed to consist of two neutrons in the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ shells. ${}^{58}\text{Fe}$ was supposed to consist of four neutrons in the above shells and two proton holes in the $1f_{7/2}$ shell. The wave functions used were

$${}^{58}\text{Fe}(\text{gs}) = 0.8|\psi_n(\text{gs}), f_{7/2}^{-2}(p) J_p=0\rangle - 0.6|\psi_n(2^+); f_{7/2}^{-2}(p) J_p=2\rangle$$

$${}^{58}\text{Fe}(0_1^+) = |\psi_n(0_1^+), f_{7/2}^{-2}(p) J_p=0\rangle$$

The wave functions $\psi_n(\text{gs})$ and $\psi_n(2^+)$ were obtained by diagonalizing the phenomenological interaction of Auerbach³⁶⁾ in the above model space. These wave functions are somewhat complicated to be given here. The ${}^{58}\text{Ni}$ wave function was obtained in the same way. Again we notice cancellations between the $J = 0$ and $J = 2$ components of the ${}^{58}\text{Fe}(\text{gs})$ wave function, but they are not as complete as before. This explains why the Gamow-Teller matrix element is now of "reasonable" size.

iii) A = 92 System: the wave function of the initial system was assumed to consist of four protons distributed in the $2p_{1/2}$ and $1g_{9/2}$ shells and was obtained by diagonalizing the phenomenological interaction of Gloeckner and Serduke³⁷⁾ which describes reasonably well the low-energy data. The wave functions of ${}^{92}\text{Zr}$ were assumed to be of the form

$$^{92}\text{Zr}(\text{gr}) = \psi_p(\text{gs}) \psi_n(\text{gs})$$

$$^{92}\text{Zr}(\text{o}_1^+) = \psi_p(\text{o}_1^+) \psi_n(\text{gs})$$

The wave functions ψ_p were obtained by diagonalizing the above-mentioned phenomenological interaction in the space of two protons distributed in the $2p_{1/2}$ and $1g_{9/2}$ shells. The wave function $\psi_n(\text{gs})$ was taken to be the ground state wave function for ^{90}Sr found in the Appendix of Ref. 38).

iv) A = 96 System: the wave function of ^{96}Ru was obtained by diagonalizing an extended version of the phenomenological interaction of Ref. 37) in a model consisting of six protons in the $2p_{1/2}$ and $1g_{9/2}$ shells and two neutrons in the $2d_{5/2}$ and $3s_{1/2}$ shells. The wave function of ^{96}Mo was obtained in the same way by considering four protons and four neutrons in the above model spaces, respectively³⁹⁾. These wave functions were obtained using the new "Democritos" shell model code and were given to the author by L. Skouras. Neutrons in the $1g_{7/2}$ orbital were included but only in perturbation theory. Because of that the Gamow-Teller matrix elements may be somewhat underestimated.

	A=48	A=82	A=130	A=128	A=58	A=96
ϵ_0	8.4	5.8	5.0	1.7	0.16	1.3
μ_0	12	20	25	23	11	12
$K_{ov}^{R.P}$	$1.6 \times 10^{13} \text{yr}$	$7.0 \times 10^{13} \text{yr}$	$8.0 \times 10^{13} \text{yr}$	$2.6 \times 10^{15} \text{yr}$	$3.6 \times 10^{20} \text{yr}$	$1.2 \times 10^{18} \text{yr}$
K_{ov}^{TAB}	$1.1 \times 10^{13} \text{yr}$	$3.1 \times 10^{13} \text{yr}$	$1.3 \times 10^{13} \text{yr}$	$4.1 \times 10^{14} \text{yr}$	$1.4 \times 10^{20} \text{yr}$	$3.1 \times 10^{17} \text{yr}$
$\tilde{K}_{ov}^{TAB}$	$2.8 \times 10^{21} \text{yr}$	$7.8 \times 10^{22} \text{yr}$	$3.2 \times 10^{22} \text{yr}$	$1.0 \times 10^{24} \text{yr}$	$3.5 \times 10^{29} \text{yr}$	$8.0 \times 10^{26} \text{yr}$
I_S	0.123	0.080	0.070	0.027	0.0036	0.019
I_A	2.42	1.262	0.974	0.151	0.0022	0.105
I_{OS}	0.297	0.280	0.273	0.184	0.0032	0.152
$K_{2\nu}^{R.P}$	$1.1 \times 10^{19} \text{yr}$	$2.7 \times 10^{20} \text{yr}$	$1.2 \times 10^{21} \text{yr}$	$6.3 \times 10^{24} \text{yr}$	$1.2 \times 10^{37} \text{yr}$	$4.0 \times 10^{28} \text{yr}$
$K_{2\nu}^{TAB}$	$7.7 \times 10^{18} \text{yr}$	$1.2 \times 10^{20} \text{yr}$	$2.0 \times 10^{20} \text{yr}$	$1.0 \times 10^{24} \text{yr}$	$3.0 \times 10^{36} \text{yr}$	$1.0 \times 10^{28} \text{yr}$
K_{χ^0}	$3.6 \times 10^{14} \text{yr}$	$1.9 \times 10^{15} \text{yr}$	$9.8 \times 10^{14} \text{yr}$	$2.4 \times 10^{16} \text{yr}$	$1.7 \times 10^{25} \text{yr}$	$4.3 \times 10^{20} \text{yr}$
$\tilde{K}_{\chi^0}$	$1.4 \times 10^{19} \text{yr}$	$7.6 \times 10^{19} \text{yr}$	$3.9 \times 10^{19} \text{yr}$	$9.6 \times 10^{20} \text{yr}$	$6.8 \times 10^{29} \text{yr}$	$2.7 \times 10^{25} \text{yr}$

TABLE 1 - Some of the kinematical functions relevant to the various processes discussed in the text. To aid the reader getting a feeling of the time scale for ov decay, we present the value $\tilde{K}_{ov}$ computed with a typical lepton violating parameter $\eta = 2 \times 10^{-5}$ ($\langle m_\nu \rangle = 10 \text{ eV}$) and $\tilde{K}_{\chi^0}$ computed with $g \approx 5 \times 10^{-3}$ (see text). The superscript R.P indicates that the approximate Fermi functions were used, while the superscript TAB indicates that the Fermi functions were taken from Ref. 24). The functions I_S , I_A and I_{OS} give a measure of the right-handed interaction in $\beta\beta$ decay. None of these quantities contains the nuclear matrix elements. ϵ_0 is the available (kinetic) energy for the two electrons and μ_0 the average excitation energy of the intermediate nuclear states in 2ν decay (both in units of $m_e c^2$).

System	Ω	$\langle f \Omega_0 i \rangle$	$\langle f \Omega_N i \rangle$	$\langle f \Omega_\pi i \rangle$	$\langle f \Omega_\Delta i \rangle$	$\langle f \Omega_1 i \rangle$	$\langle f \Omega_2 i \rangle$	$\langle f \Omega_3 i \rangle$	Gammow Teller
${}^{48}\text{Ca} \rightarrow {}^{48}\text{Ti}$	$T_1(\text{gs})$	0.63	50	72	260	3.9	0.56	-6.7	0.130
	gs	2.22	71	41	213	0.46	1.08	1.9	1.36
${}^{58}\text{Ni} \rightarrow {}^{58}\text{Fe}$	2.257 MeV	2.50	33	-6.4	17	0.10	0.08	5.1	2.09
	gs	0.33	23	20	101	0.51	0.08	-2.3	0.254
${}^{92}\text{Mo} \rightarrow {}^{92}\text{Zr}$	1.383 MeV	0.66	24	22	76	0.64	0.49	-1.2	0.096
	gs	0.37	30	37	137	1.00	0.70	-3.7	0.226
${}^{96}\text{Mo} \rightarrow {}^{96}\text{Ru}$	1.148 MeV	0.41	13	8.6	33	0.41	0.46	-0.30	0.028
	2.75 MeV	0.57	60	15	60	0.39	0.41	-1.3	—

TABLE 2 - Values of the nuclear matrix elements entering the calculation of the double $\beta\beta$ decay rates discussed in the text. They were obtained using the nuclear models briefly discussed in the Appendix.

Transitions		$T_2^1(2\nu)$ yrs	$T_2^1(\text{ov})$ yrs					
			η_ν	$\eta_{\text{N}+\eta_{\text{H}}}$	η_Δ	η_{LL}	$\eta_{\text{LL}}+\eta_{\text{RL}}$	χ^0
$\beta^+ \beta^+$	$^{58}\text{Ni} \rightarrow ^{58}\text{Fe}(\text{gs})$	1.6×10^{36}	1.4×10^{27}	1.1×10^{28}	3.8×10^{28}	6.0×10^{26}	6.0×10^{26}	1.3×10^{29}
	$^{96}\text{Ru} \rightarrow ^{96}\text{Mo}(\text{gs})$	1.6×10^{28}	1.0×10^{26}	5.6×10^{25}	1.5×10^{26}	1.0×10^{25}	0.9×10^{25}	$1.1 \times 10^{26} \text{yr}$
(e_b^-, e^+)	$^{58}\text{Ni} \rightarrow ^{58}\text{Fe}(\text{gs})$	1.9×10^{24}	2.6×10^{24}	2.1×10^{25}	6.9×10^{25}	1.1×10^{24}	1.1×10^{24}	1.9×10^{23}
	$^{92}\text{Mo} \rightarrow ^{92}\text{Zr}(\text{gs})$	1.4×10^{25}	2.0×10^{26}	1.5×10^{26}	4.6×10^{26}	2.5×10^{25}	2.3×10^{25}	2.4×10^{25}
	$^{96}\text{Ru} \rightarrow ^{96}\text{Mo}(\text{gs})$	2.6×10^{24}	1.1×10^{25}	1.2×10^{25}	1.8×10^{25}	1.0×10^{24}	8.6×10^{25}	1.5×10^{24}
	$^{96}\text{Ru} \rightarrow ^{96}\text{Mo}(\text{O}_1^+)$	3.8×10^{29}	1.0×10^{26}	6.8×10^{26}	3.2×10^{27}	4.0×10^{25}	4.0×10^{25}	2.3×10^{25}
Double Electron Capture	$^{58}\text{Ni} \rightarrow ^{58}\text{Fe}$	1.5×10^{25}	3.5×10^{34}	2.8×10^{35}	1.4×10^{36}	1.6×10^{34}	—	1.0×10^{25}
	$^{92}\text{Mo} \rightarrow ^{92}\text{Zr}$	8.5×10^{25}	4.8×10^{34}	1.7×10^{35}	7.0×10^{36}	1.6×10^{34}	—	5.2×10^{25}
	$^{96}\text{Ru} \rightarrow ^{96}\text{Mo}$	$7.7 \times 10^{24} (\text{gs})$ $2.7 \times 10^{24} (\text{O}_1^+)$	5.5×10^{30}	2.3×10^{31}	1.0×10^{32}	1.9×10^{30}	— —	$5.0 \times 10^{24} (\text{gs})$ $7.4 \times 10^{26} (\text{O}_1^+)$

TABLE 3 - The predicted half-lives for the various nuclear systems associated with the various lepton violating processes discussed in the text. The predictions are based on the lepton violating parameters extracted from the experimental limits on the $^{48}\text{Ca} - ^{48}\text{Ti}$ decay (see Section 4), i.e., $\eta_\nu = 1.4 \times 10^{-4}$, $\eta_N + \eta_H = 1.0 \times 10^{-6}$, $\eta_\Delta = 3 \times 10^{-7}$ and $g = 5 \times 10^{-3}$.

REFERENCES

- 1) See, for example:
H. Primakoff and S.P. Rosen, Phys. Rev. 184 (1969) 1925 and references therein.
- 2) For recent reviews, especially on the experimental side, see:
Yu G. Zdesenko, Sov. J. Part. Nucl. 11(6) (1980) 542;
D. Bryman and C. Piccioto, Rev. Mod. Phys. 50 (1978) 11.
- 3) For mechanisms for neutrino mass see, for example:
R. Barbieri, D.V. Nanopoulos, G. Morchio and F. Strocchi, Phys. Lett. 90B (1980) 91;
E. Witten, Phys. Rev. Lett. 91B (1980) 81.
- 4) H. Georgi, S.L. Glashow and S. Nussinov, Nucl. Phys. B193 (1981) 297;
G.B. Gelmini and M. Roncadelli, Phys. Lett. 99B (1981) 411.
- 5) Observation of $2\nu \beta\beta$ decay in the $A = 82$ system has been reported by
M.K. Moe and D. Lowental, Phys. Rev. C22 (1980) 2186.
The reported rate, however, is an order of magnitude faster than the accepted
geochemical rate.
- 6) T. Kirsten, W. Gentner and O.A. Schaeffer, Zeit. für Physik 202 (1967) 203;
T. Kirsten, O.A. Schaeffer, E. Norton and R.W. Stoenner, Phys. Rev. Lett.
20 (1968) 1300.
- 7) E.W. Hennecke, O.K. Manuel and D.D. Sabu, Phys. Rev. C11 (1975) 1378.
- 8) See, for example:
S.P. Rosen and H. Primakoff, Alpha, Beta and Gamma Ray Spectroscopy, ed. by
K. Siegbahn, North Holland Publishing Co. (1965).
- 9) A. de Rújula, Nucl. Phys. B188 (1981) 414.
- 10) M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. 102B (1981) 323.
- 11) J.D. Vergados, Phys. Rev. 24C (1981) 640.
- 12) J.D. Vergados, Phys. Lett. 109B (1982) 96; 113B (1982) 513.
- 13) P.H. Schneiner and F. von Hippel, Phys. Rev. Lett. 172 (1974) 1764;
J.D. Vergados and M. Ericson, Nucl. Phys. B195 (1982) 262.
- 14) J.D. Vergados, Phys. Rev. D25 (1982) 914.
- 15) L. Wolfenstein, Phys. Lett. 107B (1981) 72.
- 16) M. Doi et al., Prog. Theor. Phys. 66 (1981) 1739; 66 (1981) 1765.
- 17) W.C. Haxton, G.J. Stephenson and D. Strottman, Phys. Rev. D25 (1982) 2360.
- 18) R.N. Mohapatra and J.D. Vergados, Phys. Rev. Lett. 47 (1981) 1713;
R.N. Mohapatra and G. Senjanovic, Phys. Rev. 23D (1981) 165.
- 19) C.E. Piccioto and M.S. Zahir, Neutrinoless double β decay in left-right
symmetric models, University of Victoria preprint (1982).
- 20) J. Schechter and J.W.F. Valle, Phys. Rev. 25D (1982) 2951.

- 21) L. Wolfenstein, Carnegie-Mellon University preprint CMU HEG82-5 (1982).
- 22) S.L. Adler et al., Phys. Rev. D11 (1975) 3309.
- 23) J.D. Vergados, Phys. Rev. C13 (1976) 865.
- 24) H. Behrens and J. Jännecke, Numerical data and functional relations in science and technology, ed. K.H. Kelliege, Springer-Verlag (1969).
- 25) E. Mateosian and M. Goldhaber, Phys. Rev. 146 (1966) 810.
- 26) B.J. Cleveland, W.R. Leo, C.S. Wu, P.J. Callon and J.D. Ullman, Phys. Rev. Lett. 35 (1975) 757.
- 27) R.K. Burdin, P.J. Callon, J.D. Ullman and W.S. Wu, Nucl. Phys. A158 (1970) 337.
- 28) G.A. Miller and J.E. Spencer, Ann. Phys. (NY) 100 (1976) 562.
- 29) See, for example:
R.N. Mohapatra and G. Senjanovic, Ref. 18).
- 30) T. Kirsten, private communication.
- 31) D. Cline, private communication.
- 32) E. Belliotti, E. Fiorini, C. Liguori, A. Pullia, A. Sarracino and L. Zanotti, Lett. Nuovo Cimento 33 (1982) 273.
- 33) Tables of isotopes, eds. C.M. Lederer and V.S. Shirley, John Wiley (1978).
- 34) E. Bellotti, E. Fiorini, C. Liguori, A. Pullia, A. Sarrachino and L. Zannoti, XXI International Conference on High Energy Physics, Paris (July 1982).
- 35) J.D. Vergados, Study of baryon and lepton violating two proton annihilation in gauge theories, CERN preprint TH-3361 (1982).
- 36) N. Auerbach, Phys. Rev. 163 (1967) 1203 ; Nucl. Phys. 76 (1966) 321.
- 37) D.H. Gloeckner and F.J.D. Serduke, Nucl. Phys. A220 (1974) 491.
- 38) J.D. Vergados, Nucl. Phys. A166 (1971) 285.
- 39) L.D. Skouras, Private communication.

FIGURE CAPTIONS

- Figure 1 : The Majorana neutrino mediated neutrinoless double β decay when there exist left-handed currents only. (a) corresponds to light and (b) to heavy neutrinos.
- Figure 2 : A possible contribution to neutrinoless double β decay when the intermediate Majorana neutrinos are heavy. A pion in flight between two nucleons emits electrons. This process is negligible in the case of light Majorana neutrinos.
- Figure 3 : A possible contribution to neutrinoless double $\beta\beta$ decay which does not involve the mass of the neutrino, but is accompanied by the emission of a light (massless) Higgs particle. This process may be experimentally confused with the lepton allowed 2ν decay.
- Figure 4 : A possible contribution to neutrinoless double β decay into two electrons of a doubly charged Higgs particle Δ_R^{--} . Communication with the hadronic sector is achieved via the vector bosons associated with the right-handed interaction.
- Figure 5 : A similar process with that of the previous figure except that now communication with the hadronic sector is achieved via the Higgs doublets.
- Figure 6 : A possible Majorana neutrino mediated contribution to double β decay in the presence of right-handed interactions. Lepton violation is in this case not explicitly dependent on the neutrino masses. The upper diagrams are proportional to κ and the lower ones to ϵ (see subsection 2.4).
- Figure 7 : A plot of $\eta_{LL} = \langle m_\nu \rangle / m_e$ vs. η_{RL} which gives the region of these parameters consistent with the experimental limit of 0ν decay [Refs. 26) and 27)] on ^{48}Ca . The parameter η_{RL} gives the admixture of right-handed currents [case iii) of text is presented]. The parameters η_{LL} and η_{RL} were assumed real with $\eta_{LL} > 0$.

Figure 8 : The two atomic species which can be admixed via a lepton violating weak interaction. (a) has Z electrons and Z protons. (b) has $Z-2$ electrons (two holes in the $1s$ shell) and $Z-2$ protons in an excited nuclear state. Both can de-excite emitting EM radiation. The nuclear de-excitation is shown in Fig. (c). E_Y^* is the γ -ray which will signal double electron capture originating from a state with energy $\Delta m - 2b$ which has tiny admixtures of 0_{gs}^+ , 0_1^+ , 0_2^+ , etc.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8